Lecture 3: Vectorization, Indexing, Relational and Logical Operators

B0B17MTB – Matlab

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Outline

1. Indexing
2. Relational Operators
3. Logical Operators
4. Excercises

Indexing in MATLAB

- Now we know all the stuff necessary to deal with indexing in MATLAB.
- Mastering indexing is crucial for efficient work with MATLAB.
- Up to now, we have been working with entire matrices, quite often we need, however, to access individual elements of arrays.
- Two ways of accessing matrices/vectors are distinguished.
  - Access using round brackets "( )".
    - Matrix indexing: refers to position of elements in a matrix.
  - Access using square brackets "[ ]".
    - Matrix concatenation: refers to element's order in a matrix.

Indexing in MATLAB I.

- Let's consider following triplet of matrices.
  - Execute individual commands and find out their meaning.
  - Start from inner part of the commands.
  - Note the meaning of the keyword `end`.

$$\mathbf{N}_1 = \begin{bmatrix} -5 \\ 0 \\ 5 \end{bmatrix} \qquad \mathbf{N}_2 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ 2 & 3 & 5 & 7 & 11 \end{bmatrix} \qquad \mathbf{N}_3 = \begin{bmatrix} 11 & 12 & 13 & 14 \\ 22 & 24 & 26 & 28 \\ 33 & 36 & 39 & 42 \\ 44 & 48 & 52 & 56 \end{bmatrix}$$

```
N1 = (-5:5:5)';    N2 = [1:5;2:2:10;primes(11)];    N3 = (1:4)'*(11:14);
```

```
N1(1:3)
N1([1 2 3])
N1(1:2)
N1([1 3])
N1([1 3].')
N1([1 3]).'
N1([1; 3])
N1([1 3],1)
```

```
N2(1, 3)
N2(3, 1)
N2(1, end)
N2(end, end)
N2(1, :)
N2(1, :).'
N2(:, 2)
N2(:, 3:end)
```

```
N3(2:3, [1 1 1]) % like repmat
N3(2:3, ones(1,3))
N3(2:3, ones(3,1))
N3([N2(2,1:2)/2 4], [2 3])
N3([1 end], [1:4 1:2:end])
N3(:, :, 2) = magic(4)
N3([1 3], 3:4, 3) = ...
    [1/2 -1/2; pi*ones(1, 2)]
```

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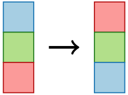
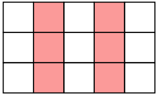
Indexing in MATLAB II.

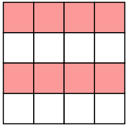
- Remember the meaning of `end` and the application of colon operator ":".
- Try to:
  - Flip the elements of the vector $\mathbf{N}_1$ without use of `fliplr`/`flipud` functions.



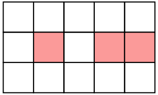
  - Select only the even columns of $\mathbf{N}_2$.



  - Select only the odd rows of $\mathbf{N}_3$.



  - Select 2nd, 4th and 5th column of 2nd row of $\mathbf{N}_2$.

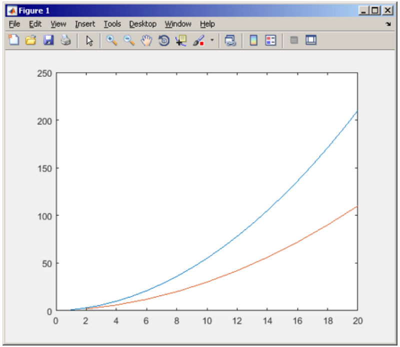


  - Create matrix $\mathbf{A}$ of size 4×3 containing numbers 1 to 12 (row-wise, from left to right).

Indexing in MATLAB III.

- Calculate cumulative sum $\boldsymbol{S}$ of a vector $\boldsymbol{x}$ consisting of integers from 1 to 20.
  - Search MATLAB help to find the appropriate function (*cumulative sum*).
  - Calculate cumulative sum $\boldsymbol{L}$ of even element of the vector $\boldsymbol{x}$.
  - What is the value of the last element of vector $\boldsymbol{L}$?

$$\boldsymbol{x} = \begin{pmatrix} 1 & 2 & \dots & 20 \end{pmatrix}$$
$$\boldsymbol{S} = \begin{pmatrix} 1 & 1+2 & \dots & 1+2+\dots+20 \end{pmatrix}$$



Indexing in Matlab IV.

