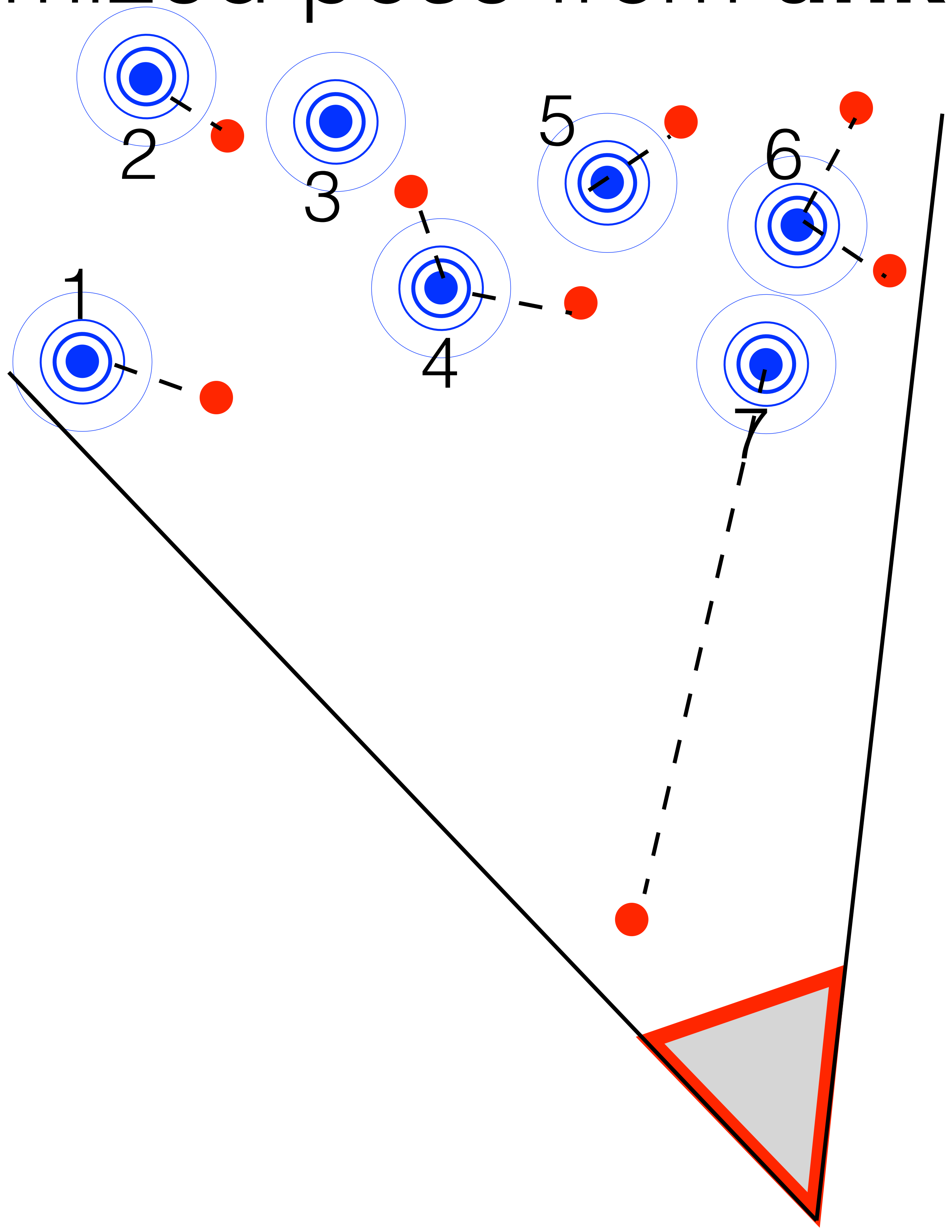
Robust regression: from ICP to RANSAC

Karel Zimmermann

ICP: Gradiently optimized pose from **unknown** correspondences with **outlier** rejection



MAP: $\mathbf{R}^\star, \mathbf{t}^\star, c(i)^\star, \arg\max_{\mathbf{R},\mathbf{t},c(i)} \prod_i K \cdot \exp\Big(-\|\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_{c(i)}\|^2 \Big)$

1. $c(i)^\star = \arg\min_{c(i)} \sum_i \left\| \mathbf{R}^\star \mathbf{p}_i + \mathbf{t}^\star - \mathbf{q}_{c(i)} \right\|^2$ … Nearest neighbour problem

2. $\mathbf{R}^\star, \mathbf{t}^\star, = \arg\min_{\mathbf{R}\in SO(3),\mathbf{t}} \sum_i \left\| \mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_{c(i)^\star} \right\|^2$ … Known absolute orientation problem

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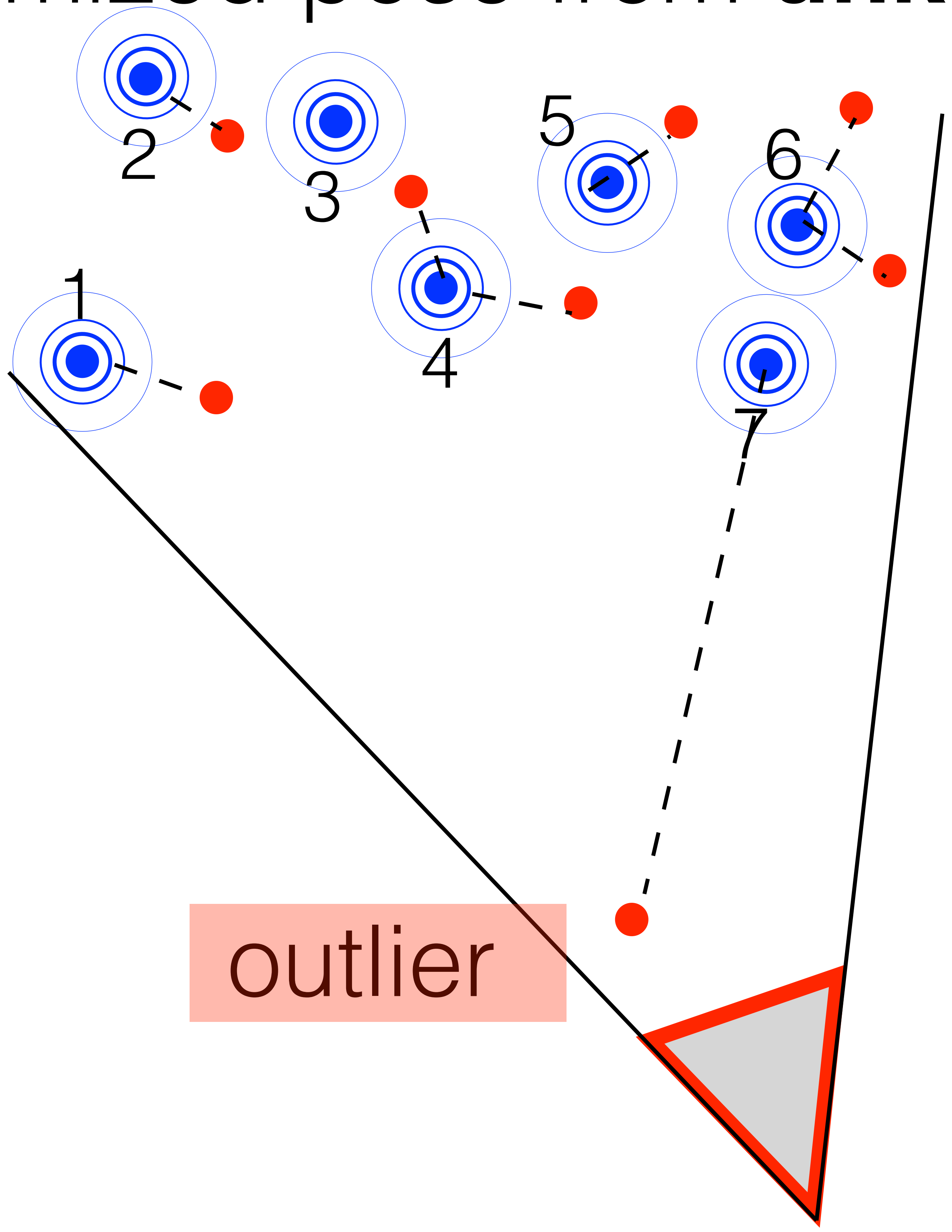

risk minimizing 7-class classification problem

scan	i	1	2	3	4	5	6	7	8
map	c(i)	1	2	4	4	5	6	6	7

MAP: $$\mathbf{R}^\star, \mathbf{t}^\star, c(i)^\star, \arg\max_{\mathbf{R},\mathbf{t},c(i)} \prod_i K \cdot \exp\left(-\|\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_{c(i)}\|^2 \right)$$

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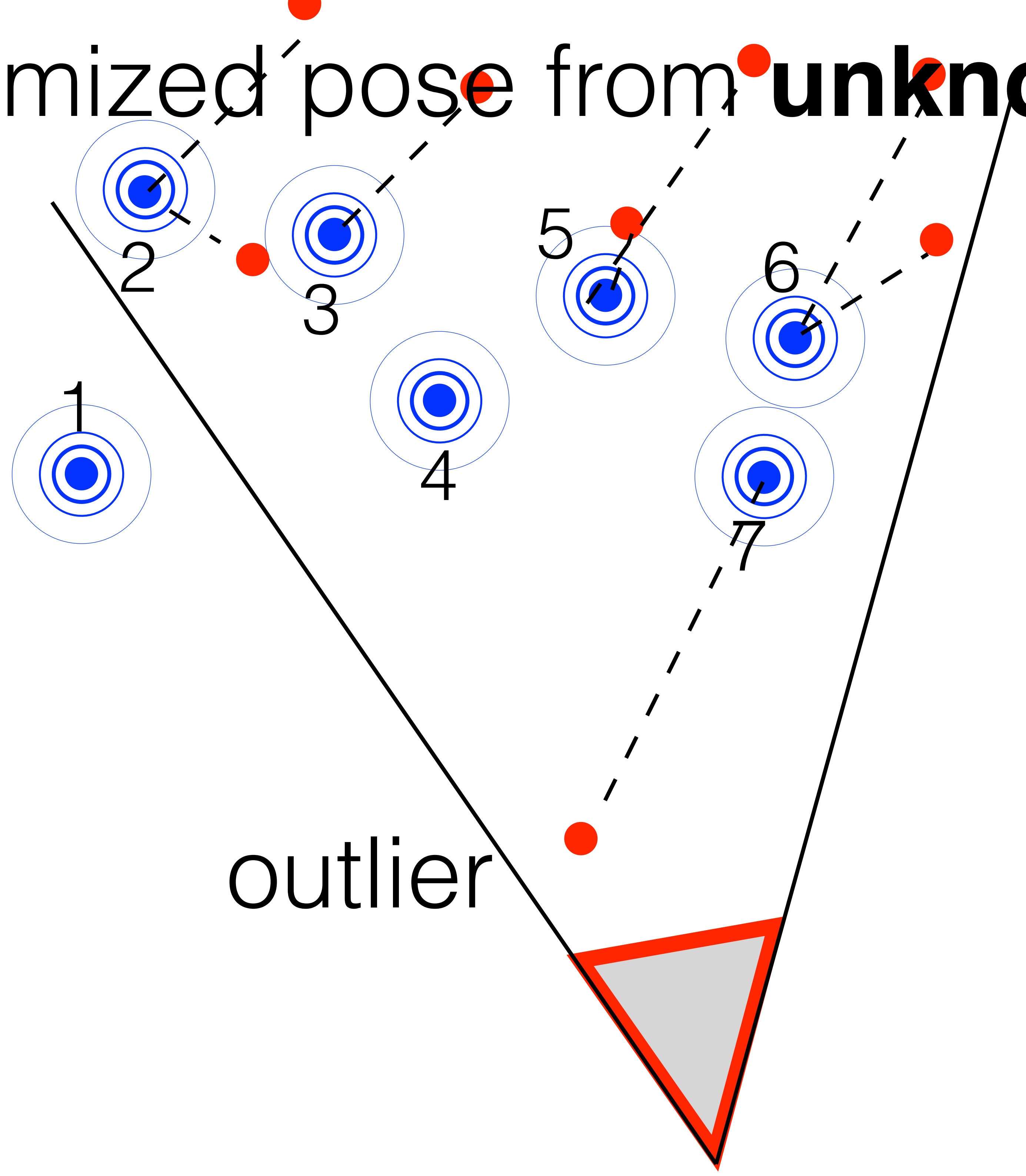

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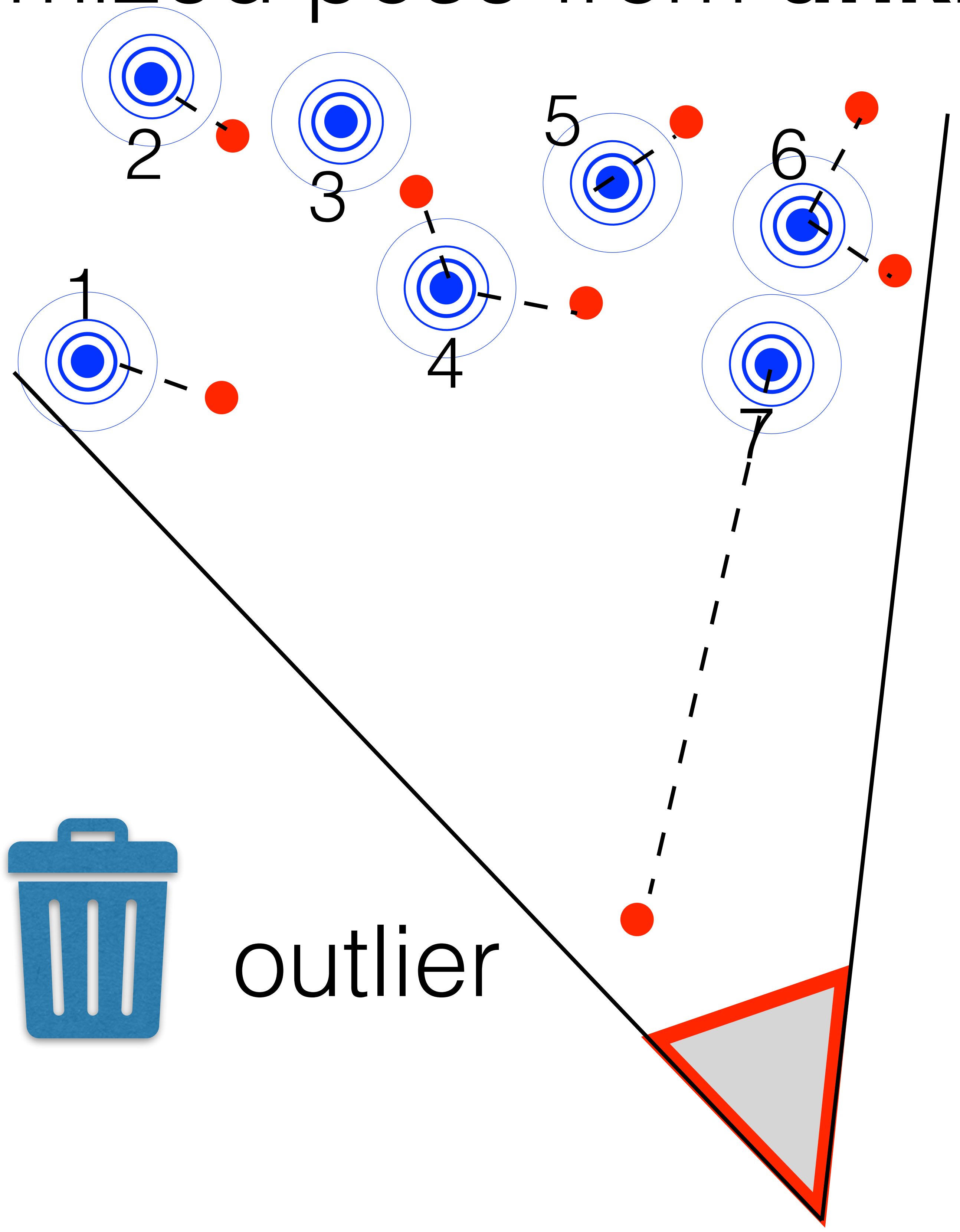

risk minimizing **8**-class classification problem

scan	**i**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
map	**c(i)**	1	2	4	4	5	6	6	7

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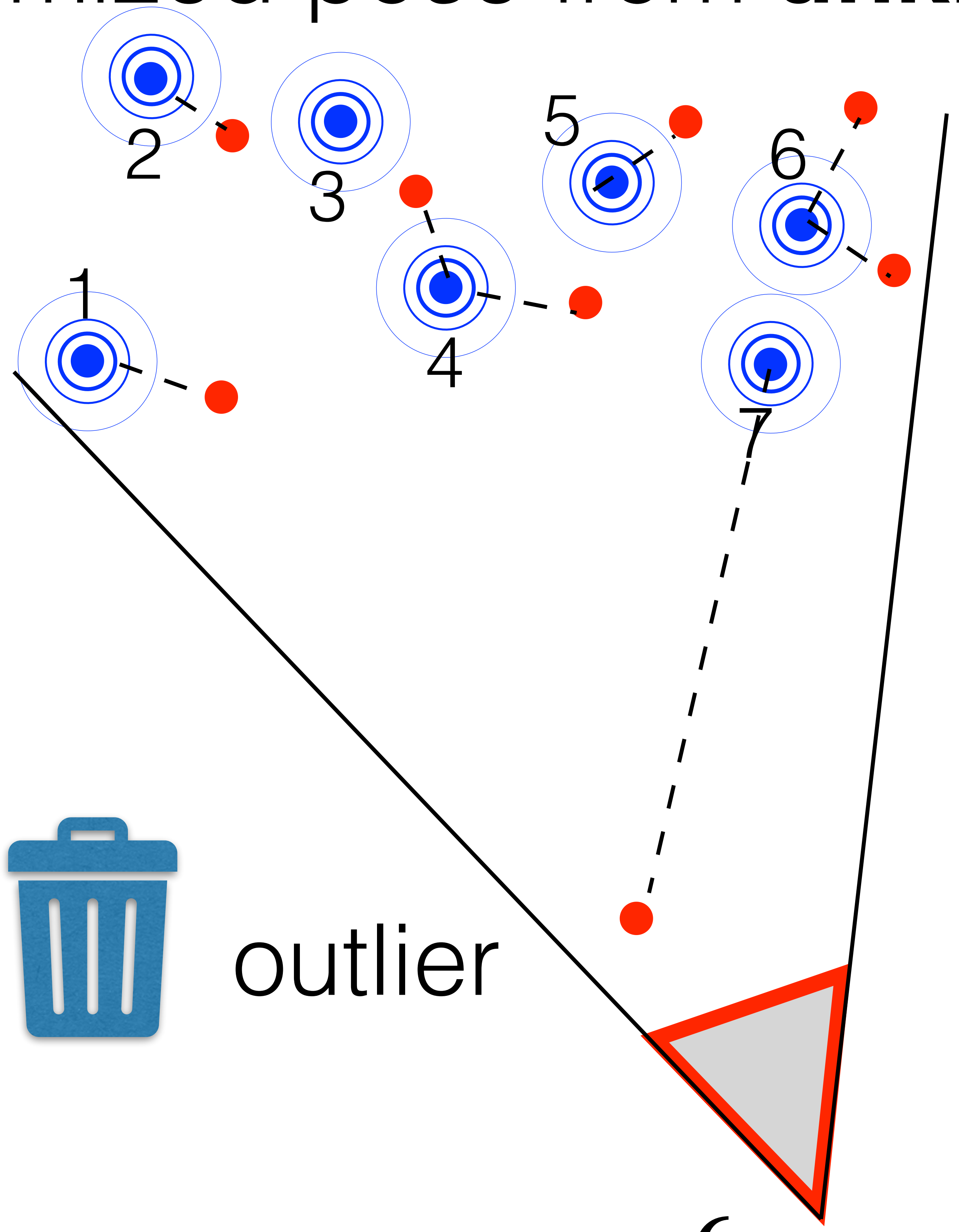

