Lecture 1: Matlab Environment, Basic Math Operators

B0B17MTB – Matlab

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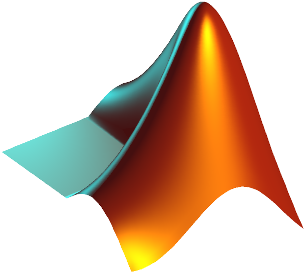
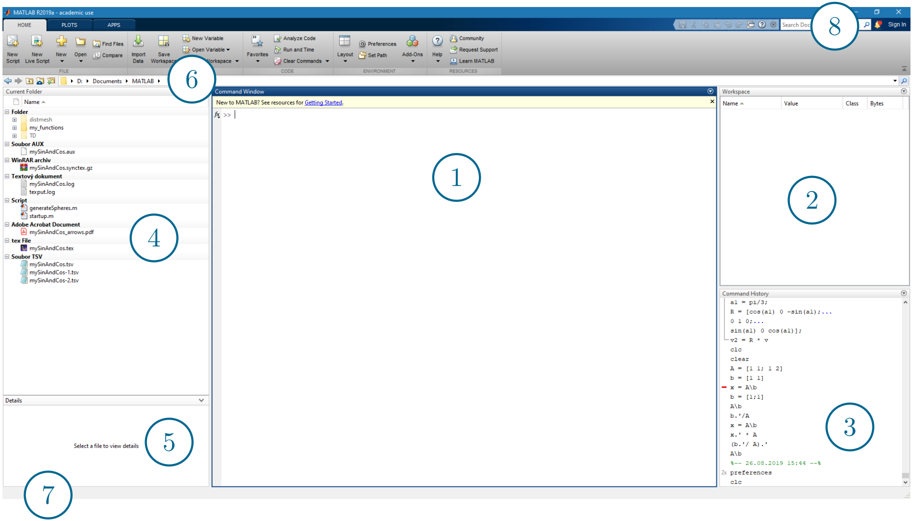
Outline

1. MATLAB Environment
2. Scalars, Vectors, Matrices
3. Basic Math Operations
4. Excercises



The MATLAB Environment



New to MATLAB? See resources for Getting Started.

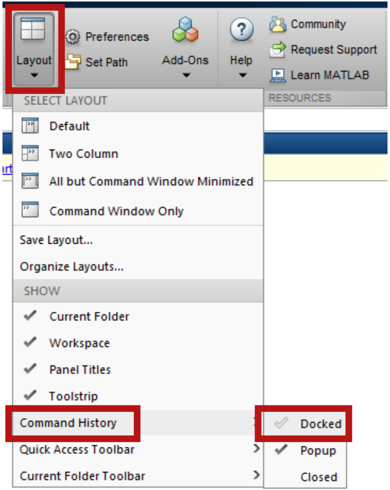
```
>>
```

Select a file to view details

```
al = pi/3;
R = [cos(al) 0 -sin(al);...
0 1 0;...
sin(al) 0 cos(al)];
v2 = R * v
clc
clear
A = [1 1; 1 2]
b = [1 1]
x = A\b
b = [1;1]
A\b
b.'/A
x = A\b
x.' * A
(b.'/ A).'
A\b
%-- 26.08.2019 15:44 --%
2x preferences
clc
```

The MATLAB Environment – Panels

1. Command Window
2. Workspace
3. Command History – *not activated, to activate* →
4. Current Folder
5. Current Folder – Details
6. Current Working Directory
7. Status (“Busy” when MATLAB is executing your code)
8. Search in documentation

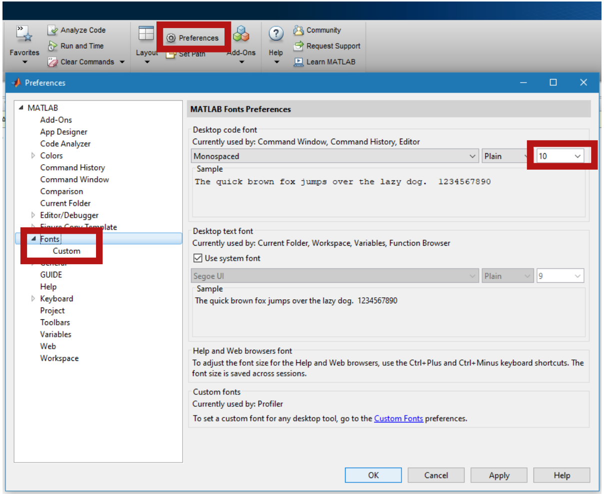


Preferences

- Command:

```
>> preferences
```

- Ribbon menu:
  - Change font size.



