Motion planning and control

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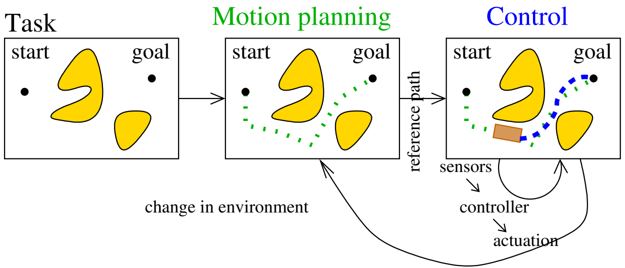
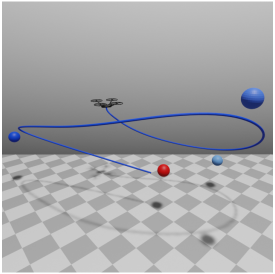




Classic decoupled planning-control approach

- A global plan is delivered by motion planning
- Robot executes the plan using a controller
  - controller uses fresh sensor data to handle control imprecission, disturbancies
  - can be extended to cope with obstacles (obstacle avoidance)
- New global plan is replanned when a global change occurs

Revision: discrete optimal planning

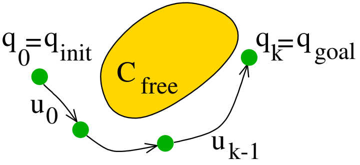
- Let $L(\pi_k)$ is the cost of the sequence $\pi_k = (u_0, \ldots, u_{k-1})$

$$L(\pi_k) = l_f(q_k) + \sum_{i=0}^{k-1} l(q_i, u_i)$$

- the final term $l_f(q_k) = 0$ if $q_k = q_{\text{goal}}$; it is ∞ otherwise

Discrete optimal planning

$$\begin{aligned} \underset{\pi_k=(u_0,\ldots,u_{k-1})}{\text{minimize}} \quad & L(\pi_k) \\ \text{subject to} \quad & q_{k+1} = f(q_k, u_k) \\ & q_0 = q_{\text{init}} \\ & q_k = q_{\text{goal}} \\ & q_k \in \mathcal{C}_{\text{free}} \end{aligned}$$



- $L(\pi_k) = \infty$ means an infeasible solution
- $L(\pi_k) < \infty$ means a feasible solution with the cost $L(\pi_k)$

Revision: discrete optimal control

- Optimal control for a discrete-time (and a finite horizon)
- initial state is x_i, goal state x_n may be given (or not)

$$\underset{u_i,\ldots,u_{N-1},(x_i),\ldots,x_n}{\text{minimize}} \quad \left(\phi(x_n, N) + \sum_{k=i}^{N-1} L_k(x_k, u_k)\right)$$

$$\text{subject to} \qquad \begin{gathered} x_{k+1} = f_k(x_k, u_k) \\ u_{lb} \leq u_k \leq u_{ub} \\ x_{lb} \leq x_k \leq x_{ub} \end{gathered}$$

Discrete optimal control (generally)

$$\underset{x \in \mathbf{R}^{n(N-i)}, u \in R^{m(N-i)}}{\text{minimize}} \qquad J(x, u)$$

$$\text{subject to} \qquad \begin{gathered} g(x, u) = 0 \\ h(x, u) \leq 0 \end{gathered}$$

equations by Z. Hurak: Discrete-time optimal control — direct approach (lectures notes of ORR)

Revision

- Formulations of optimal planning and control are same
- Both can find path/trajectory from start to goal
- What is the practical difference?

Motion planning

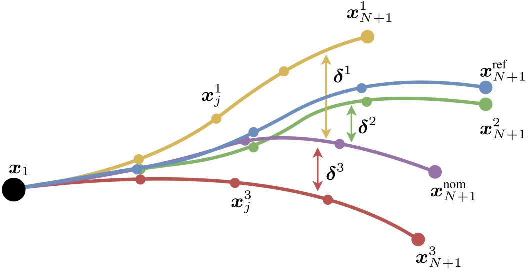
- Suitable for making long-term and complex plans
- Solution is (generally) achieved by searching $\mathcal{C}$-space
- Difficult to react on changes (robot control error, dynamic obstacles) $\rightarrow$ replanning
- Replanning requires to solve the problem from scratch $\rightarrow$ slow

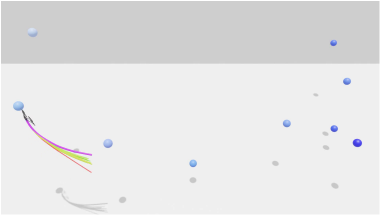
Control

- Achieved via mathematical optimization
- Difficult to find first (feasible) solution in large search space $\rightarrow$ needs a good initialization
- In robotics, usually used for reference tracking

Does not make sense to solve motion plan by control-theory methods
Does not make sense to control via planning!