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MAP: $\mathbf{R}^\star, \mathbf{t}^\star, c(i)^\star, \arg\max_{\mathbf{R},\mathbf{t},j(i)} \prod_i \begin{cases} K \cdot \exp\Big(-\|\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_{c(i)}\|^2 \Big) & c(i) \in [1,N] \\ p_{\text{outlier}} & c(i) = N+1 \end{cases}$

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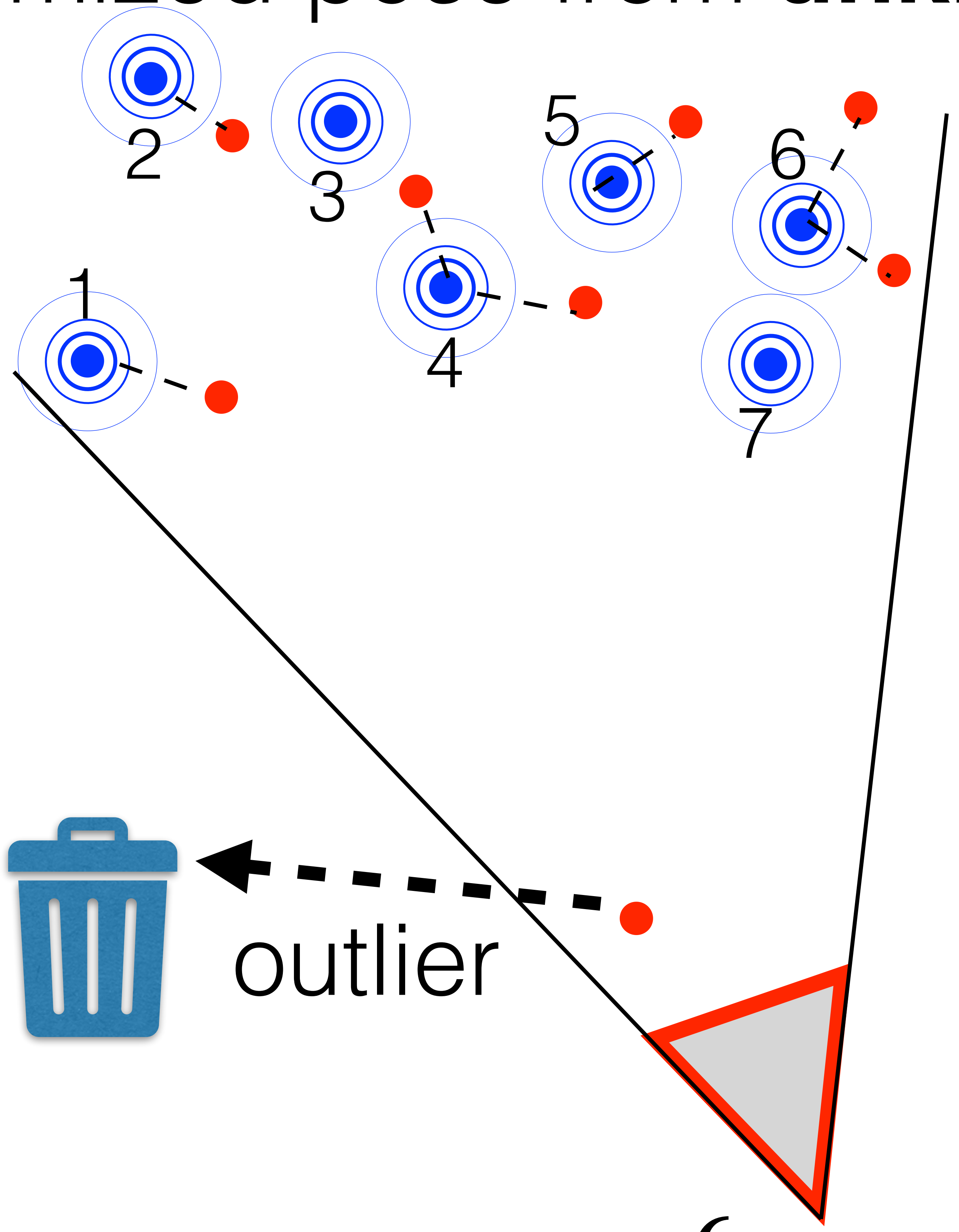

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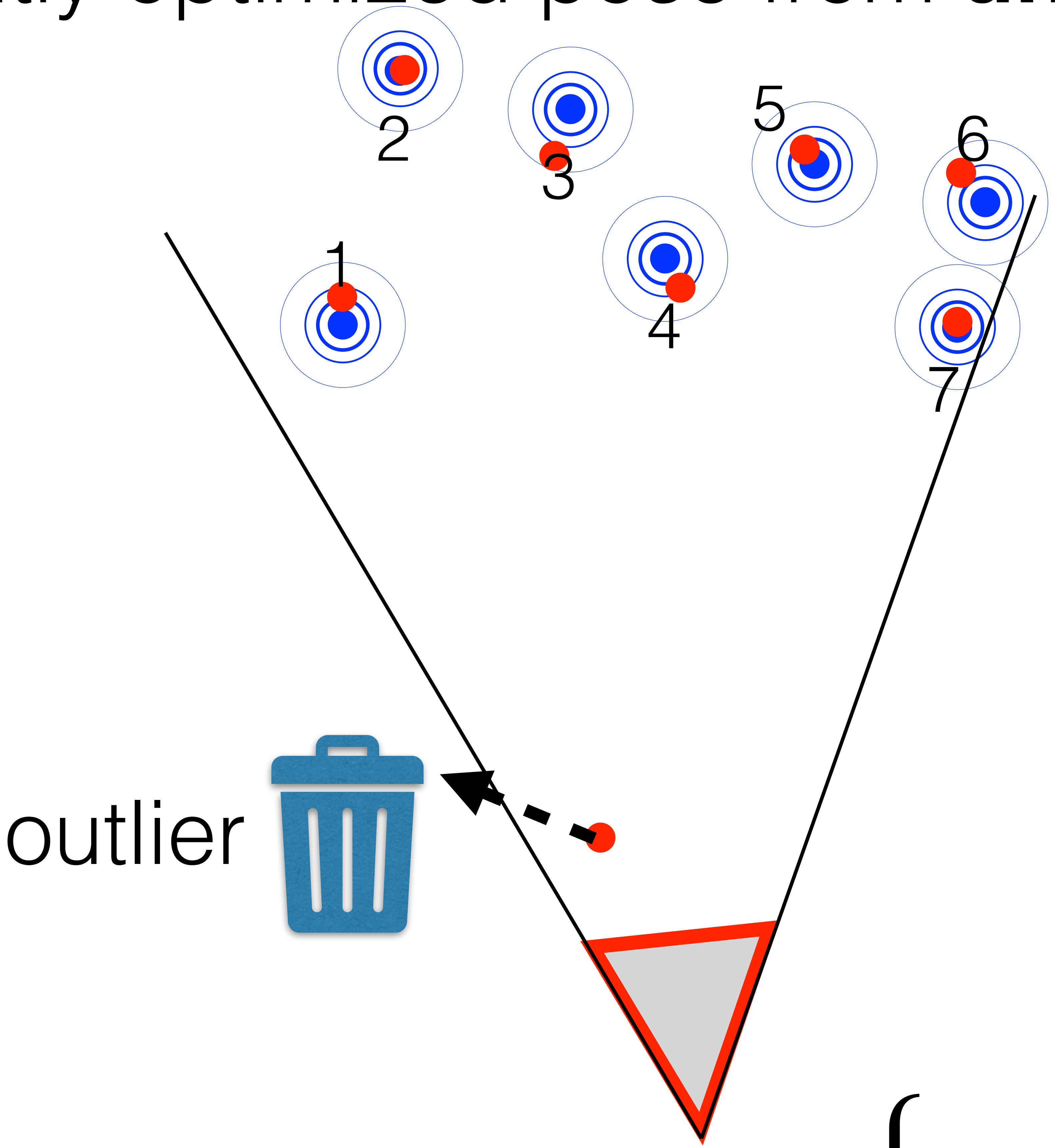

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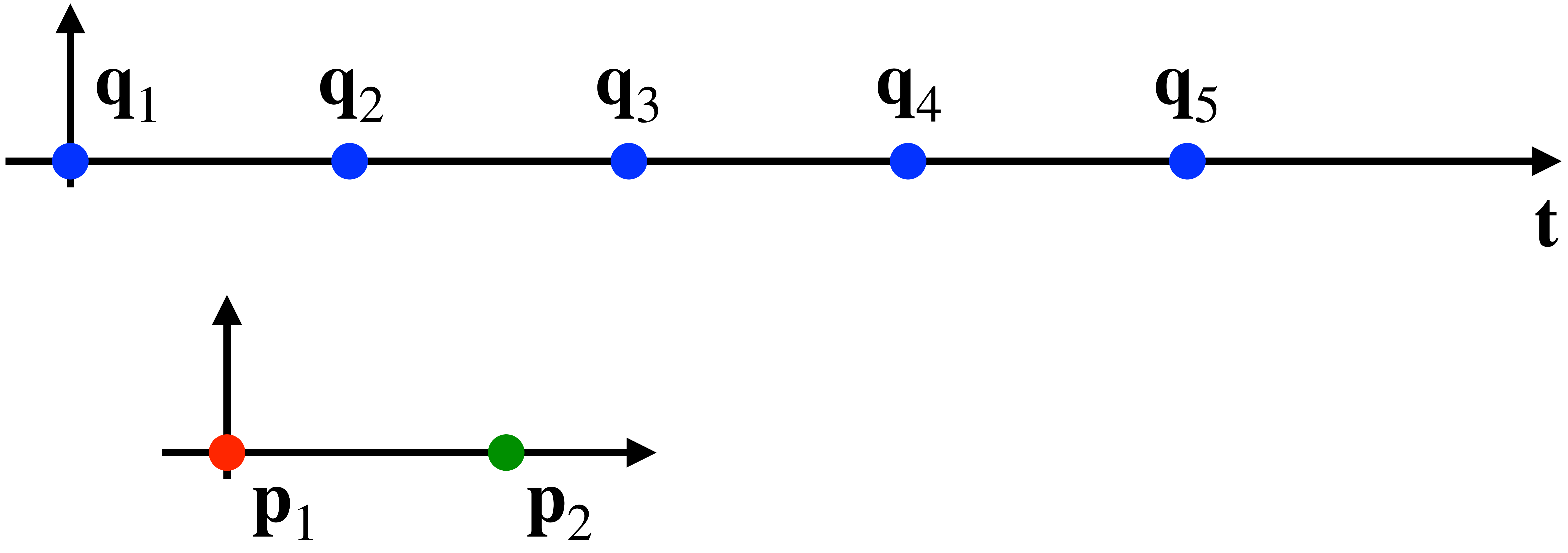
Joint global optimization of pose and correspondences



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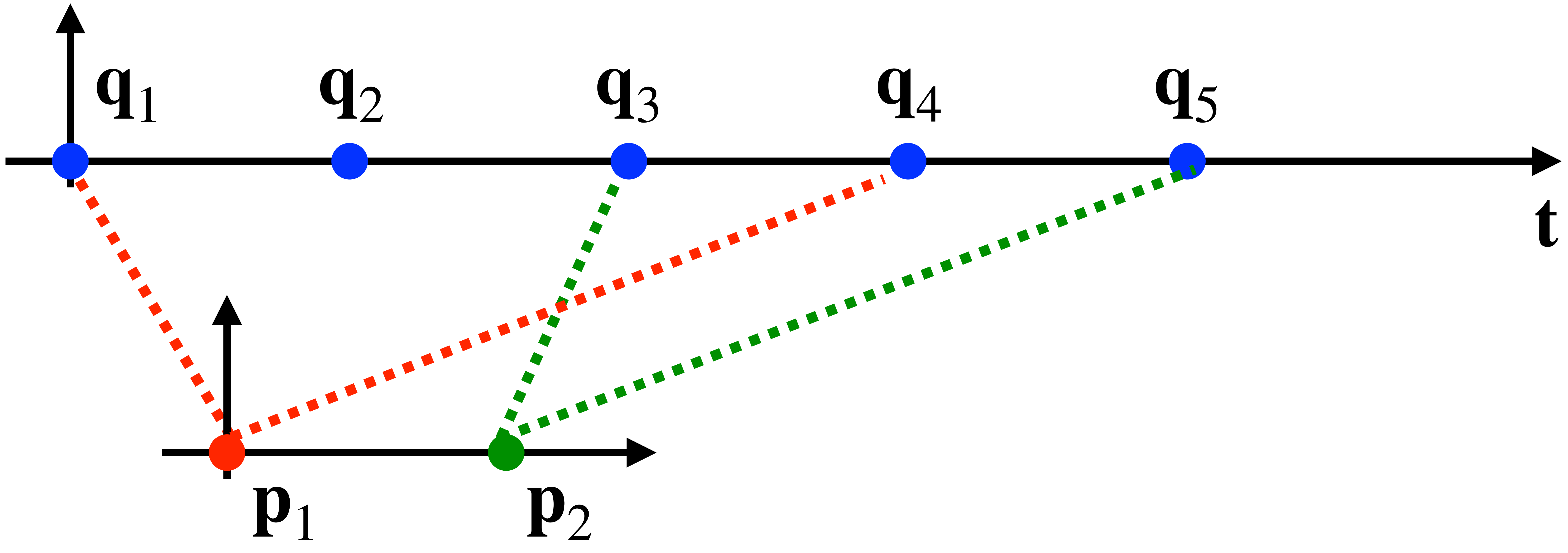

$c(1) \in \{1,4\}$ $c(2) \in \{3,5\}$

Obviously there is one global optimum

$$\mathbf{t}^\star, c(i)^\star = \underset{\mathbf{t}, c(1), c(2)}{\arg\min} \; (\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_{c(1)})^2 + (\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_{c(2)})^2$$

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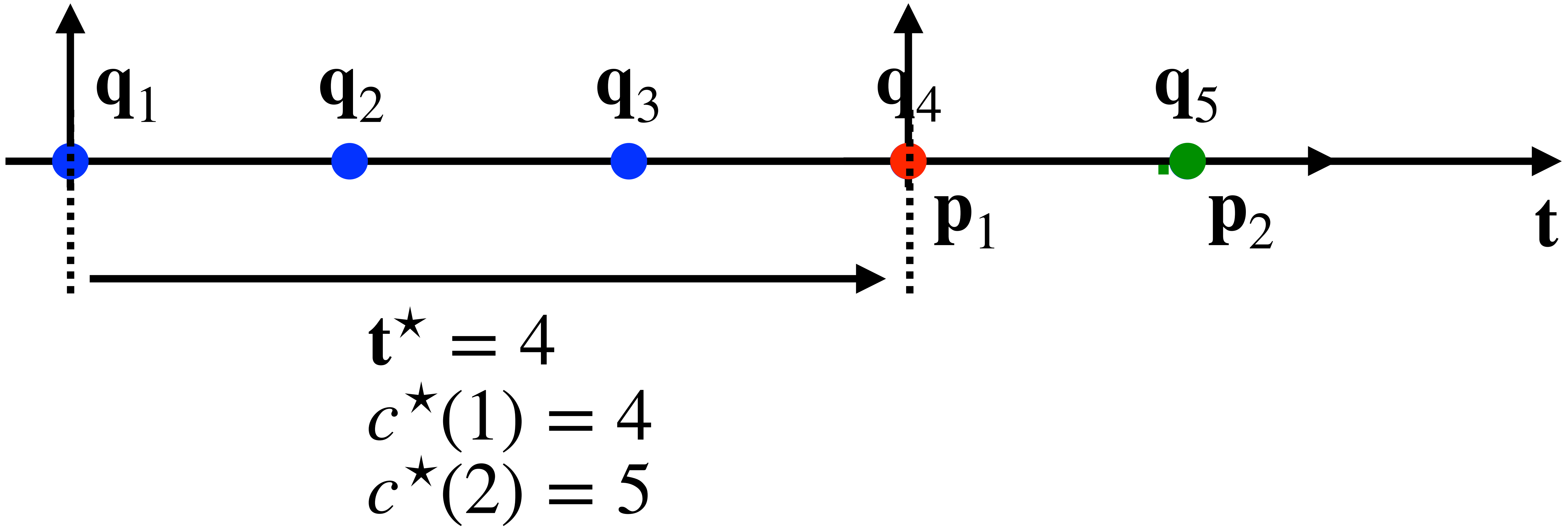

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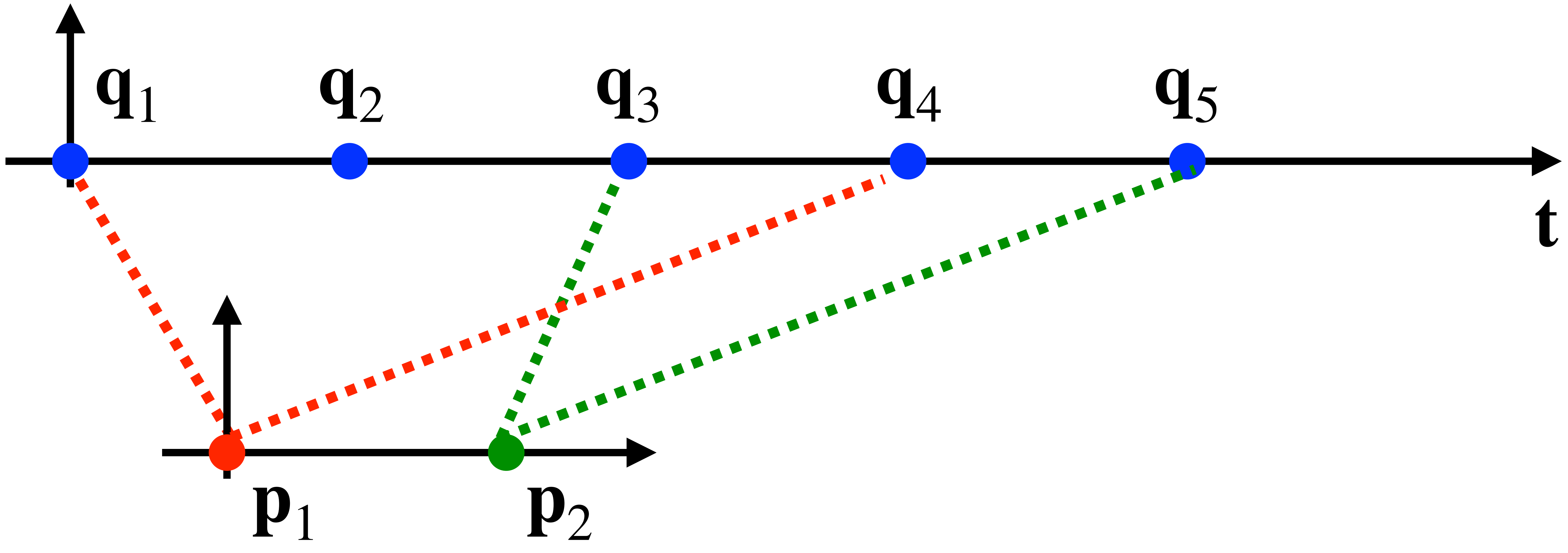