MATLAB Fonts Preferences

Desktop code font

Currently used by: Command Window, Command History, Editor

Monospaced | Plain | 10

Sample

```
The quick brown fox jumps over the lazy dog.  1234567890
```

Desktop text font

Currently used by: Current Folder, Workspace, Variables, Function Browser

☑ Use system font

Segoe UI | Plain | 9

Sample

The quick brown fox jumps over the lazy dog. 1234567890

Help and Web browsers font

To adjust the font size for the Help and Web browsers, use the Ctrl+Plus and Ctrl+Minus keyboard shortcuts. The font size is saved across sessions.

Custom fonts

Currently used by: Profiler

To set a custom font for any desktop tool, go to the Custom Fonts preferences.

Documentation

```
>> doc % opens documentation window
```

```
>> help % MATLAB help
```

```
>> demo % tutorials
```

The Help Structure

- Command:

```
>> help sin
```

- Output:

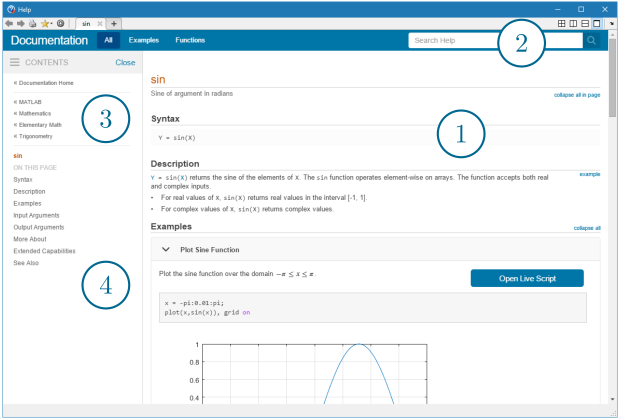
```
sin    Sine of argument in radians.
   sin(X) is the sine of the elements of X.

   See also asin, sind, sinpi.

   Reference page for sin
```

The Documentation Structure I.

- Command:

```
>> doc sin
```

1. Documentation page
2. Search field
3. Documentation contents
4. Bookmarks of this page



sin

Sine of argument in radians

Syntax

```
Y = sin(X)
```

Description

`Y = sin(X)` returns the sine of the elements of `X`. The `sin` function operates element-wise on arrays. The function accepts both real and complex inputs.

- For real values of `X`, `sin(X)` returns real values in the interval [-1, 1].
- For complex values of `X`, `sin(X)` returns complex values.

Examples

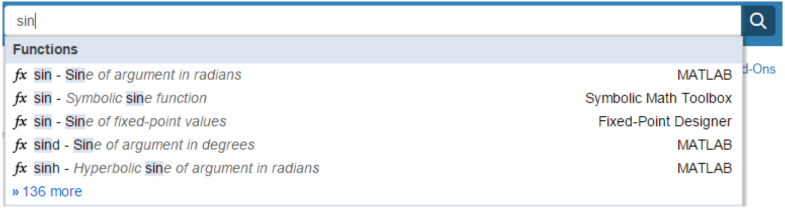
Plot Sine Function

Plot the sine function over the domain $-\pi \le x \le \pi$.

```
x = -pi:0.01:pi;
plot(x,sin(x)), grid on
```

The Documentation Structure II.