- Which one of the following returns corner elements of a matrix **A** (10×10)?

```
A([1, 1], [end, end])
A({[1, 1], [1, end], [end, 1], [end, end]})
A([1, end], [1, end])
A(1:end, 1:end)
```

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Deleting Elements of a Matrix

- Empty matrix is a crucial concept in deleting elements of a matrix.

```
T = [];
```

- We want to:
  - Remove 2nd row of a matrix **A**.

```
A(2, :) = []
```

  - Remove 3rd column of a matrix **A**.

```
A(:, 3) = []
```

  - Remove 1st, 2nd and 5th column of a matrix **A**.

```
A(:, [1 2 5]) = []
```

Adding and Replacing Elements of a Matrix

- We want to replace:
  - 3rd column of a matrix **A** (of size $M \times N$) by a vector $\boldsymbol{x}$ (length M).

```
A(:, 3) = x
```

  - 2nd, 4th and 5th row of a matrix **A** by three rows of a matrix **B** (number of columns of both **A** and **B** is the same).

```
A([2 4 5], :) = B(1:3, :)
```

- We want to swap
  - 2nd row of matrix **A** and 5th column of matrix **B** (number of columns of **A** is the same as number of rows of **B**).

```
A(2, :) = B(:, 5)
```

- Remember that always the size of matrices have to match!

Deleting, Adding and Replacing Matrices

- Which of the following deletes the first and the last column of matrix **A** (6×6)?
  - Create your own matrix and give it a try.

```
A[1, end]      = 0
A(:, 1, end)   = []
A(:, [1:end])  = []
A(:, [1 end])  = []
```

- Replace 2nd, 3rd and 5th row of matrix **A** by first row of matrix **B**.
  - Assume the number of columns of matrices **A** and **B** is the same.
  - Consider the case where **B** has more columns than **A**.
  - What happens if **B** has less columns than **A**?

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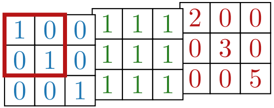
Matrix Creation, Element Replacement

- Create following 3D array:

$$\mathbf{M}(:,:,1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{M}(:,:,2) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \mathbf{M}(:,:,3) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}.$$



- Replace elements in the first two rows and columns of the first sheet of the array (*i.e.* the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ with `NaN` elements.

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Linear Indexing I.

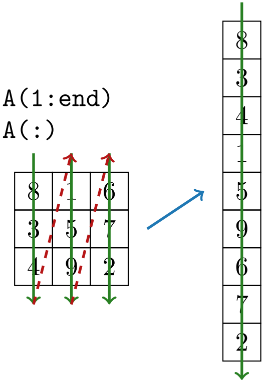
- Elements of an array of arbitrary number of dimensions and arbitrary size can be referred using simple index.
  - Indexing takes place along the main dimension (column-wise) then along the secondary dimension (row-wise) etc.

```
A = magic(3)
```

A =

8	1	6
3	5	7
4	9	2

```
A(1:end)
A(:)
```




```
A([1 5])
```

8	1	6
3	5	7
4	9	2

```
A([1 5], :)
```

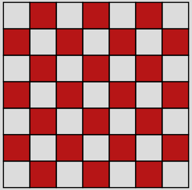
```
Index in position 1
exceeds array bounds
(must not exceed 3).
```

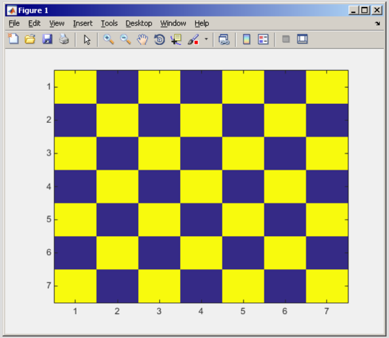
Matrix Operations

Linear Indexing II.

