Motion planning
implementation details & data structures

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Introduction

- Planning algorithms require efficient routines
  - collision detection, nearest neighbor search, random number generation
- Fast (and reliable) implementation is necessary for combinatorial and sampling-based planners

Example of important routines in sampling-based planning

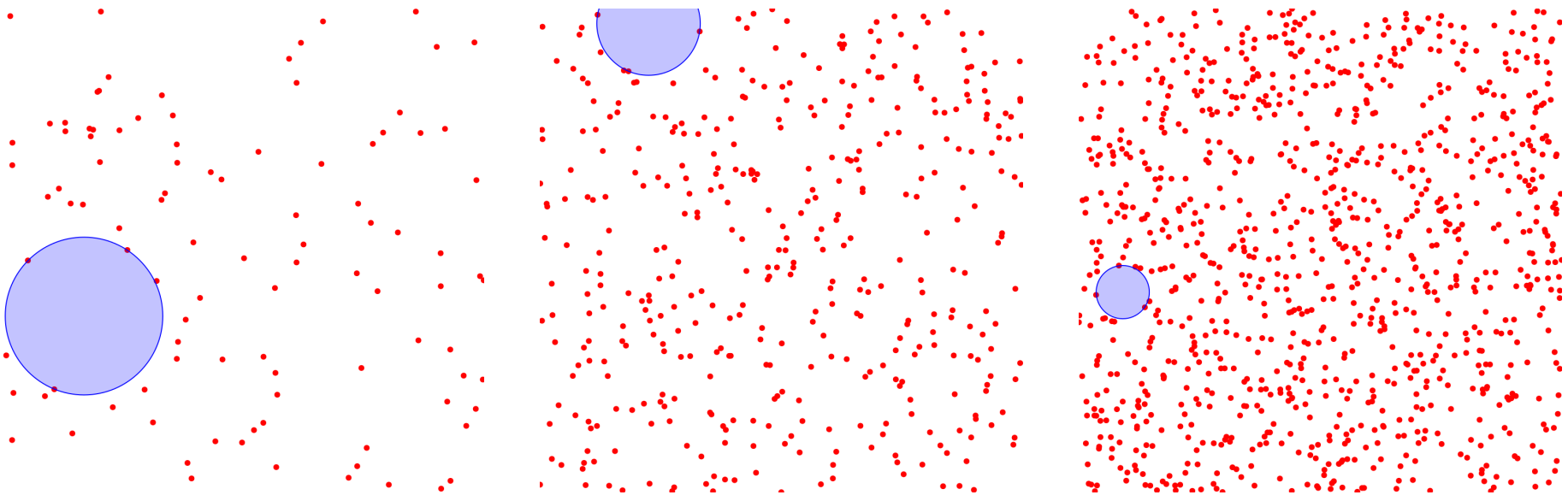
- Random number generator
- Metric
- Nearest-neighbor search
- Collision-detection

```
1  initialize tree T with q_init
2  for i = 1, ..., I_max do
3      q_rand = generate randomly in C
4      q_near = find nearest node in T towards
         q_rand
5      q_new = localPlanner from q_near towards
         q_rand
6      if canConnect(q_near, q_new) then
7          T.addNode(q_new)
8          T.addEdge(q_near, q_new)
9          if ϱ(q_new, q_goal) < d_goal then
10             return path from q_init to q_new
```

Generating random samples

Generation using standard `rand()`

- Many variants of random number generator (RNG)
- Simple RNG are implemented as Linear Congruent Generator (LCG)
- Fast, easy for usage, provides “enough” number of samples
- High dispersion (the largest empty “ball” according to the used metric)



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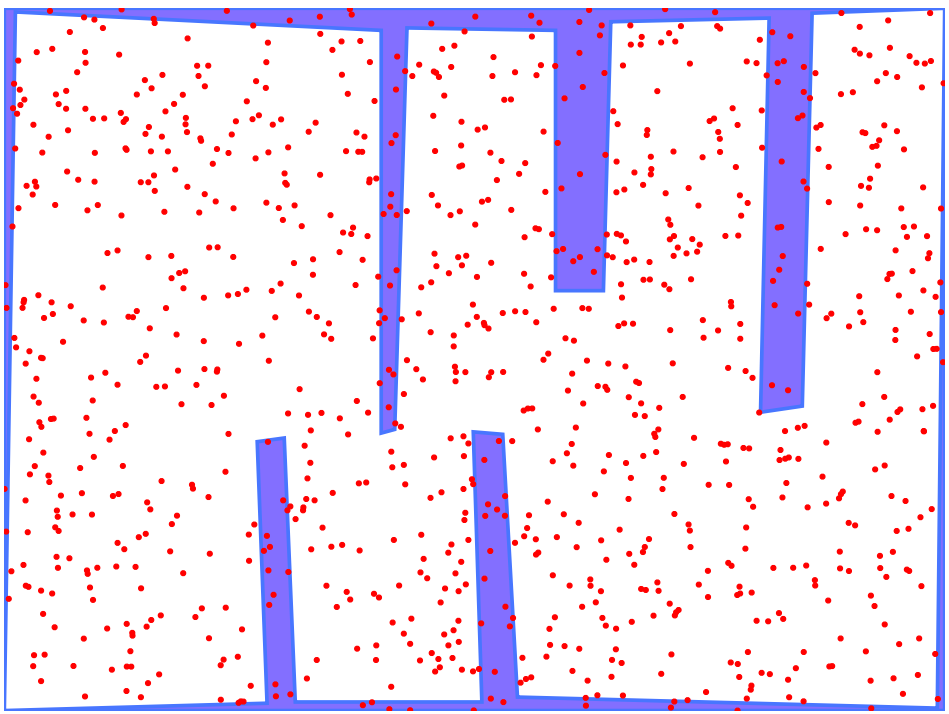

Random samples in 2D $\mathcal{C}$

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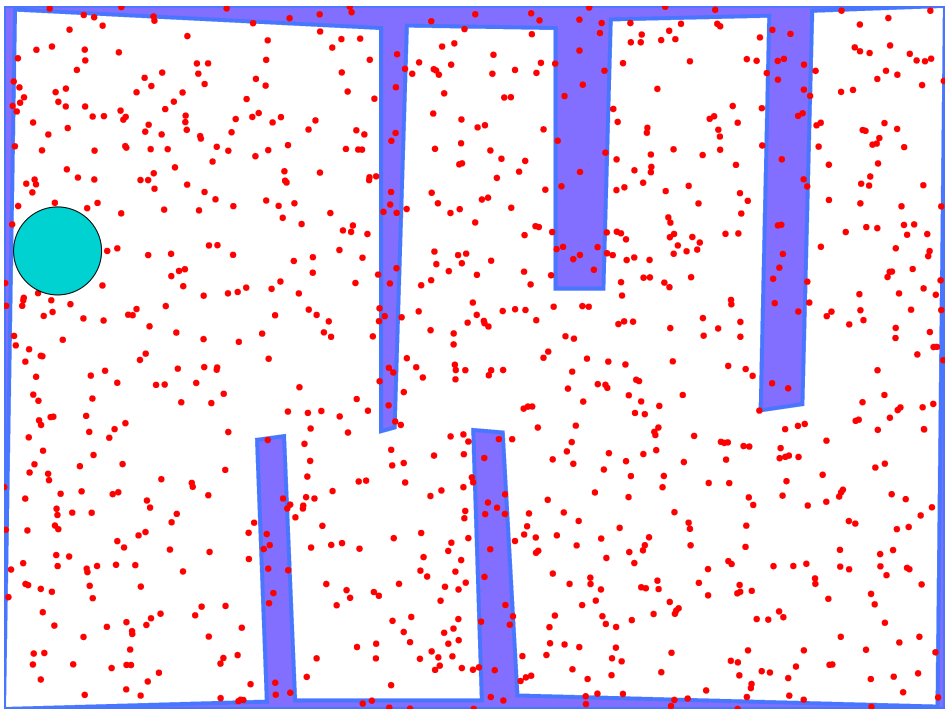

Random samples in 2D $\mathcal{C}$ + dispersion

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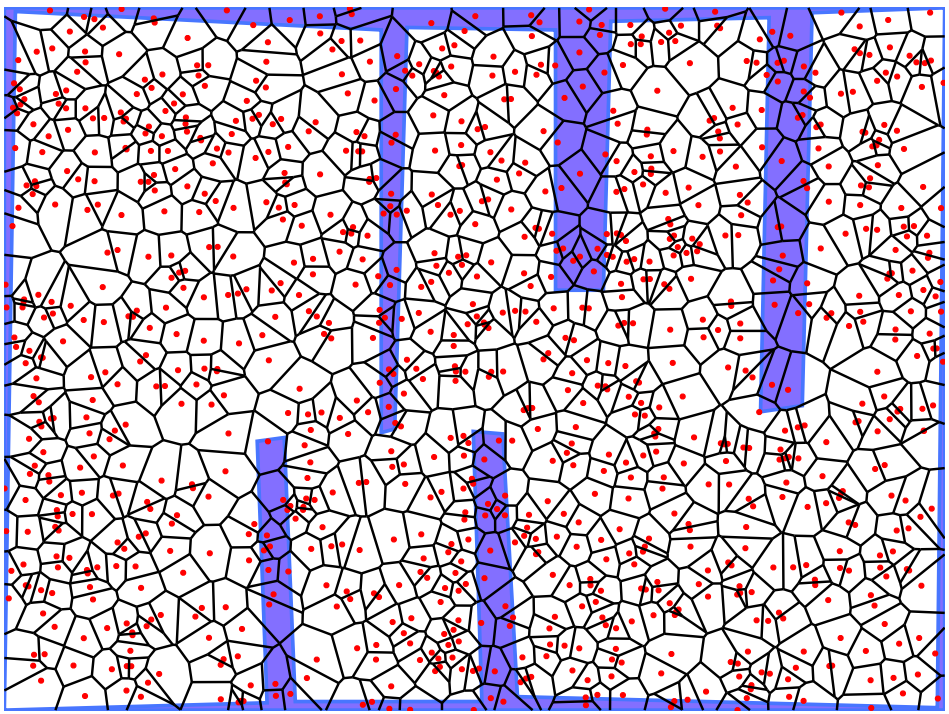

Voronoi diagram

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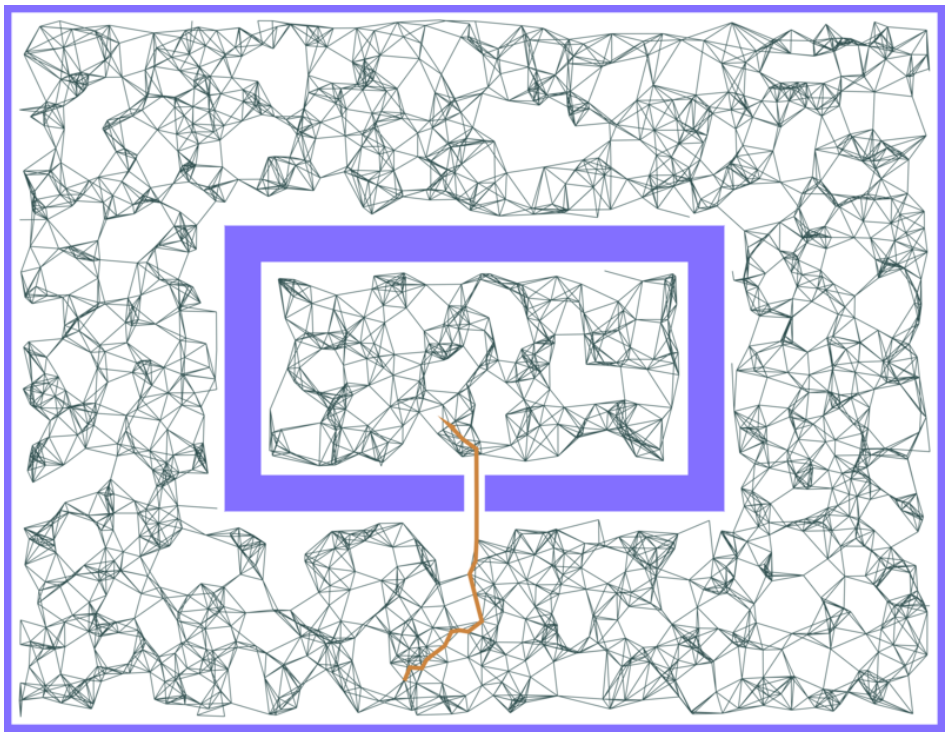

PRM roadmap, note the “holes”

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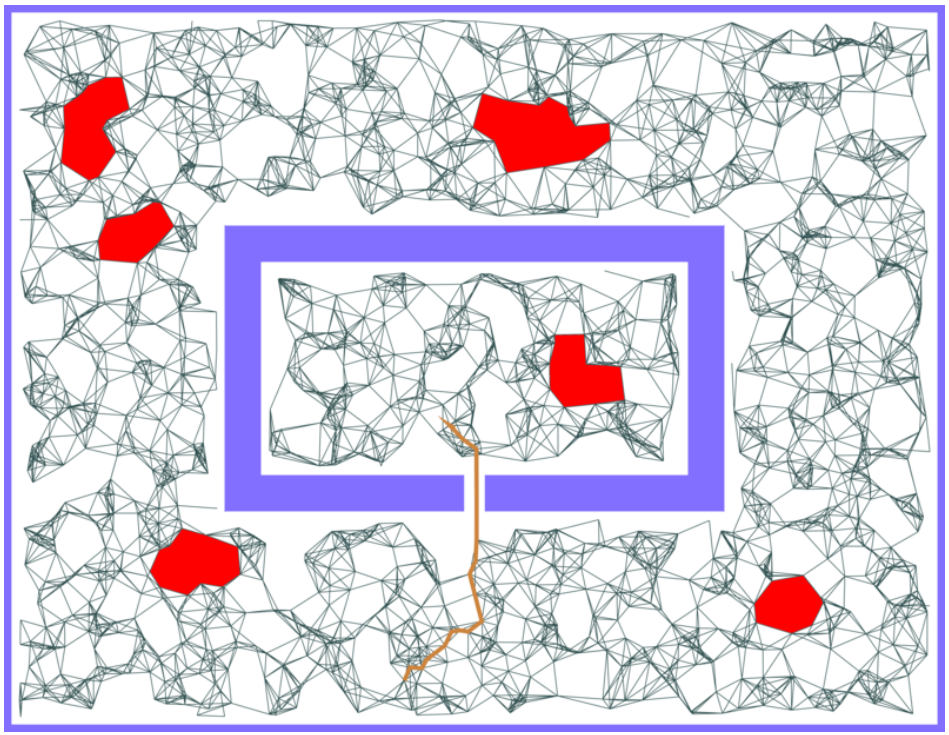

PRM roadmap, note the “holes” $\rightarrow$ due to the dispersion

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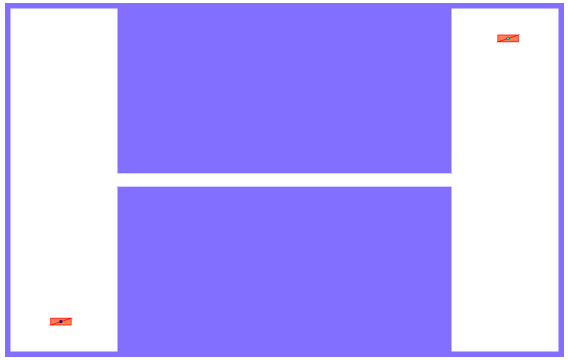
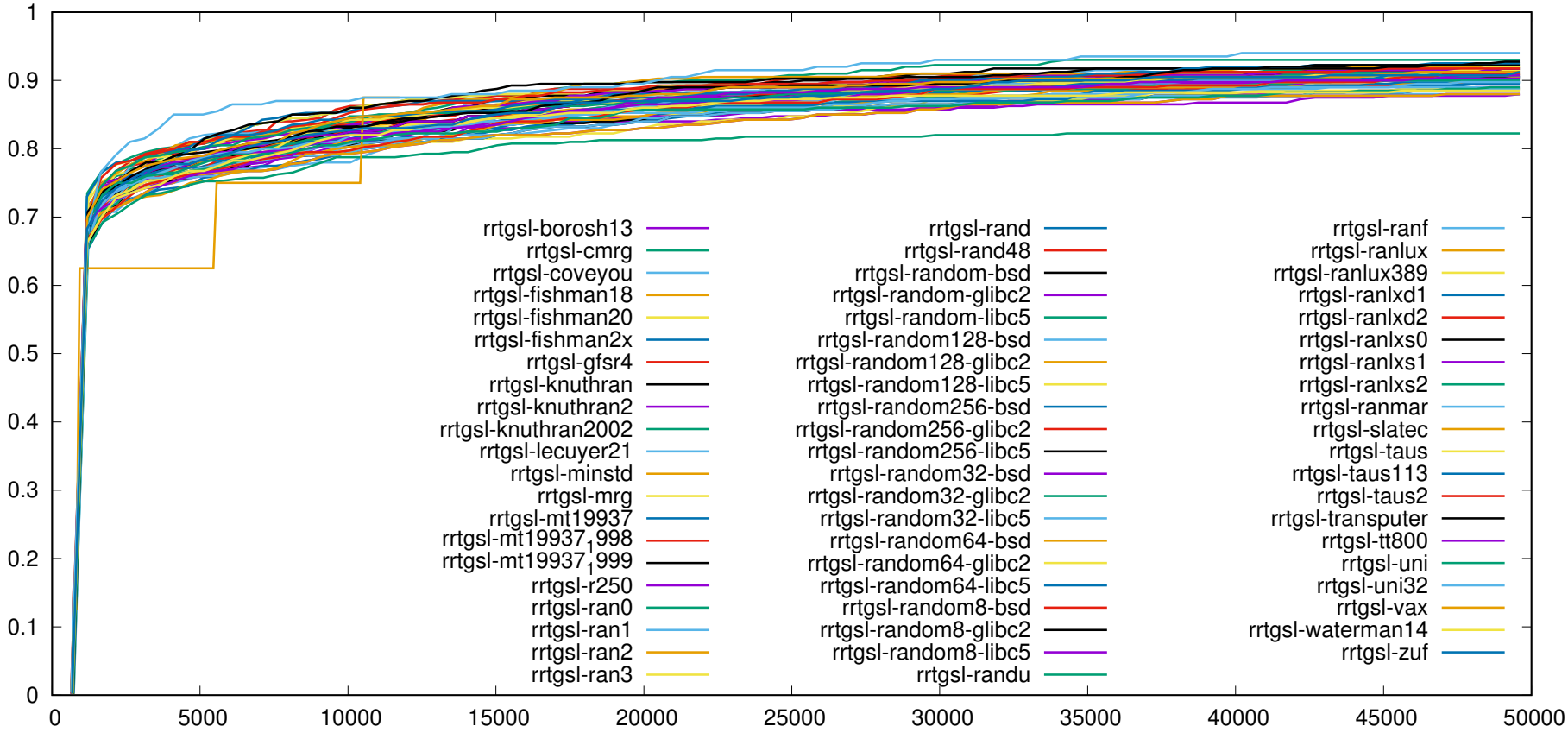
Alternatives to `rand()`

- Many libraries provide various RNG
  - e.g., Boost, GSL, Python/Numpy
- Python is using Mersenne Twister generator

Influence of RNG

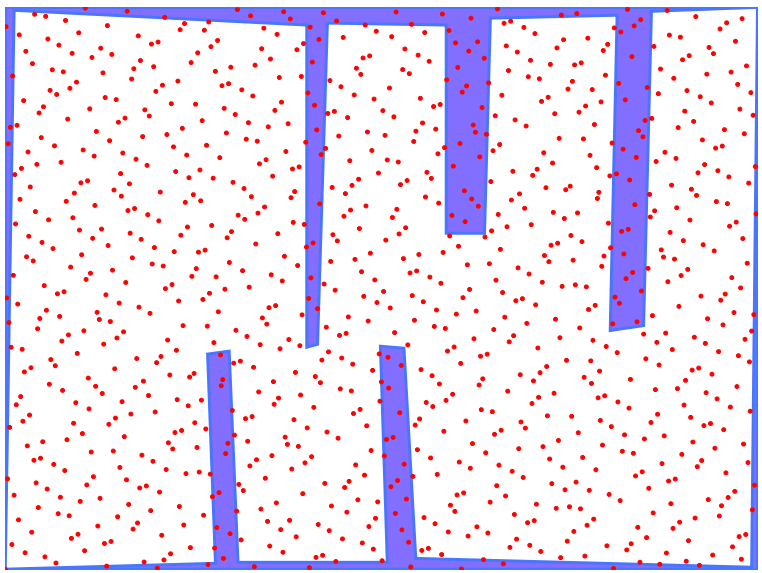
- Test scenario: square robot, 3D $\mathcal{C}$-space, narrow passage
- Generators from GSL — GNU Scientific library



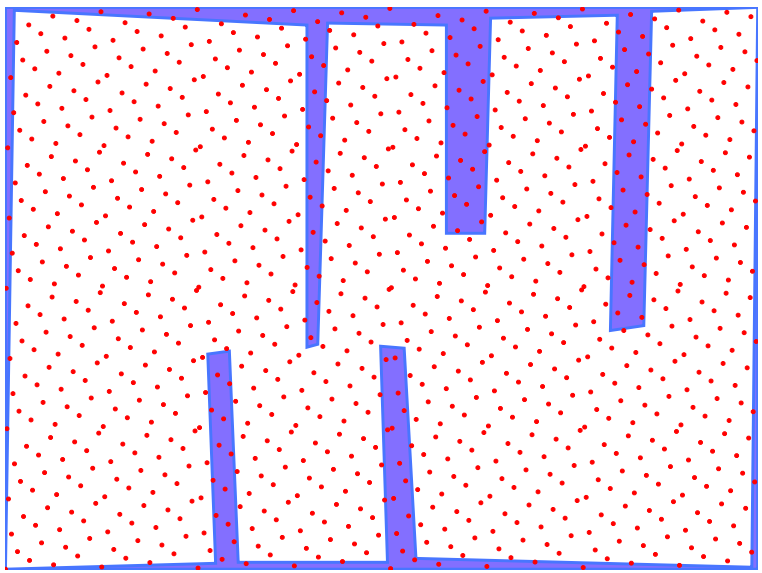


Halton/Hammersley sequences

- Deterministic sequences with low dispersion[1] [2]
- Number of samples must be known in advance
- Slower computation in comparison to basic `rand()`



Halton points



Hammersley points

[1] T.-T. Wong et al. "Sampling with Hammersley and Halton Points". In: *Journal of Graphics Tools* 2.2 (Jan. 1997), pp. 9–24. DOI: 10.1080/10867651.1997.10487471.