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Issue: Combinatorial optimization over correspondences technically intractable

$$\mathbf{t}^\star, c(i)^\star = \arg\min_{\mathbf{t},c(1),c(2)} \ (\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_{c(1)})^2 + (\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_{c(2)})^2$$

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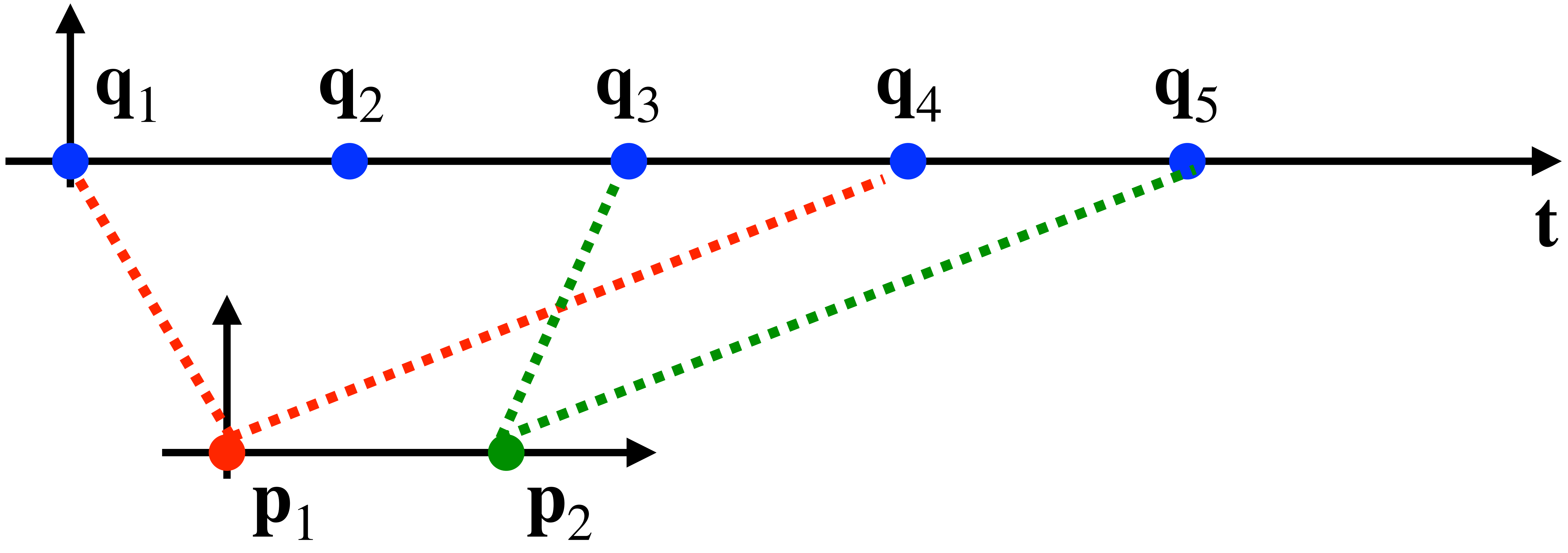

$c(1) \in \{1,4\}$ $\quad$ $c(2) \in \{3,5\}$

Treat each possible correspondence as independent measurement with gaussian n.

$$\mathbf{t}^\star, c(i)^\star = \arg\min_{\mathbf{t}} \boxed{(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_1)^2 + (\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_4)^2} + \boxed{(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_3)^2 + (\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_5)^2}$$

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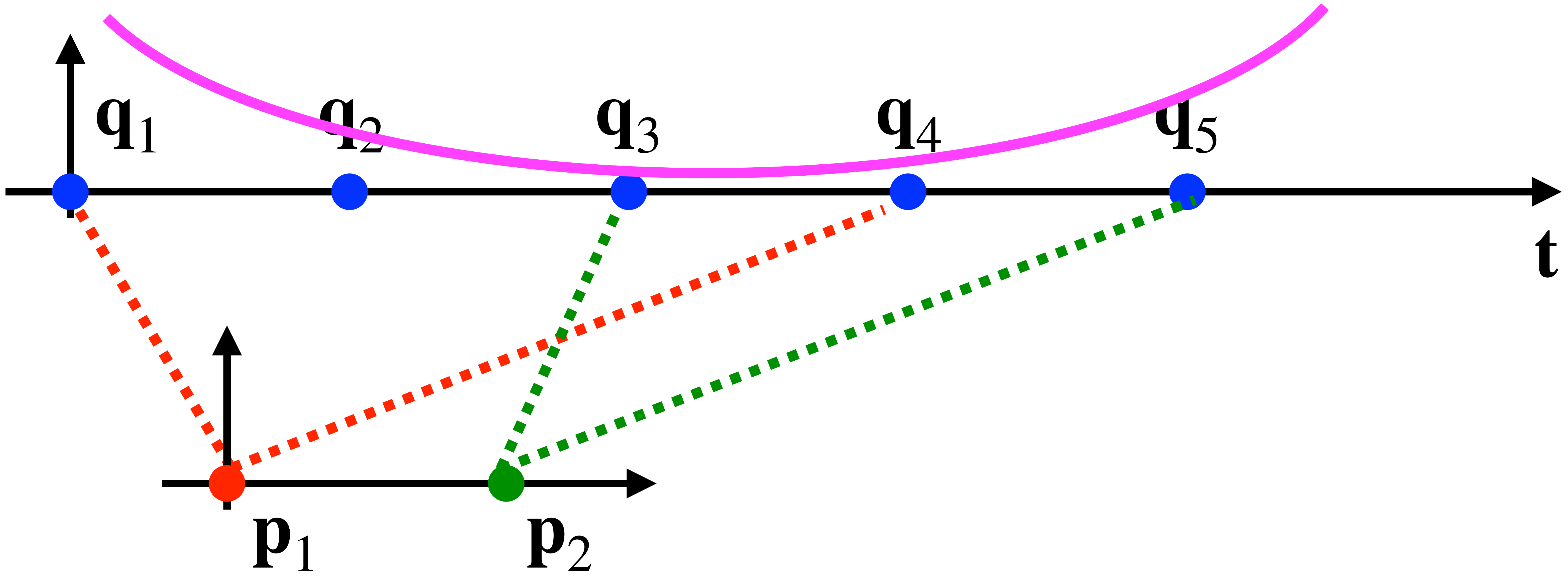

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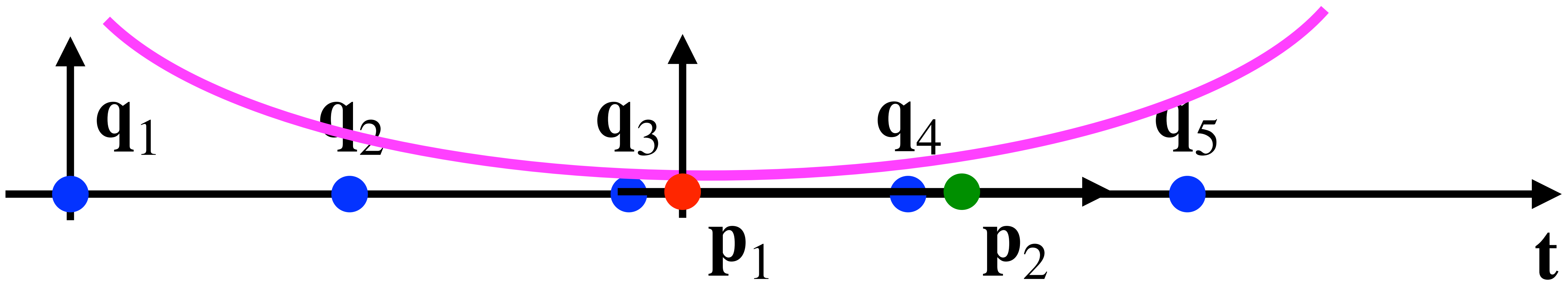

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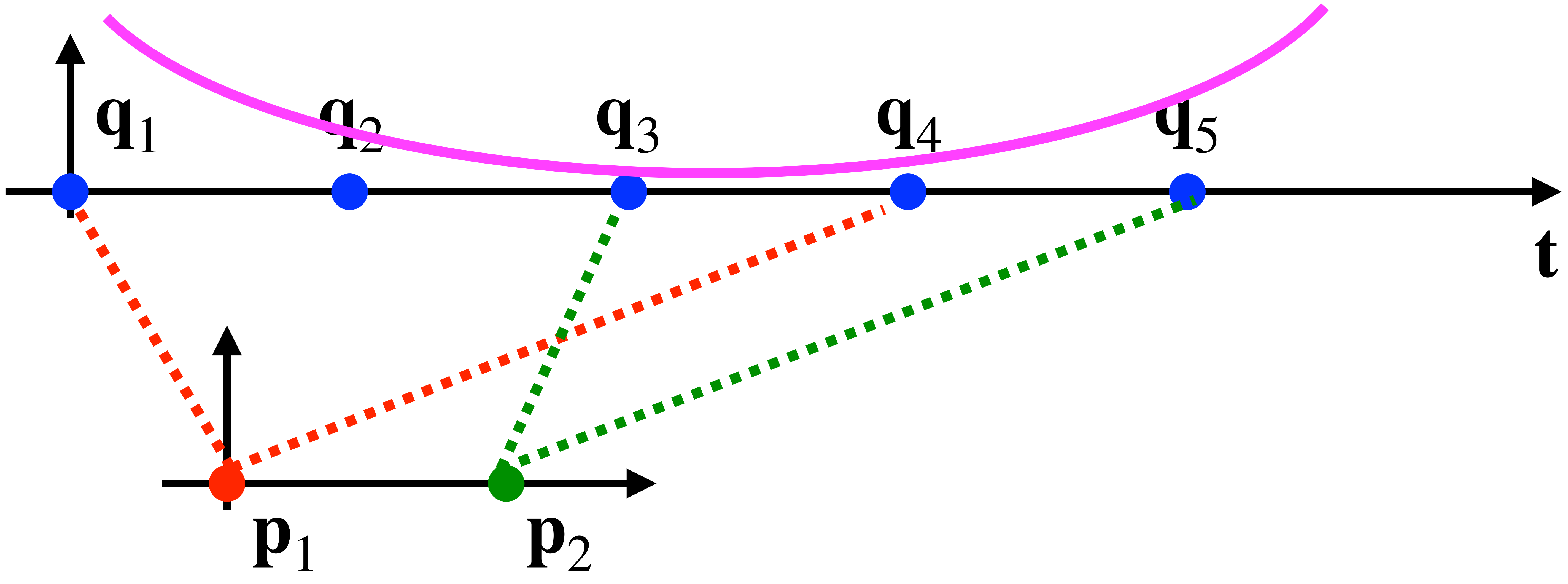

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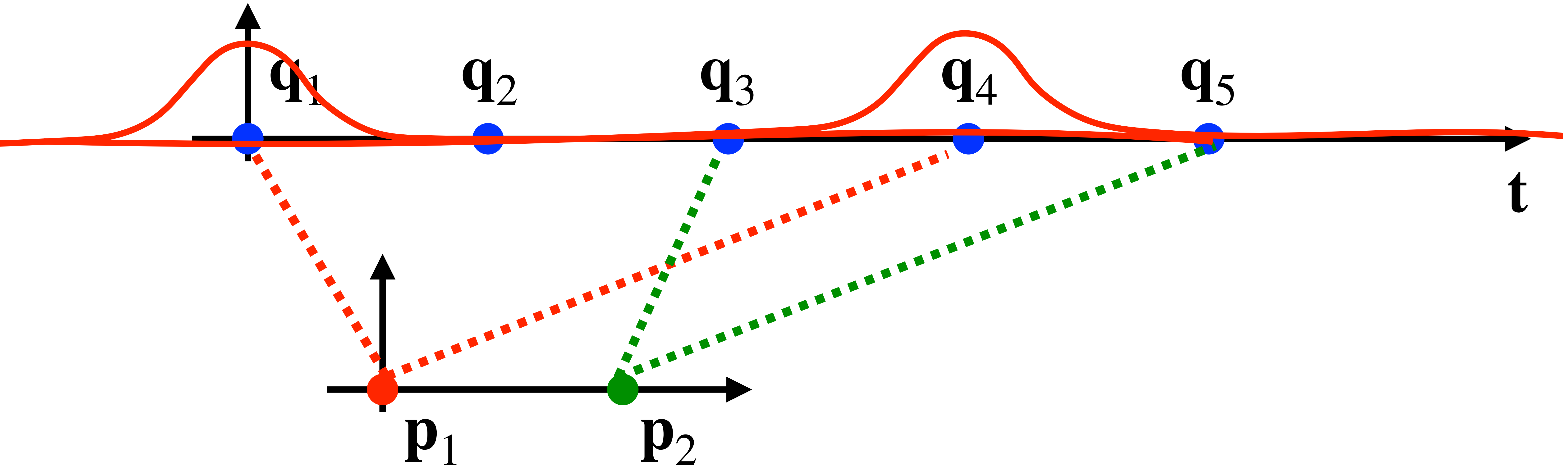

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Issue: Gaussian is too aggressive !

i.e. when $\mathbf{p}_1$ aligned correctly with $\mathbf{q}_4$, penalty from $\mathbf{q}_1$ is huge !!!!

$$\mathbf{t}^\star = \arg\min_{\mathbf{t}} (\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_1)^2 + (\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_4)^2 + (\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_3)^2 + (\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_5)^2$$

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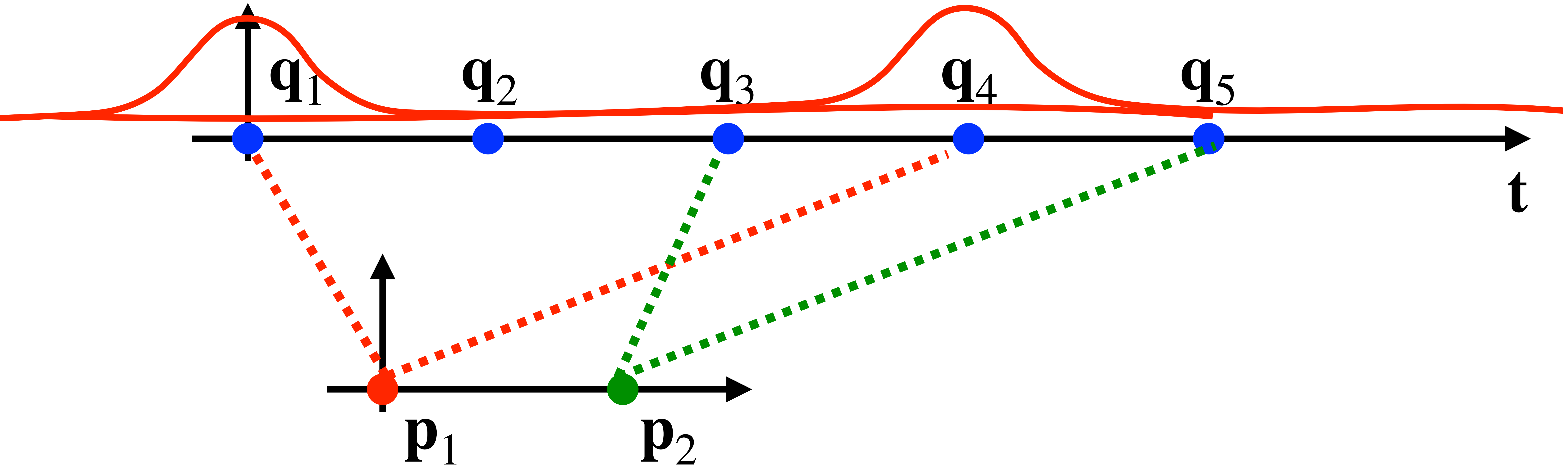

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Remedy: Heavy-tail Gaussian

$$\mathbf{t}^\star = \arg\min_{\mathbf{t}} \; \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_1) + \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_4) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_3) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_5)$$

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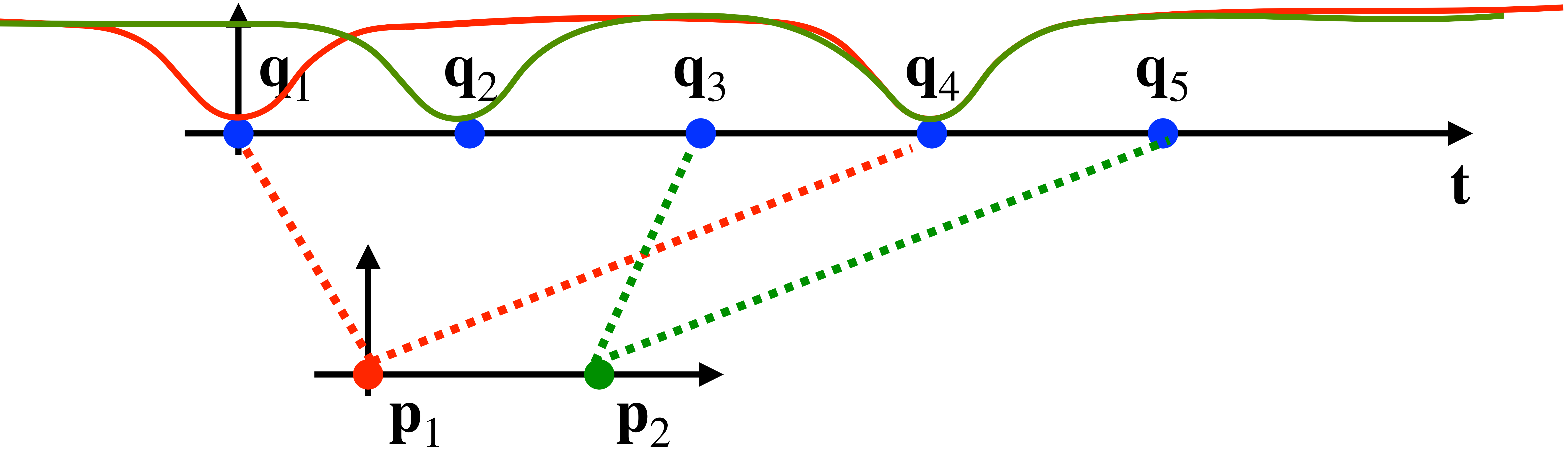