- Consider following matrix: `M = ones(7)`.
  - We set all the red-highlighted elements to zero:

```
M(2:2:end) = 0;
imagesc(M);
```





Matrix Indexing Using Own Values

- Create matrix **A**

```
N = 4;
A = magic(N);
```

- First think about what will be the result of the following operation and only then carry it out

```
B = A(A);
```

  - Does the result correspond to what you expected?
  - Can you explain why the result looks the way it looks?
  - Notice the interesting mathematical properties of the matrices **A** and **B**.
  - Are you able to estimate the evolution? `C = B(B)`
- Try similar process for $N = 3$ or $N = 5$.

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Linear Indexing III. - `ind2sub`, `sub2ind`

- `ind2sub` recalculates linear index to subscript corresponding to size and dimensions of the matrix
  - Applicable to an array of arbitrary size and dimension.

```
ind = 3:6;
[rw, col] = ind2sub([3, 3], ind)
% rw  = [3 1 2 3]
% col = [1 2 2 2]
```

1	4	7
2	5	8
3	6	9

⟶

1, 1	1, 2	1, 3
2, 1	2, 2	2, 3
3, 1	3, 2	3, 3

- `sub2ind` recalculates subscripts to linear index.
  - Applicable to an array of arbitrary size and dimension.

```
ind2 = sub2ind([3, 3], rw, col)
% ind2 = [3 4 5 6]
```

1, 1	1, 2	1, 3
2, 1	2, 2	2, 3
3, 1	3, 2	3, 3

⟶

1	4	7
2	5	8
3	6	9

Linear Indexing IV.

- For a two-dimensional array, find a formula to calculate linear index from position given by `row` (row) and `col` (column).
  - Check with a matrix **A** of size 4×4, where
    - `row = [2, 4, 1, 2]`,
    - `col = [1, 2, 2, 8]`,
  - and therefore
    - `ind = [2, 8, 5, 14]`.

```
A = zeros(4);
A(:) = (1:16)
```

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Linear Indexing V.

- Consider following matrix:

```
A = magic(4);
```

- Use linear indexing so that only the element with highest value in each row of **A** was left (all other valueas set to 0); call the new matrix **B**.

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Relational Operators I.

- To find out, to compare, whether "something" is greater than, less than, equal to etc.
- The result of the comparison is always either
  - positive (`true`), logical one "1",
  - negative (`false`), logical zero "0".
- All relation operators are vector-wise.
  - It is also possible to compare vector vs. vector, matrix vs. matrix, ...
- Often in combination with logical operators (see later)
  - Multiple relational operators can be applied to complex expressions.

`>`	greater than
`>=`	greater than or equal to
`<`	less than
`<=`	less than or equal to
`==`	equal to
`~=`	not equal to

Relational Operators II.

- Having the vector $\mathbf{G} = \begin{pmatrix} \frac{\pi}{2} & \pi & \frac{3\pi}{2} & 2\pi \end{pmatrix}$, find elements of $\boldsymbol{G}$ that are
  - greater than π,
  - less than or equal to π,
  - not equal to π.
  - Try similar operations for $\mathbf{H} = \mathbf{G}^{\mathrm{T}}$
- Try to use relational operators in case of matrices and scalars as well.
  - Find out whether $\mathbf{V} \geq \mathbf{U}$:
    - $\mathbf{V} = \begin{pmatrix} -\pi & \pi & 1 & 0 \end{pmatrix}$,
    - $\mathbf{U} = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$.

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Relational Operators III.

- Find out the results of following relations.
  - Try to interpret the results.

```
2 < 1 ~= 1 % ???
```

```
r = 1/2;
0 < r < 1 % ???
```

```
(1 > A) <= true
```

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Logical Operators I.

- To to find out, whether particular condition is fulfilled.
- The result is always either
  - positive (`true`), logical one “1”,
  - negative (`false`), logical zero “0”.
- `all`, `any` is used to convert logical array into a scalar.
- MATLAB interprets any numerical value except 0 as `true`.
- All logical operators are vector-wise.
  - It is also possible to compare vector vs. vector, matrix vs. matrix, . . .
- Function `is*` extends possibilities of logical expressions.
  - We will see later

&	and
—	or
~	not
	xor
	all
	any

Logical Operators II.