[2] https://extremelearning.com.au/unreasonable-effectiveness-of-quasirandom-sequences/

Metric

- Sampling-based planners require a metric $\varrho(q_1, q_2)$, $q_1, q_2 \in \mathcal{C}$
- Often used are L_p metrics:

$$\varrho(x, x') = \left(\sum_{i=1}^{n} |x_i - x'_i|^p \right)^{1/p}$$

- L_2 is Euclidean metric
- L_1 is Manhattan metric
- Metric for 1D rotation:

$$\varrho(\theta_1, \theta_2) = \min\left(|\theta_1 - \theta_2|, 2\pi - |\theta_1 - \theta_2|\right)$$

- Metrics can be combined, let's assume that $\mathcal{C} = X \times Y$ with ϱ_X and ϱ_Y:

$$\varrho(q, q') = \left(c_x \varrho_x(x, x')^p + c_y \varrho_y(y, y')^p \right)^{1/p}$$

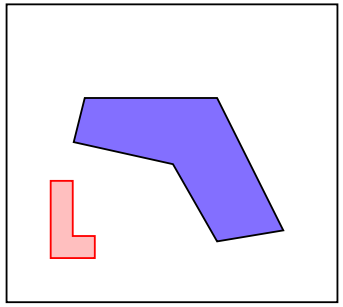
- where $c_x, c_y \geq 0$ are weights

Remind that for $a, b, c \in X$ in a metric space X and metric ϱ: $\varrho(a, b) \geq 0$; $\varrho(a, b) = 0$ if and only if $a = b$; $\varrho(a, b) = \varrho(b, a)$; $\varrho(a, b) + \varrho(b, c) \geq \varrho(a, c)$

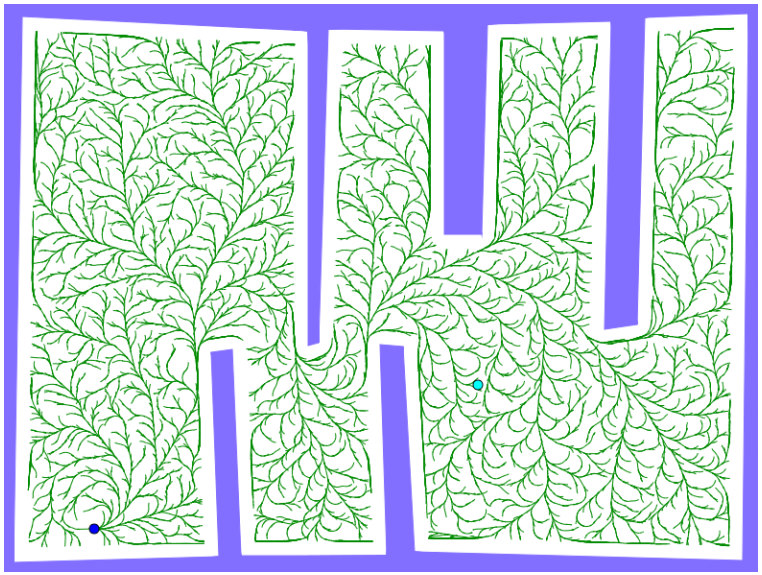
Metric

- 2D object, translation + rotation $\rightarrow q = (x, y, \varphi) \in C$

$$\varrho(q, q') = \sqrt{c_1((x - x')^2 + (y - y')^2) + c_2 \varrho_\theta(\varphi, \varphi')}$$



- where $\varrho_\theta(\varphi, \varphi')$ is the metric for 1D rotation
- The weights for translation c_1 is "usually" bigger than c_2, so the effect of the rotation is suppressed
- Wrong setting of weights can worse motion planning



Correct

Big weight on y- distance

Notes

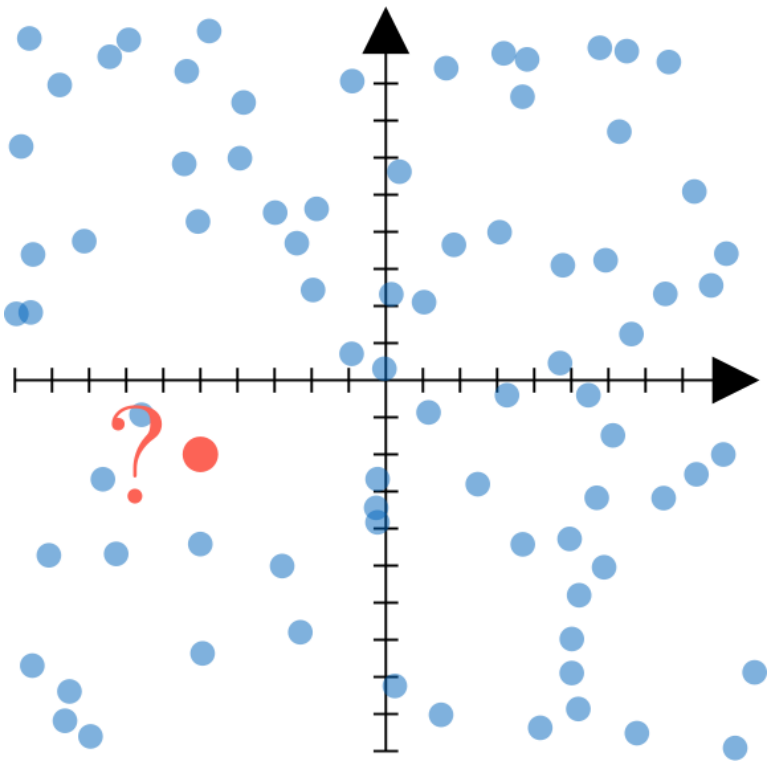
- Standard metrics (e.g., L_p etc) do not consider obstacles
- They are optimistic — return shortest distances than the robot really need to travel
- Selection of metric is further limited/prescribed by nearest-neighbor search
- Many nearest-neighbor search data structures are designed for specific metric only!

Nearest-neighbor search

- Given a set S, find a nearest point towards a query q
- Alternatives:
  - Find k nearest neighbors
  - Find all neighbors in the range r
- Naïve $\mathcal{O}(n)$ search is too slow!

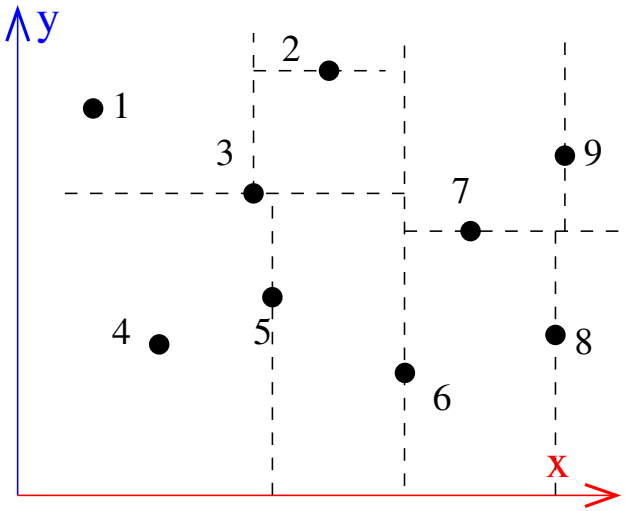
Challenges

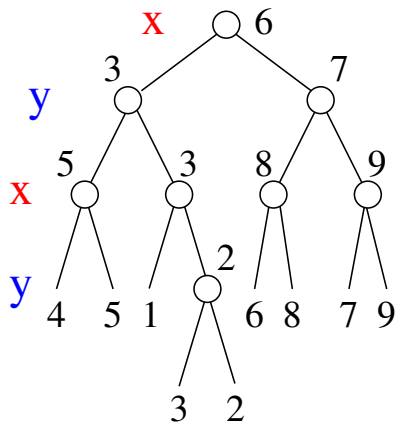
- Fast query time
- Consider arbitrary metrics
- Dimensionality of S
- Fast preprocessing
- Fast insertion
- Low space requirements



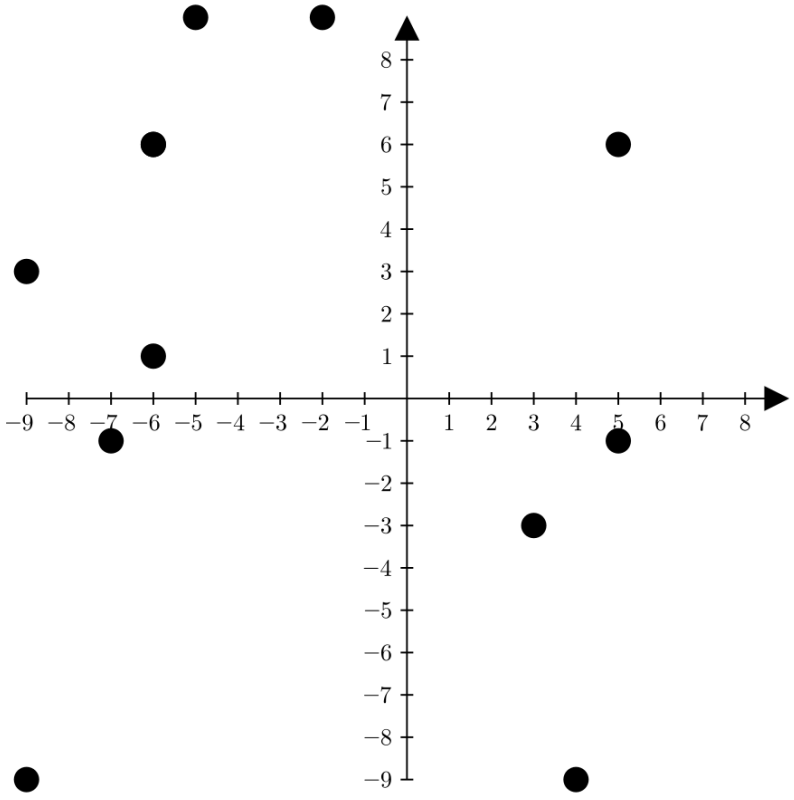
KD-trees: construction

- KD-tree is a binary tree, nodes represent a decision value
- Each level (of node) is for a different dimension



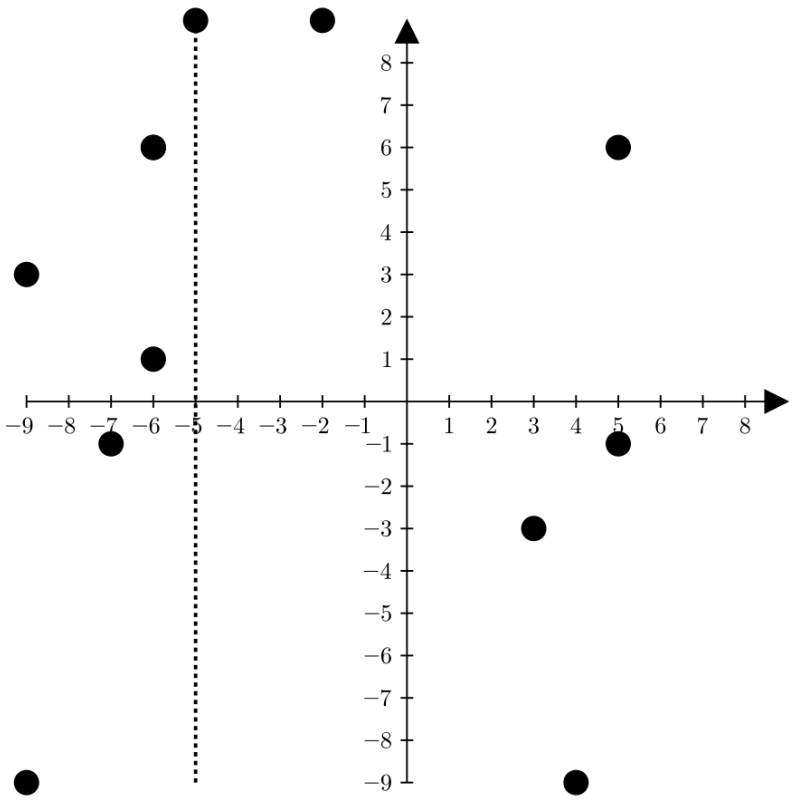


- Search is $\mathcal{O}(\log n)$ for n points in the KD-tree
- Construction $\mathcal{O}(dn \log n)$, n is number of points, d is dimension



Construction of kd-trees

- Compute median in the given axis, make a new node (decision)
- Split points to two sets based on the decision
- Recursively build left and right subtrees, each subtree works with the next dimension



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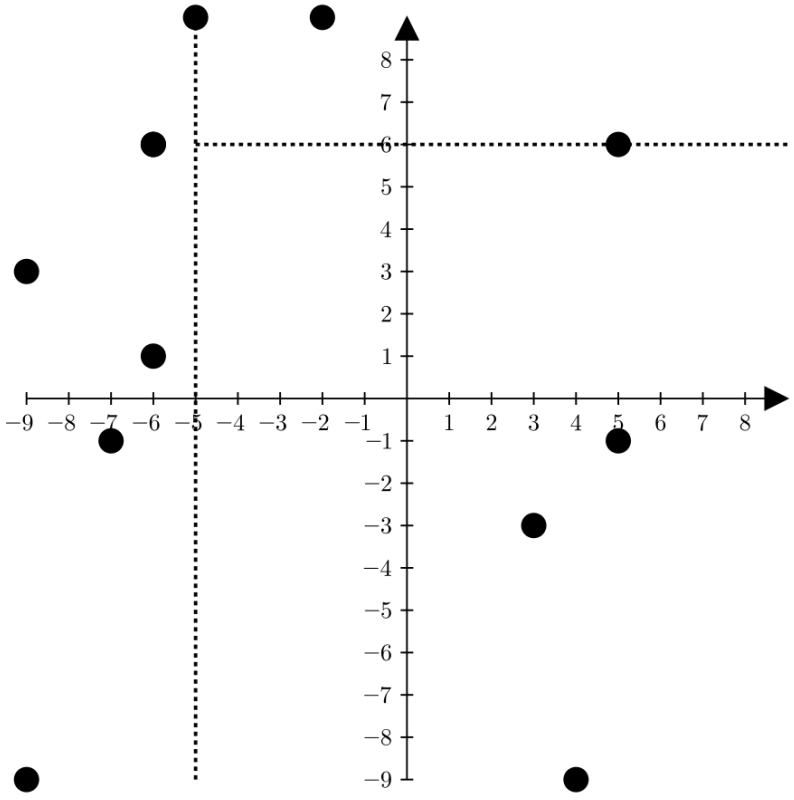

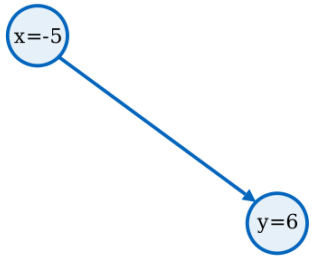

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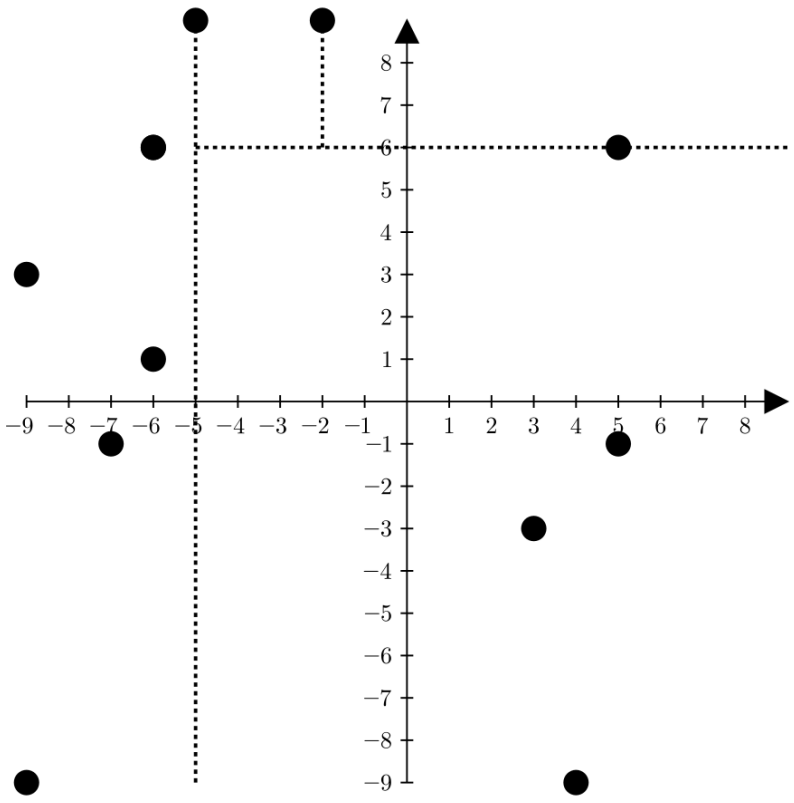

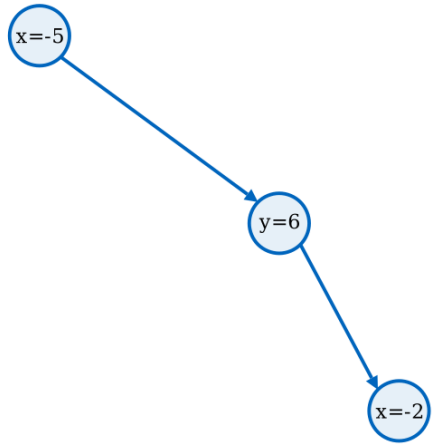

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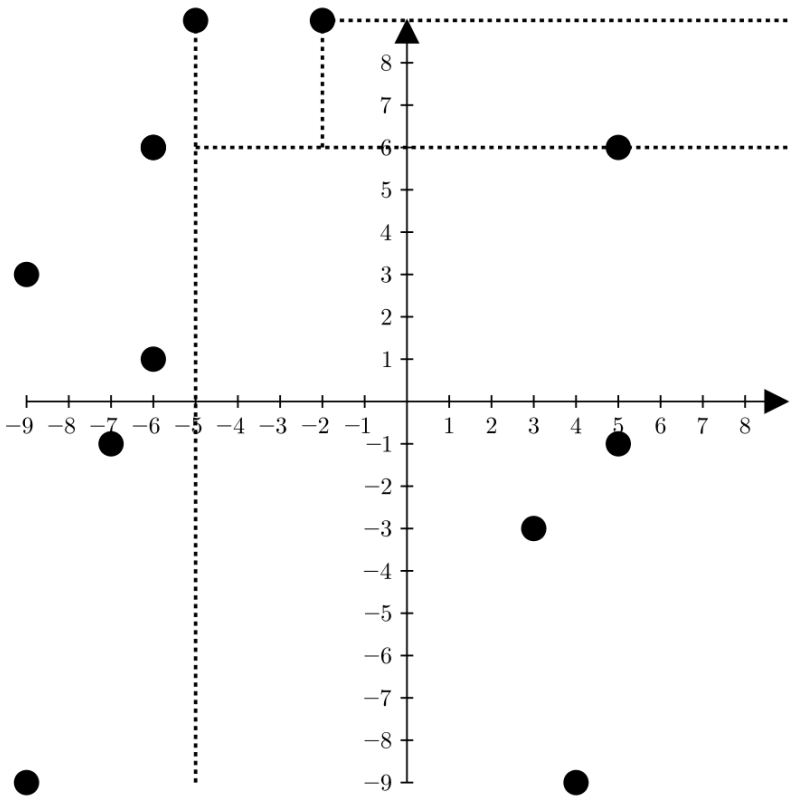

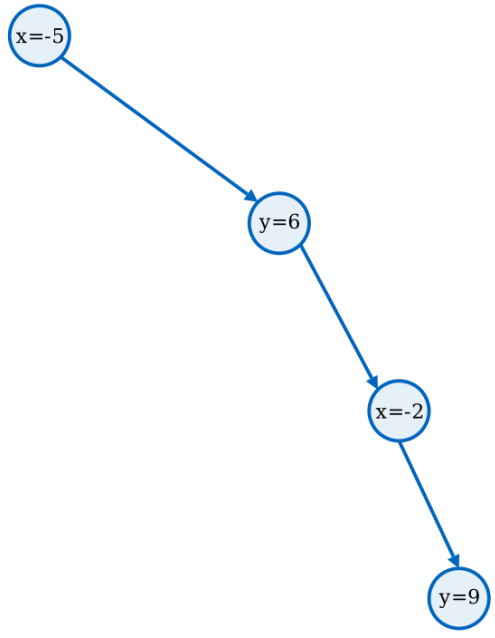

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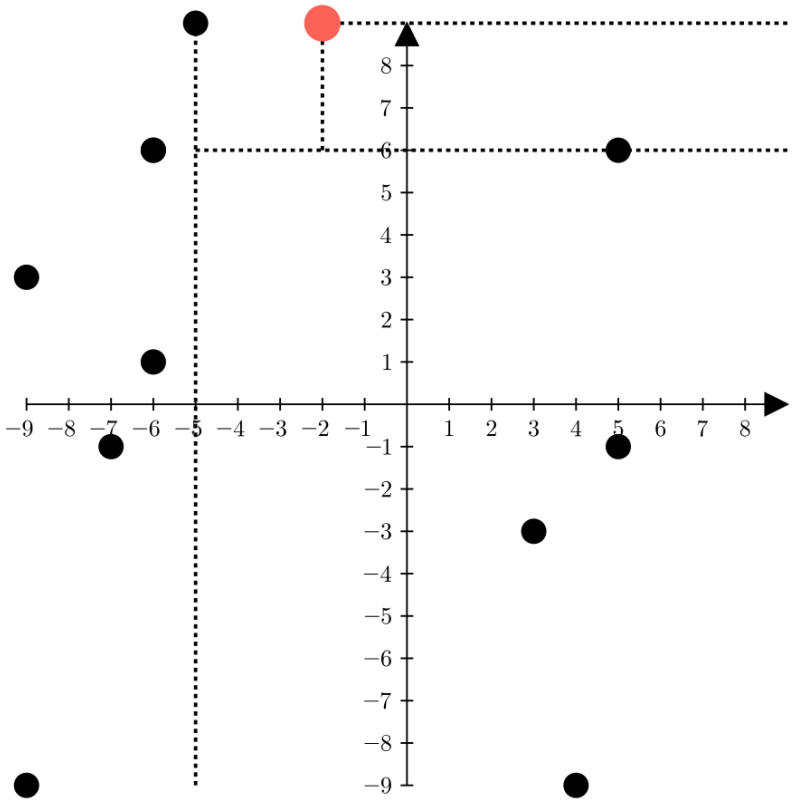

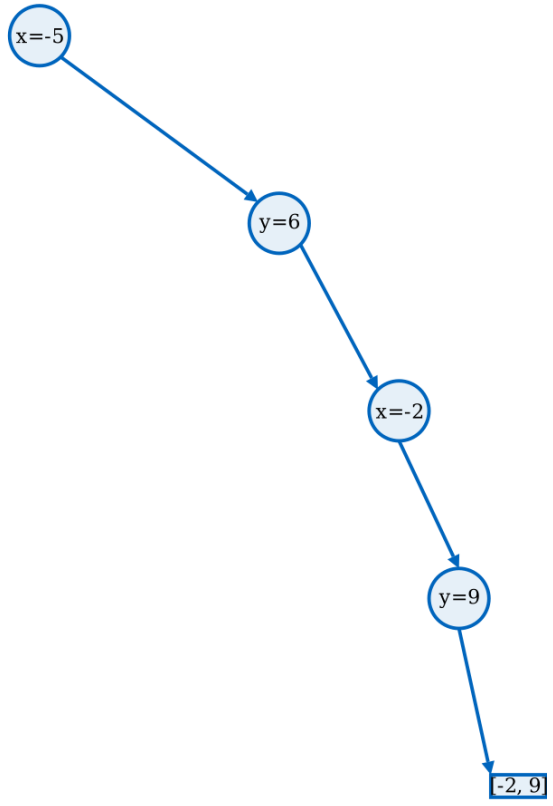

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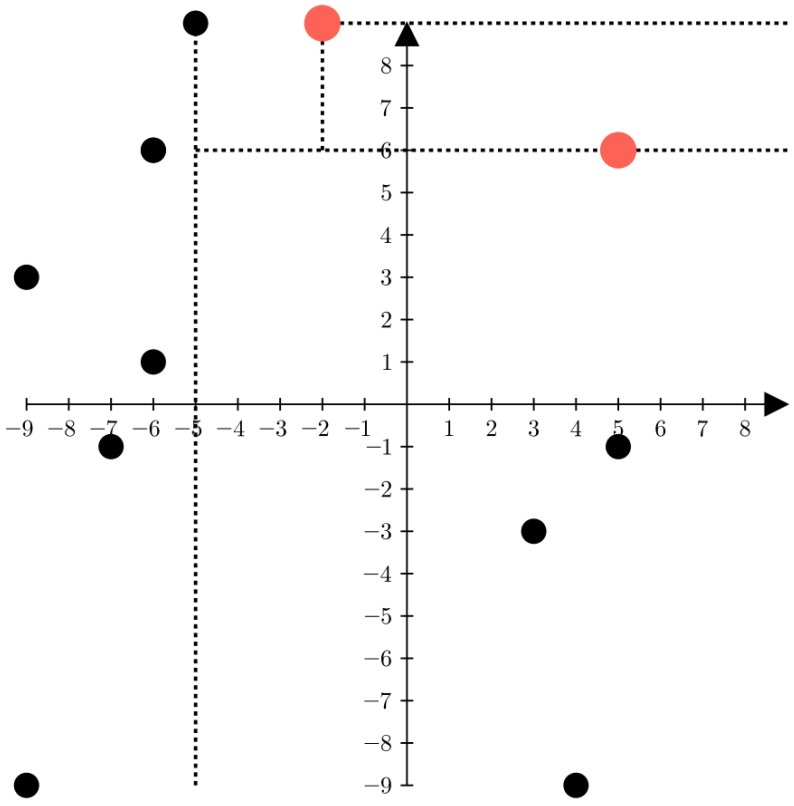