- Check the origin of the function.
  - Several functions with the same name may exist.
- Functions types by origin:
  - MATLAB core functions – most of them build-in, some are available for editing (not recommended!).
  - Functions from installed toolboxes.
  - User-created functions.
- Calling priority for functions will be discussed later.
- During this course, always open a function from core installation.



sin

Functions

fx sin - *Sine of argument in radians* MATLAB

fx sin - *Symbolic sine function* Symbolic Math Toolbox

fx sin - *Sine of fixed-point values* Fixed-Point Designer

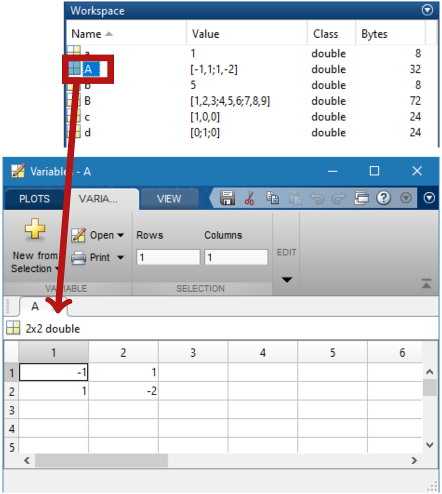
fx sind - *Sine of argument in degrees* MATLAB

fx sinh - *Hyperbolic sine of argument in radians* MATLAB

» 136 more

Workspace Browser

- List of variables.
- Deleting/modification of existing variables.
- Saving/loading.
- Values, Class and Memory information.
  - Other information can be added: size, min, max, ...
  - All information can be obtained using MATLAB functions that we learn later, *e.g.*, `min`, `max`, `max`, `length`.
- Fast data plotting option (in ribbon).



Workspace

Name	Value	Class	Bytes
a	1	double	8
A	[-1,1;1,-2]	double	32
b	5	double	8
B	[1,2,3;4,5,6;7,8,9]	double	72
c	[1,0,0]	double	24
d	[0;1;0]	double	24

Variables - A

Rows: 1

Columns: 1

A

2x2 double

	1	2	3	4	5	6
1	-1	1				
2	1	-2				
3						
4						
5						

MATLAB Commands

- Matlab is cAsE sEnSiTiVe!
  - Almost entirely, with certain exceptions (properties of graphics objects, . . .).
  - Pay attention to typos and variable names (see later).
    - New versions of MATLAB offer certain options.

```
>> AA = [1 1 1]
>> Aa
```

  - Beware of different syntax in Mathematica.
    - Following syntax is incorrect both in MATLAB and Mathematica:

```
>> Sin(pi/2) % function names start with lower case
>> cos[pi/2] % function input is in parentheses ()
```

    - Will be discussed in the next lectures.

Matrix Operations

Naming Conventions

- Names of variables can have max. 63 characters starting with letter (`>> namelengthmax`)
  - Letters and numbers are allowed, other symbols (colon ":", hyphen "-" and others) are not.
  - Underscore is allowed in the variable name "_" (not at the beginning, though!).
- Lowercase letters in the names of scalars and variables (`a = 17.59;`).
- Matrix names usually start with a capital letter (`A = [ .. ];`).
- Iteration variables, variables used in `for` cycles usually named `m`, `n`, `k`, etc.
  - It is advisable to avoid `i` and `j` (complex unit).
- Chose the names to correspond to the purpose of the variable.
- Avoid, if possible, standalone letter "`l`" (to be confused with one "`1`") and predefined variables in MATLAB environment (see later).
- Choose names corresponding to the meaning of each particular variable.
- Avoid using names of existing functions or scripts (overloading can occur).
- The same conventions are valid for names of functions and scripts.

Variable Names

- Examples of valid variable names:

```
a, A, b, c, x1, x2, M_12, test1, matrix_A, fx, fX
```

- Examples of invalid variable names:

```
1var      % starts with a number (not possible in MATLAB)
matrix A  % contains space
coef.a    % possible only if coef is of type 'struct'
Test-1    % algebraic expressing: ans = Test - 1
f(y)      % makes sense when using symbolic expressinos
```

- Examples of valid numbers in MATLAB,

```
3, -66, +0.0015, .015, 1e2, 1.6025e-10, 05.1
```

Functions who, whos

- Function **who** lists all variables in MATLAB Workspace.
  - Wide variety of options.
- Functions **whos** lists the variable names + dimension, size and data type of the variables or displays content of a file.
  - Wide variety of options.

```
>> whos('-file', 'matlab.mat');
```

```
>> a = 15; b = true; c = 'test'; d = 1 + 5j;
>> who
>> whos
>> Ws = whos;
```

Workspace – Output Deletion

- To clean (erase) command window:

```
>> clc
```

- To clean one (or more) variable(s):

```
>> clear      % whole Workspace is deleted
>> clear XX   % variable XX is deleted
>> clear XX YY % variables XX and YY are deleted
>> clear z*   % everything starting with 'z' is deleted
```

- `clear` has a number of other options (graphics, I/O)

Program Flow

Command History Window

- Command History window stores all commands from the Command Window.
- Command History is accessible though ↑ or ↓.
- it is possible to filter out past commands by, *e.g.*:
  `>> A = [` + ↑.
- It is possible to copy-and-paste entire Command History:
  SHIFT / CTRL / CTRL + A → CTRL + C.