- Assume a vector of 10 random numbers ranging from -10 to 10.

```
a = 20*rand(10, 1) - 10
```

  - Following command returns `true` for elements fulfilling the condition.

```
a < -5 % relation operator
```

  - Following command returns values of those elements fulfilling the condition (logical indexing).

```
a(a < -5)
```

  - Following command puts value of -5 to the position of elements fulfilling the condition.

```
a(a < -5) = -5
```

  - Following command sets value of the elements in the range from -5 to 5 equal to zero (opposite to thresholding).

```
a(a > -5 & a < 5) = 0
```

  - Thresholding function (values below -5 set equal to -5, values above 5 set equal to 5).

```
a(a < -5 | a > 5) =
    sign(a(a < -5 | a >
    5))*5
```

Logical Operators III.

- Determine which of the elements of the vector $\mathbf{A} = \begin{bmatrix} \frac{\pi}{2} & \pi & \frac{3\pi}{2} & 2\pi \end{bmatrix}$ fulfill following condition.
  - Which elements are equal to π or are equal to 2π.
    - Pay attention to the type of the results (=logical values `true`/`false`).
  - Which elements are greater than $\frac{\pi}{2}$ and at the same time are not equal to 2π.
- Group elements from the previous condition with vector $\mathbf{A}$.

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Logical Operators IV.

- Create a row vector in the interval from 1 to 20 with step of 3.
  - Create the vector filled with elements from the previous vector that are greater than 10 and at the same time less than 16. Use logical operators.

Logical Operators V.

- Create matrix **M** (`M = magic(3)`) and answer following questions using functions `all` and `any`.
  - In which of the columns all elements are greater than 2?
  - In which of the rows there is at least one element greater than or equal to 8?
  - Does the matrix **M** contain only positive numbers?

$$\texttt{any(}\begin{bmatrix}0&1&1\\1&1&0\\0&1&1\end{bmatrix}\texttt{)=}\begin{bmatrix}1&1&1\end{bmatrix}, \texttt{all(}\begin{bmatrix}0&1&1\\1&1&0\\0&1&1\end{bmatrix}\texttt{)=}\begin{bmatrix}0&1&0\end{bmatrix}, \texttt{any(all(}\begin{bmatrix}0&1&1\\1&1&0\\0&1&1\end{bmatrix}\texttt{))=any(}\begin{bmatrix}1&1&1\end{bmatrix}\texttt{)=}1$$

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Logical Operators VI.

- In the case we need to compare scalar values only then "short-circuited" evaluation can be used.
- Evaluation keeps on going until the point where it makes no sense to continue
  - *e.g.*when evaluating

```
clear;
a = true;
b = false;
a && b && c && d
```

- There are no problems with undefined variables `c` and `d`, because the execution is terminated before evaluating those variables.

- However:

```
clear;
a = true;
b = true;
a && b && c && d
```

- This is terminated with error ...

Logical Operators VII.

- Find out the result of the following operation and interpret it.

```
~(~[1 2 0 -2 0])
```

- Test whether variable b is not equal to zero and then test whether at the same time $a/b > 3$.
  - Following operation tests whether both conditions are fulfilled while avoiding division by zero!
  - However: `1/0 > 3 %Inf > 3` $\rightarrow 1$

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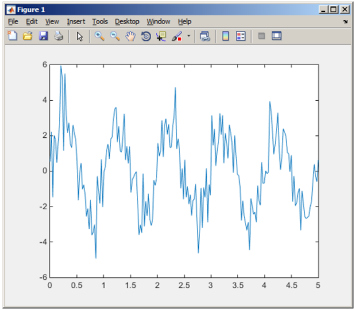
Exercises

Exercise I.