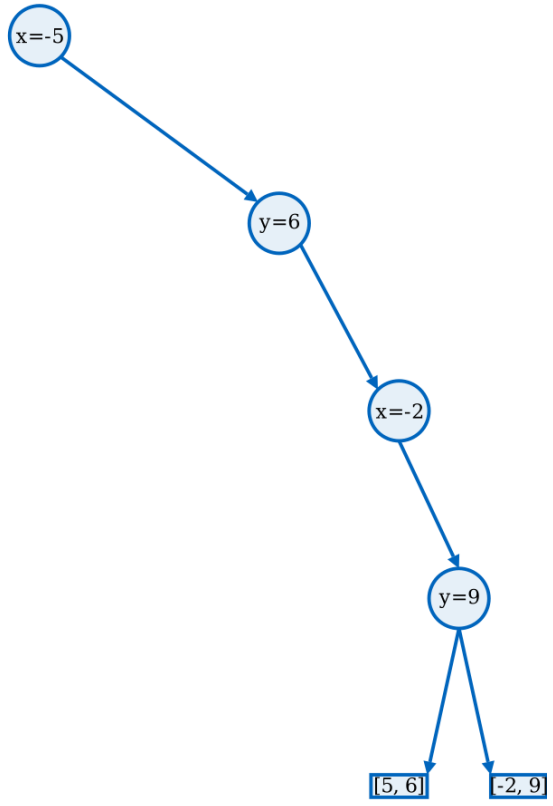

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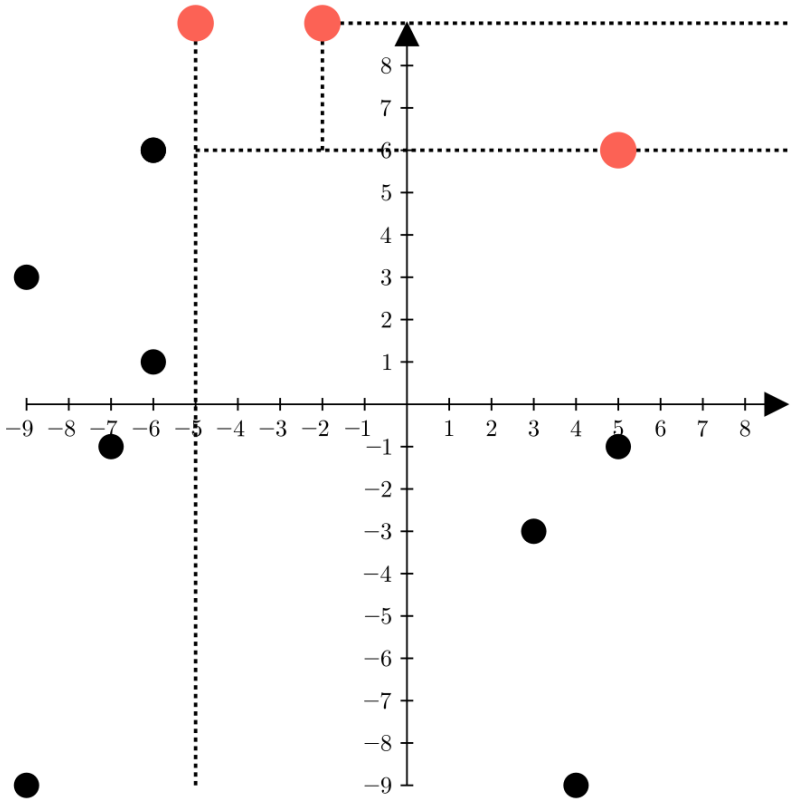

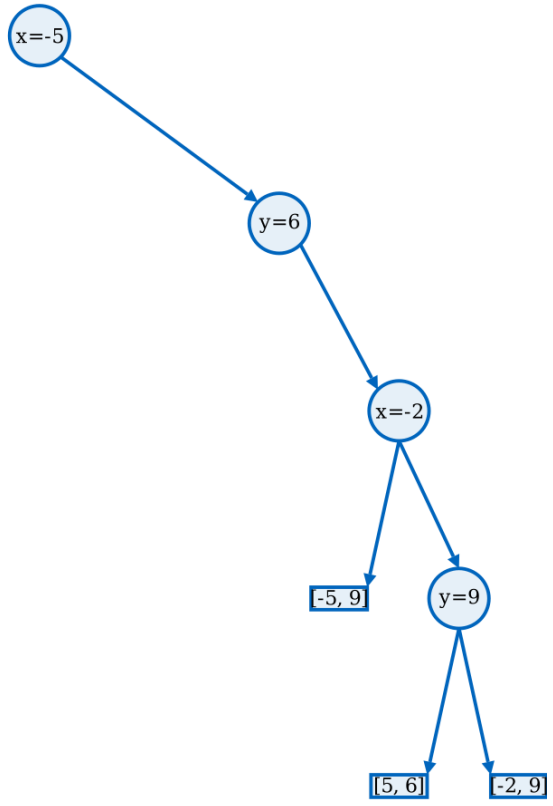

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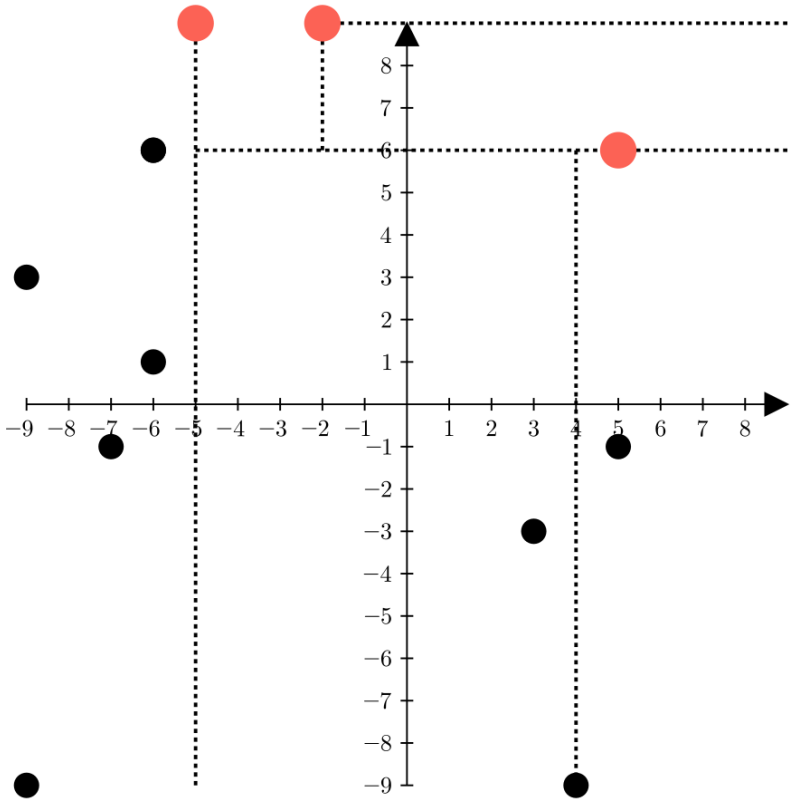

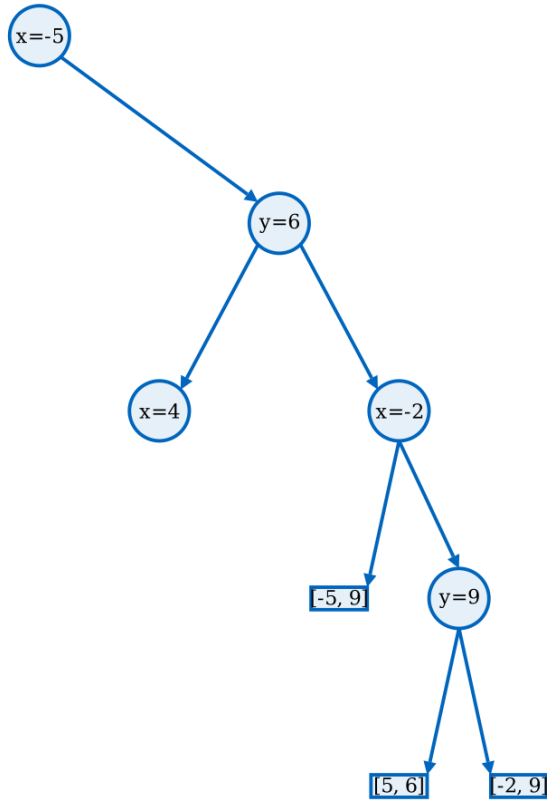

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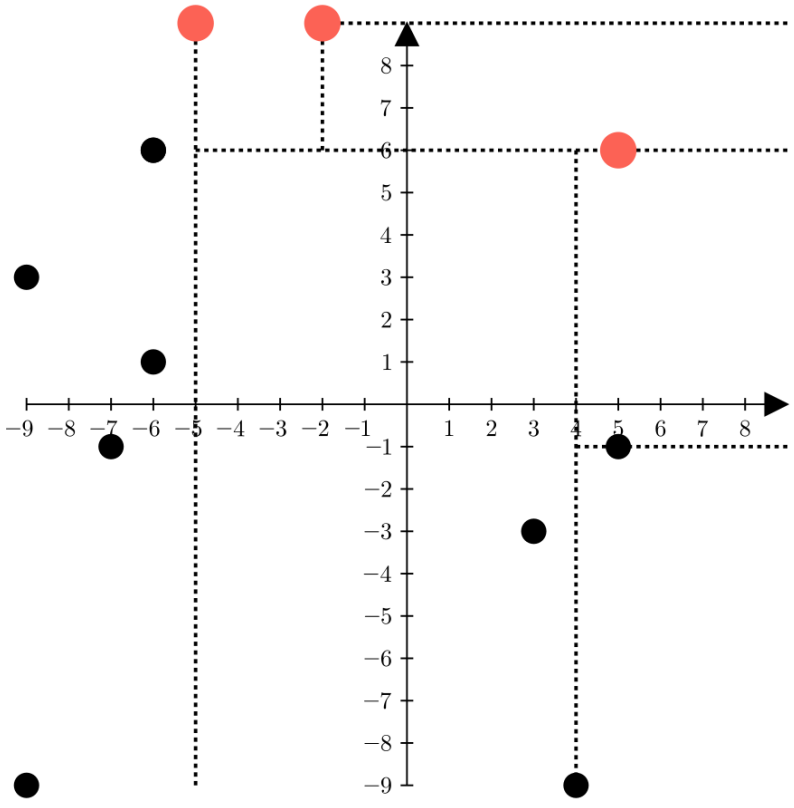

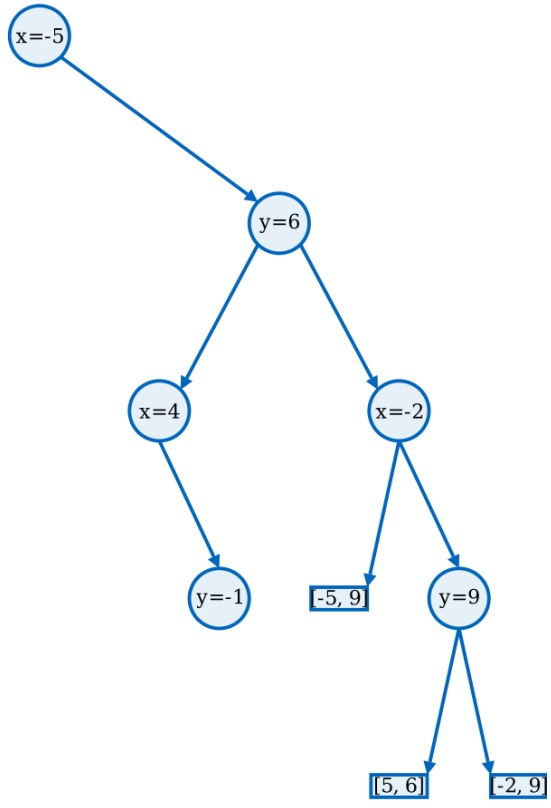

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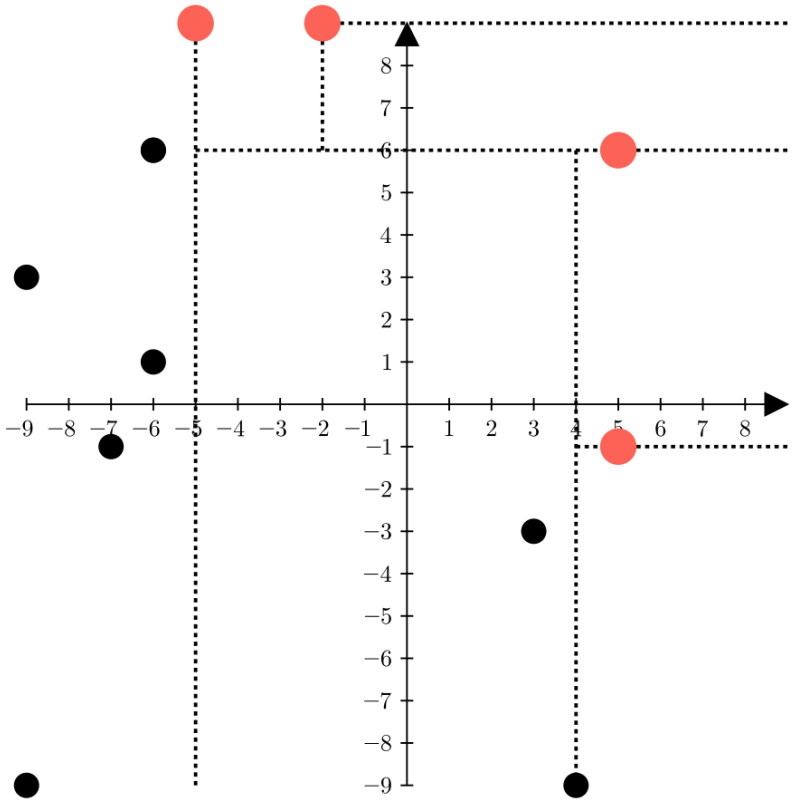

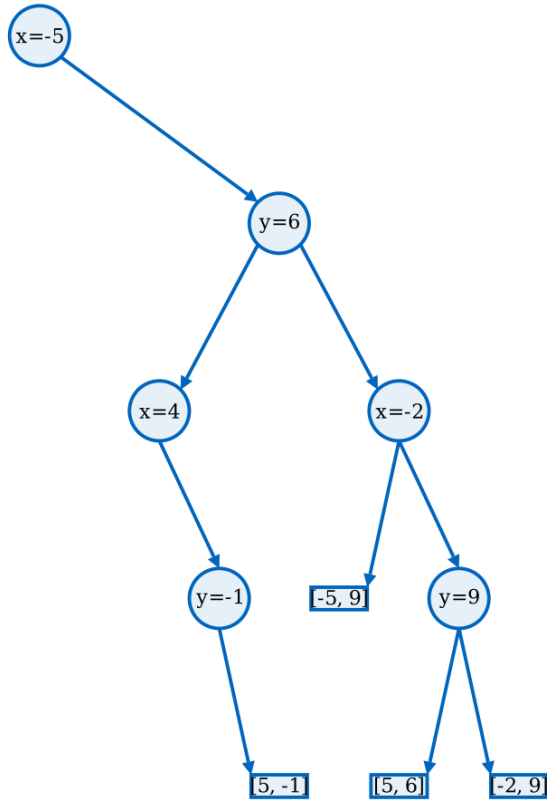

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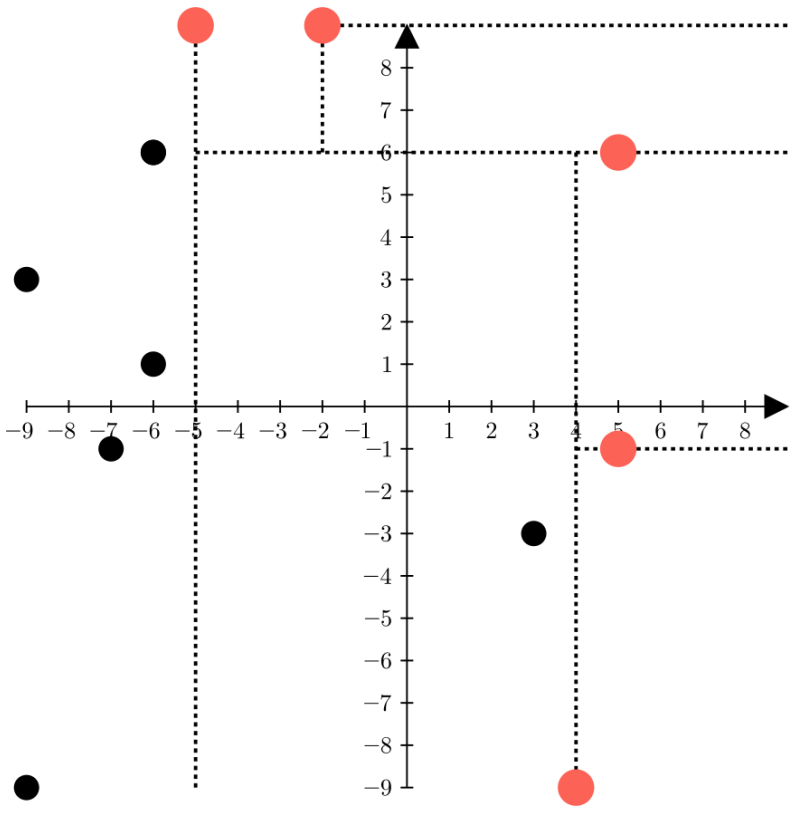

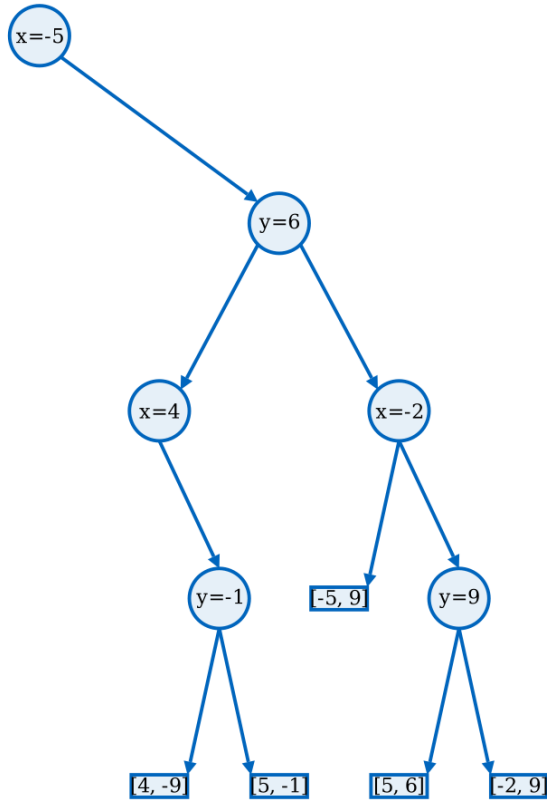

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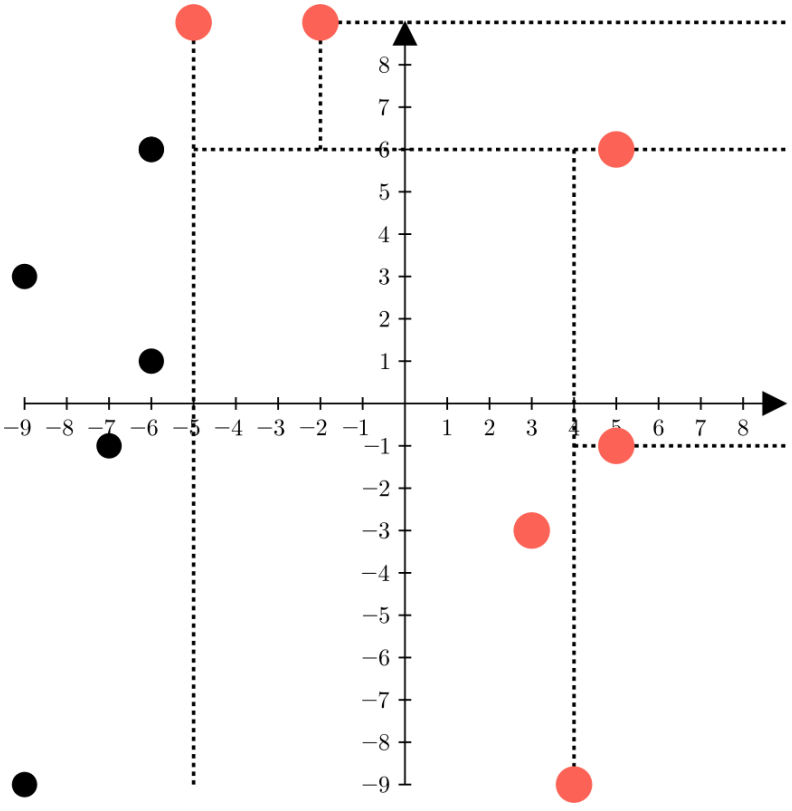

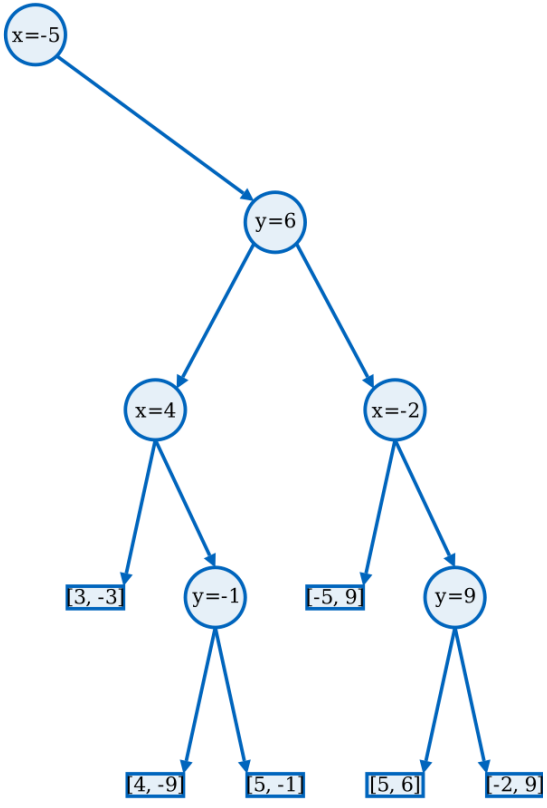

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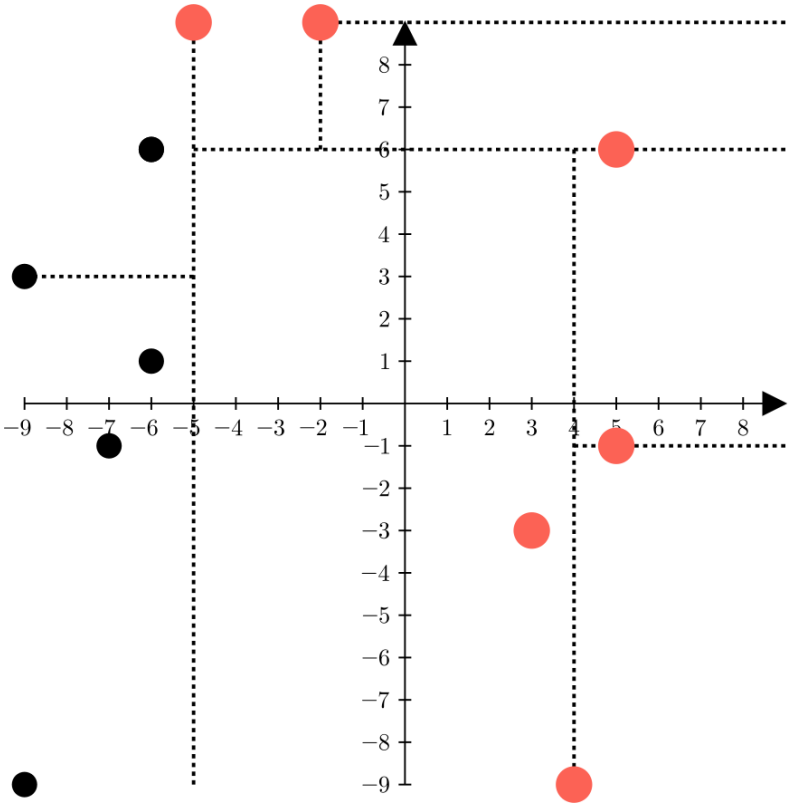