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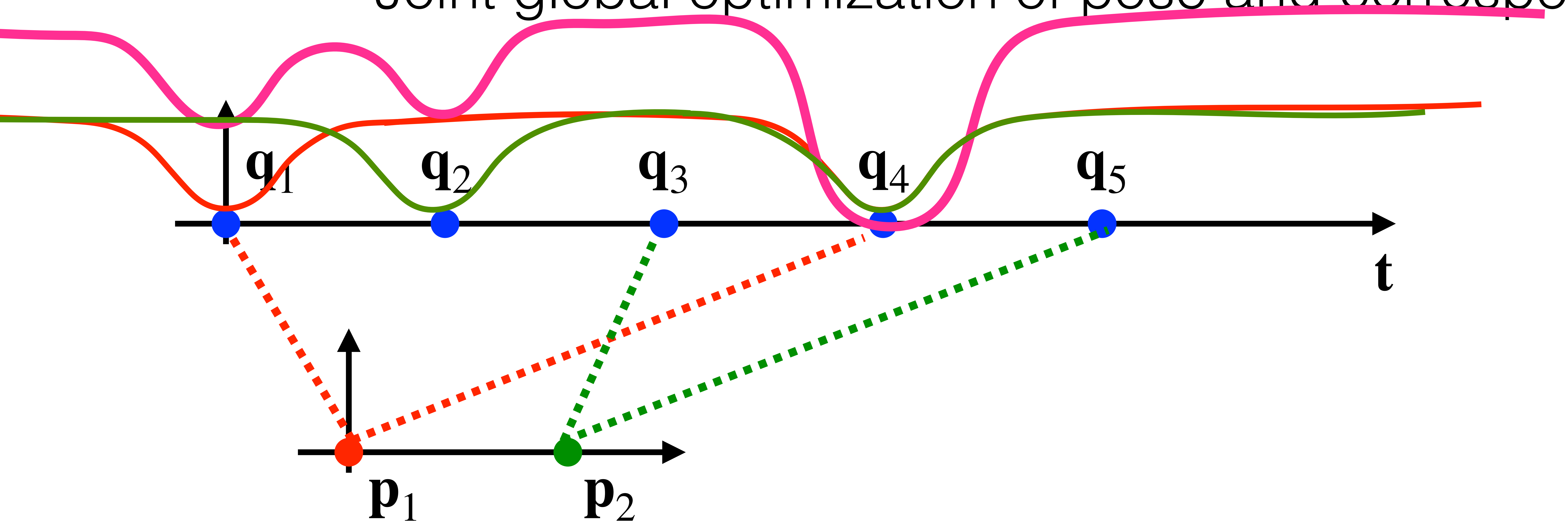

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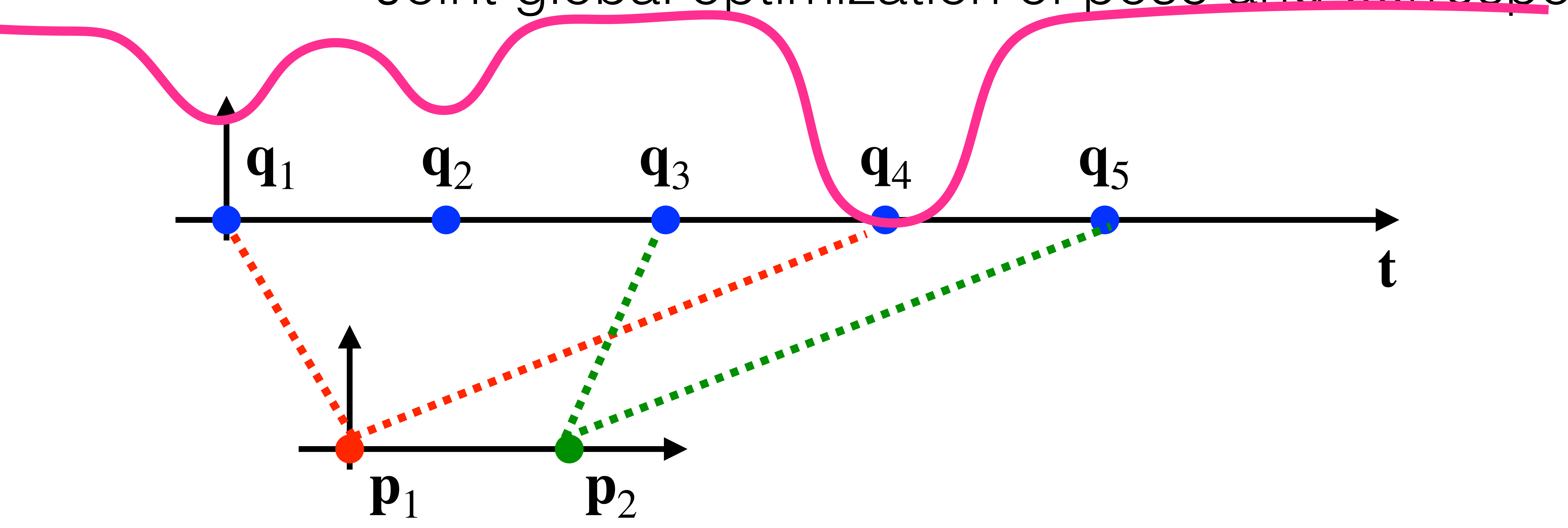


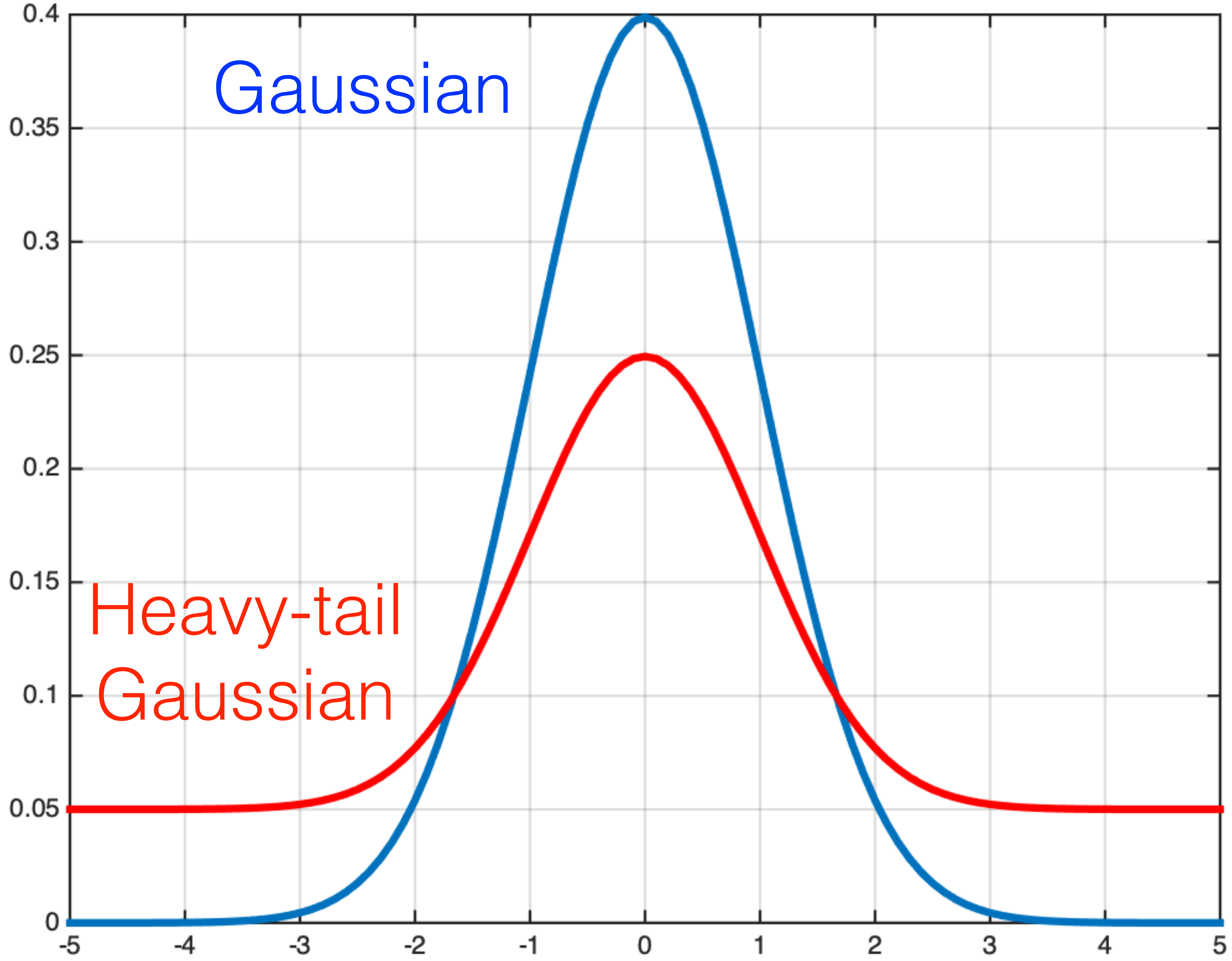
$c(1) \in \{1,4\}$ $c(2) \in \{3,5\}$

Issue: the underlying problem is not optimization-friendly

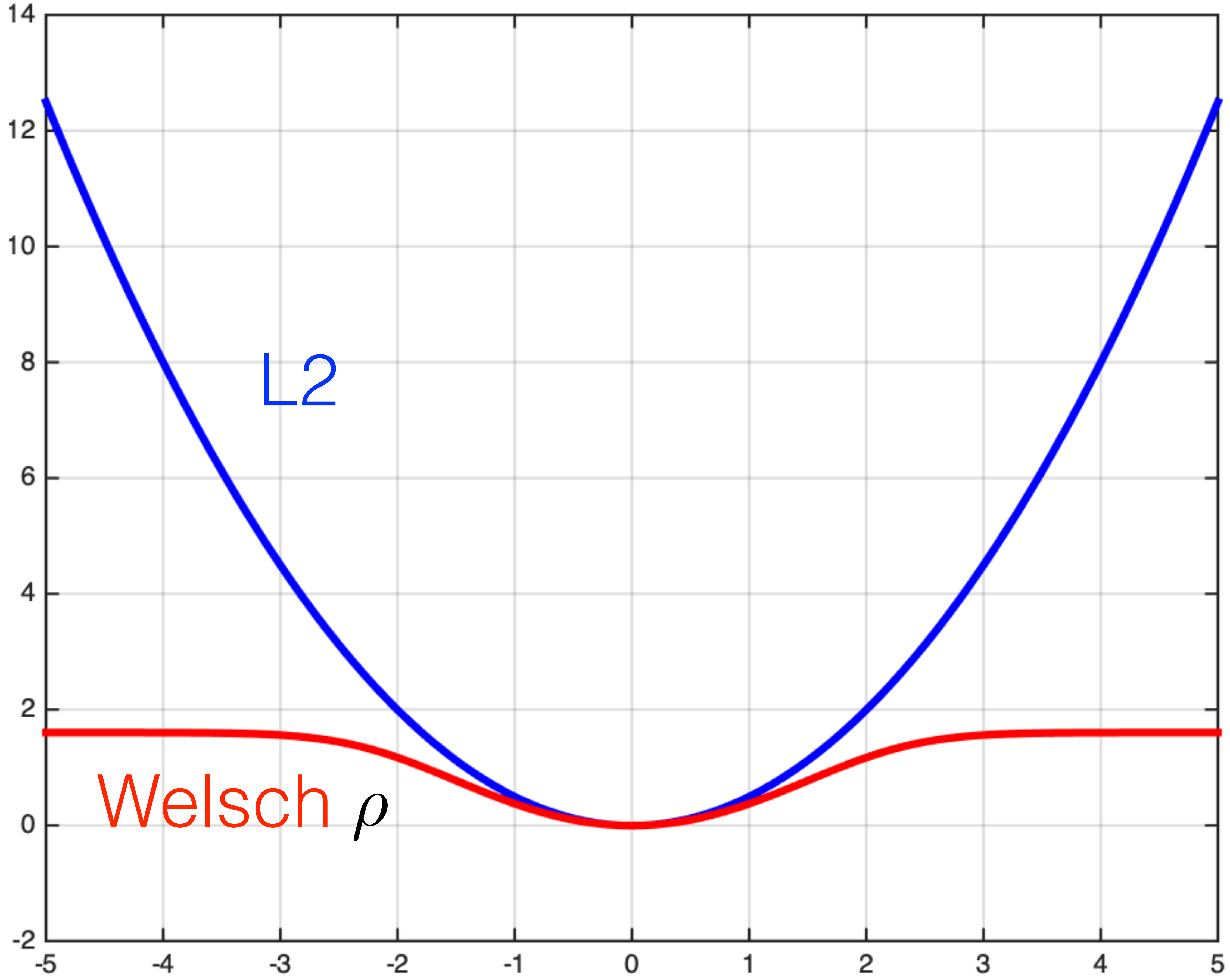
$$\mathbf{t}^\star = \arg\min_{\mathbf{t}} \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_1) + \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_4) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_3) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_5)$$

RANSAC vs robust regression

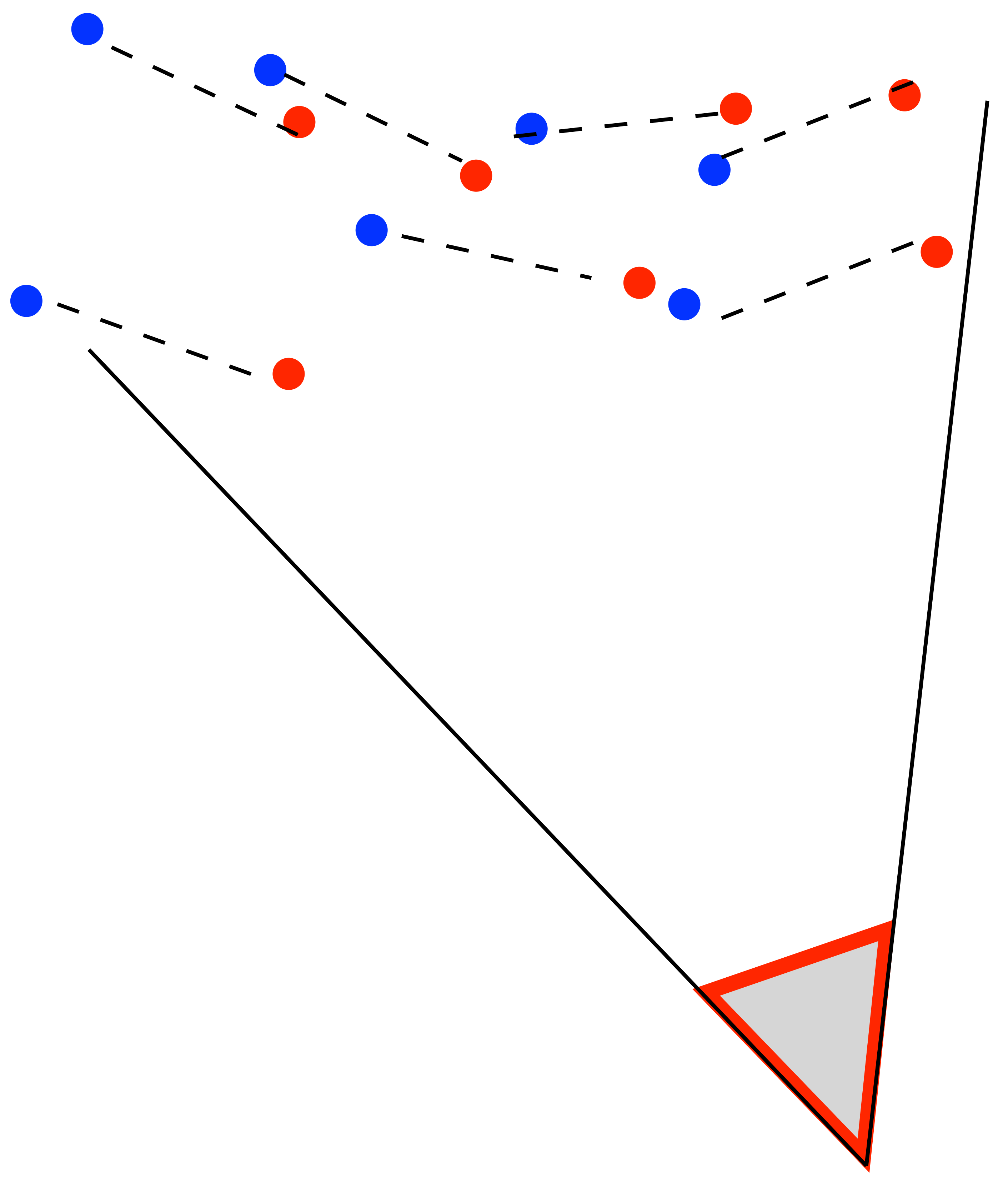
Gaussian vs Heavy-tail gaussian



Corresponding losses



Alignment of two pointclouds

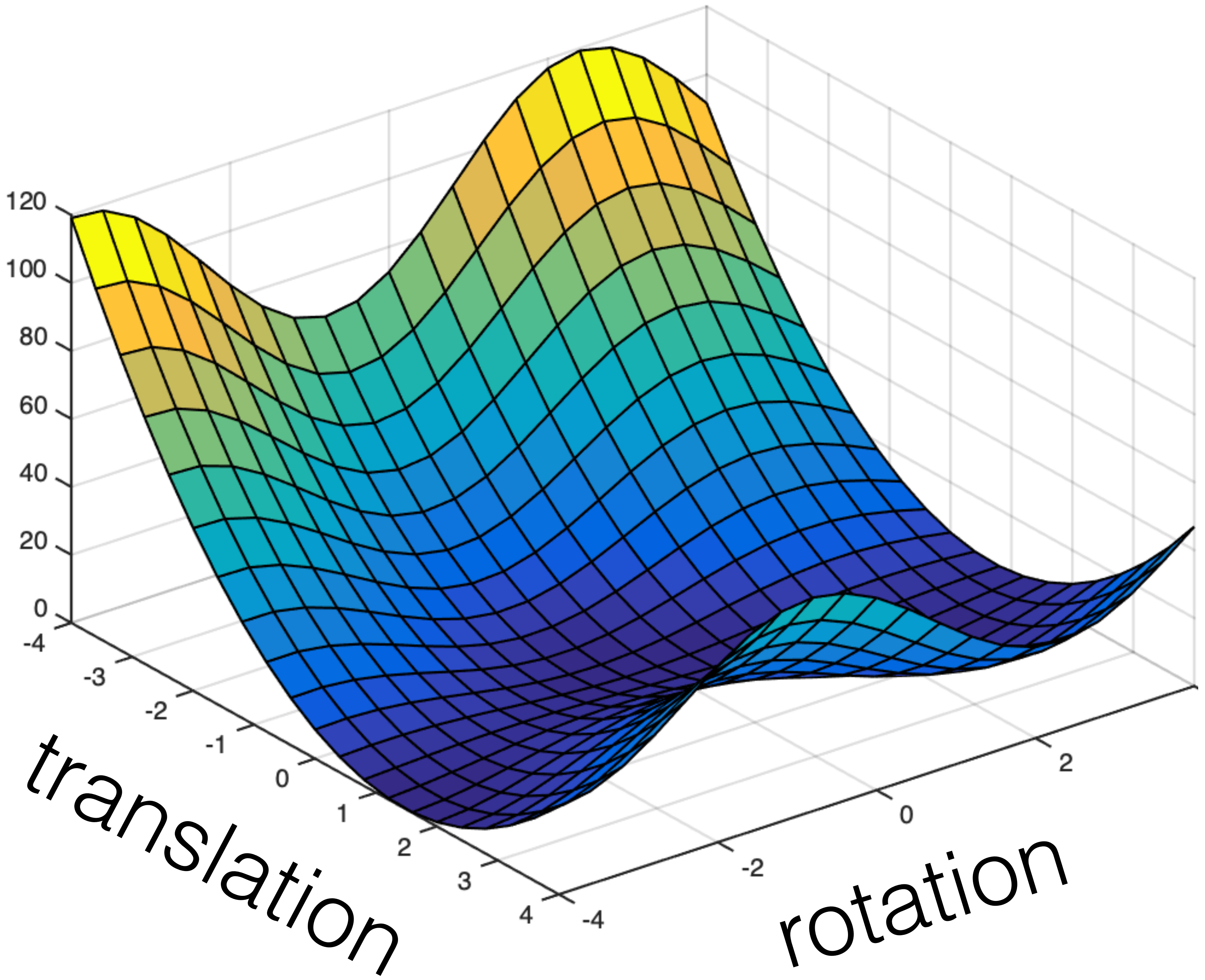


$$\mathbf{R}^*, \mathbf{t}^* = \arg\min_{\mathbf{R}\in SO(3), \mathbf{t}\in\mathcal{R}^3} \sum_i \|\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i\|_2^2$$ ~~**L2 regression**~~

$$\mathbf{R}^*, \mathbf{t}^* = \arg\min_{\mathbf{R}\in SO(3), \mathbf{t}\in\mathcal{R}^3} \sum_i \boldsymbol{\rho}(\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i)$$ **Robust regression**