- Consider signal: $s(t) = \sqrt{2\pi}\sin(2\omega_0 t) + n(\mu,\sigma)$, $\omega_0 = \pi$, where the mean and standard deviation of normal distribution n are: $\mu = 0$ (`mu = 0`), $\sigma = 1$ (`sigma = 1`).
  - Create time dependence of the signal spanning over $N = 5$ periods of the signal using $V = 40$ samples per period.
  - One period is $T = 1 : t \in [kT, (k+N)T], k \in \mathbb{Z}^0$ (choose k equal for instance to 0).
  - The function $n(\mu,\sigma)$ has following MATLAB syntax:

```
n = mu + sigma*randn(1, N*V); % noise
```

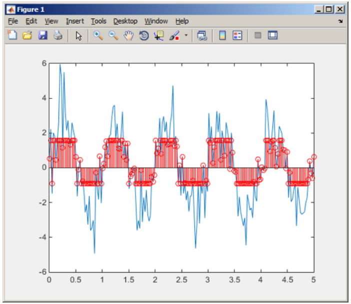


Exercise II.

- Apply threshold function to generated signal from the previous exercise to limit its maximum and minimum value:
  - The result is vector `sp_t`.
  - Use function `min` and `max` with two input parameters (see MATLAB help for details).
  - Use the following code to check your output:

$$s_p(t) = \begin{cases} s_{\min} \Leftrightarrow s(t) < s_{\min} \\ s_{\max} \Leftrightarrow s(t) > s_{\max} \\ s(t) \ldots \text{otherwise} \end{cases} \qquad s_{\min} = -\frac{9}{10} \qquad s_{\max} = \frac{\pi}{2}$$

```
close all;
plot(t, s_t); hold on;
stem(t, sp_t, 'r');
```



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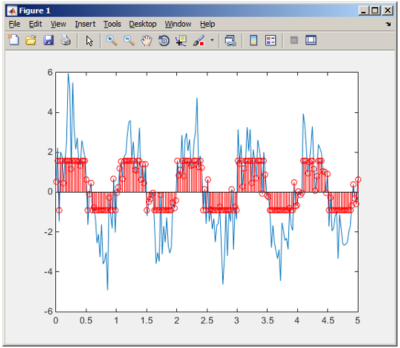
Exercise III.

- Recall the signal from Exercise I.
  - Try again to limit the signal by values $s_{\min}$ and $s_{\max}$.
  - Use relational operators (>/<) and logical indexing (`s(a>b) = c`) instead of functions `min` and `max`.
    - Solve the task item-by-item.




```
N = 5; V = 40;
t = linspace(0, N, N*V);
s_t = randn(1, N*V) +
    sqrt(2*pi)*sin(2*pi*t);
```

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Exercise IV.a

- Create a script to calculate compound interest[1].
  - The problem can be described as

$$P = \frac{rA\left(1+\frac{r}{n}\right)^{nk}}{n\left(\left(1+\frac{r}{n}\right)^{nk}-1\right)},$$

    where P is regular repayment of debt A, paid n-times per year in the course of k years with interest rate r (decimal number).
  - Create a new script and save it.
  - At the beginning delete variables and clear Command window.
  - Implement the formula first, then proceed with inputs (`input` and outputs (`disp`).
  - Try to vectorize the code, *e.g.*for various values of n, r or k.
  - Check your results (for $A = 1000$, $n = 12$, $k = 15$, $r = 0.1$ is $P = 10.7461$).

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[1]Interest from the prior period is added to principal.

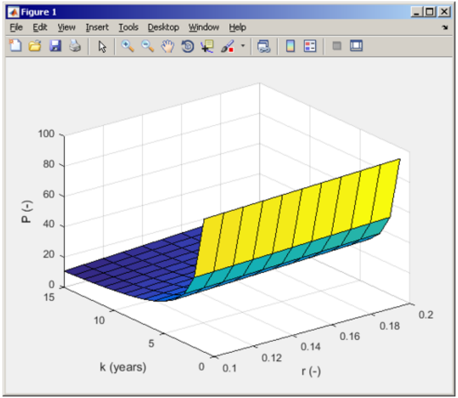
Exercise IV.b

- Try to vectorize the code, both for r and k.
- Use scripts for future work with MATLAB.
  - Bear in mind, however, that parts of the code can be debugged using command line.

$$P = \frac{rA\left(1+\frac{r}{n}\right)^{nk}}{n\left(\left(1+\frac{r}{n}\right)^{nk}-1\right)}$$