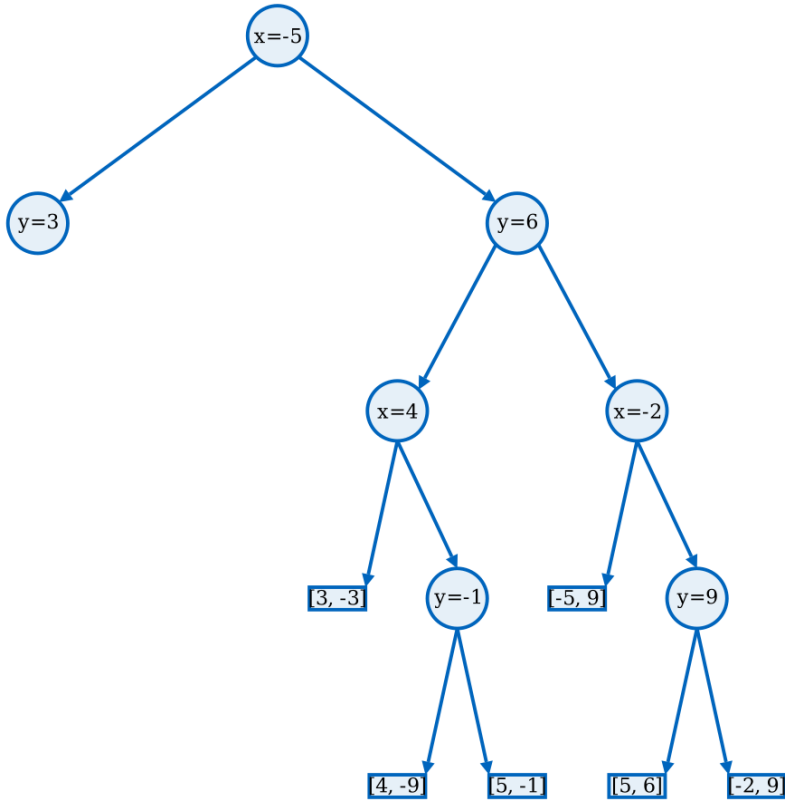

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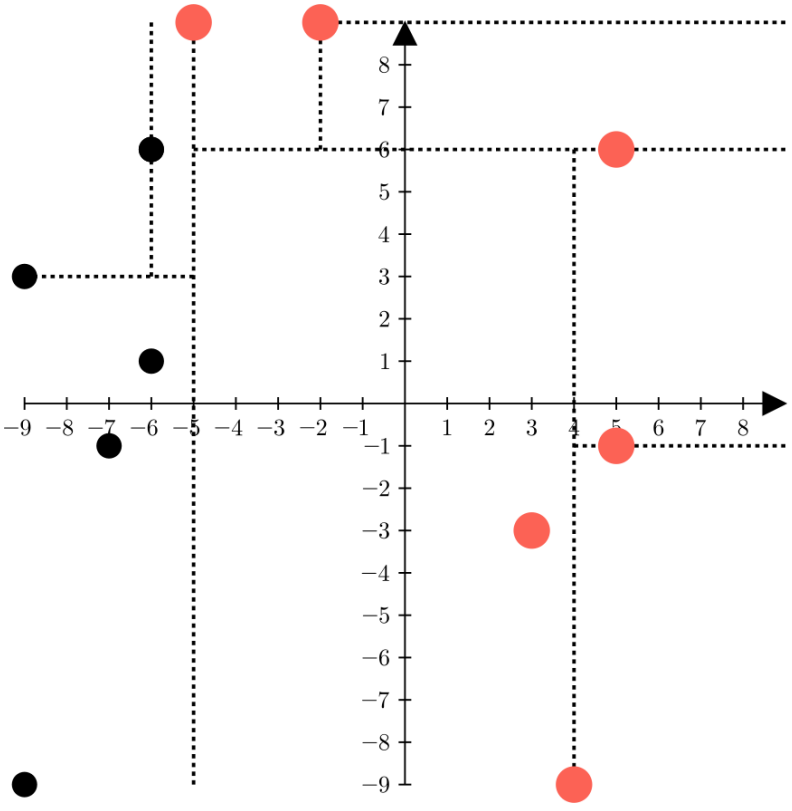

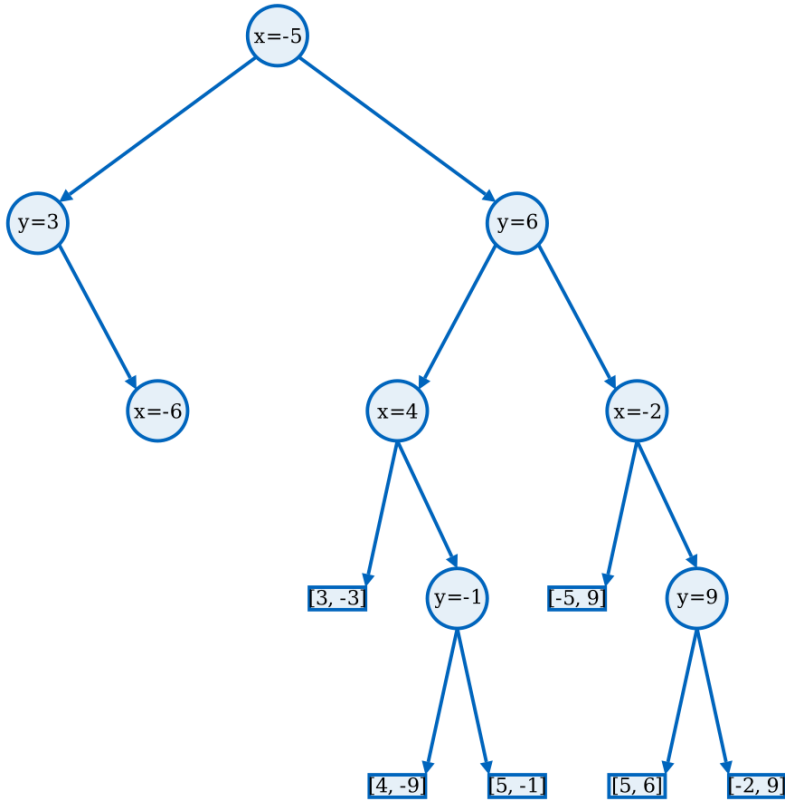

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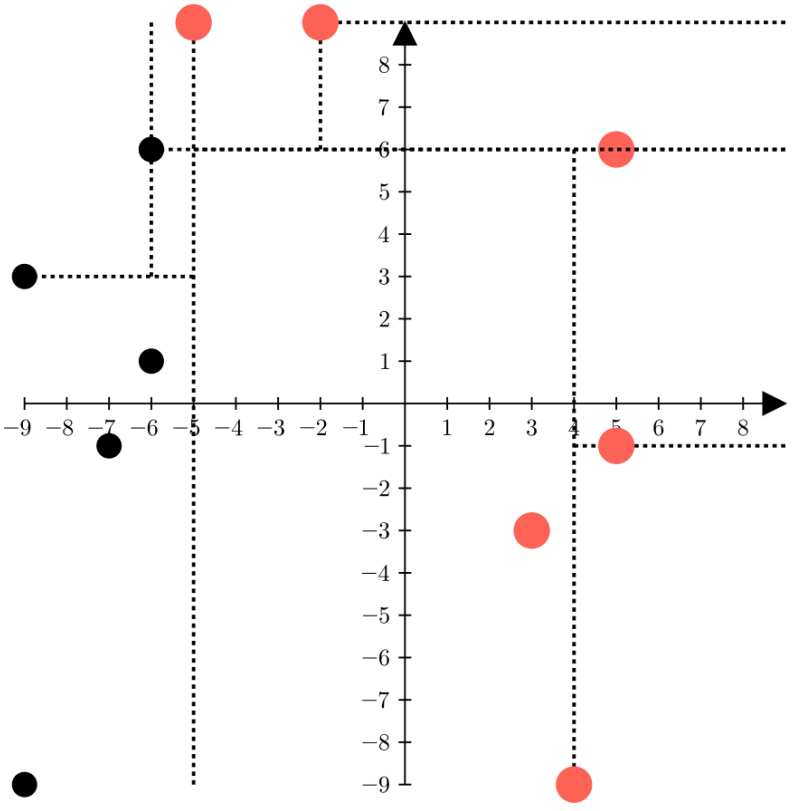

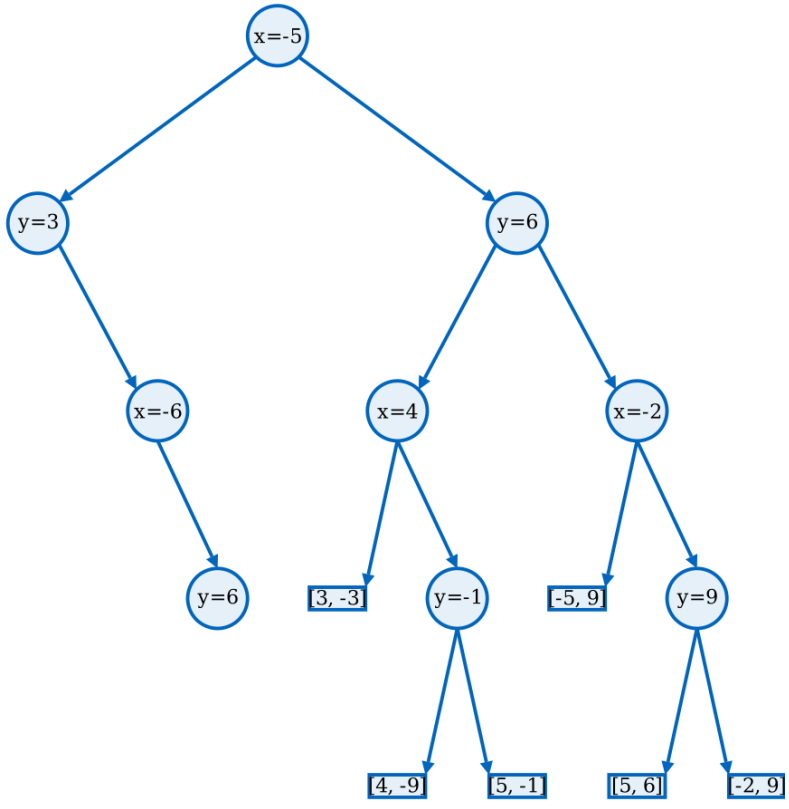

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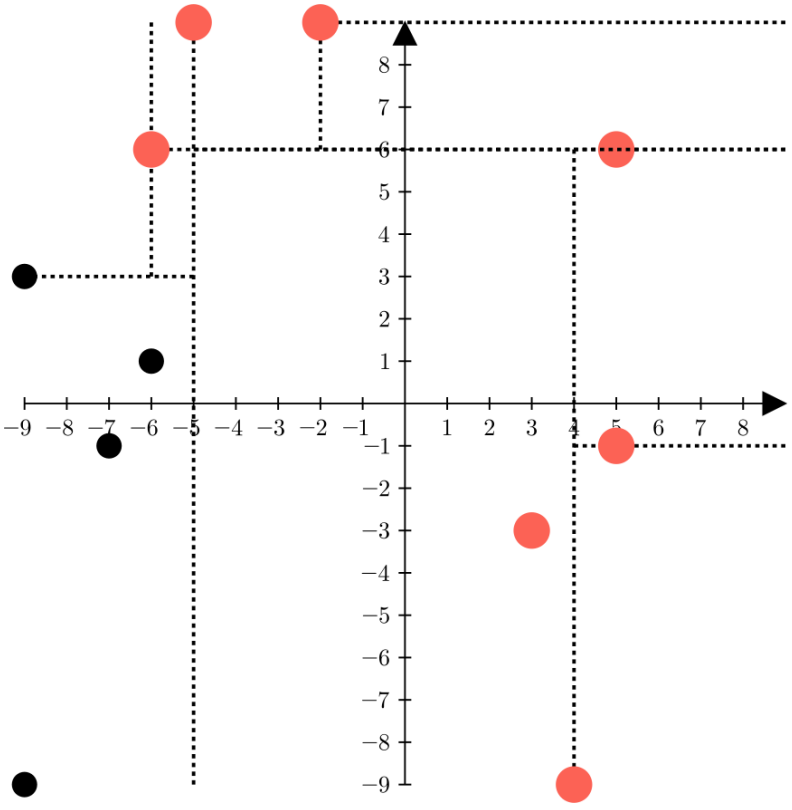

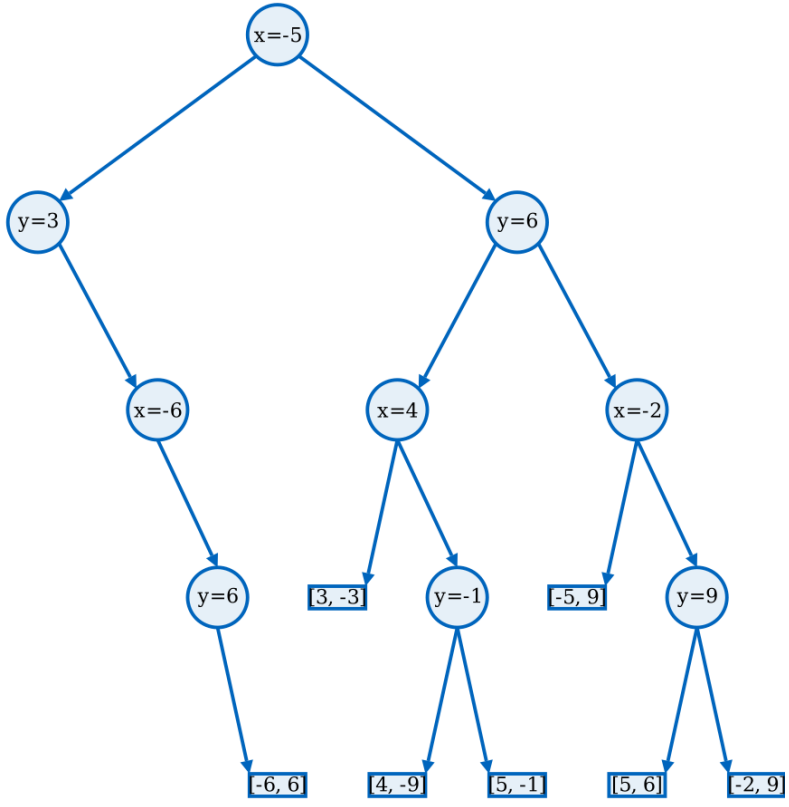

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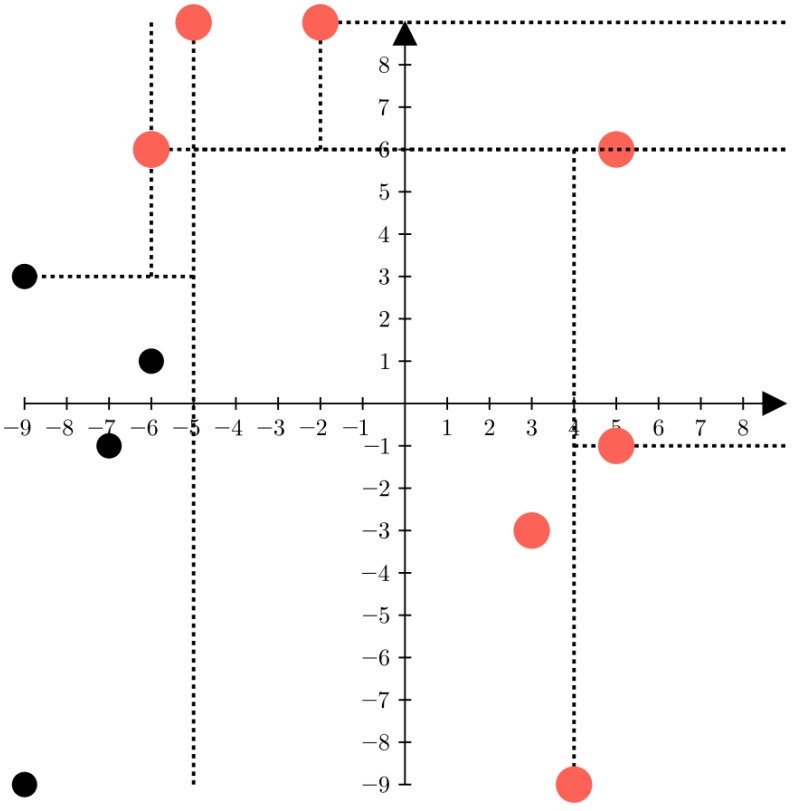

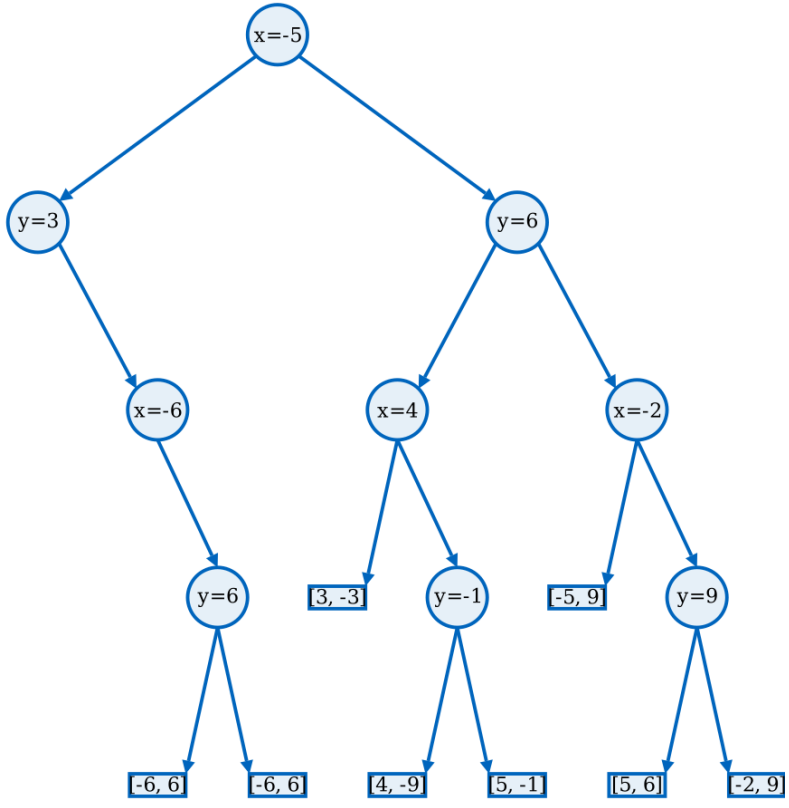

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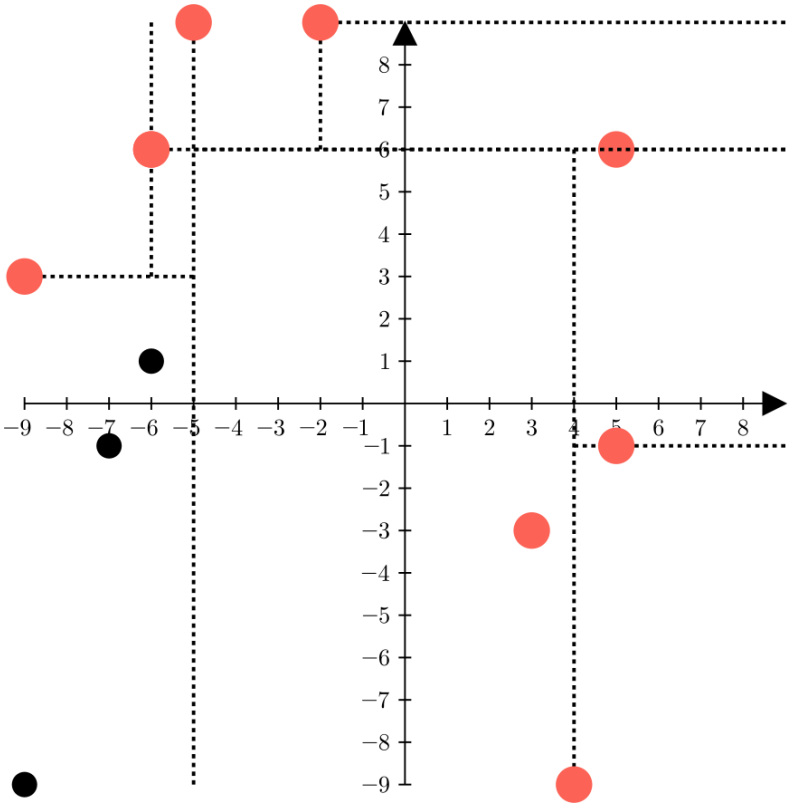

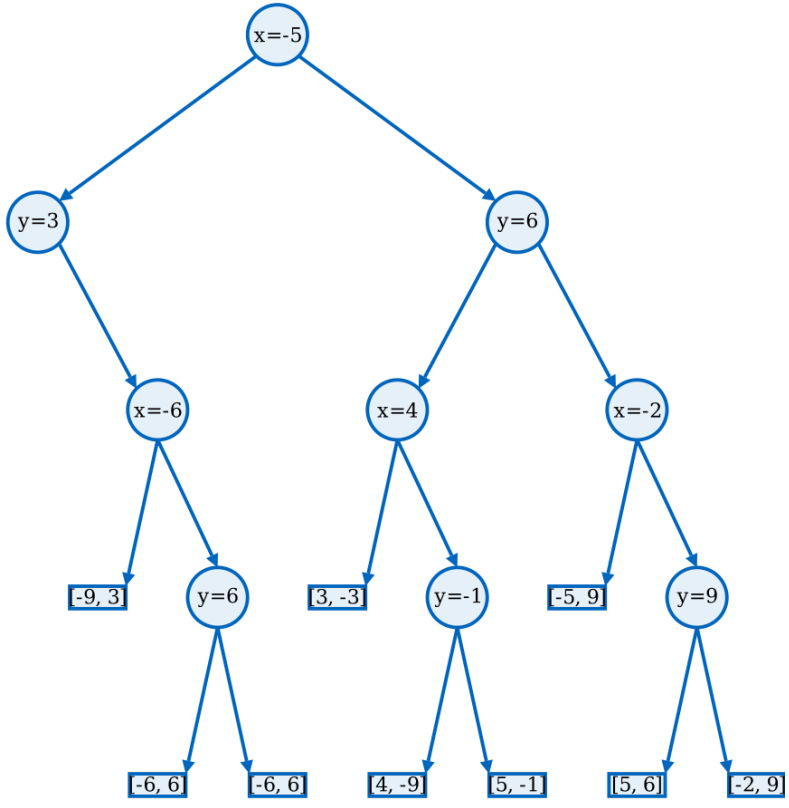

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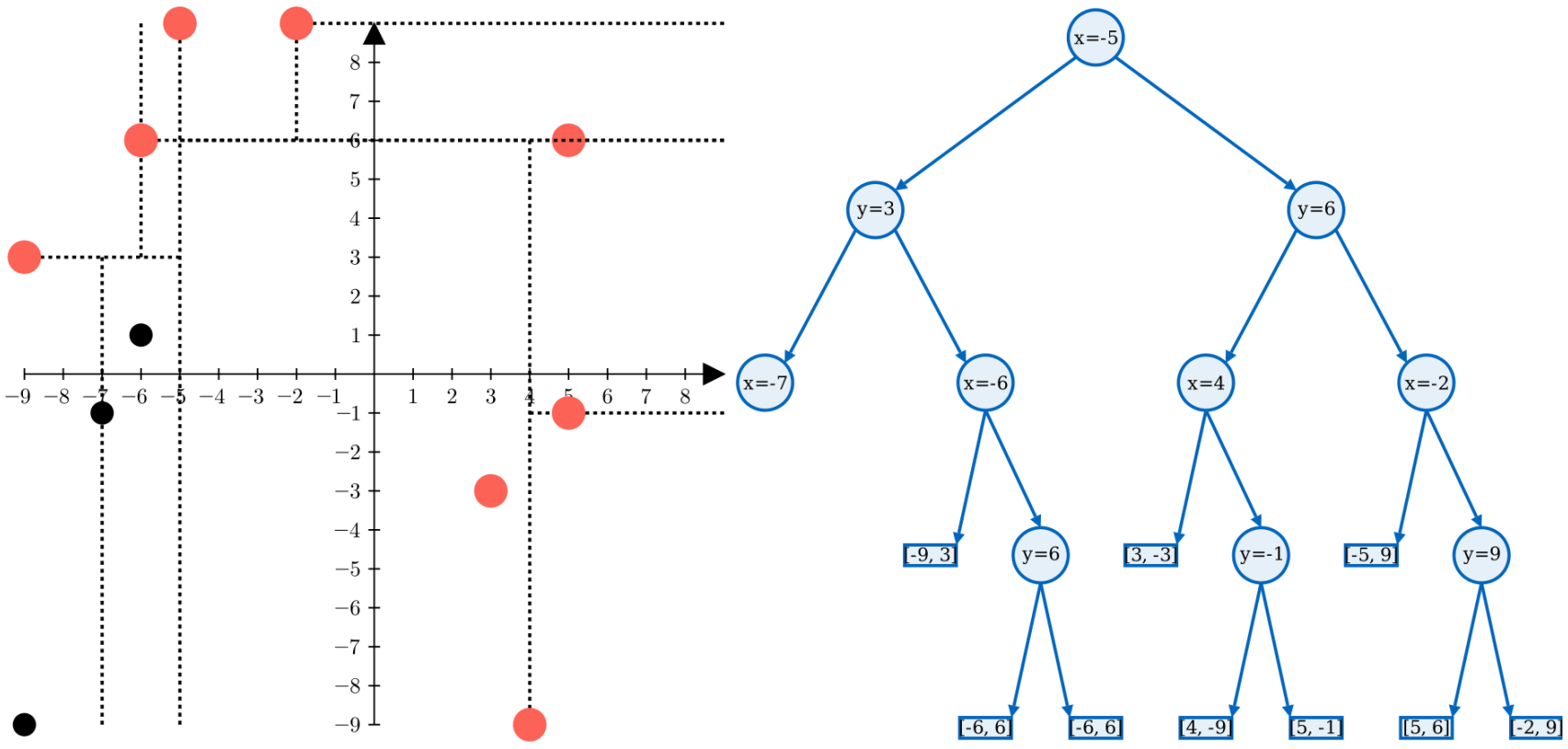