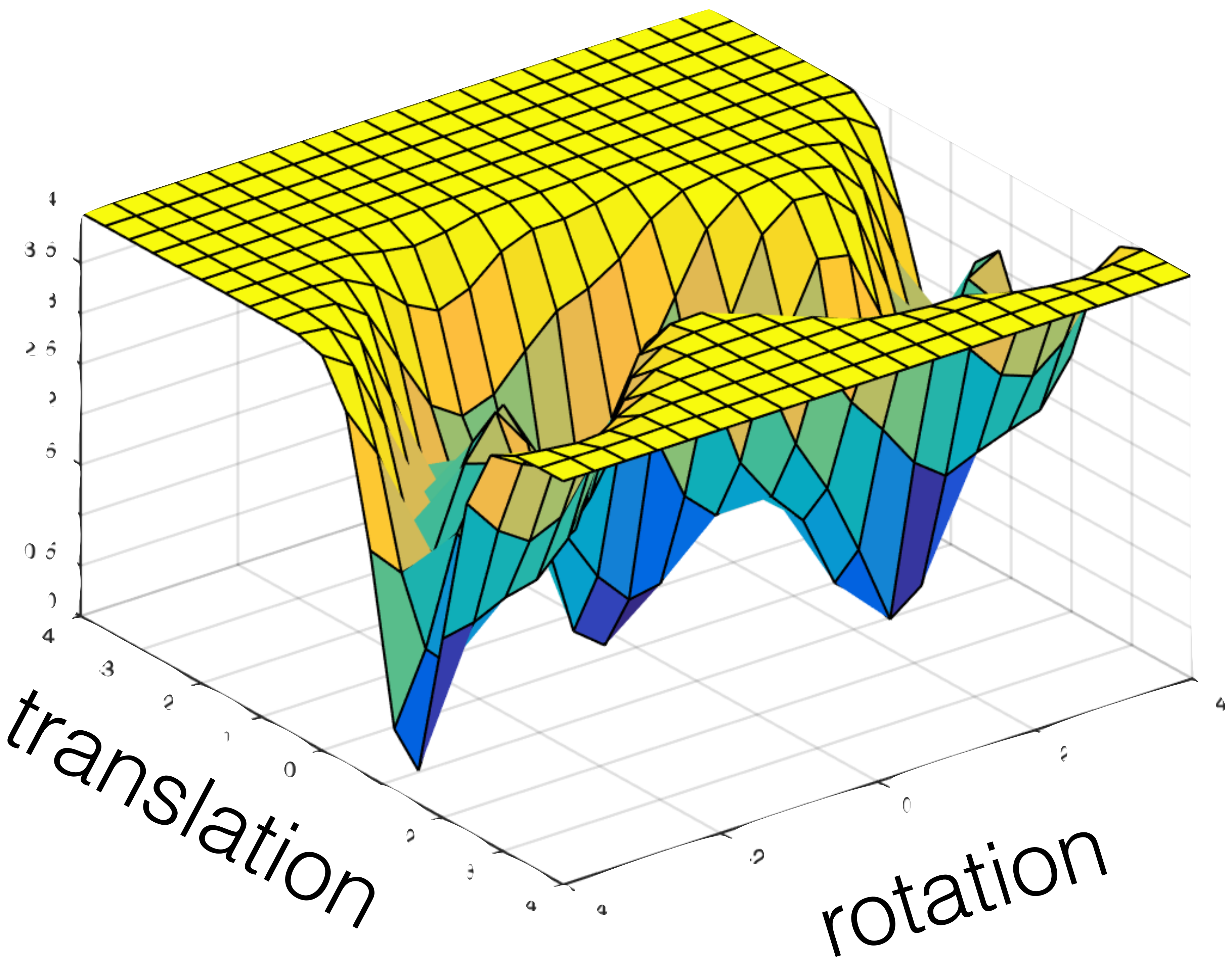
Gradient optimisation of robust loss

L2 landscape



- Convex in translation space
- Non-convex but smooth in SO3

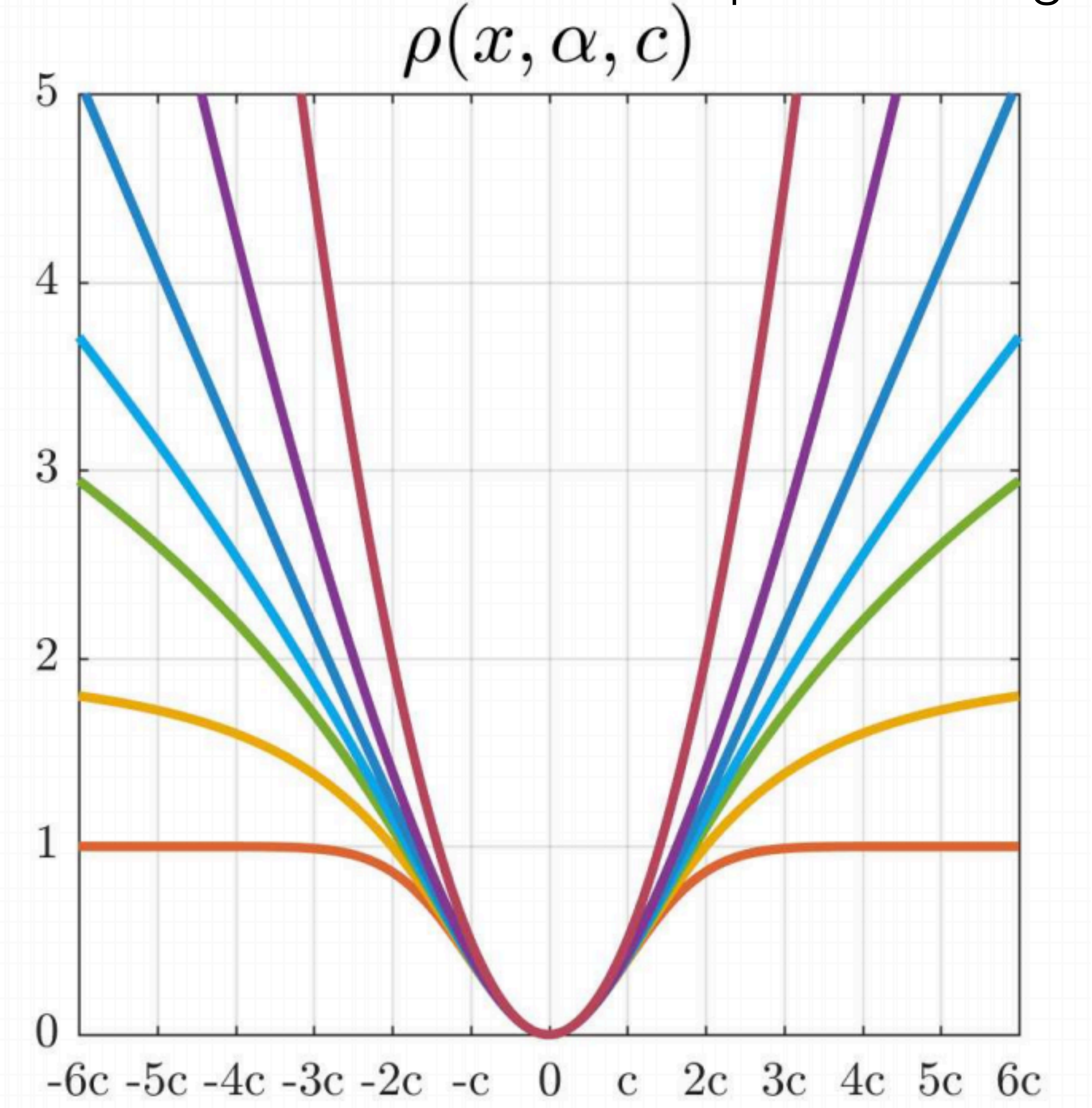
Welsch landscape



- Non-convex: Large narrow plateaus with zero gradient
- Good initialization required

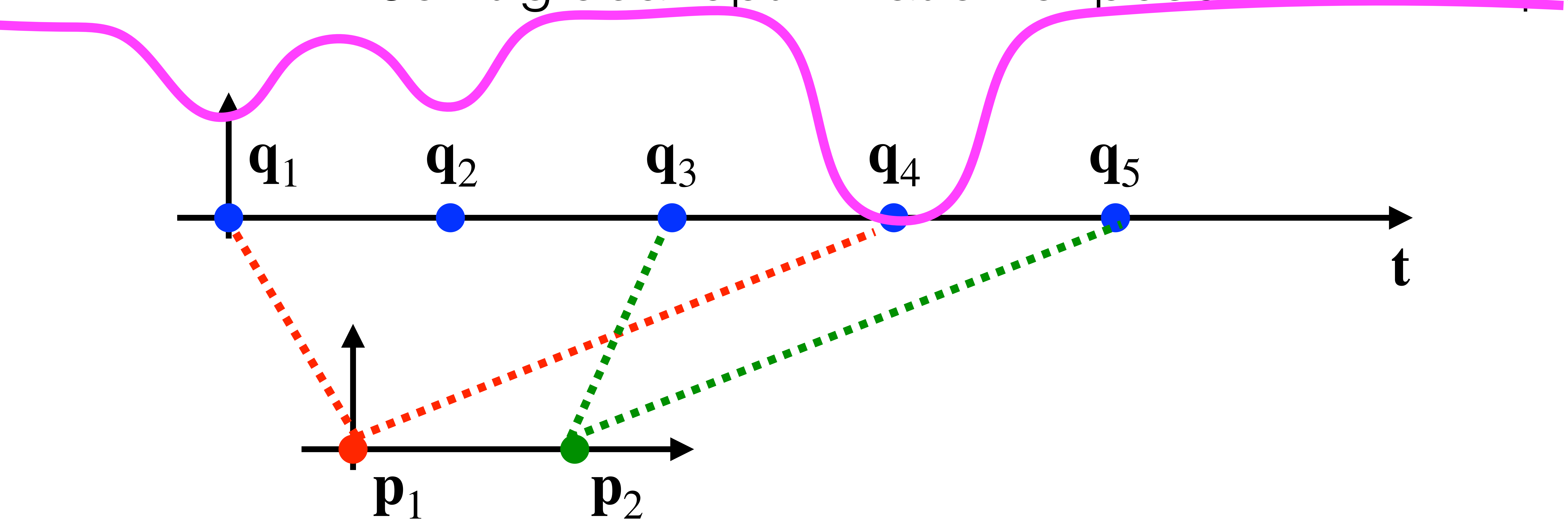
Shape of robust regression functions [Barron CVPR 2019]

https://arxiv.org/abs/1701.03077



$$\rho\left(x, \alpha, c\right) = \frac{|\alpha - 2|}{\alpha} \left(\left(\frac{\left(x/c\right)^2}{|\alpha - 2|} + 1 \right)^{\alpha/2} - 1 \right)$$

Joint global optimization of pose and correspondences



$c(1) \in \{1,4\}$ $c(2) \in \{3,5\}$

Remedy: Smart or close initialization!

$$\mathbf{t}^\star = \arg\min_{\mathbf{t}} \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_1) + \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_4) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_3) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_5)$$

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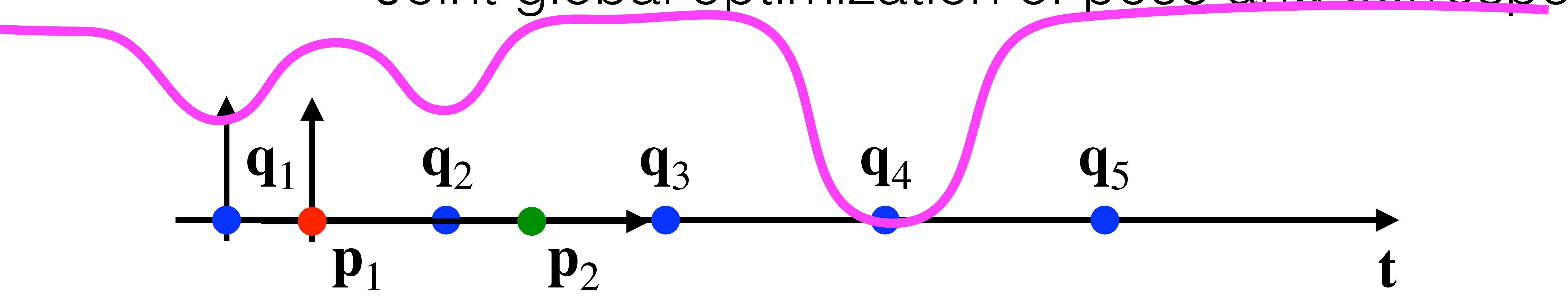

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Idea 0: Randomly initialize initial translation $\mathbf{t}_0$ + gradient optimization

Issue: Basin of attraction small => too many initializations needed

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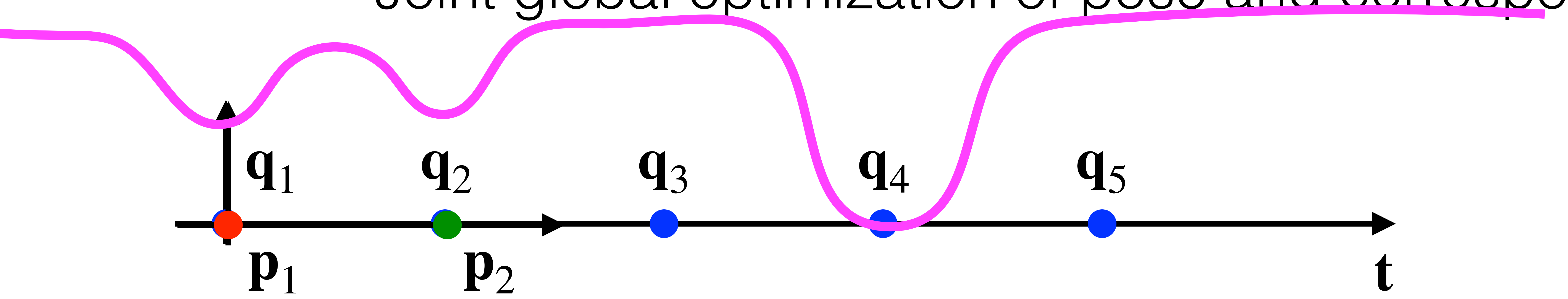

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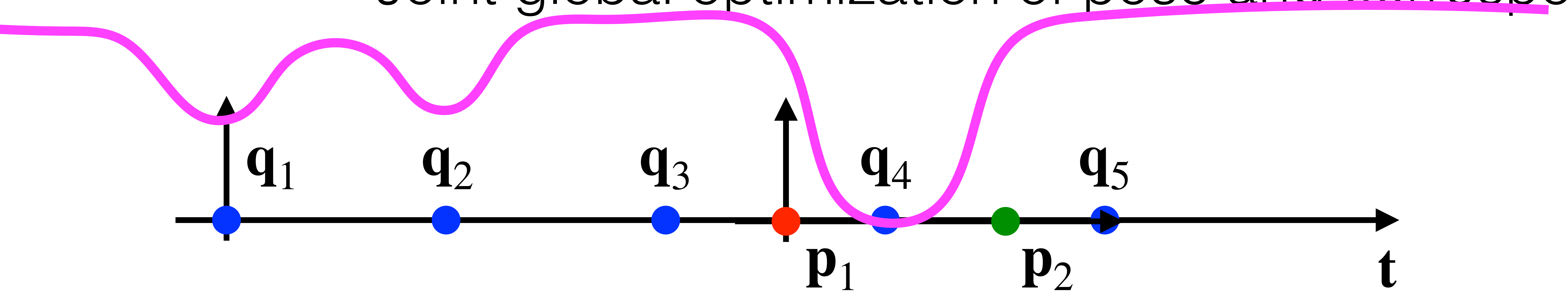
Joint global optimization of pose and correspondences



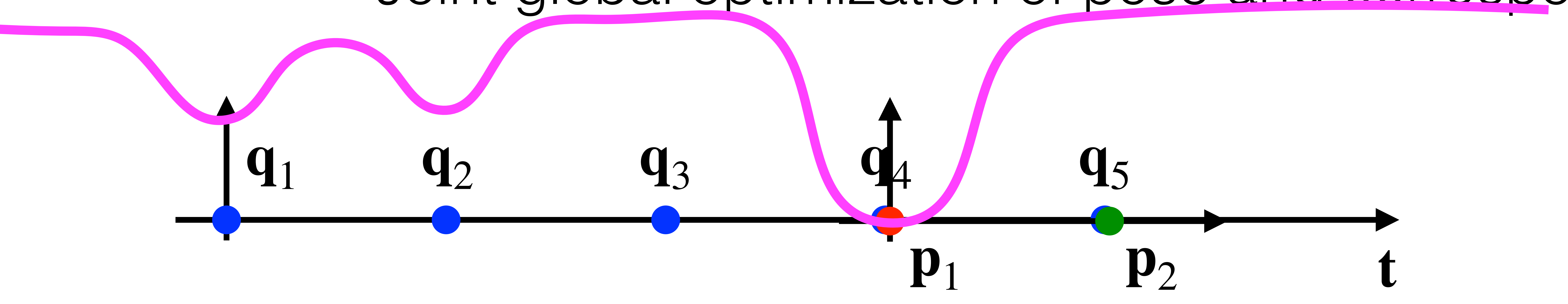
$c(1) \in \{1,4\}$ $c(2) \in \{3,5\}$

Idea 0: Randomly initialize initial translation $\mathbf{t}_0$ + gradient optimization