Matrices in MATLAB

- Matrix is a basic data structure in MATLAB.
- There are following variables types depending on size:
  - scalar: 1×1
  - vector: $M \times 1$ or $1 \times N$
  - matrix: $M \times N$
  - array (multidimensional matrices): $M \times N \times P \times Q \times R \times \ldots$
- Matrices can be complex.
- It can contains text as well (beware the length).

- M-by-N matrix:

$$\begin{array}{cc} a_{i,j} & \xrightarrow{\begin{array}{c}N\text{ columns}\\ j\text{ changes}\end{array}} \\ \begin{array}{c}M\text{ rows}\\ i\text{ changes}\end{array}\Bigg\downarrow & \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots \\ a_{3,1} & a_{3,2} & a_{3,3} & \cdots \\ a_{4,1} & a_{4,2} & a_{4,3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \end{array}$$

Matrix Creation

- Following techniques are available:
  - element-by-element entering (suitable for small matrices only),
  - colon notation ":" to define elements of series,
  - generation by built-in functions,
  - generation of matrices in m-files,
  - import and export from/to external files(**.mat**, **.txt**, **.xls**, ...).

Matrix Construction Element-by-element I.

- Test following commands to construct matrices by element enumeration.
  - Suitable for small matrices only.

```
>> a1 = -1
>> a2 = [-1] % brackets are redundant
```

```
>> v1 = [-1 0 1]
>> v2 = [-1; 0; 1]
```

```
>> M1 = [-1 0 1; -2 0 2]
>> M2 = [-1 -2; 0 0 ; 1 2]
>> M3 = [[-1 -2]; [0 0]] % inner brackets are redundant
```

$$a_1 = a_2 = -1$$

$$\mathbf{v}_1 = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$\mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{M}_1 = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \end{bmatrix}$$

$$\mathbf{M}_2 = \begin{bmatrix} -1 & -2 \\ 0 & 0 \\ 1 & 2 \end{bmatrix}$$

$$\mathbf{M}_3 = \begin{bmatrix} -1 & -2 \\ 0 & 0 \end{bmatrix}$$

Matrix Construction Element-by-element II.

- Construct following matrices:
  - Matrix values are defined inside square brackets `[]`,
  - semicolon “;” separates individual rows of a matrix.

$$\mathbf{A} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

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Matrix Construction

- Semicolon placed at the end of a command suppresses display of the output in Command Window.

```
>> a = 1
>> b = 5;
```

- When there is more than one command on the same line, comma is used to separate each of the commands.

```
>> a = 1, b = 5
>> a = 1; b = 5;
```

  - Note: it is possible to copy and paste code including “>>”

- Row vs column vector:

```
>> c = [1 0 0]
>> d = [0; 0; 1]
```

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Basic Math Operators I.

- Operator types:
    - arithmetics:
        - matrix,
        - vector,
    - relational,
    - logical and other (to be mentioned later . . .).
- Other operations using MATLAB functions:
    - complex conjugate,
    - sum, determinant, square root,
    - and hundreds of other functions . . .

`+`	addition
`-`	subtraction
`*`	multiplication
`^`	power
`'`	transpose
`\`	left matrix division
`/`	right matrix division
`.`	dot notation

Matrix Operations

Operator Precedence in MATLAB

- According to the following table:
  - see MATLAB → Language Fundamentals → Operators and Elementary Operations → Arithmetic

1	parentheses	()					
2	transpose, power	'	.'	^	.^		
3	unary plus, unary minus, logical negation	+	-	~			
4	multiplication, division	*	.*	/	\	./	.\
5	addition, subtraction	+	-				
6	colon operator	:					
7	relation operators	<	>	<=	>=	==	~=
8	logical AND (element-wise)	&					
9	logical OR (element wise)	|					
10	logical AND (short-circuit)	&&					
11	logical OR (short-circuit)	||					

Basic Math Operators II.