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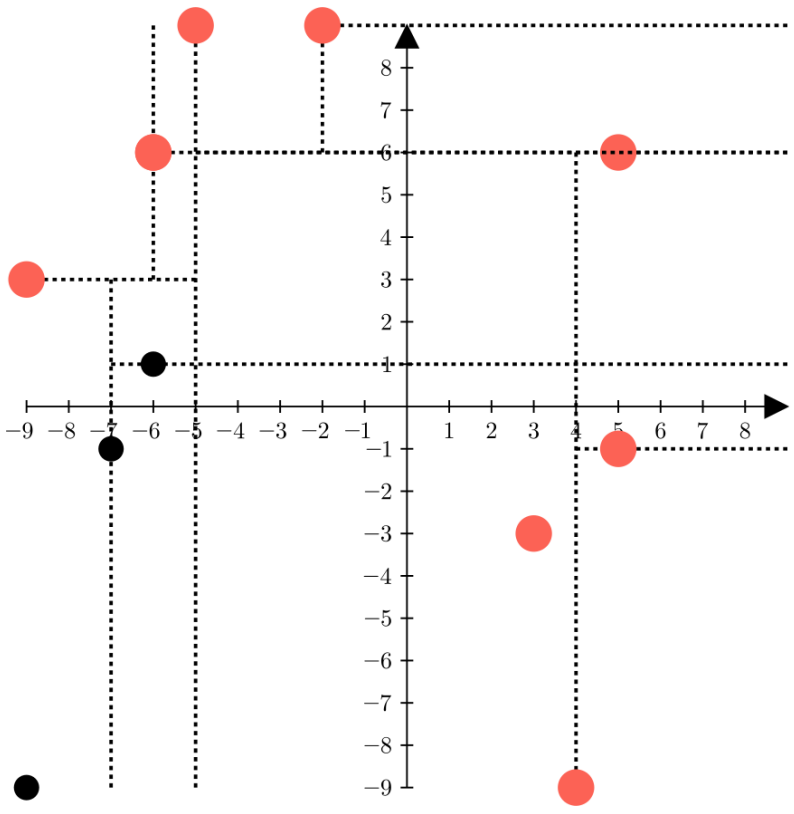

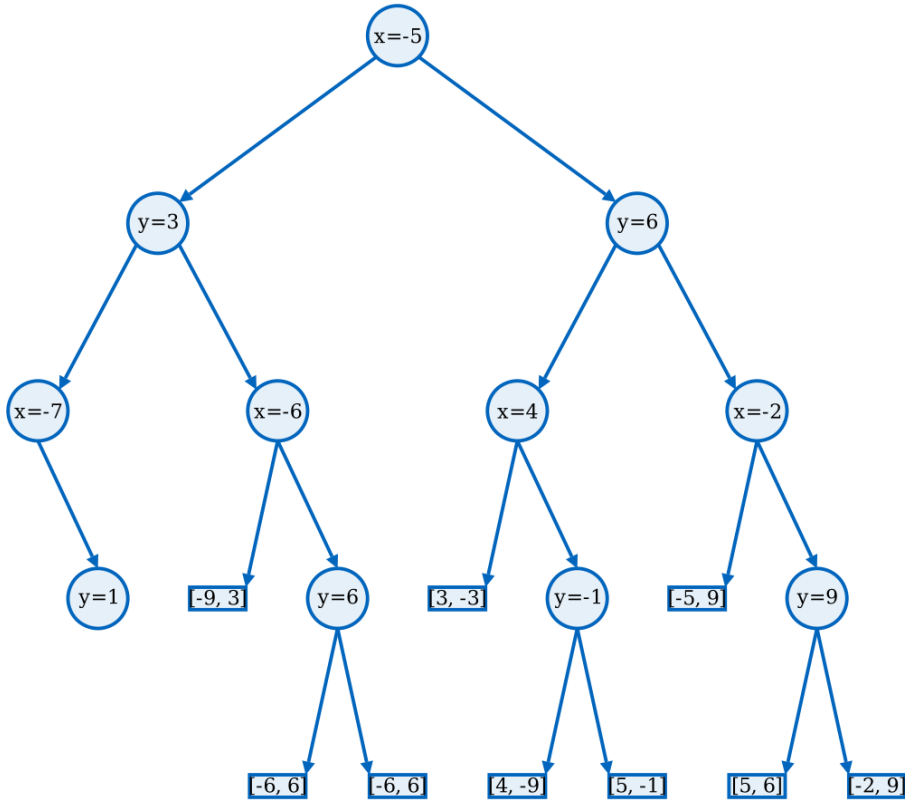

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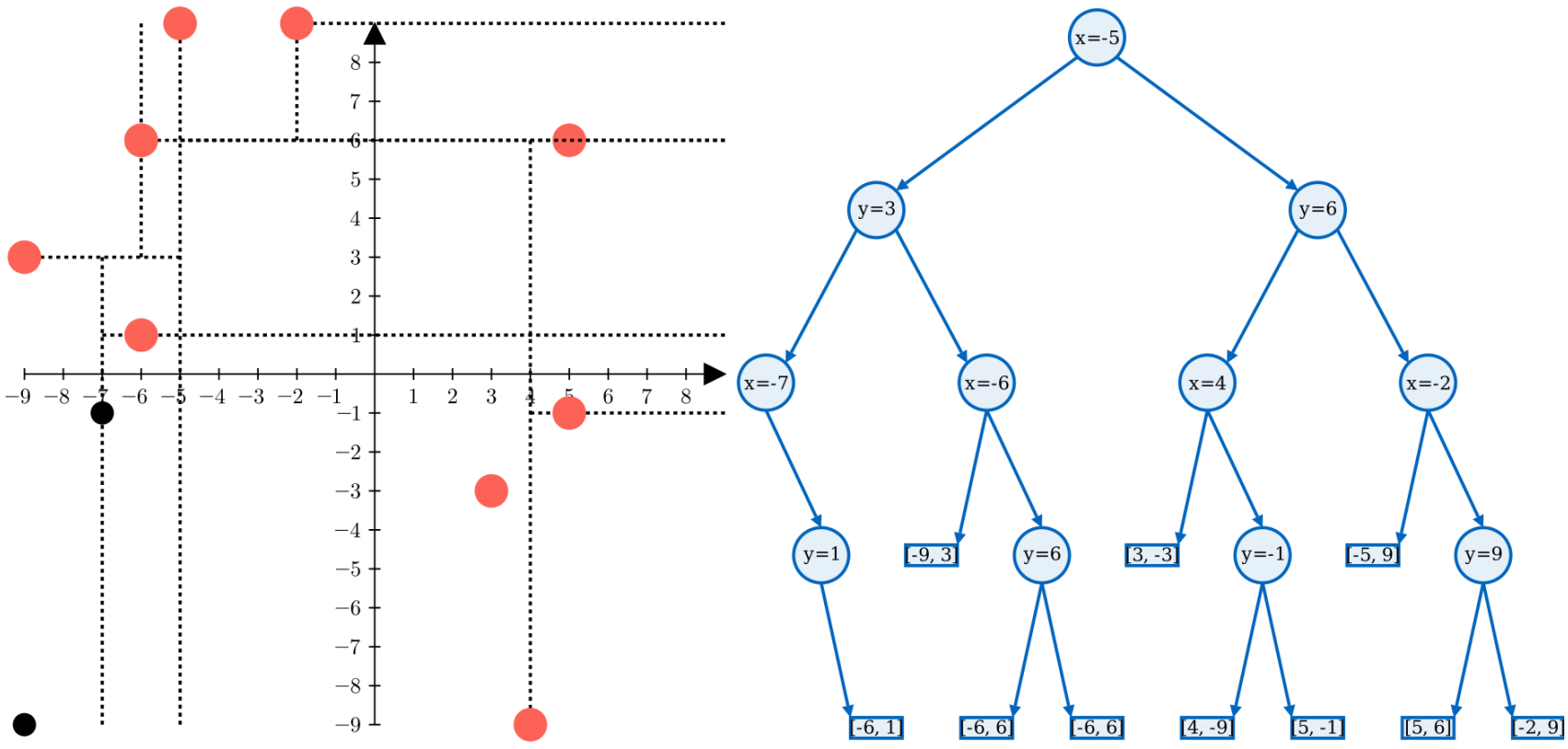

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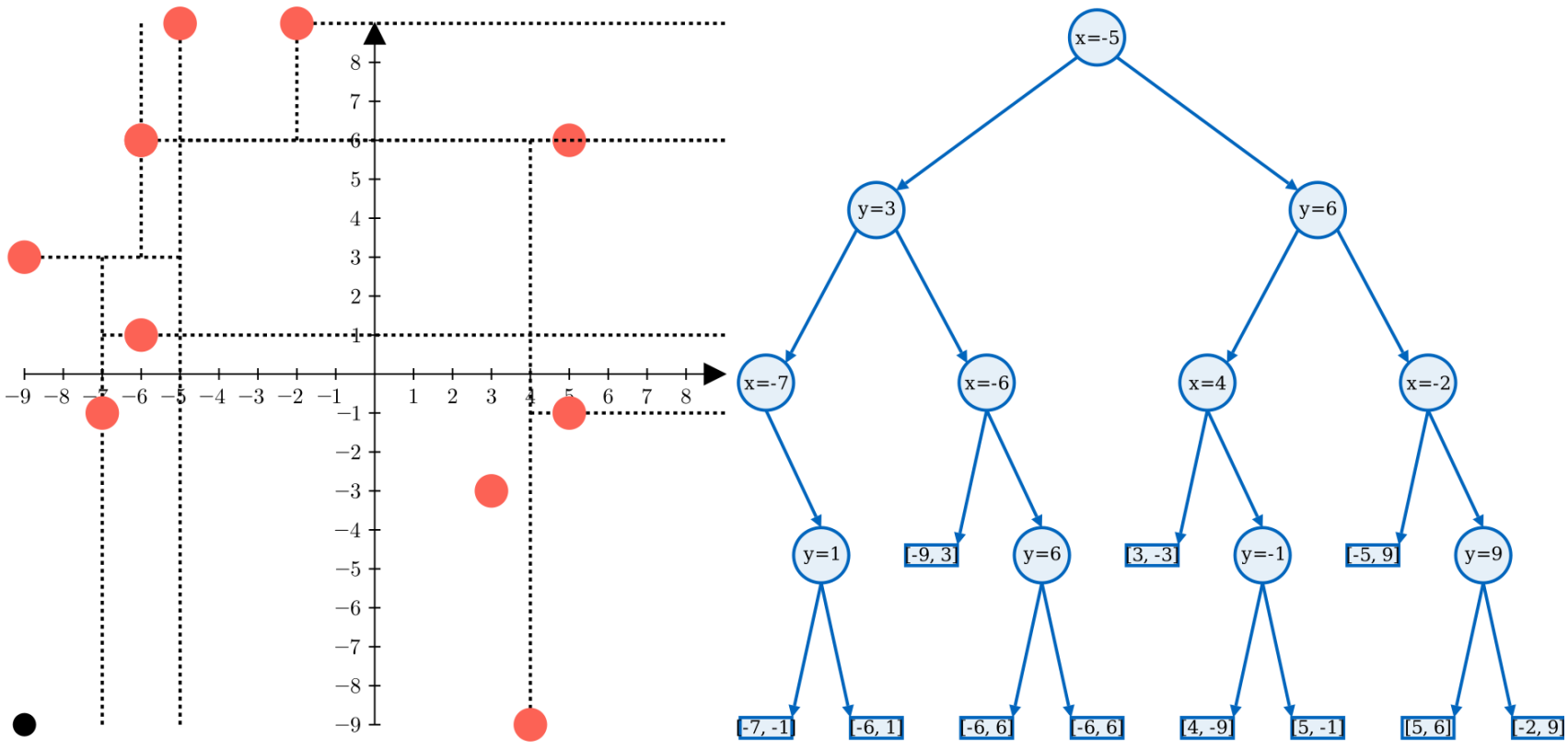

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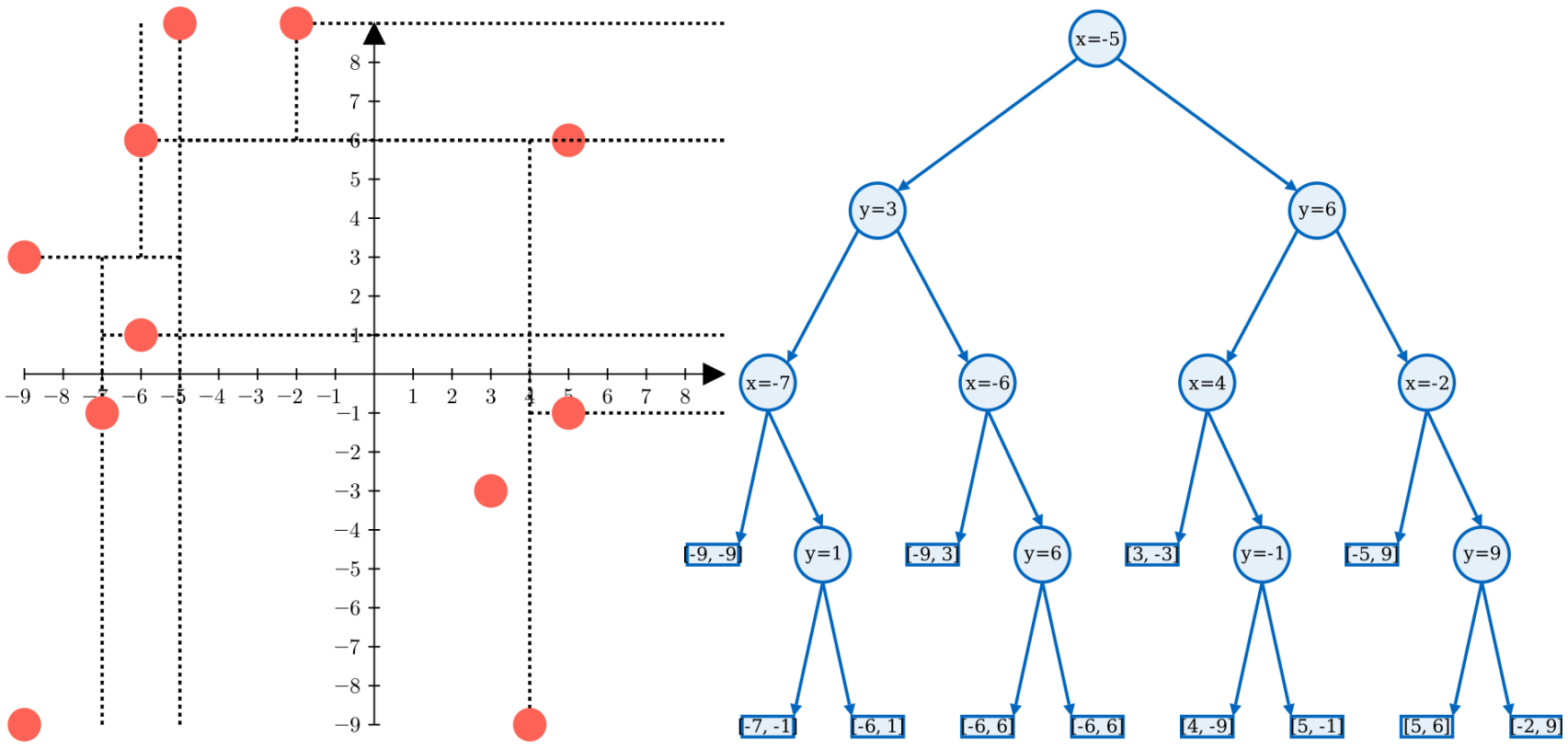

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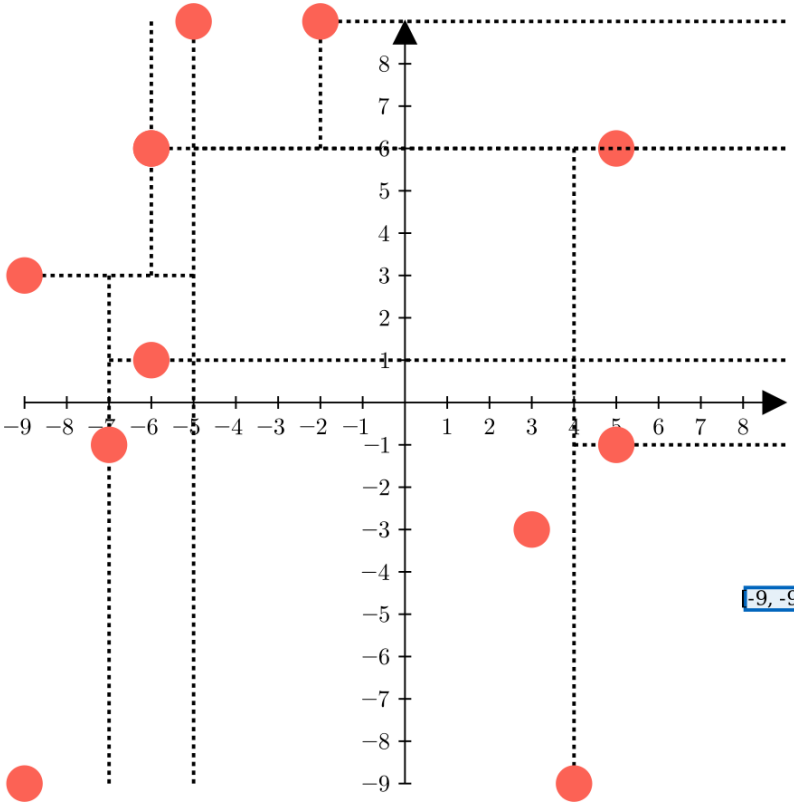
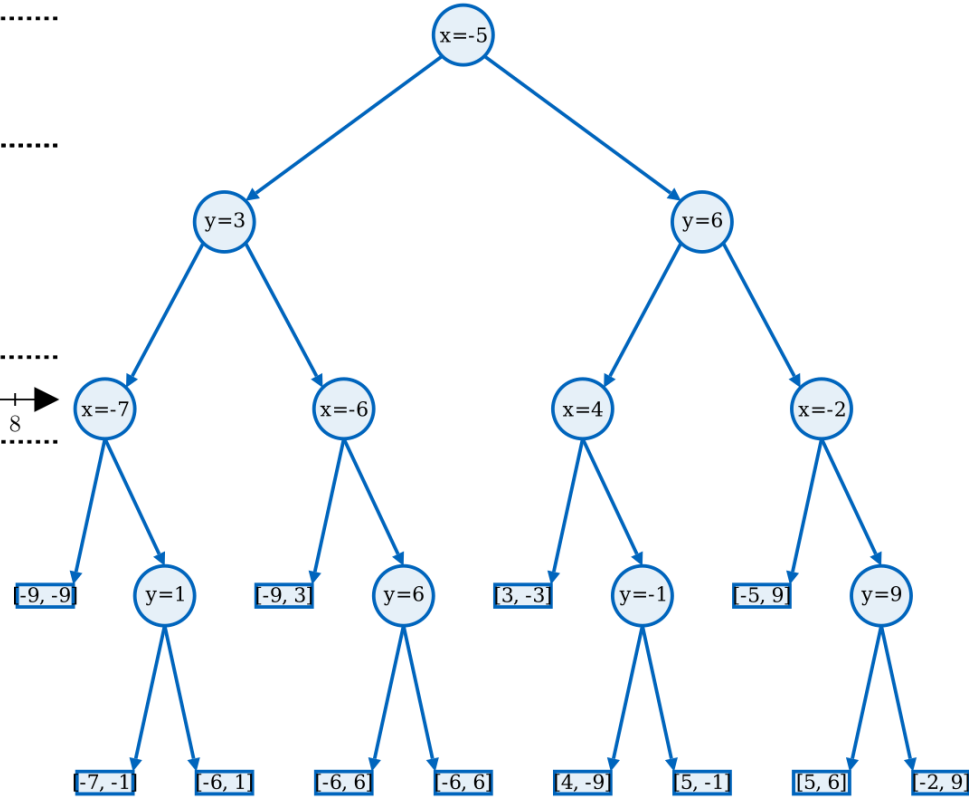
KD-Tree: search

- Input is a point
- Traverse the nodes till the leaf (based on decision in each node)
- This locates a region that **may contain** the nearest neighbor
- Search also **all surrounding regions** to ensure finding the nearest neighbor



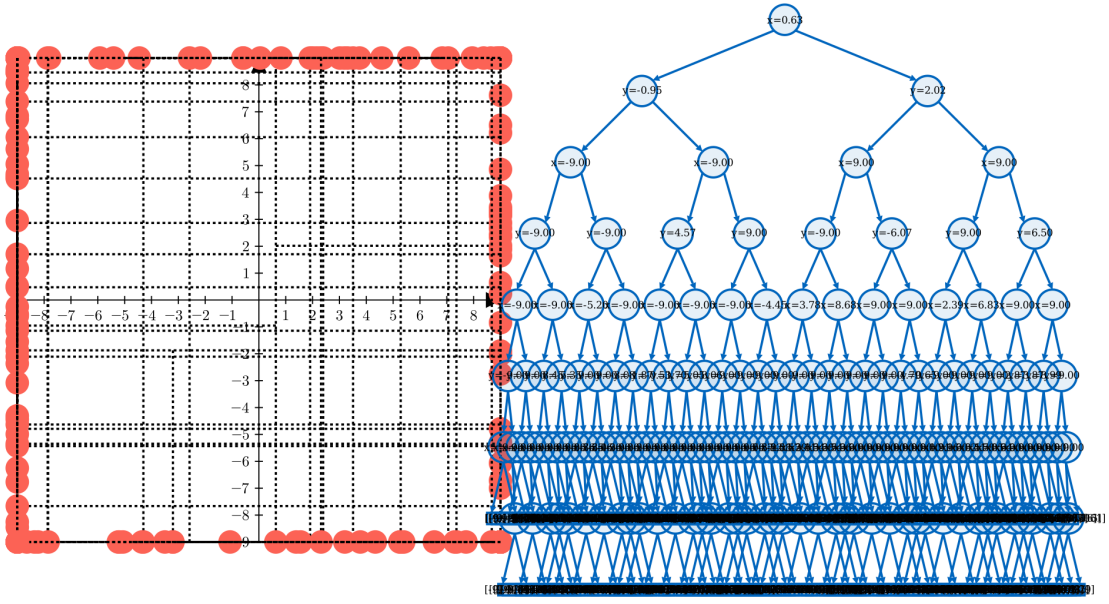


Usefull operations

- Inserting new item in $\mathcal{O}(\log n)$
- Removing existing item in $\mathcal{O}(\log n)$

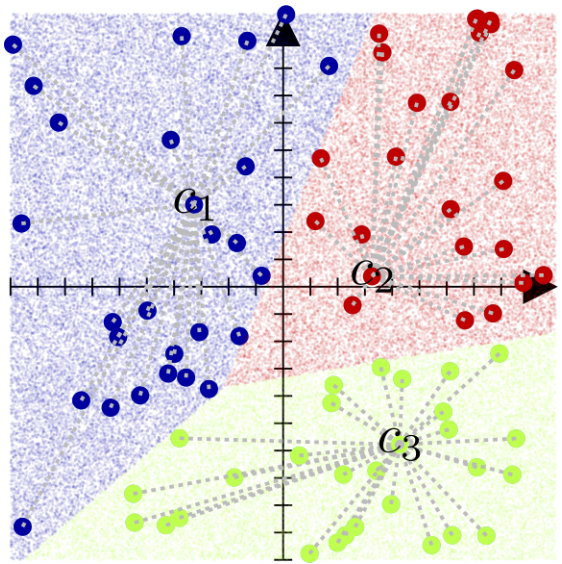
KD-Tree issues

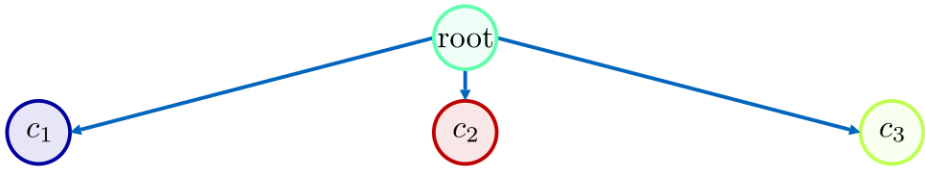
- Not suitable for other than Euclidean metrics
- Ineffective for large dimensions k
- It needs $n \gg 2^k$ data to achieve $\mathcal{O}(\log n)$ performance, otherwise it performs almost linear search
- Ineffective for non-uniform data



GNAT — Geometric Nearest-neighbor Access Tree[3]

- Similarly to KD-tree, it is based on a decision tree
- In each node, multiple pivots (points) are defined
- Decision at the node is to find the nearest pivot
- Decisions then repeat on lower levels until nearest point is found

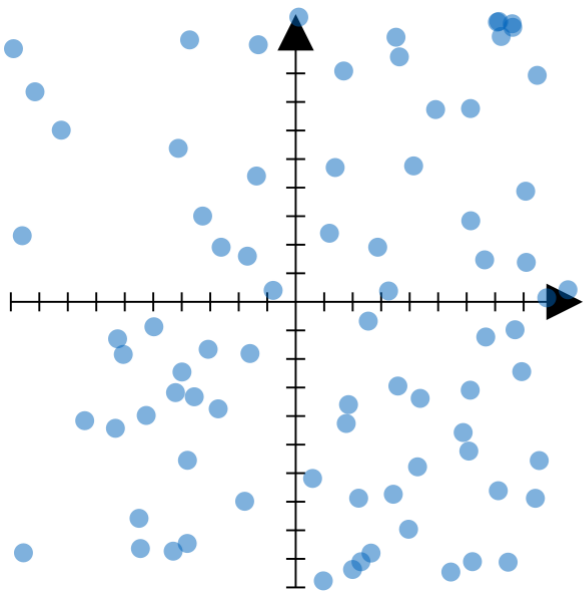




[3] K. Fredriksson. *Geometric Near-neighbor Access Tree (GNAT) revisited*. 2016. arXiv: 1605.05944 [cs.DS]. URL: https://arxiv.org/abs/1605.05944.