Issue: Basin of attraction small => too many initializations needed

$$\mathbf{t}^\star = \arg\min_{\mathbf{t}} \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_1) + \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_4) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_3) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_5)$$

Joint global optimization of pose and correspondences



$c(1) \in \{1,4\}$ $\quad$ $c(2) \in \{3,5\}$

Idea 0: Randomly initialize initial translation $\mathbf{t}_0$ + gradient optimization

Issue: Basin of attraction small => course-of-dim => many initializations needed

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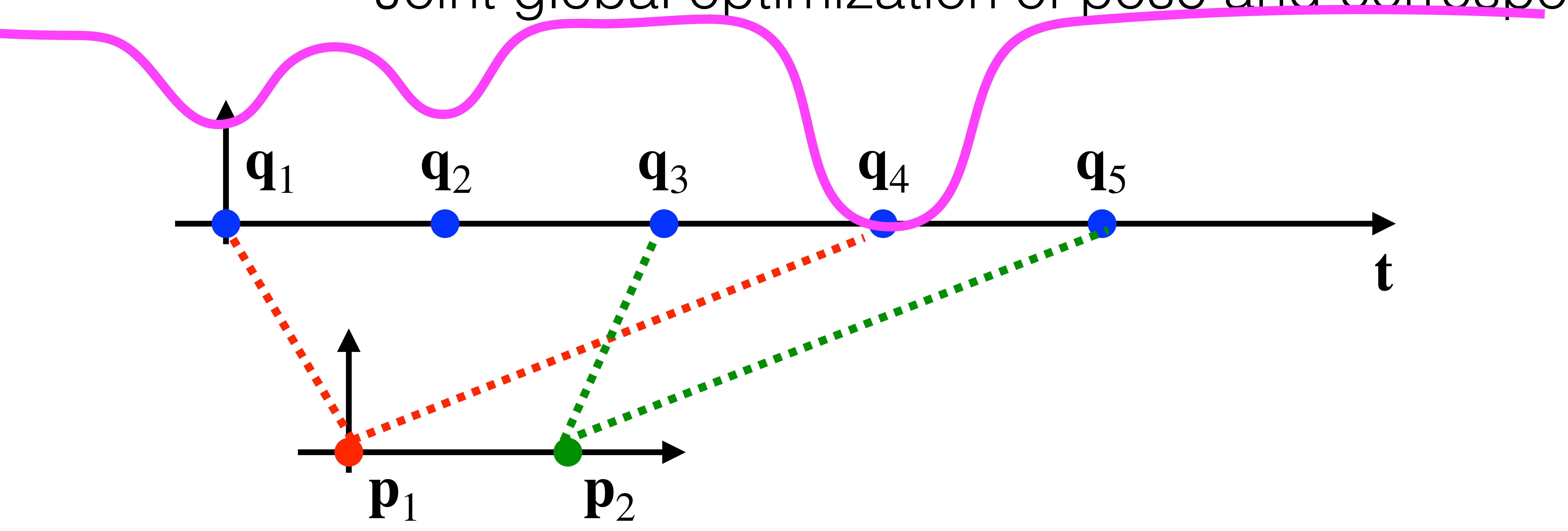
Joint global optimization of pose and correspondences



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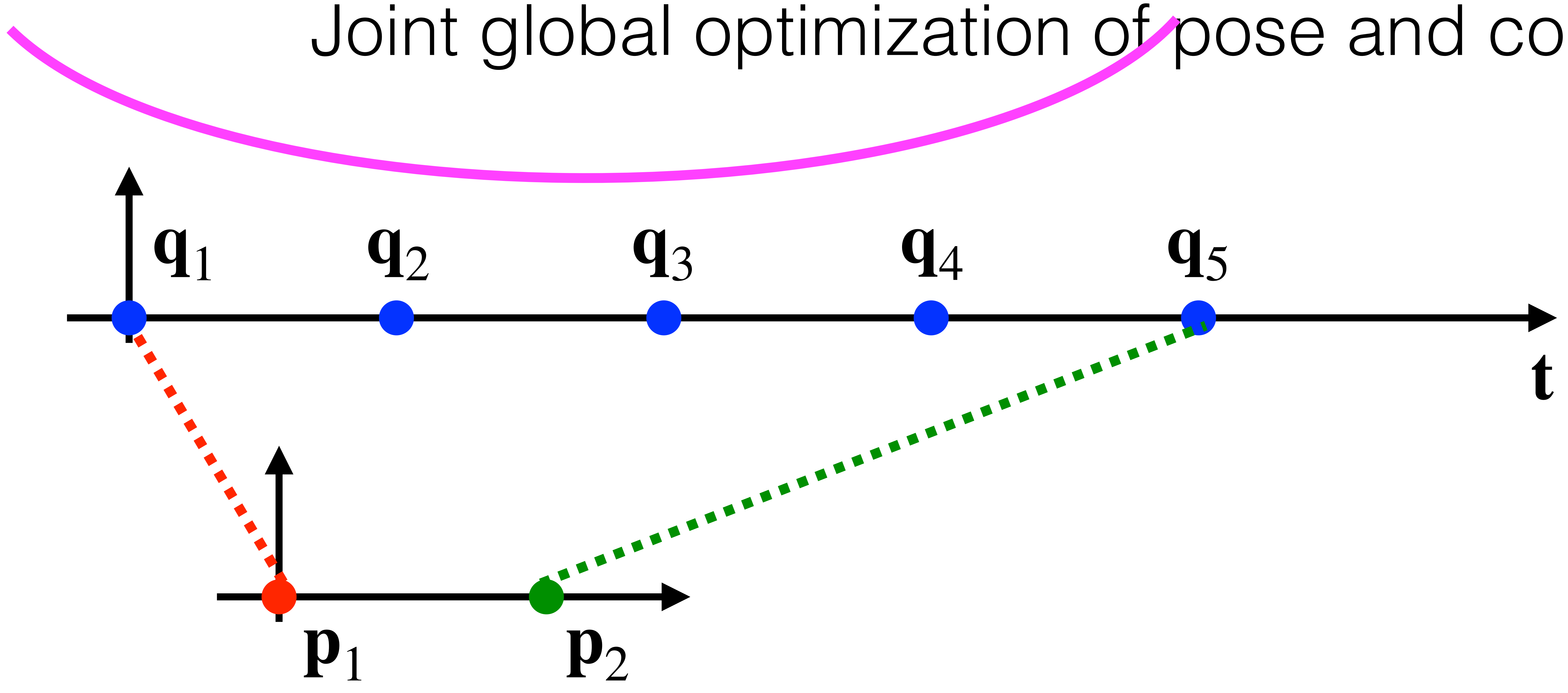
Joint global optimization of pose and correspondences

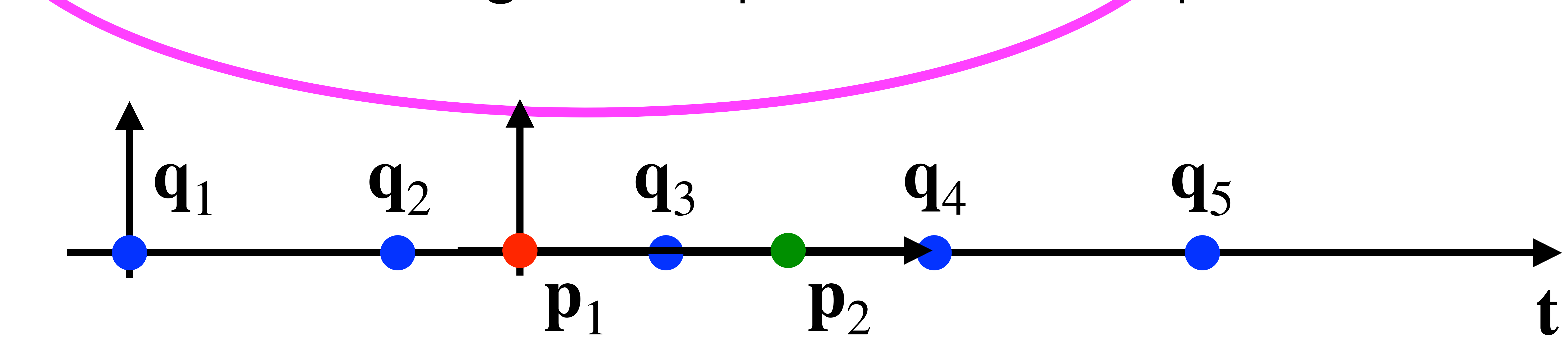


$c(1) \in \{1,4\}$ $c(2) \in \{3,5\}$

Idea 1: Randomly initialize 1-to-1 correspondences + gradient optimization

$$\mathbf{t}^\star = \arg\min_{\mathbf{t}} \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_1) + \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_4) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_3) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_5)$$

Joint global optimization of pose and correspondences



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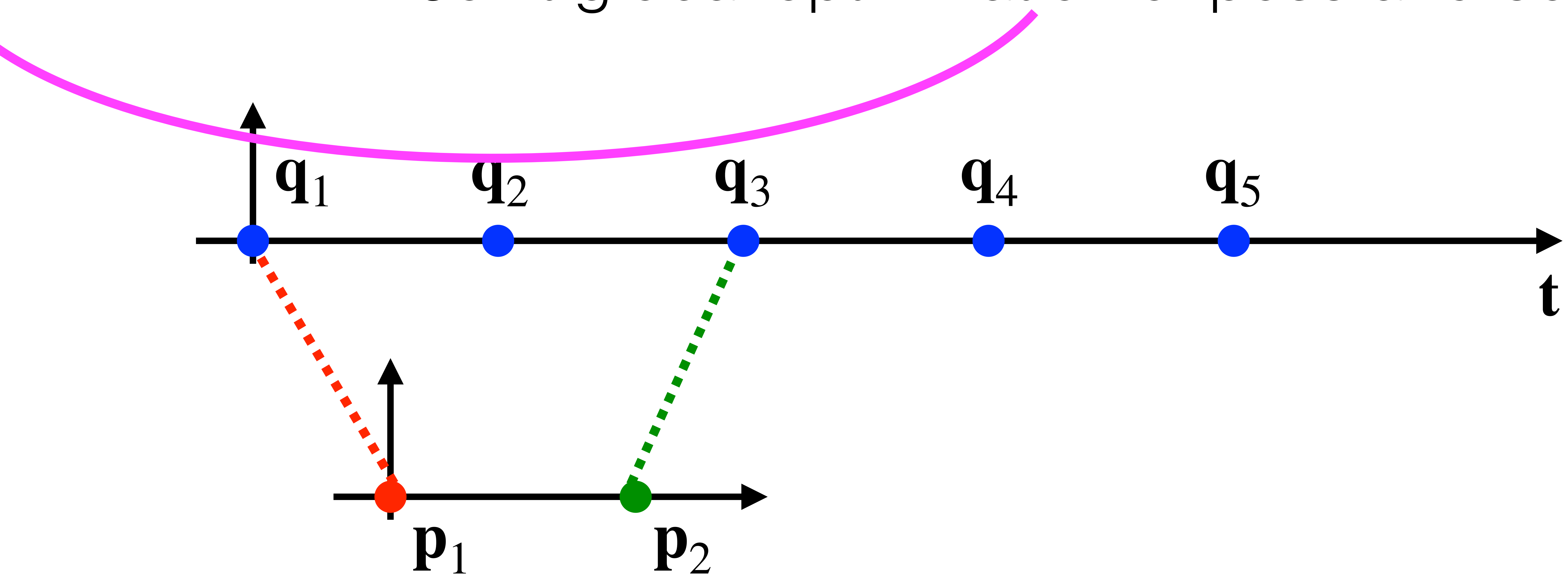
Idea 1: Randomly initialize 1-to-1 correspondences + gradient optimization

$$\mathbf{t}^{\star} = \arg\min_{\mathbf{t}} \; \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_1) + \cancel{\rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_4)} + \cancel{\rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_3)} + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_5)$$

Joint global optimization of pose and correspondences



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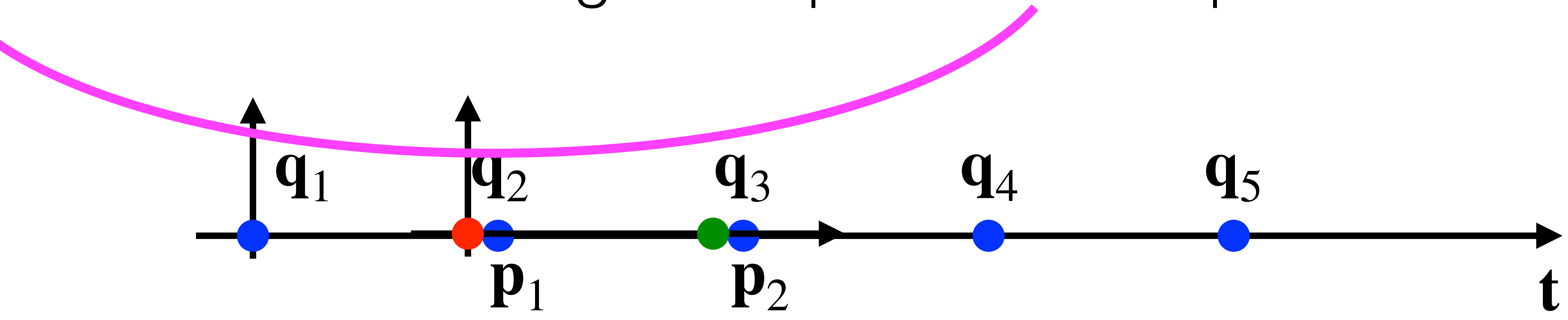
Idea 1: Randomly initialize 1-to-1 correspondences + gradient optimization

$$\mathbf{t}^\star = \arg\min_{\mathbf{t}} \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_1) + \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_4) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_3) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_5)$$

Joint global optimization of pose and correspondences

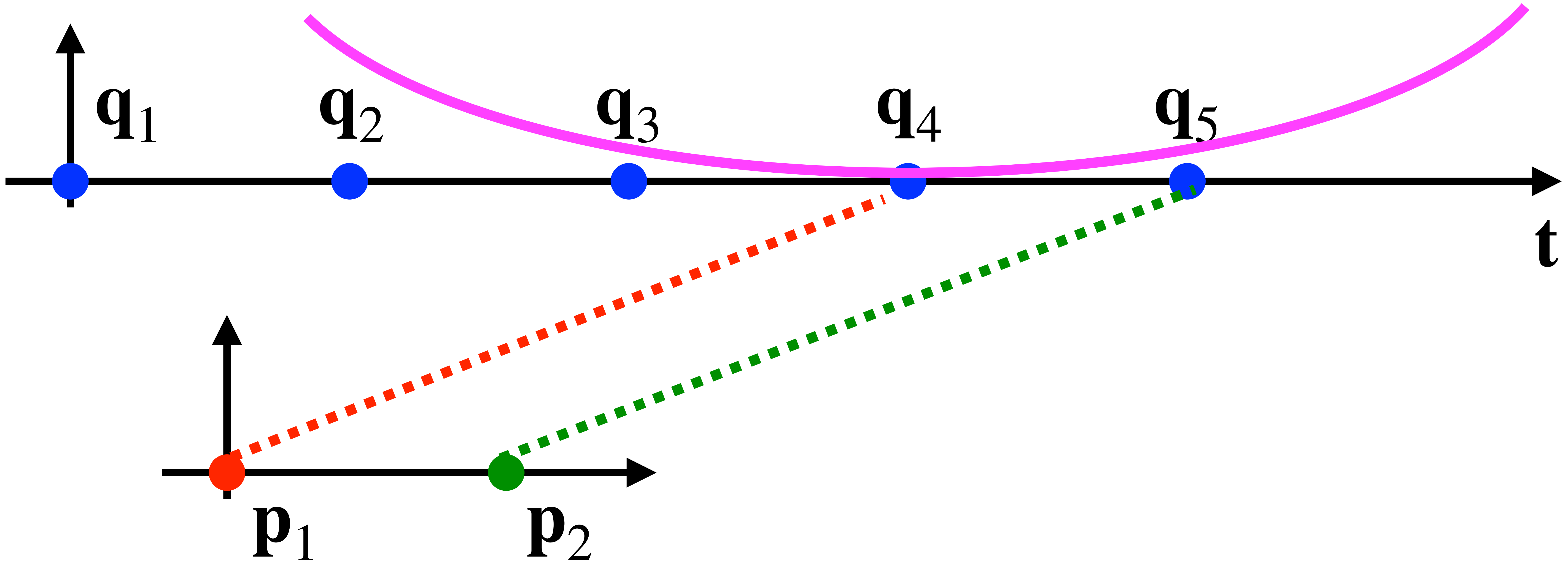


$c(1) \in \{1,4\}$ $\quad$ $c(2) \in \{3,5\}$

Idea 1: Randomly initialize 1-to-1 correspondences + gradient optimization

$$\mathbf{t}^\star = \arg\min_{\mathbf{t}} \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_1) + \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_4) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_3) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_5)$$

Joint global optimization of pose and correspondences



$c(1) \in \{1,4\}$ $c(2) \in \{3,5\}$

Idea 1: Randomly initialize 1-to-1 correspondences + gradient optimization

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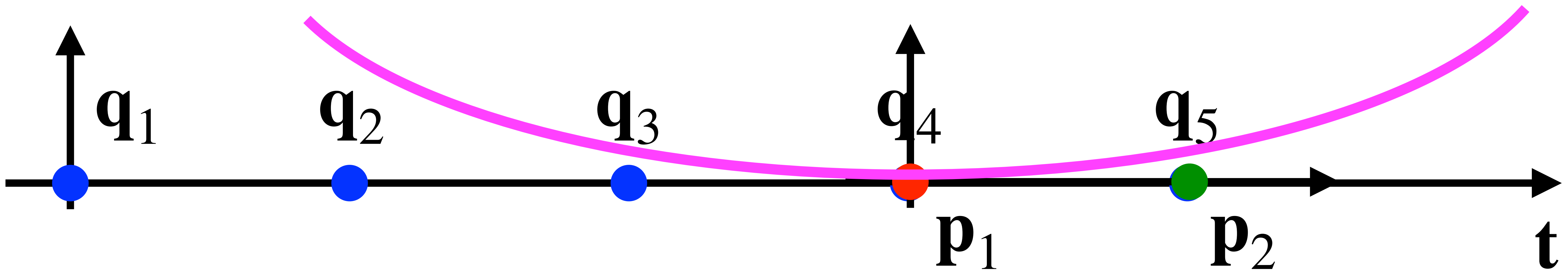
Joint global optimization of pose and correspondences