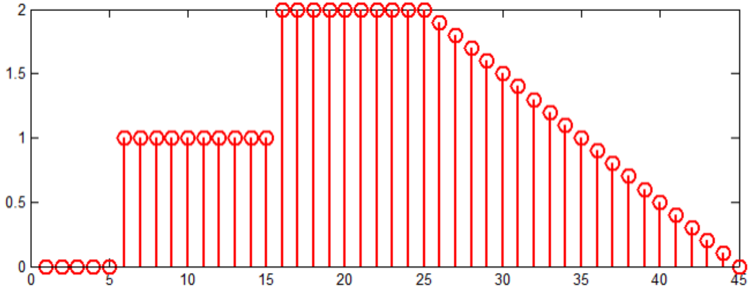
Exercise IV.c

- Vectorized code for both r and k.
  - The compatible size array feature used.
  - `surf` created 3D surface plot.



Exercise V.a

- Generate vector containing following sequence.
  - Note the x-axis (interval, number of samples).
  - Split the problem into several parts to be solved separately.
  - Several ways how to solve the problem.
  - Use `stem(x)` instead of `plot(x)` for plotting.
- Try to generate the same signal beginning with zero …



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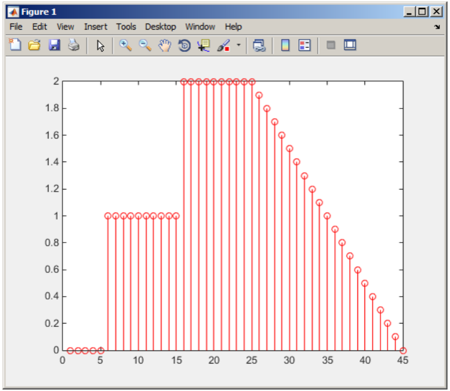
Exercise V.b

- Generate vector containing following sequence.



- One of possible solutions:

- Or:

Exercise VI.

- Consider following matrix $\mathbf{A} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \end{bmatrix}$.
- Write an expression testing whether all elements of $\mathbf{A}$ are positive and at the same time all elements of the first row are integers.
- Compare with:

```
all(all(A > 0)) && all(mod(A(1, :), 1) == 0)
```

  - What is the difference?

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Exercise VII.a

- Reflection coefficient S_{11} of a one-port device of impedance Z is given by:

$$S_{11} = 10\log_{10}\left(\left|\frac{Z - Z_0}{Z + Z_0}\right|^2\right),$$

where $Z_0 = 50\Omega$ and $Z = R + \mathrm{j}X$.

  - Calculate and depict the dependence of S_{11} for $R = 30\Omega$ and X on the interval $[1, 1000]$ with 100 evenly spaced points in logarithmic scale.
  - Use the code below and correct errors in the code. Correct solution generates plot depicted on the next slide.

```
500 = Z0;                  % reference impedance
R   == 30;                 % real part of the impedance
X   = Logspace(0, 3, 1e2); % reactance vector
clear;
Z   = i*(R + 1i*X);        % impedance
S11 = 10*log(abs(Z-Z0)./(Z+Z0))^2); % reflection coeff. in dB
semilogx(S11, X)           % plotting using log. x-axis
```

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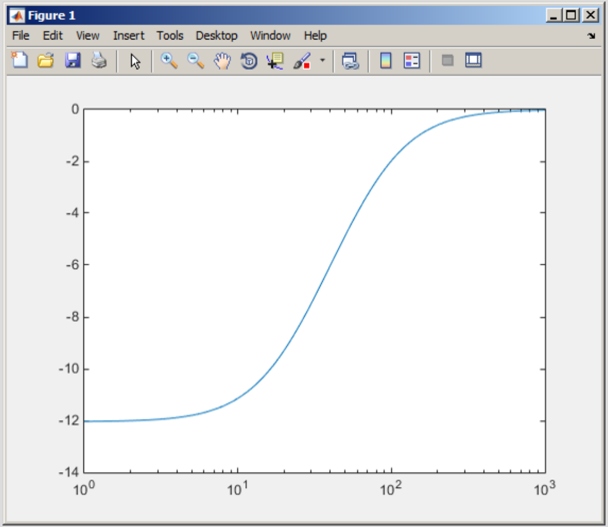
Exercise VII.b



Questions?

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