Construction of GNAT

- Select m pivots $c_1, \ldots c_m \in S$
- Assign each point in S to the nearest pivot, D_{c_i} (clusters)
- For each cluster D_{c_i}:
  - $R_{i,j} = [\min_{x \in X} \varrho(c_i, x), \max_{x \in X} \varrho(c_i, x)]$, $X = D_{c_j} \cup \{c_j\}$
  - $R_{i,j} = [\mathrm{low}, \mathrm{high}]$ are ranges of distances between c_i and data points of other clusters
- Build recursively the children c_i with its points D_{c_i}



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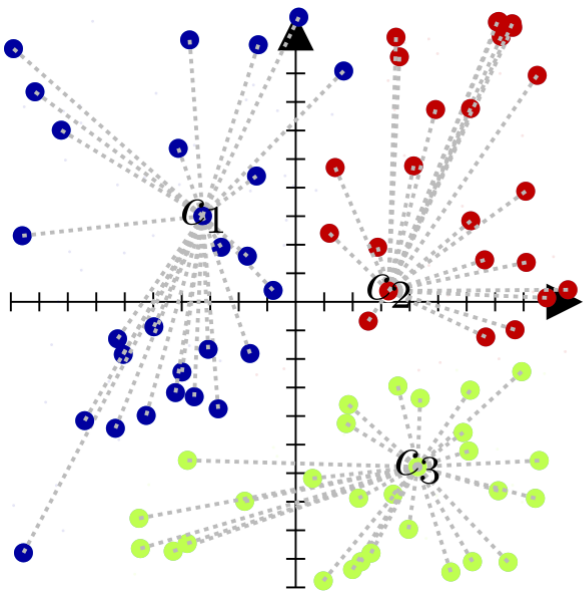

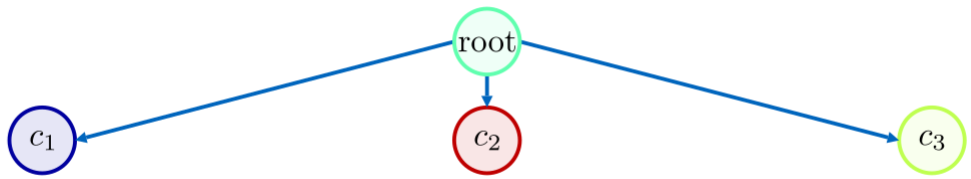

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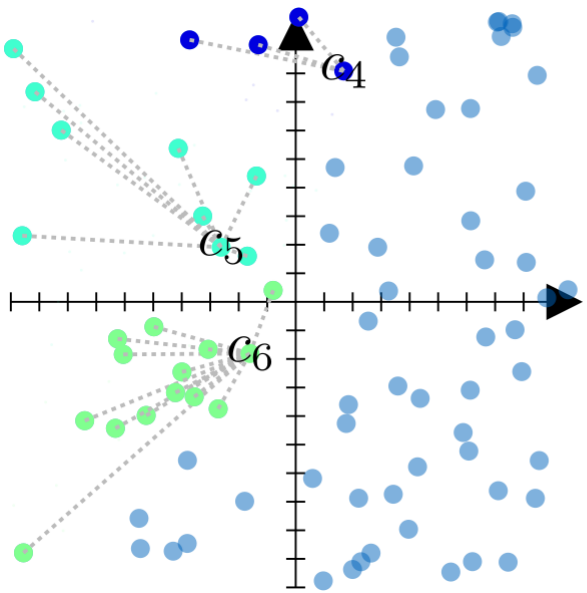

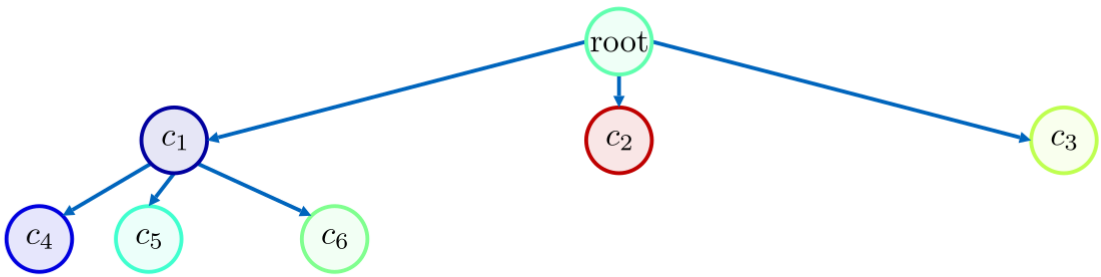

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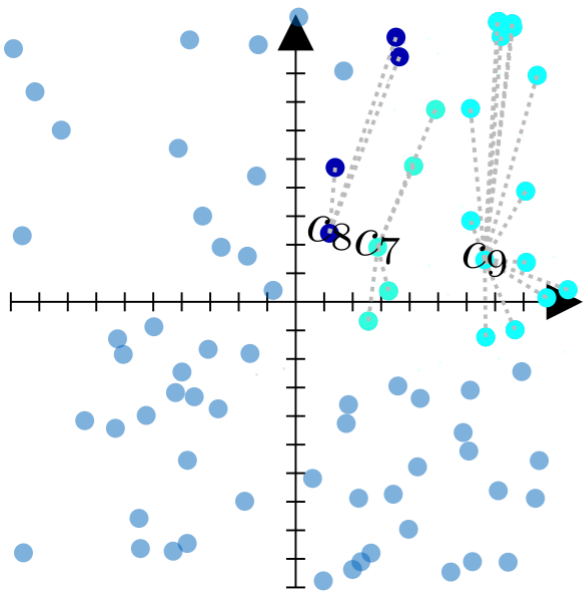

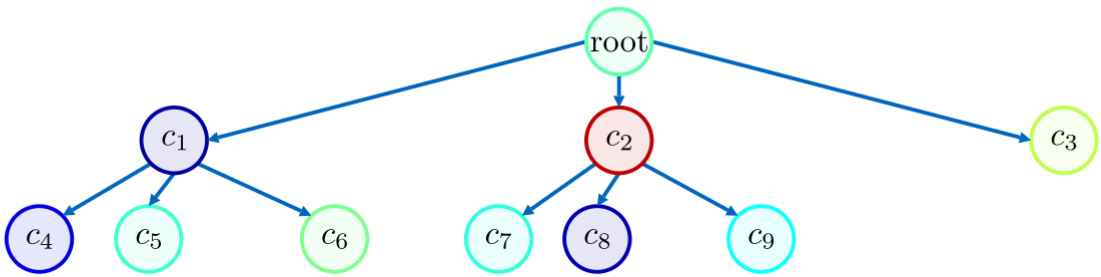

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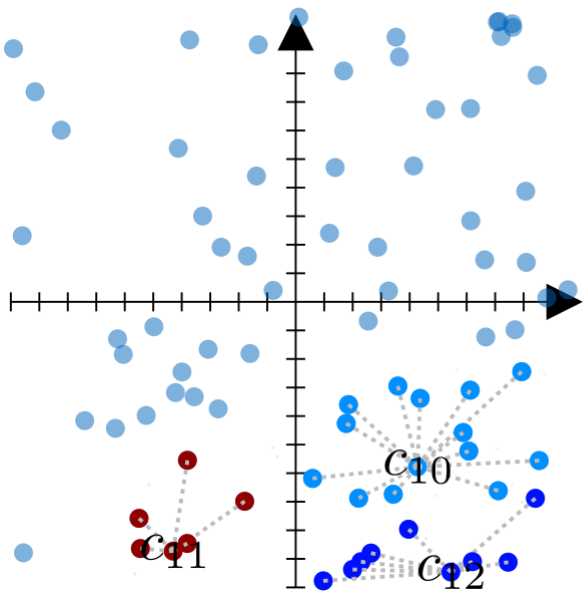
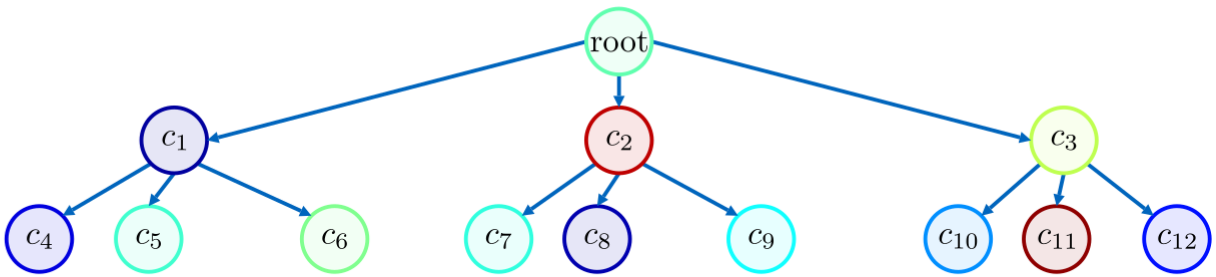
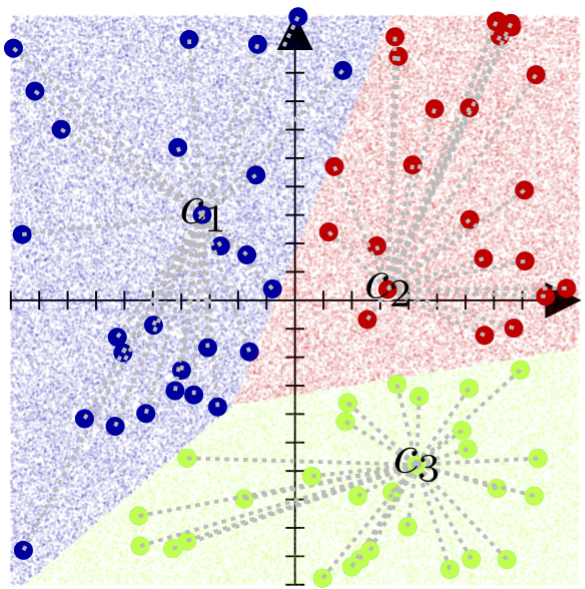
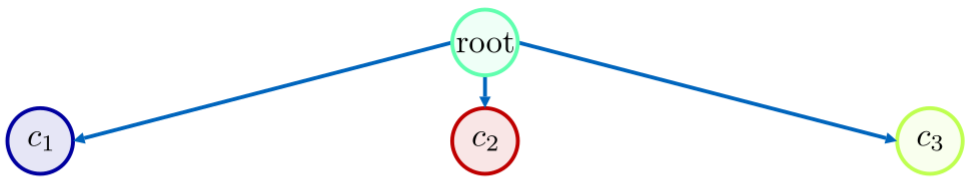




Nearest neighbor search towards a query point q

1. Start at root
2. Select a pivot c_i
3. If $e = \varrho(q, c_i) \leq r$, report c_i
4. If $[e - r, e + r] \cap R_{i,j} = \emptyset$, we can prune node of cluster c_j
5. Repeat steps 2–4 for all clusters $i = 1, \ldots, m$ at the current level
6. For each non-pruned cluster c_i, search its corresponding subtree





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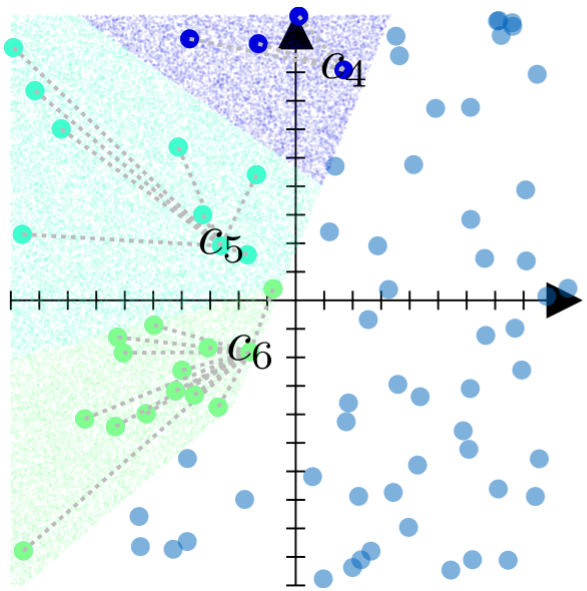

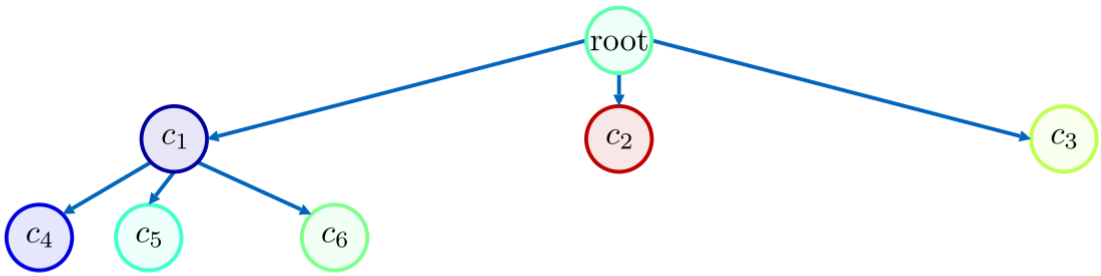

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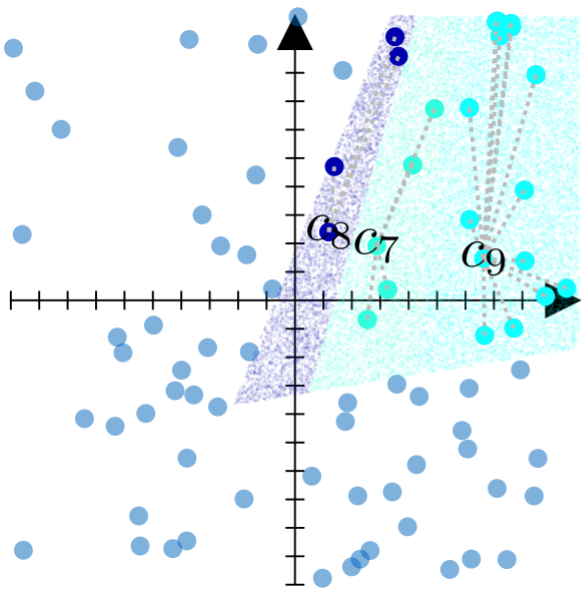

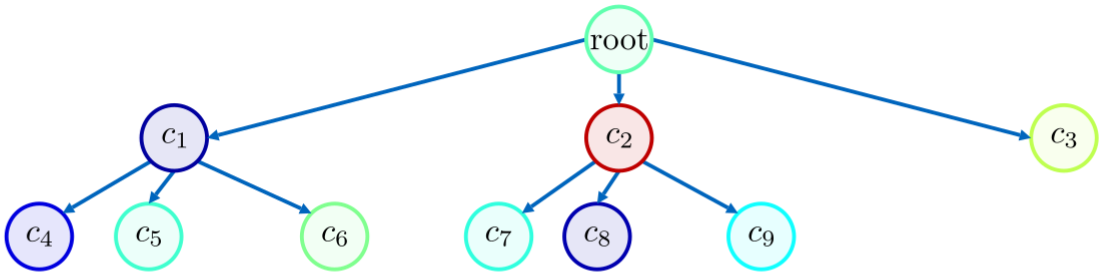

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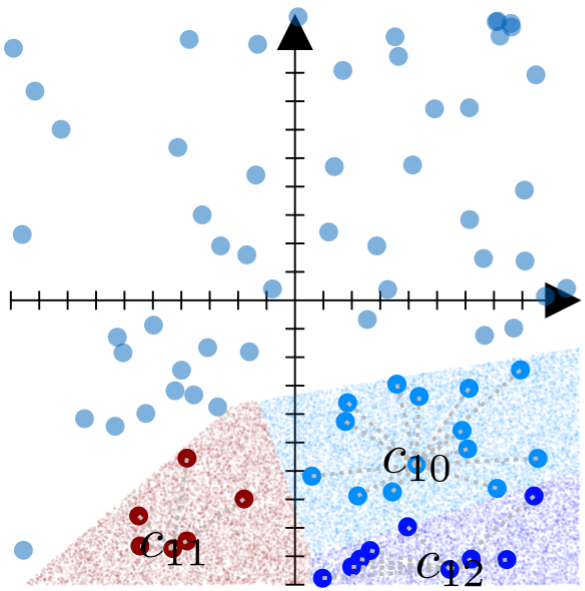
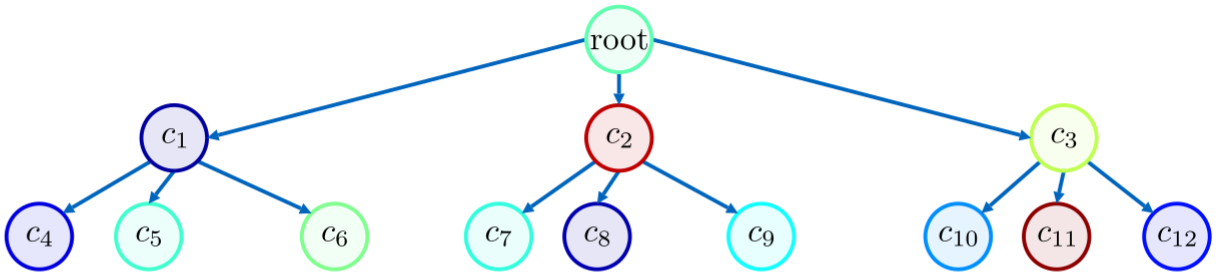


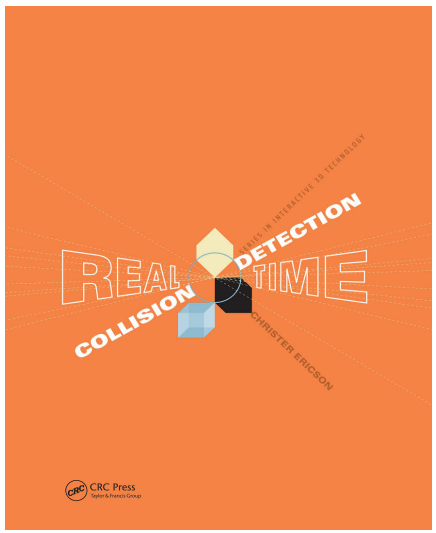


GNAT: properties

- Assume m clusters at each node
- Construction (average) $\mathcal{O}(nm \log_m n)$, worst case $\mathcal{O}(n^2)$
- Space complexity $\mathcal{O}(mn)$
- Search: time complexity is hard to analyze, experiments show that it's $\sim$ logarithmic
- Practically, GNAT performs better for larger d than KD-trees
- GNAT works with arbitrary metric
- GNAT does not degenerate with non-uniform distributions

Collision detection

- The most important part of all planners
- Combinatorial: for identification of points in/outside obstacles
- Sampling-based: testing collision between complex robot and obstacle geometries
- Good reference is the book[4]



[4] C. Ericson. *Real-time collision detection*. Crc Press, 2004.

Collision detection

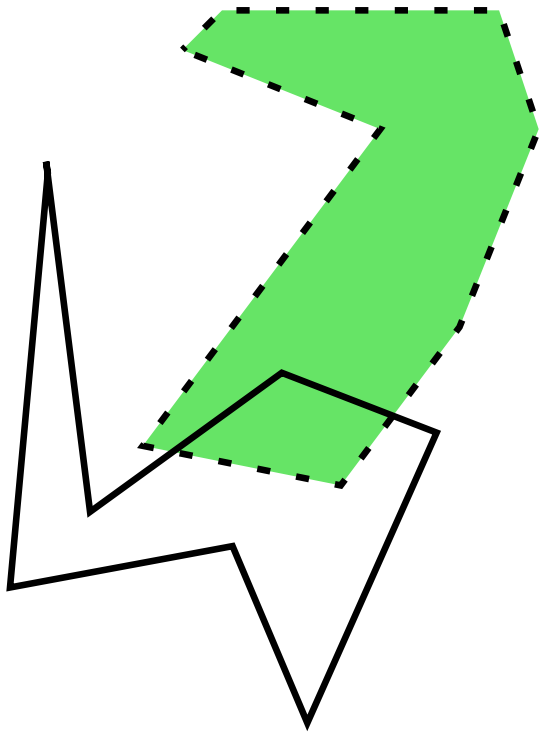
- Determines if/how objects collide/overlap/intersect/touch
- “Collision detection” covers two different techniques:

Collision-detection:

- True/False answer
- Fast, suitable for sampling-based planners

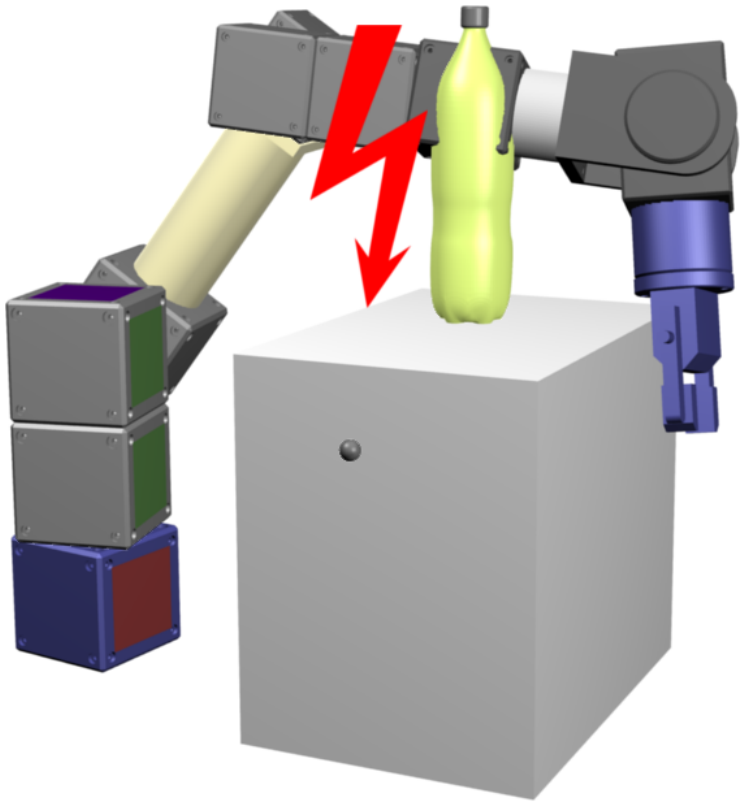
Collision-determination:

- Report details about collisions
- Identify which objects intersect
- Enumerate involved primitives
- Optionally computes the “penetration vector”, or point of intersection
- Slower than collision-detection



Consider the manipulator at collision

- How can you react with collision-detection?
- How collision-determination helps to overcome the problem?