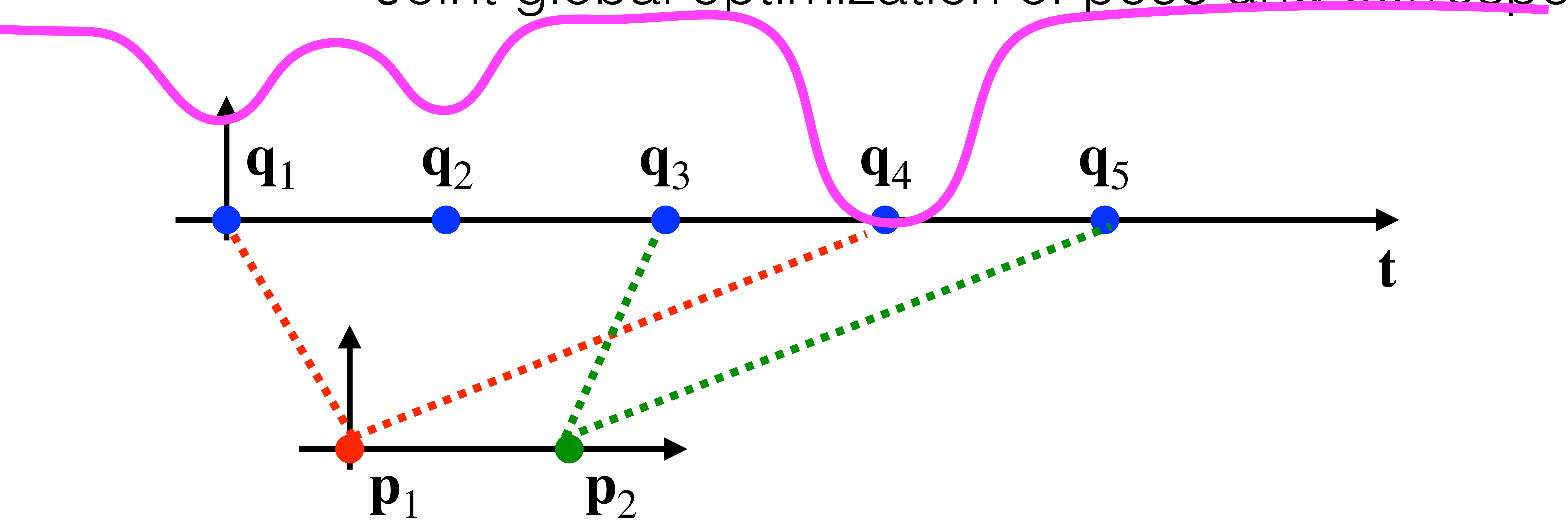


$c(1) \in \{1,4\}$ $\quad$ $c(2) \in \{3,5\}$

Idea 1: Randomly initialize 1-to-1 correspondences + gradient optimization
Issue: Too many attemps needed (10 points => 100 combinations)

$$\mathbf{t}^\star = \arg\min_{\mathbf{t}} \; \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_1) + \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_4) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_3) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_5)$$

Joint global optimization of pose and correspondences

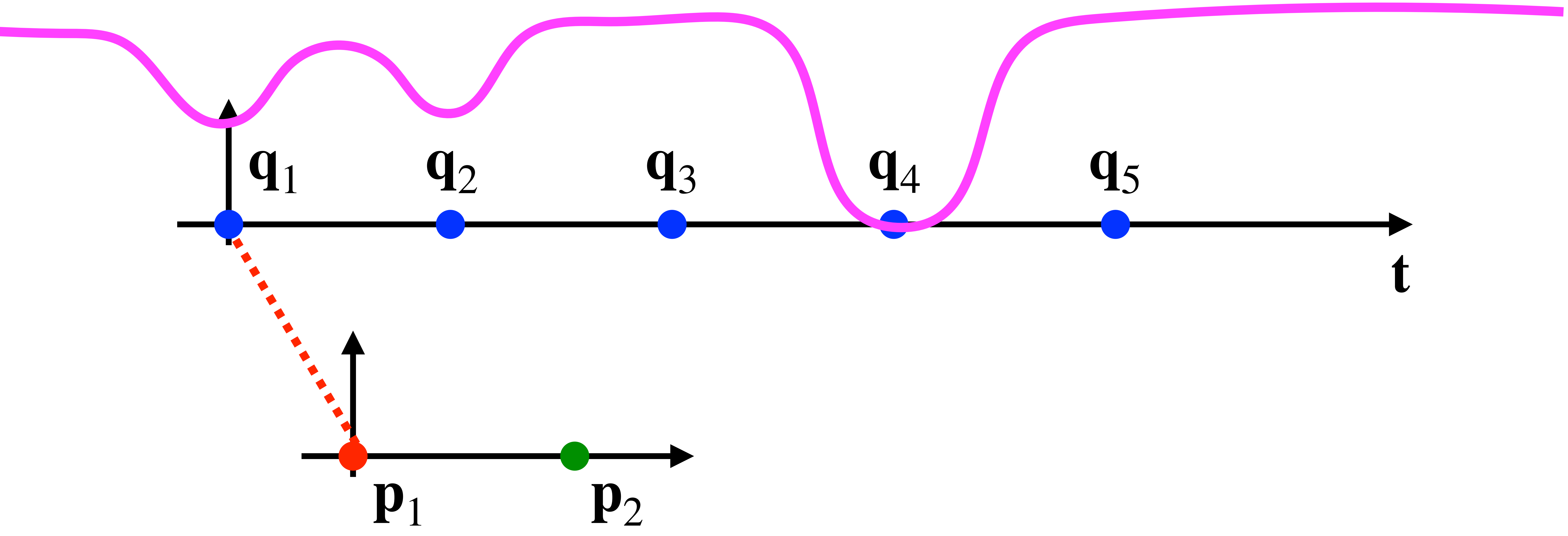


$c(1) \in \{1,4\}$ $c(2) \in \{3,5\}$

Idea 2: (a) Randomly initialize minimum subset of 1-to-1 correspondences

(b) Evaluate the value of original criterion function

$$\mathbf{t}^\star = \arg\min_{\mathbf{t}} \; \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_1) + \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_4) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_3) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_5)$$



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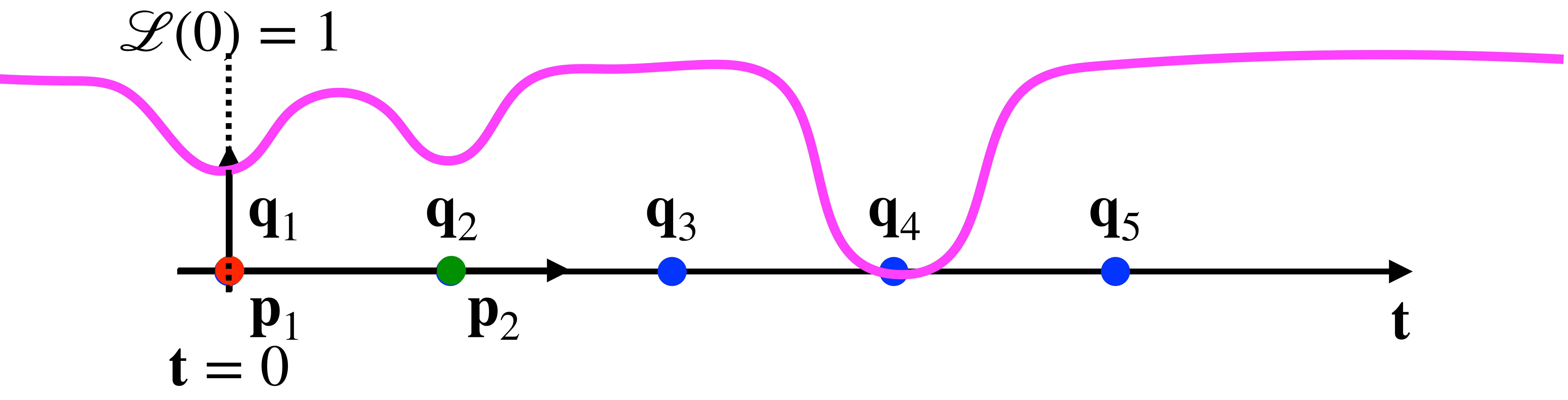

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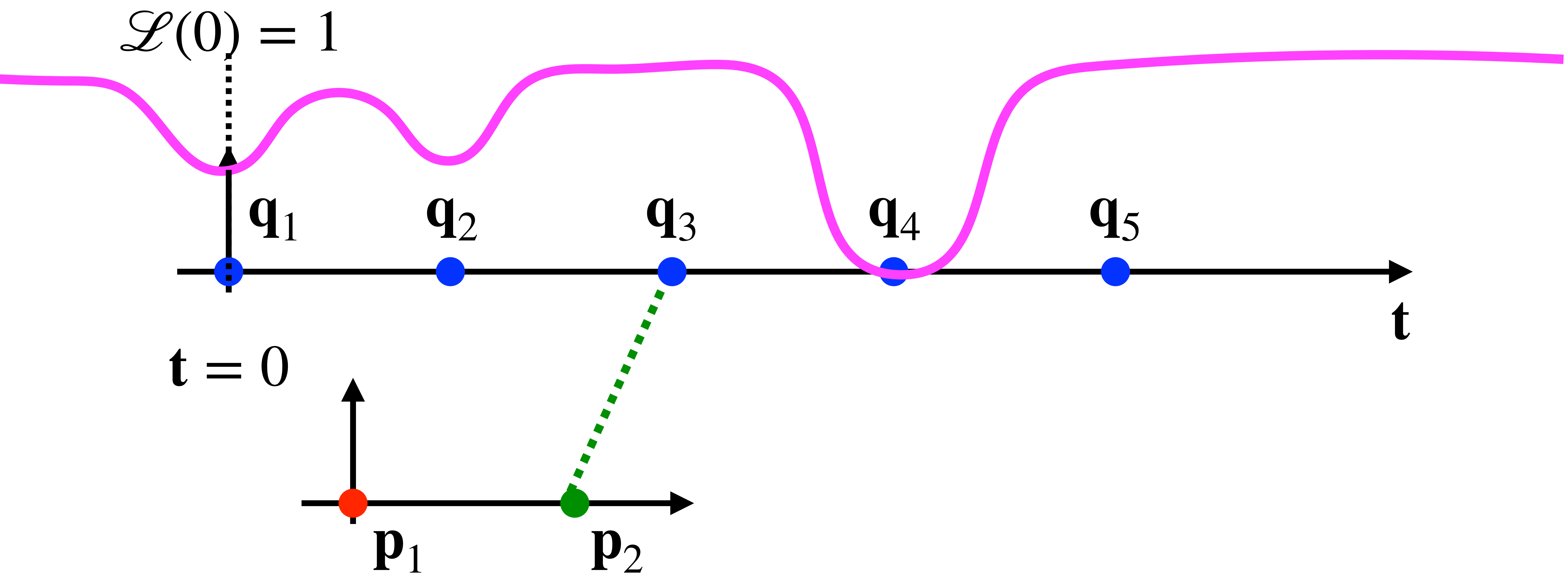

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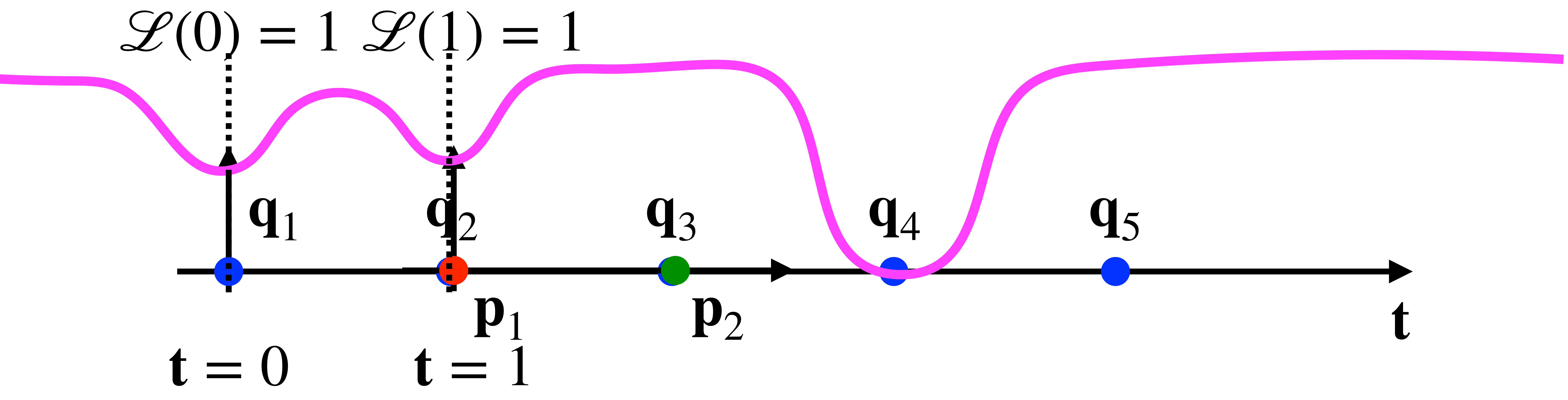

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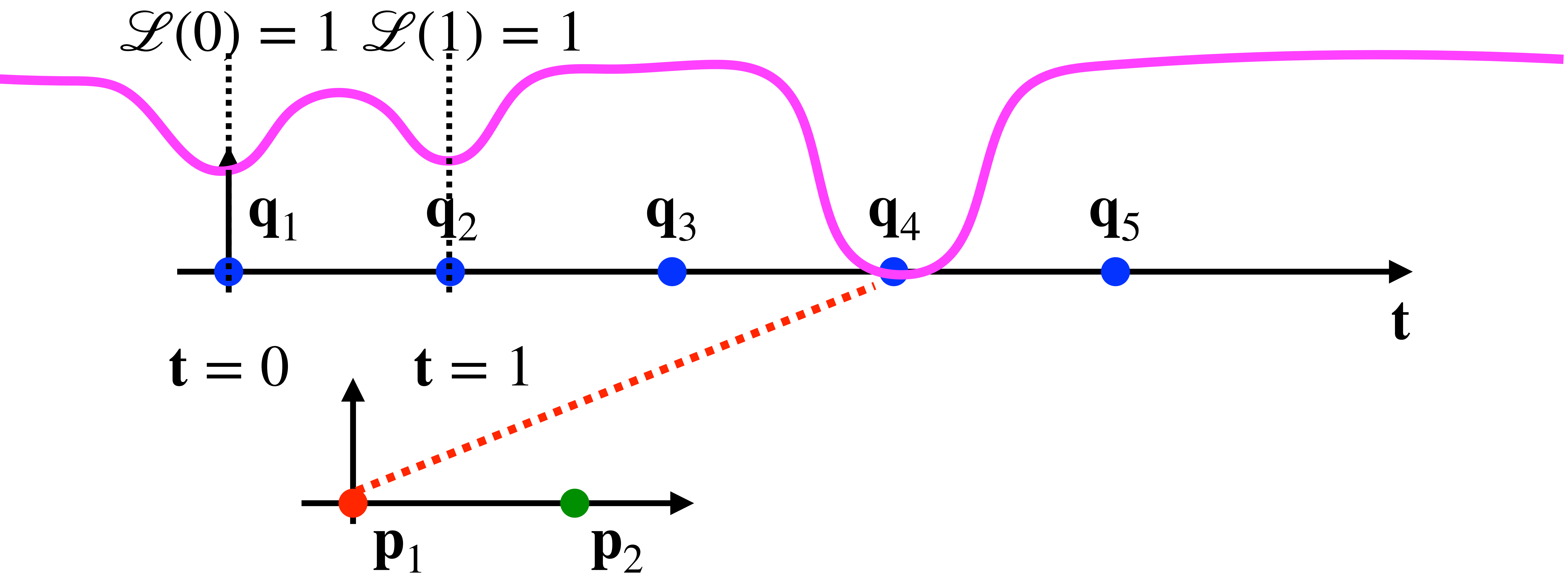

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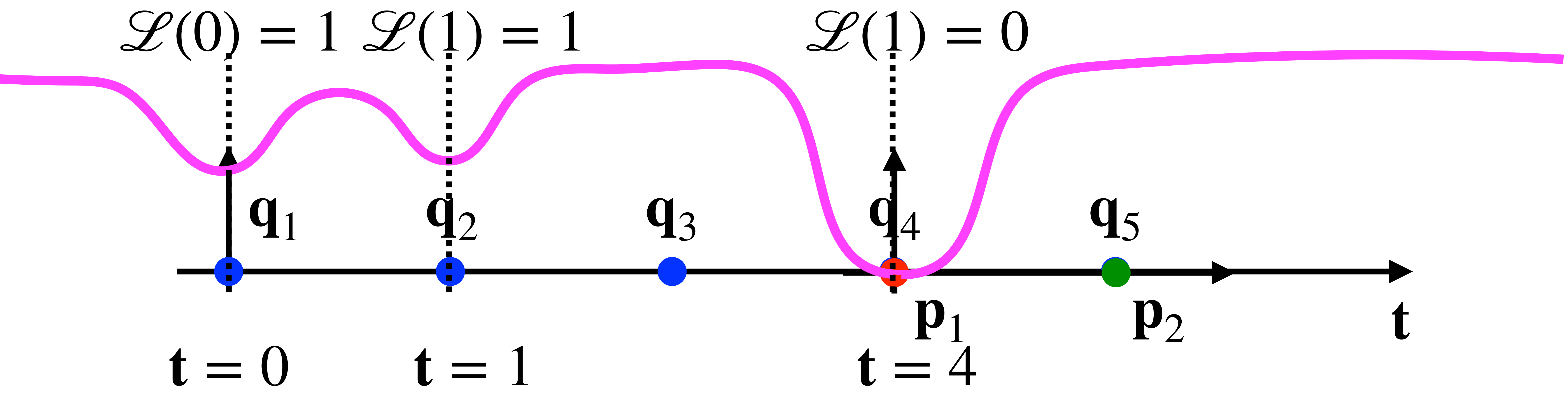


Advantages: (1) always initialized in a local optima (10 points=> 10 local optima)

