

Camera and stereo examples

1. **3D PCL \Rightarrow RGB:** Consider a perspective camera with the following intrinsic camera parameters \mathbf{K} , camera rotation matrix \mathbf{R} , and translation vector \mathbf{t} :

$$\mathbf{K} = \begin{bmatrix} 500 & 0 & 500 \\ 0 & 500 & 250 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \mathbf{t} = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix},$$

- a) Construct camera projection matrix $\mathbf{P} \in \mathcal{R}^{3 \times 4}$:

$$\mathbf{P} =$$

- b) Project point

$$\mathbf{q} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$$

into the camera. What are pixel coordinates $\mathbf{u} \in \mathcal{R}^2$ of the projection?

$$u_1 =$$

$$u_2 =$$

2. **RGBD \Rightarrow 3D PCL:** Consider the RGBD perspective camera with the previously defined parameters:

$$\mathbf{K} = \begin{bmatrix} 500 & 0 & 500 \\ 0 & 500 & 250 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \mathbf{t} = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix},$$

You are given pixel coordinates $\mathbf{u} = [500; -50]$. Depth estimated at this coordinates by the RGBD sensor is $D(\mathbf{u}) = 3$. What is corresponding 3D point $\mathbf{q} \in \mathcal{R}^3$?

Hint: $q_z = D(\mathbf{u})$

3. **Stereo reconstruction from 2D-2D correspondences:** Consider a stereo pair created from two perspective cameras with the following intrinsic camera parameters \mathbf{K} , camera rotation matrices $\mathbf{R}_1, \mathbf{R}_2$ and translation vectors $\mathbf{t}_1, \mathbf{t}_2$:

$$\mathbf{K} = \begin{bmatrix} 500 & 0 & 500 \\ 0 & 500 & 250 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \mathbf{t}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{t}_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix},$$

You are given one 2D-2D correspondence $\mathbf{u} = [1500, 1250]^\top$ (pixel coordinates in camera \mathbf{P}), $\mathbf{v} = [1500, 250]^\top$ (pixel coordinates in camera \mathbf{S}). Estimate position of the corresponding 3D point $\mathbf{q} \in \mathcal{R}^3$.

- a) Construct camera projection matrix of both matrices $\mathbf{P}, \mathbf{S} \in \mathbb{R}^{3 \times 4}$:

$$\mathbf{P} =$$

$$\mathbf{S} =$$

- b) Formulate the problem of the 3D reconstruction as a solution of the following set of linear equations $\mathbf{A}_{[4 \times 3]} \mathbf{q}_{[3 \times 1]} = \mathbf{b}_{[4 \times 1]}$

$$\mathbf{A} =$$

$$\mathbf{b} =$$

- c) Use arbitrary language or tool (e.g. python/linalg, matlab, C/C++, mathematica) to solve the problem

$$\mathbf{x} =$$

4. **Stereo epipolar line:** Consider the same stereo pair created from two perspective cameras \mathbf{P} , \mathbf{S} with the same previously specified parameters.

$$\mathbf{K} = \begin{bmatrix} 500 & 0 & 500 \\ 0 & 500 & 250 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \mathbf{t}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{t}_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix},$$

Verify if these two pixel coordinates can create 2D-2D correspondence $\mathbf{u} = [1000, 750]^\top$ (pixel coordinates in camera \mathbf{P}), $\mathbf{v} = [1000, 250]^\top$ (pixel coordinates in camera \mathbf{S}).

- a) Estimate relative pose of camera \mathbf{S} wrt camera \mathbf{P} :

$$\mathbf{R} =$$

$$\mathbf{t} =$$

- b) Compute fundamental matrix.

$$\text{Hint: } (\mathbf{R} \times \mathbf{t}) = \mathbf{R} \cdot \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$$

$$\mathbf{F} =$$

- c) Check if point \mathbf{v} lies in the epipolar line generated by point \mathbf{u}

YES - these two points can create 2D-2D correspondence (they correspond to a single 3D point).

NO - they cannot create 2D-2D correspondence

5. **Lidar calibration:** Given set of 3D-3D correspondences $\{(\mathbf{p}_i, \mathbf{q}_i), \mid i = 1 \dots N\}$ between two lidar scans. Suggest a suitable optimization problem, which estimates rotation \mathbf{R} and translation \mathbf{t} , between these two lidars. Specify also domain of \mathbf{R} and \mathbf{t} .
Hint: Do not copy it from slides, just think about it and you will figure it out.