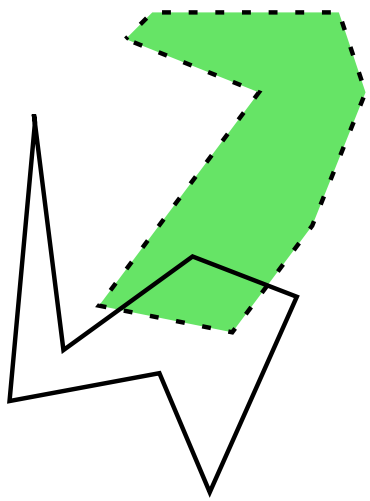


Geometric primitives

- Points, Lines, Circles, Triangles, Spheres, Cylinders, Rectangles
- Objects are constructed from these primitives
- The primitives determines which CD algorithm can be chosen
- CD relies on intersection tests between the primitives
- Convex shapes are always better, CD is faster with them

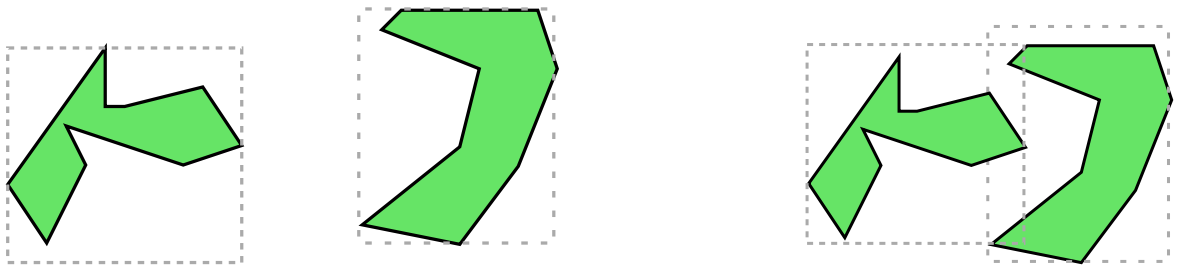
Collision detection between n and m primitives

- Naïve CD: $\mathcal{O}(mn)$
- This can be too slow!



Bounding volume

- Reduce complexity of CD[56] by replacing the original object by a simpler object that contains the original one
- Represent an object by a Bounding Box (BB)
- If two BBs do not overlap, object inside cannot collide (fast test)
- If two BBs collide, further test is made using internal objects (slow test)



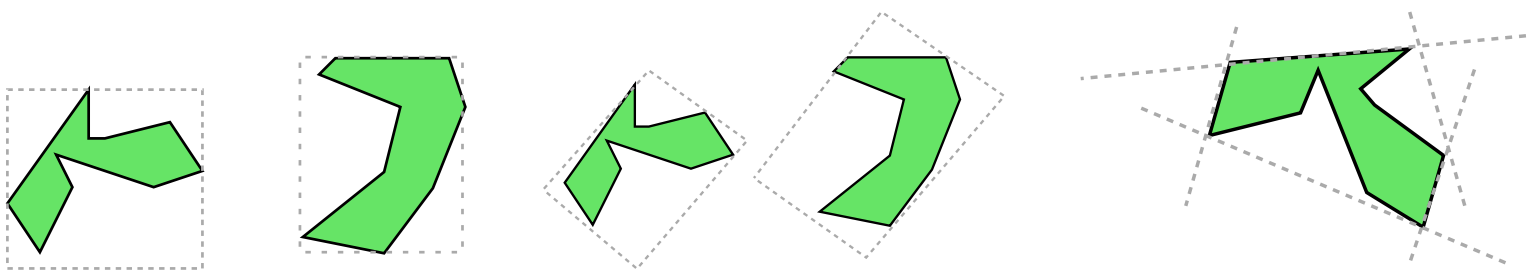
- BB should be geometrically simple to enable fast BB-vs-BB tests
- Spheres/circles, ellipses, rectangles
- BB should be as tight as possible to minimize false-positives

[5] R. R. Man et al. "A Survey of Collision Detection". In: *Mechanical, Electronic and Engineering Technologies (ICMEET 2014)*. Vol. 538. Applied Mechanics and Materials. Trans Tech Publications Ltd, June 2014, pp. 360–363. DOI: 10.4028/www.scientific.net/AMM.538.360.

[6] P. JimÄŠnez et al. "3D Collision Detection: A Survey". In: *Computers & Graphics* 25.2 (Apr. 2001), pp. 269–285. DOI: 10.1016/S0097-8493(00)00130-8.

Rectangles as bounding boxes

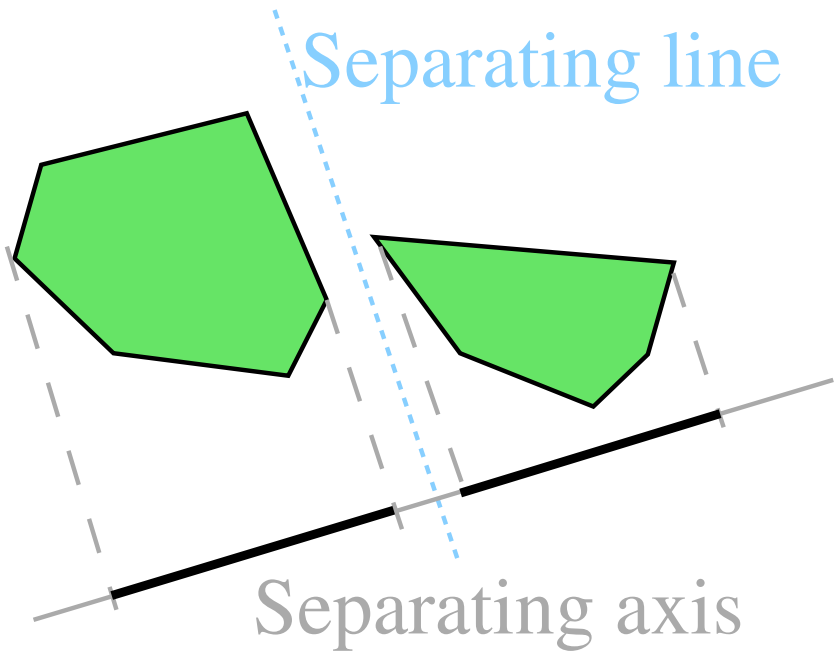
- **AABB — Axis Aligend Bounding Box**
  - Faces of bounding box are parallel to the coordinated system
  - Very fast detection of overlap of two BBs
  - Not suitable for 'rotated' objects that lead to large BB
- **OBB — Oriented Bounding Box**
  - Faces of BB are oriented according to the object
  - Lower volume of BB, less false-positives
  - Slower detection of BBs overlap than for AABB
- **k-DOP — *k* Discrete Oriented Polytope**
  - Boolean intersection along *k* directions
  - Axes of DOPs do not have to be orthogonal
  - Generalization of AABB/OBB (e.g., AABB in 2D is 4-DOP)



AABB OBB k-DOP

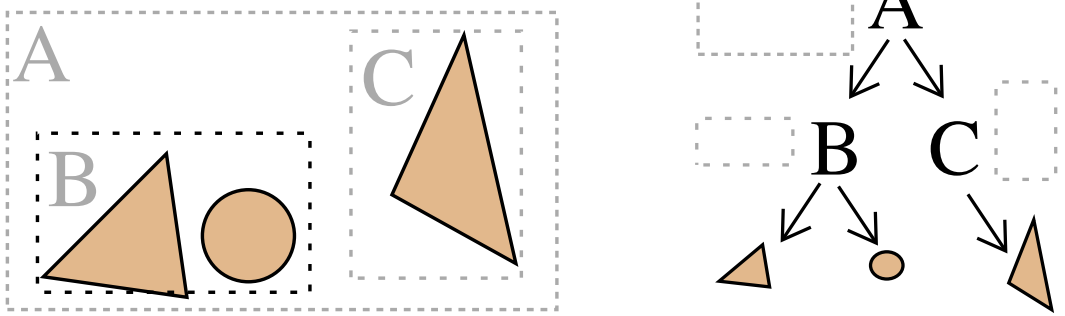
Separation axis theorem

- Is used to determine overlap of two convex objects
- Two convex polytopes do not overlap if there exists a line onto which the projection of the two objects do not overlap
- Separating line can be determined by testing all combinations of lines/faces of both objects
- Convex objects!



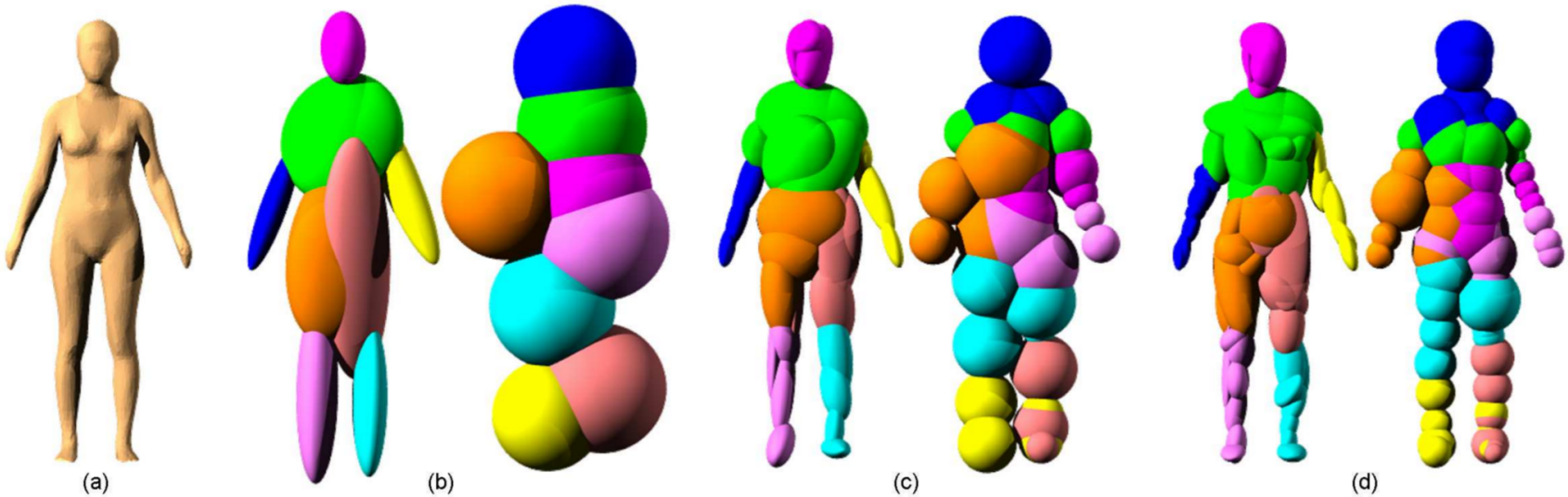
Bounding Volume Hierarchy (BVH)

- Original objects are recursively split to subsets
- BVH is a tree structure of bounding-boxes (BB) for each subset
- A Node in BVH is either a BB or a geometric object



Typical usages of BVH

- Collision detection[7], physical-based simulations[8]
- Compact representation of scenes[9], computer graphics[10]



[7] A. Mandalika et al. "Generalized Lazy Search for Robot Motion Planning: Interleaving Search and Edge Evaluation via Event-based Toggles". In: *CoRR* abs/1904.02795 (2019). arXiv: 1904.02795.

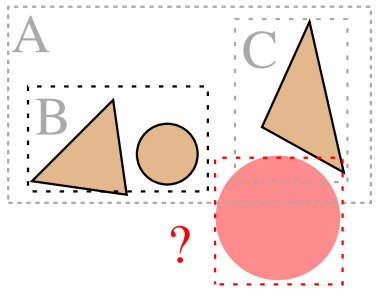
[8] X. Li et al. "Efficient Collision Detection Using Hybrid Medial Axis Transform and BVH for Rigid Body Simulation". In: *Graphical Models* 128 (July 2023), p. 101180. DOI: 10.1016/j.gmod.2023.101180.

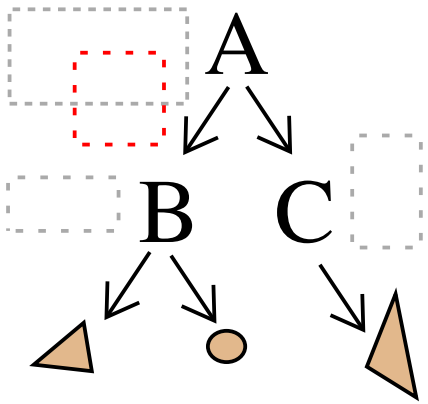
[9] S. Liu et al. "Approximating solid objects by ellipsoid-tree". In: *2009 11th IEEE International Conference on Computer-Aided Design and Computer Graphics*. 2009, pp. 134–139. DOI: 10.1109/CADCG.2009.5246919.

[10] D. Meister et al. "A survey on bounding volume hierarchies for ray tracing". In: *Computer Graphics Forum*. Vol. 40. 2. Wiley Online Library. 2021, pp. 683–712.

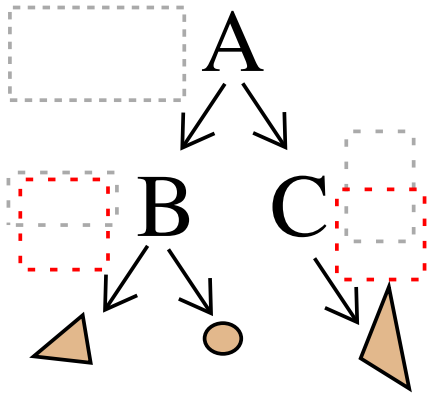
Collision detection using BVH

- **Broad phase**
  - Traverse BVH from the root
  - At each level, evaluate overlaps between BBs
  - If BBs do not overlap, return no-collision
  - If BBs overlap, continue to child nodes
- **Narrow phase**
  - for two overlapping BBs, perform collision detection of their internal objects
- Hierarchical CD: $\mathcal{O}(\log n)$ for n geometric primitives
- Building of BVH (depends on its type) takes at least $O(n)$

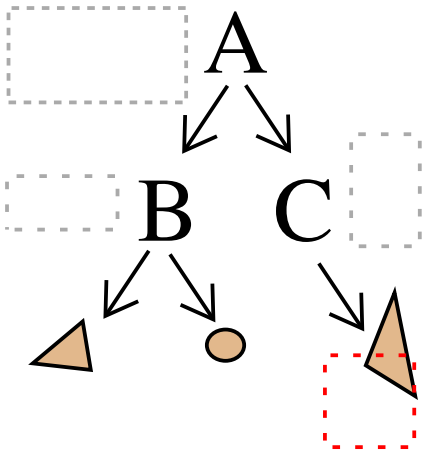




(BB vs BB)



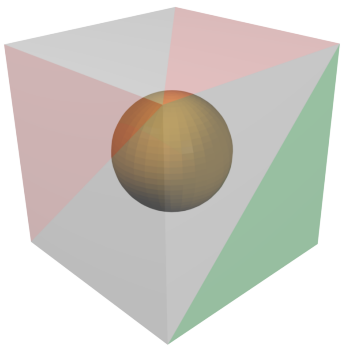
(BB vs BB)



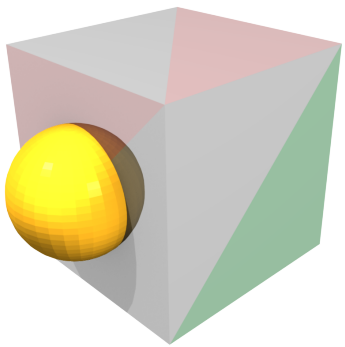
(Circle vs Triangle)

Notes on collision detection I

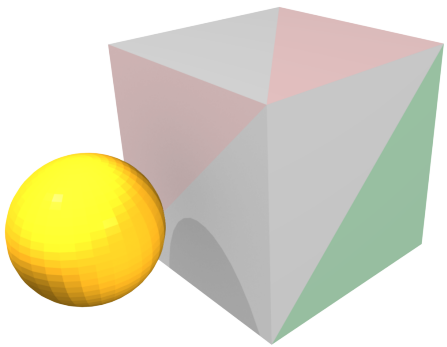
- Usual representation for 2D objects:
  - Combination of boxes/spheres, polygons, triangulated polygons
- Usual representation for 3D objects:
  - Combination of 3D geometric primitives (boxes,spheres,cylinders), triangle mesh
- Note: triangle meshes are hollow $\rightarrow$ detection of 'object inside object' is not possible



No collision



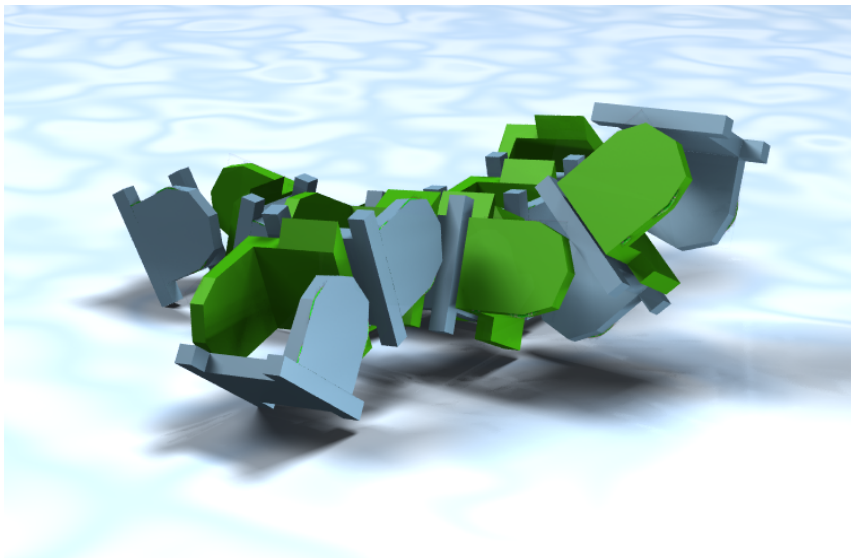
Collision



No collision

Notes on collision detection II

- CD between objects of the same type is usually faster than between objects of different types
- It's a good practice to represent the robot by a combination of basic primitives than using a full CAD model



Collision-detection
$\sim$ 100 triangles/robot

Visualization, from CAD
$\sim$ 10k triangles + textures/robot

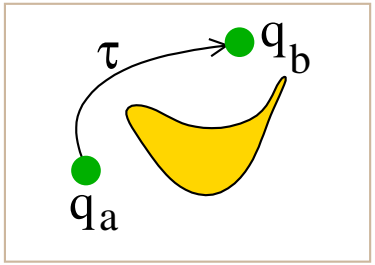
Local planners

- Sampling-based planners rely on a “local planner”
- Given configurations $q_a \in \mathcal{C}_{\text{free}}$ and $q_b \in \mathcal{C}_{\text{free}}$, local planner attempts to find a path τ:

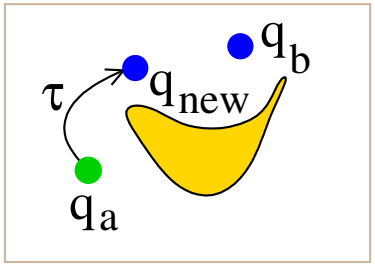
$$\tau : [0, 1] \rightarrow \mathcal{C}_{\text{free}}$$

such that $\tau(0) = q_a$ and $\tau(1) = q_b$, and τ must be collision free!

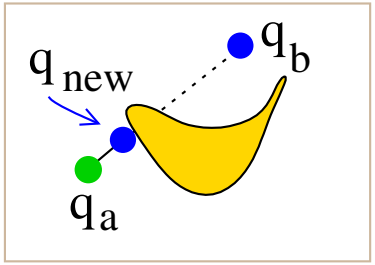
- Two-point boundary value problem (BVP)



Exact local planner



Approximate

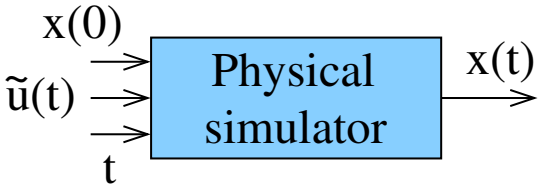


Straight-line

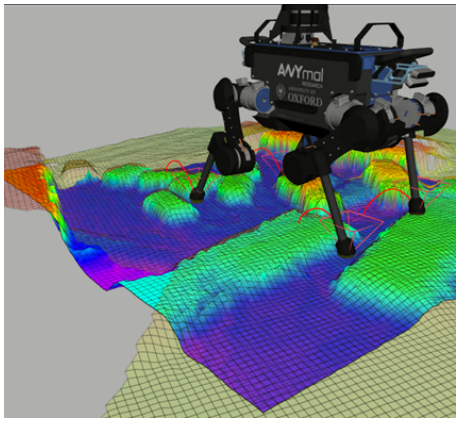
Types of local planners (revision)

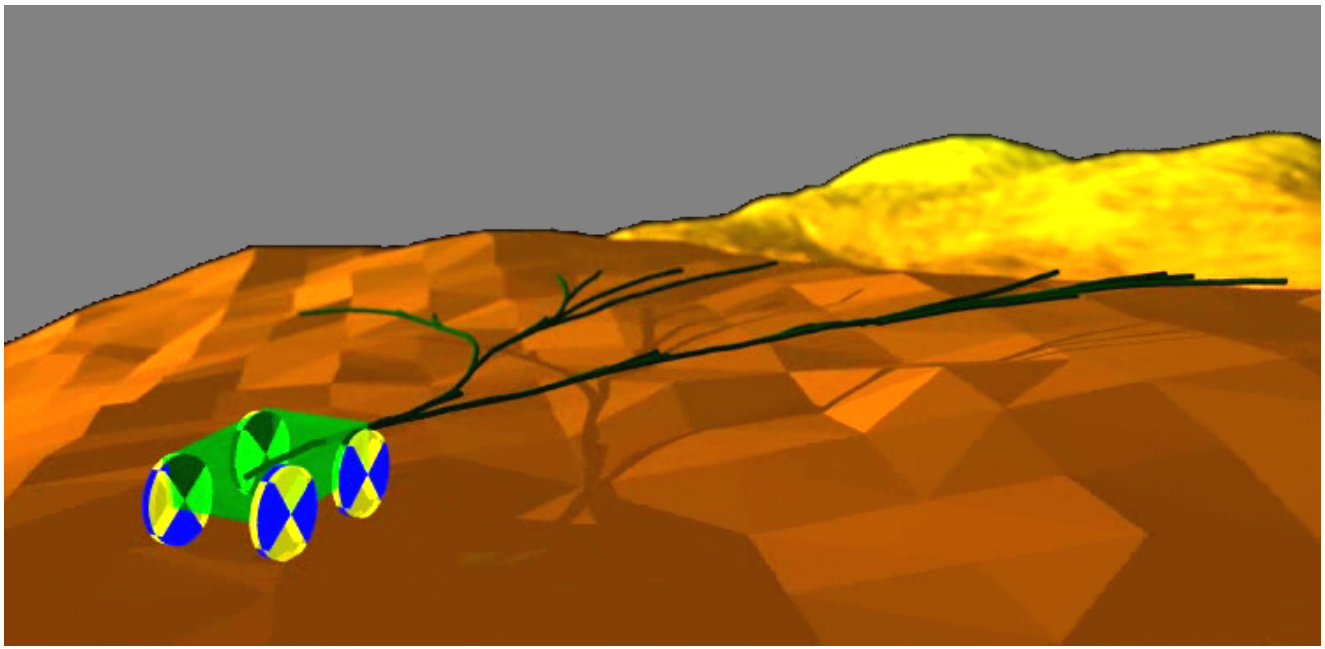
- Exact: analytic solution to BVP, e.g., Dubins or Reeds Shepp, straight-line (sometimes)
- Approximate: τ from q_a with q_{new} that is near-enough from q_b, e.g., straight-line
- Black-box models: physical simulation, e.g., for situations that cannot be solved analytically