- Type in following commands:
  - Zero can be omitted with a decimal number beginning with zero (not recommended).

```
>> a3 = -2/4
>> a4 = -0.5
>> a5 = -.5
```

  - What is the difference between a_3, a_4 and a_5?
  - Beware the precedence of operators (wee see in the next slides):

```
>> 3*5*6
>> a1 = 15
>> a2 = 10;
>> a2/a3
>> a2/a3*a4
>> a2/(a3*a4)
```

  - Explain the difference between `a2/a3*a4` and `a2/(a3/a4)`.
  - Verify the rules of operator precedence from the previous slide.

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Lengthy commands in Matlab

- It is suitable to structure command blocks for clarity:
  - next line: SHIFT + ENTER

```
>> A = [1 1 1]; B = [2 2 2]; % SHIFT + ENTER
C = [2 3 2];
```

- Three dots notation:
  - For continuation of the same command on the next line.
  - Compare results:

```
>> A1 = [ 1 1 ...
2 3]
```

```
>> A2 = [ 1 1
2 3]
```

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Basic Math Functions I.

- Math functions in MATLAB are generally divided in three groups:
  - Scalar:
    - Function operates over individual elements of a matrix,
    - *e.g.*: `sin`, `sqrt`, `log`, `factorial`.
  - Vector:
    - Function operates over individual rows/columns of a matrix,
    - *e.g.*: `sum`, `max`.
  - Matrix:
    - Function operates over a whole matrix,
    - *e.g.*: `det`, `trace`.

Basic Math Functions II.

- Using MATLAB help, calculate the following expression: $a\sin^2(\alpha) + a\cos^2(\alpha) - a$
  - Use numerical values your own choice.

- Verify following logarithmic identity: $\log_{10}(a) + \log_{10}(b) - \log_{10}(ab) = 0$

- Find sum of all elements in individual rows of the following matrix:

$$T = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ 6 & 7 & 7 & 9 \\ 0.2 & 0.3 & 0.4 & 0.5 \end{bmatrix}$$

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Basic Math Functions III.

- Assume following vectors $\mathbf{u} = (1, 2, 3)$ and $\mathbf{v} = (3, 2, 1)$.
  - Calculate:

$$\begin{array}{cc} \mathbf{u}\mathbf{v}^{\mathrm{T}} & \mathbf{v}\mathbf{u}^{\mathrm{T}} \\ \mathbf{v}^{\mathrm{T}}\mathbf{u} & \mathbf{u}^{\mathrm{T}}\mathbf{v} \\ \mathbf{u} \cdot \mathbf{v} & \mathbf{u} \times \mathbf{v} \end{array}$$

  - Following functions are needed:
    - `transpose` (`.'`) of a matrix,
    - `dot` scalar product,
    - `cross` product.
  - What is the result of the above mentioned operations?

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$\mathbf{A}^{\mathrm{T}} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

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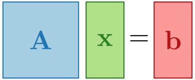
Matrix Division in MATLAB

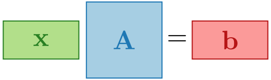
- Two cases are distinguished:
  - left division (`\` – `mldivide`),
  - right division (`/` – `mrdivide`).
- Solution of a linear system of equations:
  - **A** is an invertible (regular) matrix,
  - **b** is a column (row) vector.



$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

```
>> x = A \ b
```



$$\mathbf{x}\mathbf{A} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{b}\mathbf{A}^{-1}$$

```
>> x = b / A
```

Basic Math Functions IV.