- Control on a finite prediction horizon
- Generate several trajectories (rollouts) using foward motion model
- Evaluate each rollout by a cost functions
- Compute new nominal trajectory as a weighted sum of the rollouts
- Control robot using the first step of the nominal trajectory, and repeat





MPPI: principle

- Compute K rollouts from the initial state $\mathbf{x}_0$

$$\mathbf{x}^k = (\mathbf{x}_0^k, \ldots, \mathbf{x}_j^k, \ldots, \mathbf{x}_{N-1}^k, \mathbf{x}_N^k)$$
$$\mathbf{u}^k = (\mathbf{u}_0^k, \ldots, \mathbf{u}_j^k, \ldots, \mathbf{u}_{N-1}^k)$$
$$\delta^k = (\delta\mathbf{u}_0^k, \ldots, \delta\mathbf{u}_j^k, \ldots, \delta\mathbf{u}_{N-1}^k)$$
$$\mathbf{x}_{j+1}^k = \mathbf{x}_j^k + \mathbf{f}_{\mathrm{RK4}}(\mathbf{x}_j^k, \mathbf{u}_j^k, \Delta t)$$

$$\delta\mathbf{u}_j^k \in \mathcal{N}(0, \Sigma)$$
$$\mathbf{u}_j^k = \mathbf{u}_j^{\mathrm{nom}} + \delta\mathbf{u}_j^k$$
$$k = 1, \ldots, K$$
$$j = 0, \ldots, N-1$$

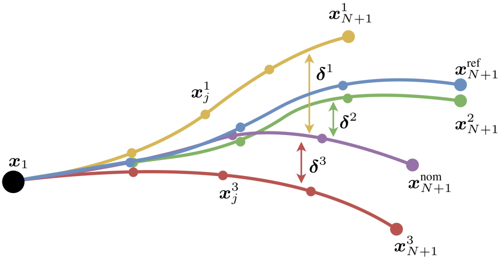
- Integration of forward motion model (e.g. using 4-th order Runge-Kutta method $\mathbf{f}_{\mathrm{RK4}}(\mathbf{x}_j^k, \mathbf{u}_j^k, \Delta t)$)

- Rollout evaluation

$$S_k = \mathrm{ComputeCost}(\mathbf{x}^k, \mathbf{u}^k)$$

- Compute new nominal trajectory

$$\mathbf{u}_j^{\mathrm{nom}} := \mathbf{u}_j^{\mathrm{nom}} + \sum_{k=1}^{K} \omega_k \cdot \delta\mathbf{u}_j^k$$



MPPI weights computation

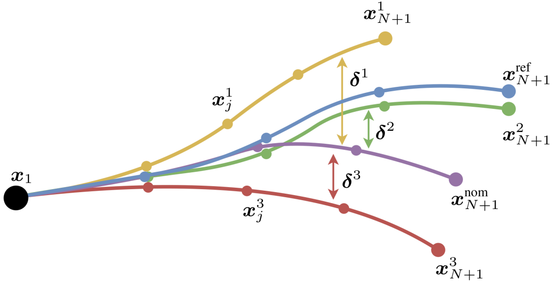
- Each rollout k is evaluated using $S_k = \text{ComputeCost}(\mathbf{x}^k, \mathbf{u}^k)$
- Smaller cost is better
- Weights ω_k

$$\omega_k = \frac{1}{\eta} \exp\left(-\frac{1}{\lambda}\left(S_k - \rho\right)\right)$$

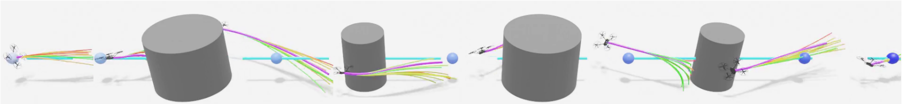
$$\eta = \sum_{k=1}^{K} \exp\left(-\frac{1}{\lambda}\left(S_k - \rho\right)\right)$$

$$\rho = \min\{S_1, \ldots, S_K\}$$

- λ scales contribution of the rollouts to final weights



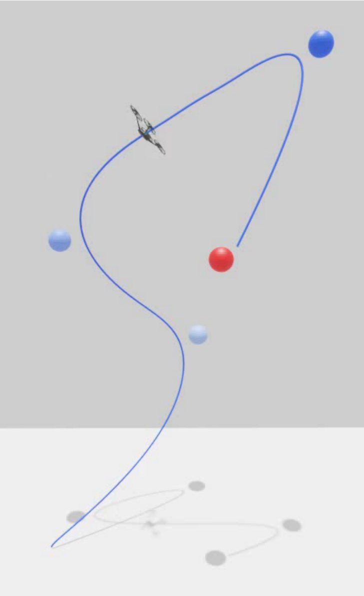
- Fast motion control
- Allows non-linear forward motion model
- Allows arbitrary cost functions (do not need to be convex)
  - useful e.g. to handle obstacles, formation coordination,
  - reference trajectory following
- When λ is low, $\omega = (0, \ldots, 0, 1, 0, \ldots, 0)$, where 1 is for the best rollout
- When λ is hight, $\omega = (\frac{1}{K}, \ldots, \frac{1}{K})$



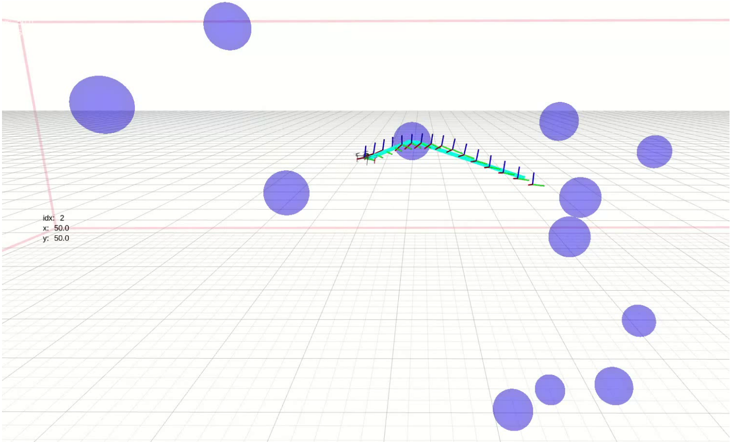
Motion planning + MPPI

- Motion planning provides reference trajectory $\mathbf{u}^{\text{nom}}$
- MPPI updates control inputs based on actual situation

MPPI examples



MPPI examples



MPPI examples

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