Local planner: System simulator



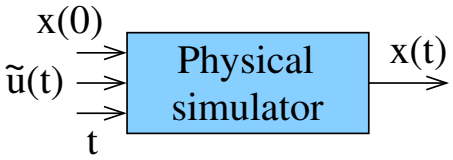
- Let's assume a non-trivial scenario, e.g.,
  - mobile robot moving on a undulating terrain
  - or a legged robot walking on stones
- Analytic motion model is not easy to derive
- Instead, we can use a (physical) simulation
- Simulation is used as a "black-box"

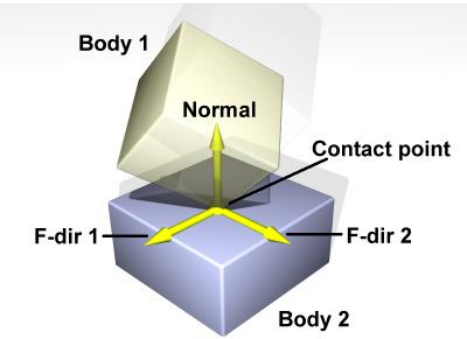


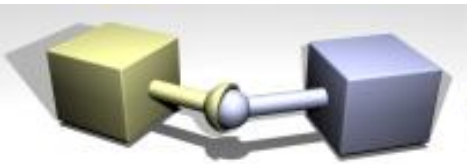


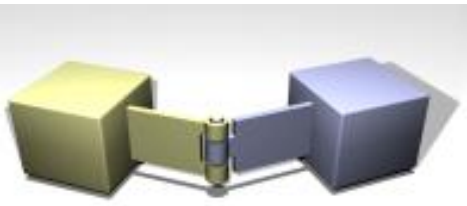
Physical simulation

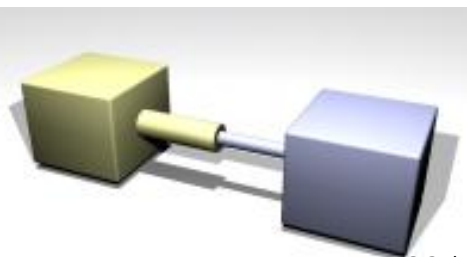
- Motion model of objects based on Newton physics
- Complex objects (robots) are composed of basic primitives
  - Spheres, Boxes, Cylinders
  - Analytic collision determination
- Each object has shape, mass and mass-density
- Objects are connected using static/movable joints
- Each join has limits/maximal moments, speed (+ internal states)
- Internal state s_i of object i: position, rotation, velocity, angular velocity

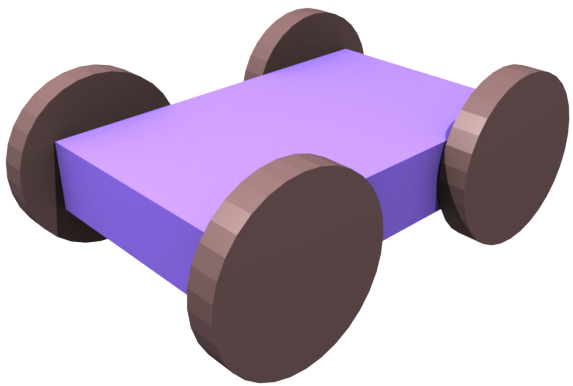






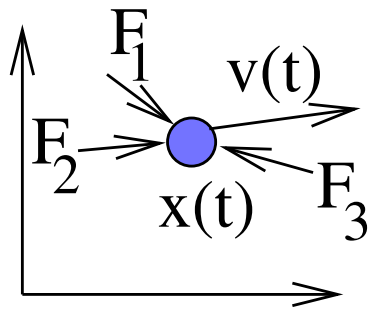


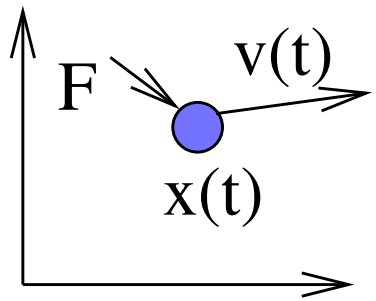


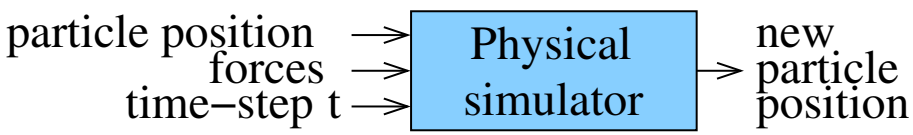


Particle

- Position $x(t)$, velocity $v(t)$, and mass m
- Various forces $\mathbf{F_i}$ are applied on the particle
- Particle movement is not constrained
- $\mathbf{F} = \sum \mathbf{F_i}$ is the total (net) force
- $\mathbf{F} = m\mathbf{a}(\mathbf{t})$, $\mathbf{a}(\mathbf{t}) = \dot{\mathbf{v}}(t)$, $\mathbf{v}(\mathbf{t}) = \dot{\mathbf{x}}(t)$
- Simulator computes $\mathbf{a}(t) \rightarrow \mathbf{v}(t) \rightarrow \mathbf{x}(t)$
- Integration over time-step ε (resolution of the simulation)
- Requires integration (Euler method, Midpoint, Runge-Kutta,...)
- Particle has no rotation



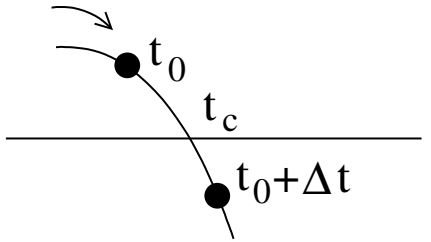




Simulating collisions

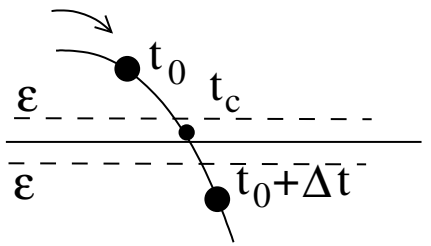
Colliding contact

- Particle is falling to a table
- Integration goes by step Δt: $t_0, t_0 + \Delta t, t_0 + 2\Delta t, \ldots$
- Integration is terminated if collision happens
- The time of collision t_c is estimated
- Change of velocities of colliding bodies is computed
- Simulation is started from t_c with new velocities
- This ensures instant change of velocities after the collision



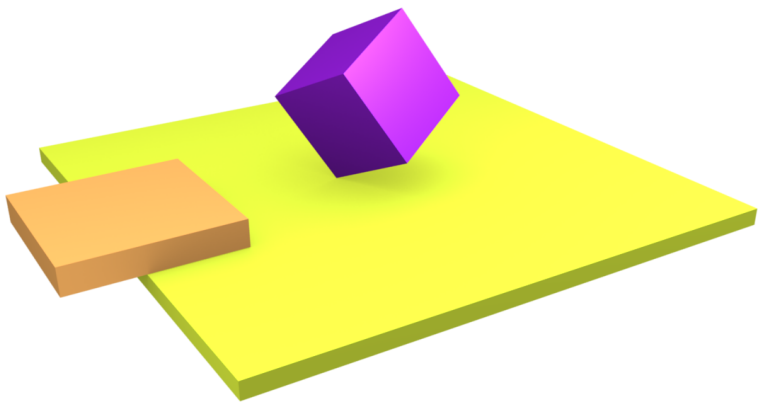
Detection of t_c assuming $t_0 < t_c < t_0 + \Delta t$

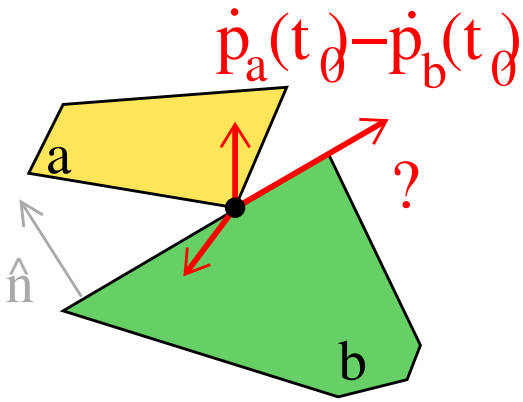
- Integration by intervals determined by the bisection method
- Alternatively, obtain collision depth from CD and accept t_c if the penetration depth is less than ε



Contacts

- Vertex/face
  - Vertex of one object is in contact with face of the other one
  - The Normal vector of the face determine the ‘normal of the contact’ $\hat{n}$
- Edge/Edge
  - Two edges *ea* and *eb* (each from different object) are in collision
  - $\hat{n} = ea \times eb$ (*ea* and *ev* are unit vectors)

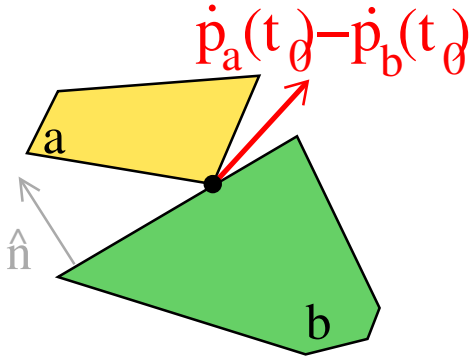




- Contacts $p_a(t_0)$ and $p_b(t_0)$, their velocity is $\dot{p}_a(t_0)$ and $\dot{p}_b(t_0)$
- $v_{rel} = \hat{n}(t_0) \cdot (\dot{p}_a(t_0) - \dot{p}_b(t_0))$
- Value of v_{rel} determines the type of collision

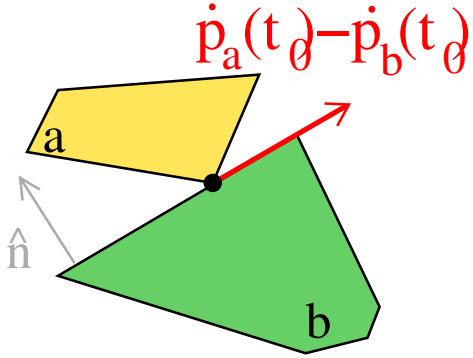
Separating

- $v_{rel} > 0$: bodies moving apart
- No reaction is needed



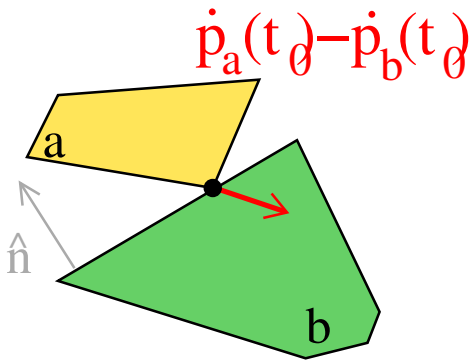
Contact/resting

- $v_{rel} = 0$
- No reaction is needed



Colliding

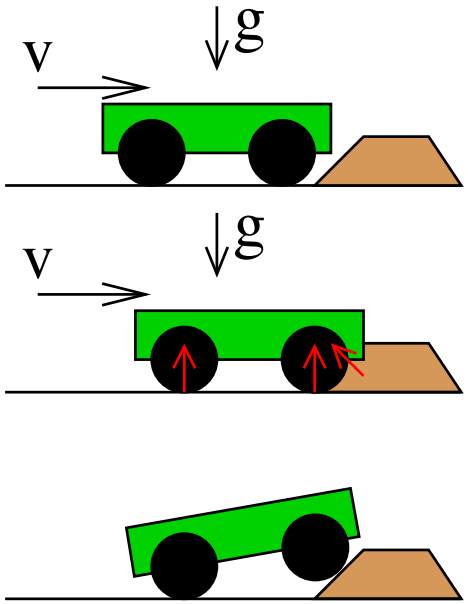
- $v_{rel} < 0$
- Compute the separating (penetration) vector, apply force to separate the objects
- Penetration vectors are not unique for non-convex objects
- The possible source of unstable simulation



Main loop of simulator

Particle

1. Create objects, create joints, . . .
2. User callback (read/set variables, display, . . .)
3. Apply forces
4. Update velocities and positions
5. Detect collisions
6. Solve constraints
7. Goto 2



Physical engines (sw. libraries)

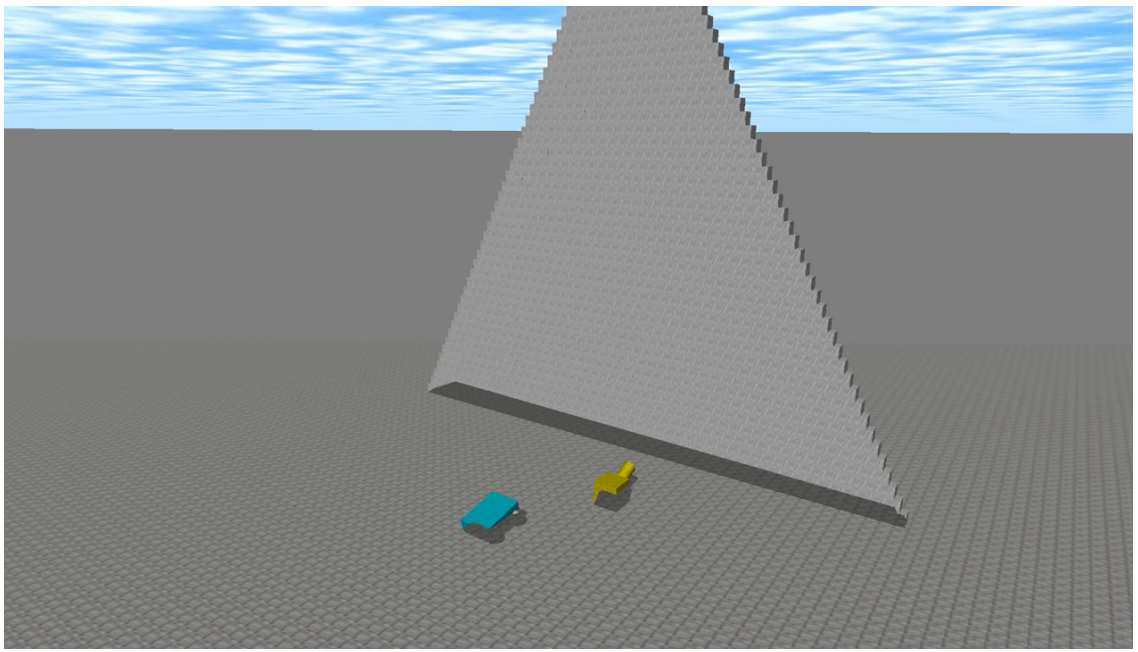
- Box2D, Chimpunk physics engine (2D)
- ODE, Bullet, Newton Game Physics (3D)

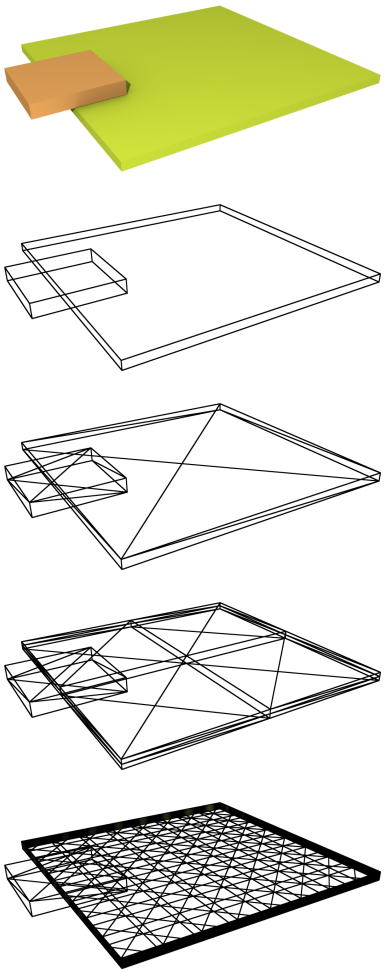
Robotic simulators (usually with GUI)

- They use physical engine inside, but offer more functionalities:
- Visualization, tools for interactive design of robots, import/export from URDF, sensors
- Gazebo, V-Rep (now CoppeliaSim), Webots, Player/Stage

Common issues of physical simulation

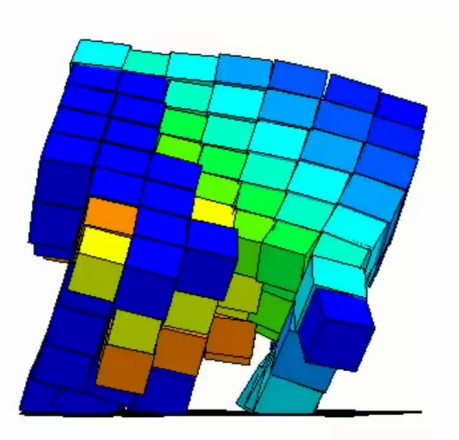
- If wrongly set up, it can “explode” or “freeze”
- Wrongly set up hinges, unrealistic masses, no gravity
- Too complex geometries $\rightarrow$ too complex (slow) collision detection
- Wrong friction parameters
- It’s better to prefer convex shapes (or composition of them) if possible

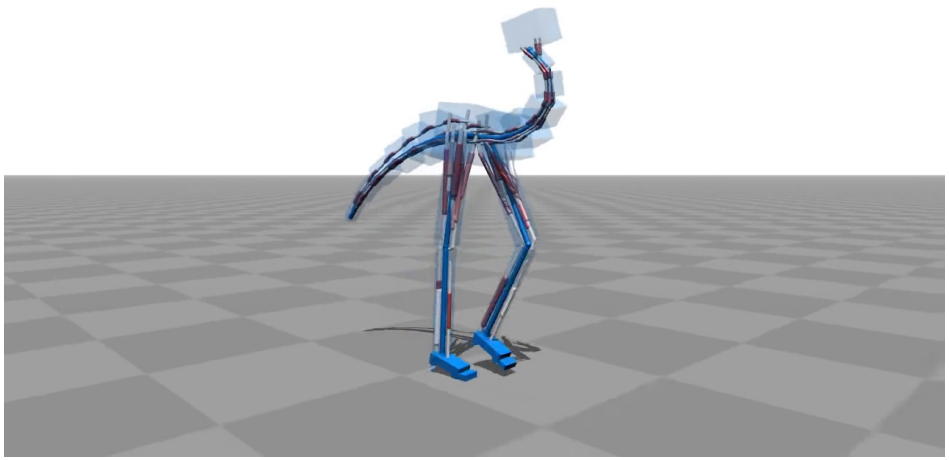




Physical simulation in other fields

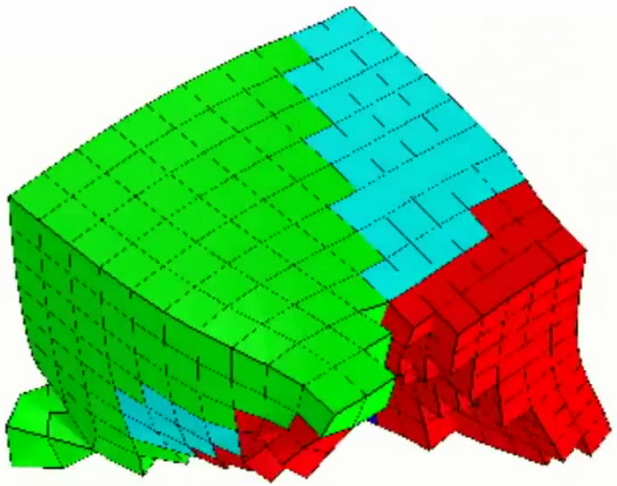
- 3D design/CAD simulation — design a machine and see how it works
- Virtual reality — e.g. for realistic object manipulation
- Computer games — realistic behavior of objects (without programming it)
- Evolving robots — evolutionary approaches to design robots or their parts, simulation serves as the fitness function evaluator





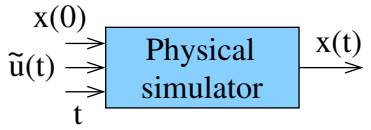
Physical simulation in other fields

In 2013, we saw simulated robots made of soft voxel cells evolve the ability to run.



Cheney, N., MacCurdy, R., Clune, J., & Lipson, H. (2013). Unshackling evolution: evolving soft robots with multiple materials and a powerful generative encoding. In Proceeding of the fifteenth annual conference on genetic and evolutionary computation (pp. 167-174). ACM.

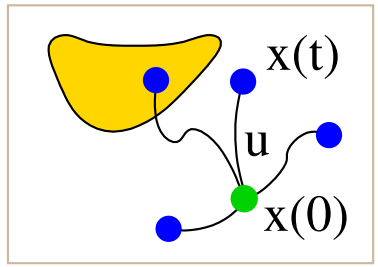
Physical simulation as local planner



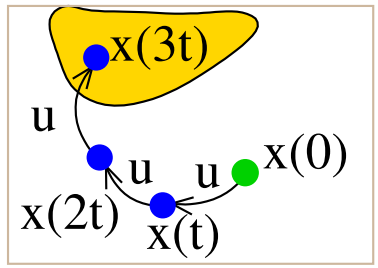
- Most of physical simulators (ODE, Bullet and their derivatives) assume time-linear simulation
- In motion planning, we need a non-linear simulation
- We need to “restart” simulation for each tree expansion

RRT with system simulator

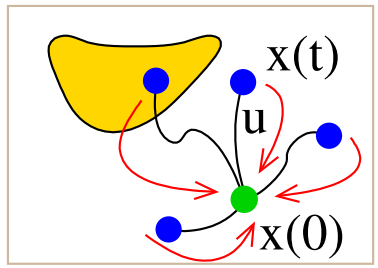
- Each node contains: $x = (s_i), i = 1, \ldots n$ (simulator state)
- Tree expansion from node $x_{\text{near}} = x(0)$ using input u
  - Set simulator to state x_{near} (restart)
  - Apply control inputs u (usually joint moments)
  - Run simulation for time Δt
  - Read simulator state x
  - Add node x to the tree
- Usually several control inputs $u \in \mathcal{U}$ is tested



We need



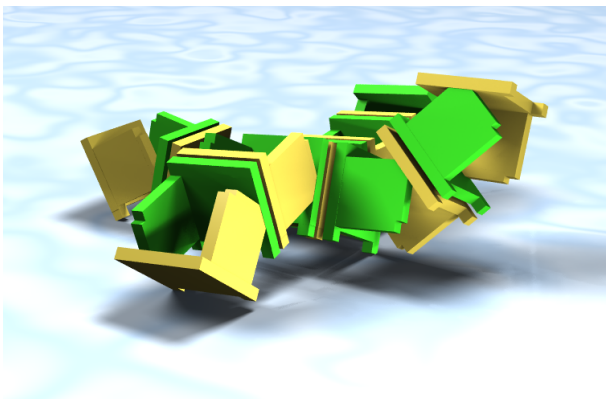
No restarts



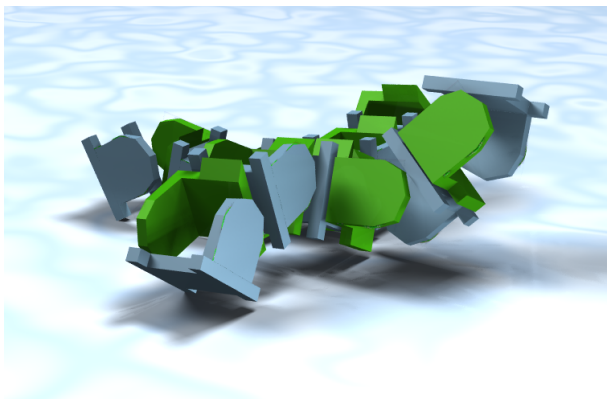
With restarts

Physical simulation: tricks

- Try to minimize number of objects/joints
- Avoid using triangle mesh for collision-detection
  - Physical simulation needs collision determination (penetration vector)
  - CD may be unstable on (non-convex) meshes, simulation can “explode”
- If possible, approximate robots by boxes/spheres/cylinders $\rightarrow$ fast and stable collision detection
- Use separate models for physics and visualization



Mass
10 boxes/robot



Collision-detection
$\sim$ 100 triangles/robot

Visualization
$\sim$ 10k triangles + textures/robot