- Find the sum of diagonal elements (trace of a matrix) of the matrix **T** with elements coming from normal distribution with mean equal to 10 and standard deviation equal to 4.

```
>> T = 10 + 4*randn(7, 7);
```

- Find determinant of matrix **U**.

$$\mathbf{U} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & -2 & -1 \end{bmatrix}$$

```
>> U = [1 2 3; 0 2 0; ...
0 -2 -1];
```

- Solve the linear system of equations:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 6 \\ 4x_1 + 5x_2 + 6x_3 &= 15 \\ 7x_1 + 8x_2 + x_3 &= 16 \end{aligned}$$

$$\begin{aligned} \mathbf{A}\mathbf{x} &= \mathbf{b} \\ \mathbf{x} &= \mathbf{A}^{-1}\mathbf{b} \end{aligned}$$

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Predefined Values in MATLAB

- MATLAB contains several predefined values:
  - `eps` – precision of single/double numbers (Determines the shortest distance between two single/double numbers).
  - `ans` – *answer* – most recent answer.
  - `NaN` – *not a number* (every expression containing `NaN` is `NaN`)
    - `NaN` can be used advantageously in some cases.
  - `Inf` – *infinite number* (variable `Inf` can be used in calculation:))
    - Pay attention to `Inf` propagation throughout your code (use allowed operations only).
  - `i`, `j` – complex unit.
    - They are all basically functions (without input parameter).
  - Check results of the following expressions:

```
>> t1 = 10/0    % t1 = Inf
>> t2 = 0/0     % t2 = NaN
>> t3 = t1*5    % t3 = Inf
>> t4 = t1 + t2 % t4 = NaN
```

  - `pi`, `intmin`, `intmax`, `realmin`, `realmax`, . . . (functions)

Format of Command Line Output

- Up to now we have been using basic setup.
- MATLAB offers number of other formatting options
  - Use `format style`.
  - Output format does not change neither the computation accuracy nor the accuracy of stored results (`eps`, `realmax`, `realmin`, ...still apply).

`style`	format description
`short`	fixed 4 decimal points are displayed
`long`	15 decimal points for double precision, 7 decimal points for single precision
`shortE`	floating-point format (scientific notation)
`longE`	-//-
`bank`	two decimal points only (eur – cents)
`rat`	MATLAB attempts to display the results as a fraction
`compact`	suppressed the display of blank lines
and others...	note: omitting `style` parameter restores default setup

Format of Command Line Output

- Try following output format settings:
  - Each format is suitable for different type of problems.

```
>> s = [-5 1/2 1/3 10*pi sqrt(2)];
>> format long ; s
>> format rat ; s
>> format bank ; s
>> format hex ; s
>> format +; s
>> format; s
```

- There exist other formats with slight differences.
  - Check doc format
- Later, we will learn how to use formatted conversion into strings (commands sprintf and fprintf).

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Complex Numbers I.

- More entry options in MATLAB.

```
>> C1 = 1 + 1j % prefered
>> C2 = 1 + 5i % prefered
>> C3 = 1 + 5*i % NO!
>> C4 = sqrt(-1)
>> C5 = complex(1, 2)
>> C6 = 1e1i
>> C7 = exp(1j*pi/4)
```

- Frequently used functions:

`real,imag`	real and imaginary part of a complex number
`conj`	complex conjugate
`abs`	absolute value of a complex number
`angle`	angle in complex plane [rad]
`complex`	constructs complex number from real and imaginary components
`isreal`	checks if the input is a complex number (more on that later)
`i, j`	complex unit
`cplxpair`	sort complex numbers into complex conjugate pairs

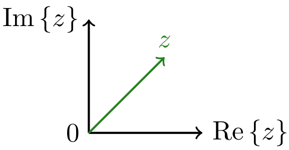
Complex Numbers II.

- Create complex number $z = 1 + 1\mathrm{j}$ and its complex conjugate $s = z^*$.
- Switch between Cartesian and polar form (find $|z|$ and φ).

$$
\begin{aligned}
z &= \mathrm{Re}\left\{z\right\} + \mathrm{Im}\left\{z\right\} = a + \mathrm{j}b \\
z &= |z|\,\mathrm{e}^{\mathrm{j}\varphi}, |z| = \sqrt{a^2 + b^2} \\
z &= |z|\left(\cos\varphi + \mathrm{j}\sin\varphi\right)
\end{aligned}
$$

- Verify Moivre's theorem:

$$
\begin{aligned}
z^n &= \left(|z|\,\mathrm{e}^{\mathrm{j}\varphi}\right)^n \\
z^n &= |z|^n\left(\cos\left(n\varphi\right) + \mathrm{j}\sin\left(n\varphi\right)\right)
\end{aligned}
$$



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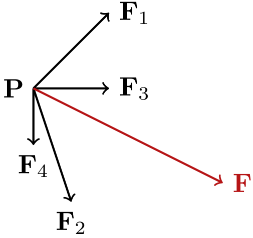
Exercises

Exercise I.

- Following forces were localized at point $\mathbf{P}$ in xy plane:

$$\begin{array}{ll} \mathbf{F}_1 = [2, 2] & \mathbf{F}_3 = [2, 0] \\ \mathbf{F}_2 = [1, -3] & \mathbf{F}_4 = [2, -1.5] \end{array}$$

  - What is the direction of the resultant force $\mathbf{F}$?



- Normalize the resulting vector.

$$\mathbf{n}_\mathrm{F} = \frac{\mathbf{F}}{|\mathbf{F}|} = \frac{\mathbf{F}}{\sqrt{F_x^2 + F_y^2 + F_z^2}}$$

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Exercise II.

- Type-in following commands:

```
>> clear, clc;
>> w1 = [1 2 3 4]
>> w2 = [-2 -3 -4]
>> w3 = [-2; -3; -4]
>> w4 = w1^2, w5 = w2 - w1
```

  - Compare differences.
  - What is the cause of error in calculation of `w4` and `w5`?

- Try also:

```
>> w3*3, w1 - 3
>> w1 + [5 5 5 5]
>> w6 = 5*w1 - [3 5 6] - w2
```

- Calculate the norm (magnitude) of vector `w1`.
  - Try more options.
  - How to modify the calculation in the case of a complex vector?

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Exercise III.

- Calculate roots of the quadratic function:

$$-2x^2 - 5x = 3.$$

  - First, rearrange the terms of the function.

$$2x^2 + 5x + 3 = 0 \;\Rightarrow\; a = 2, b = 5, c = 3$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 24}}{4}$$
$$x_1 = -1,\; x_2 = -\tfrac{3}{2}$$

  - MATLAB provides particular function for calculation of roots a function, try to search it out.

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Exercise IV.

- Think over how many ways there are to calculate the length of hypotenuse when two legs of a triangle are given.
  - Make use of various MATLAB operators and functions.
  - Consider also the case where the legs are complex numbers.

Exercise V.

- Create an arbitrary vector $\mathbf{v}$ and rotate it around arbitrary angle α in xz plane using rotation matrix $\mathbf{R}$.

$$\mathbf{v}' = \mathbf{R}\mathbf{v}$$

$$\mathbf{R} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

200

Exercise V.

- Use the following code and round the resulting number to:

```
>> r = 1 + 10*rand(1)
```

  - nearest integer,
  - nearest integer greater than r,
  - nearest integer lower than r,
  - zero,
  - zero with precision of 2 decimal digits.
- Find remainder after r is divided by 0.1.
  - *modulus* vs. *remainder after division*

300

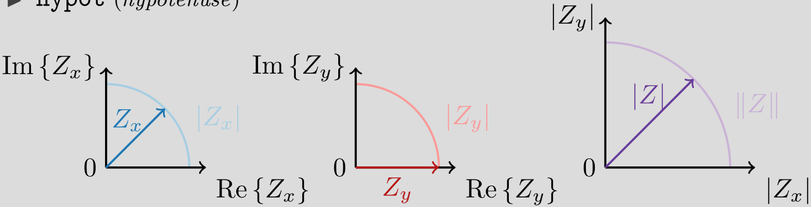
Exercise VI.

- Find out the magnitude of a complex vector (avoid indexing).
  - Use `abs` and `sqrt`.

$$\mathbf{Z} = \begin{bmatrix} 1 + 1\mathrm{j} & \sqrt{2} \end{bmatrix}$$

$$\|\mathbf{Z}\| = ?, \quad \mathbf{Z} \in \mathbb{C}^2$$

  - Alternatively, use following functions:
    - `norm`
    - `dot` (*dot product*)
    - `hypot` (*hypotenuse*)



$\operatorname{Im}\{Z_x\}$
Z_x
$|Z_x|$
0
$\operatorname{Re}\{Z_x\}$
$\operatorname{Im}\{Z_y\}$
$|Z_y|$
0
Z_y
$\operatorname{Re}\{Z_y\}$
$|Z_y|$
$|Z|$
$\|Z\|$
0
$|Z_x|$

Questions?

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