(2) no gradient optimization needed

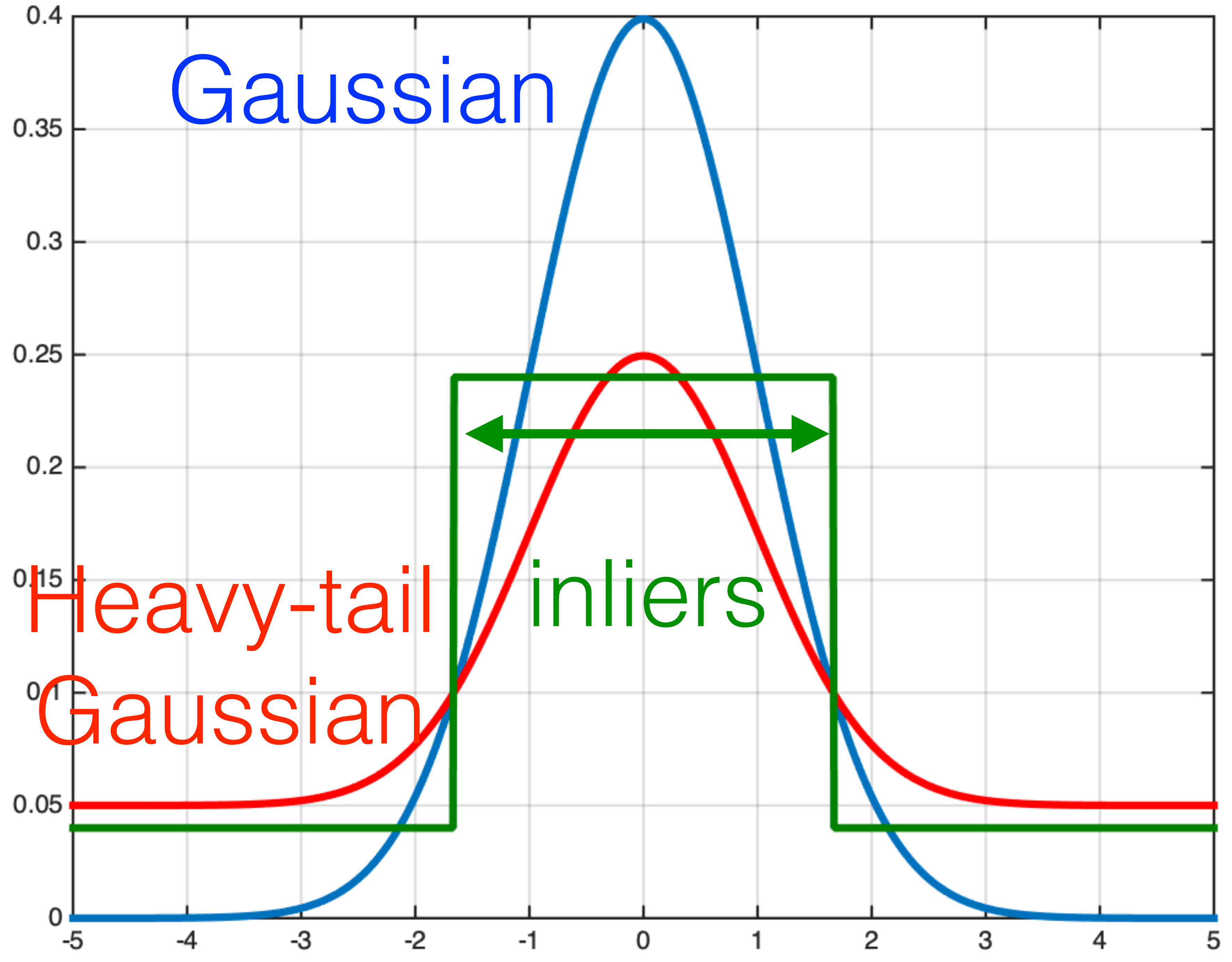
$c(1) \in \{1,4\}$ $c(2) \in \{3,5\}$

Idea 2: (a) Randomly initialize minimum subset of 1-to-1 correspondences

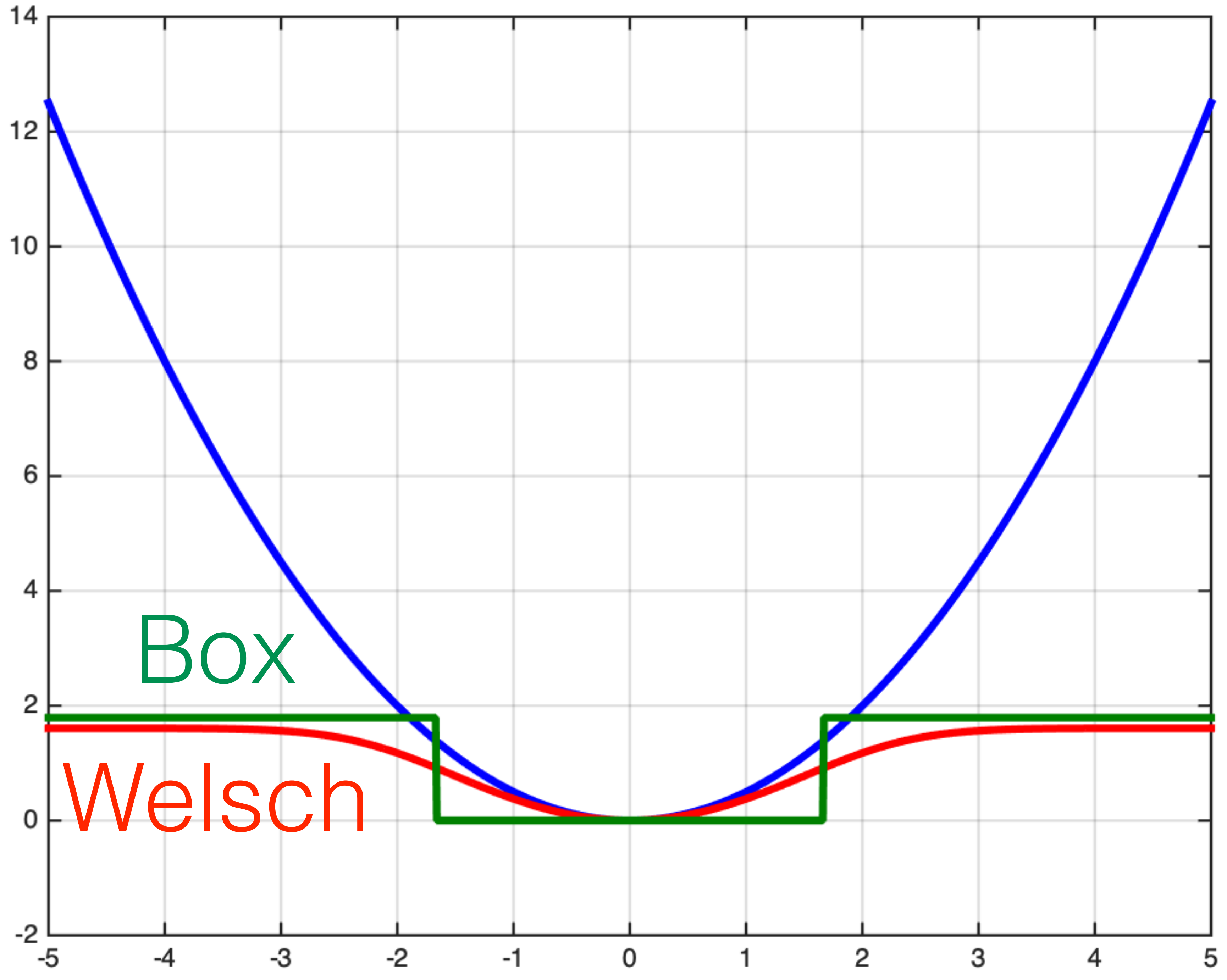
(b) Evaluate the value of original criterion function

$$\mathbf{t}^\star = \arg\min_{\mathbf{t}} \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_1) + \rho(\mathbf{p}_1 + \mathbf{t} - \mathbf{q}_4) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_3) + \rho(\mathbf{p}_2 + \mathbf{t} - \mathbf{q}_5)$$

ICP SLAM - outlier detection procedure



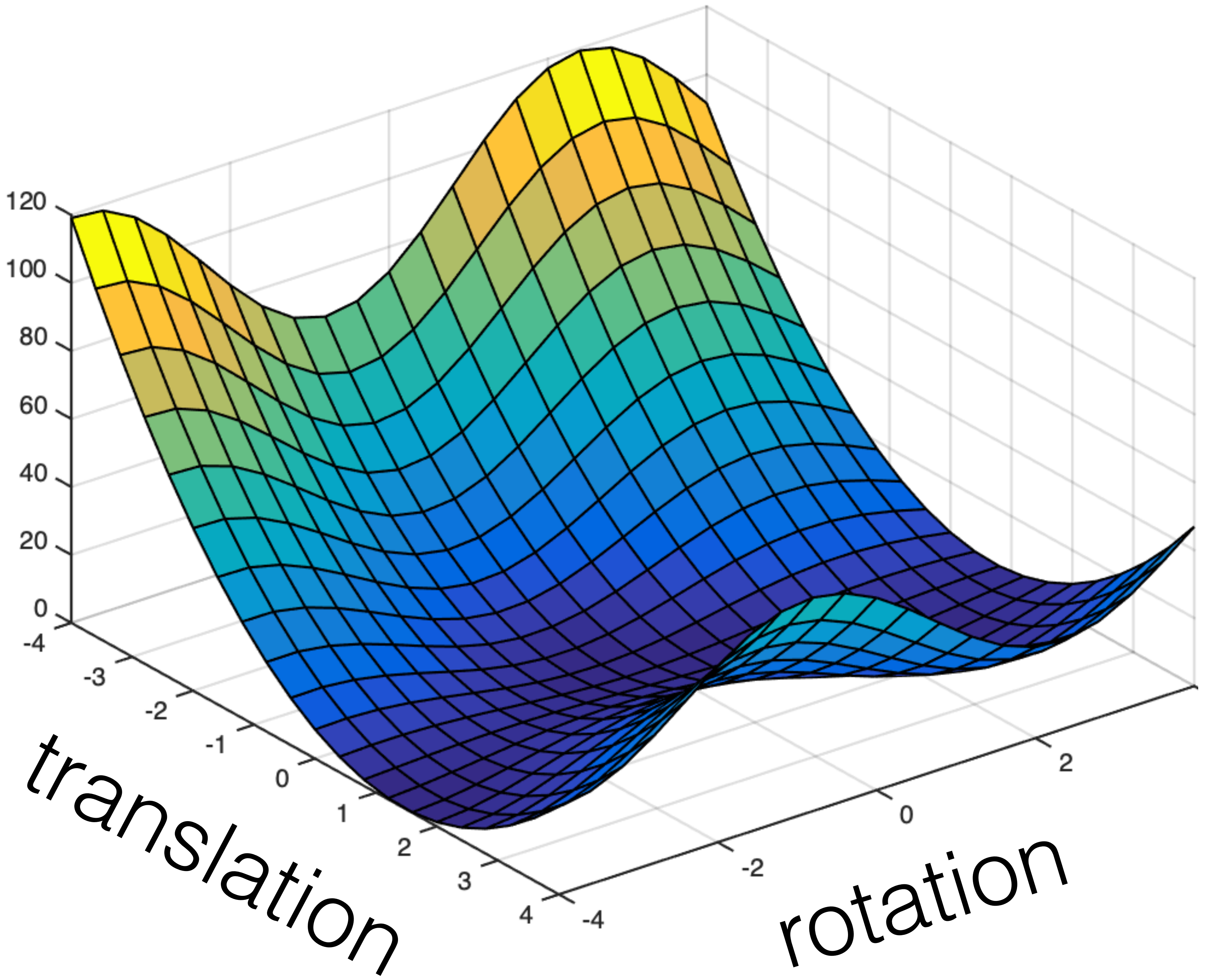
Probability distributions



Corresponding losses

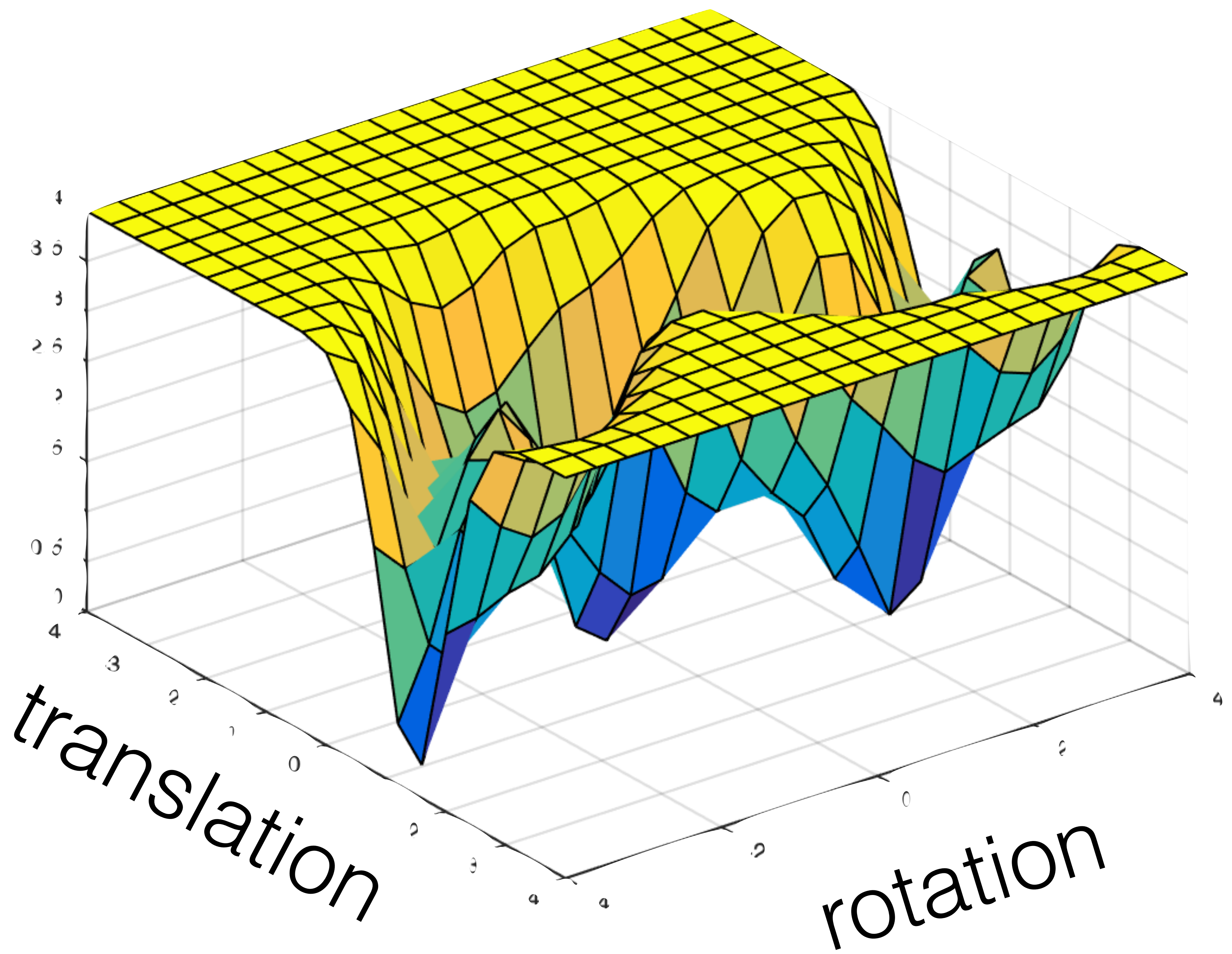
ICP SLAM - gradient optimisation of robust loss

L2 landscape



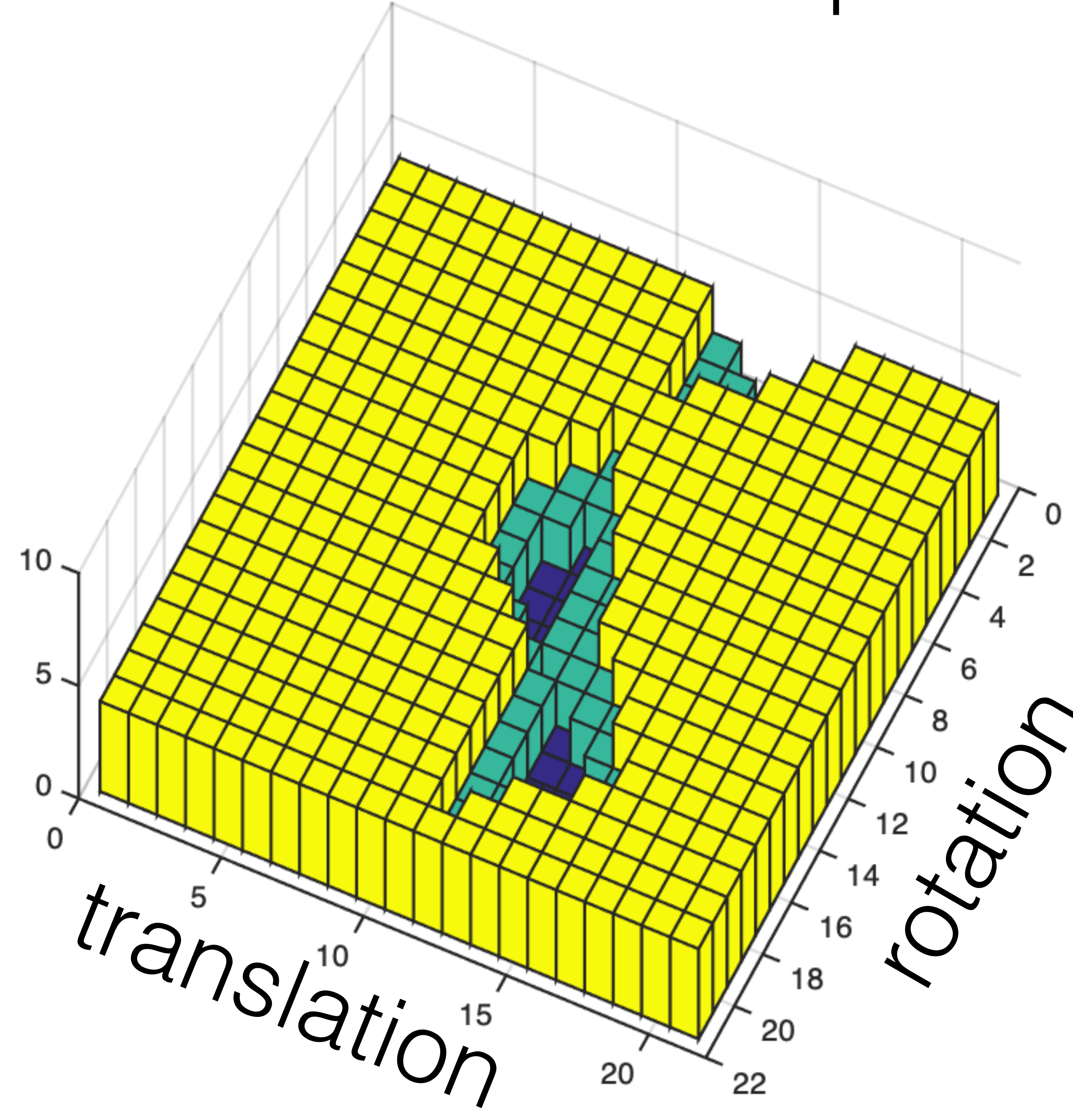
- Convex in translation space
- Non-convex but smooth in SO3

Welsch landscape



- Non-convex+Large narrow plateaus with zero gradient
- Any gradient optimization requires good initialization

Box loss landscape

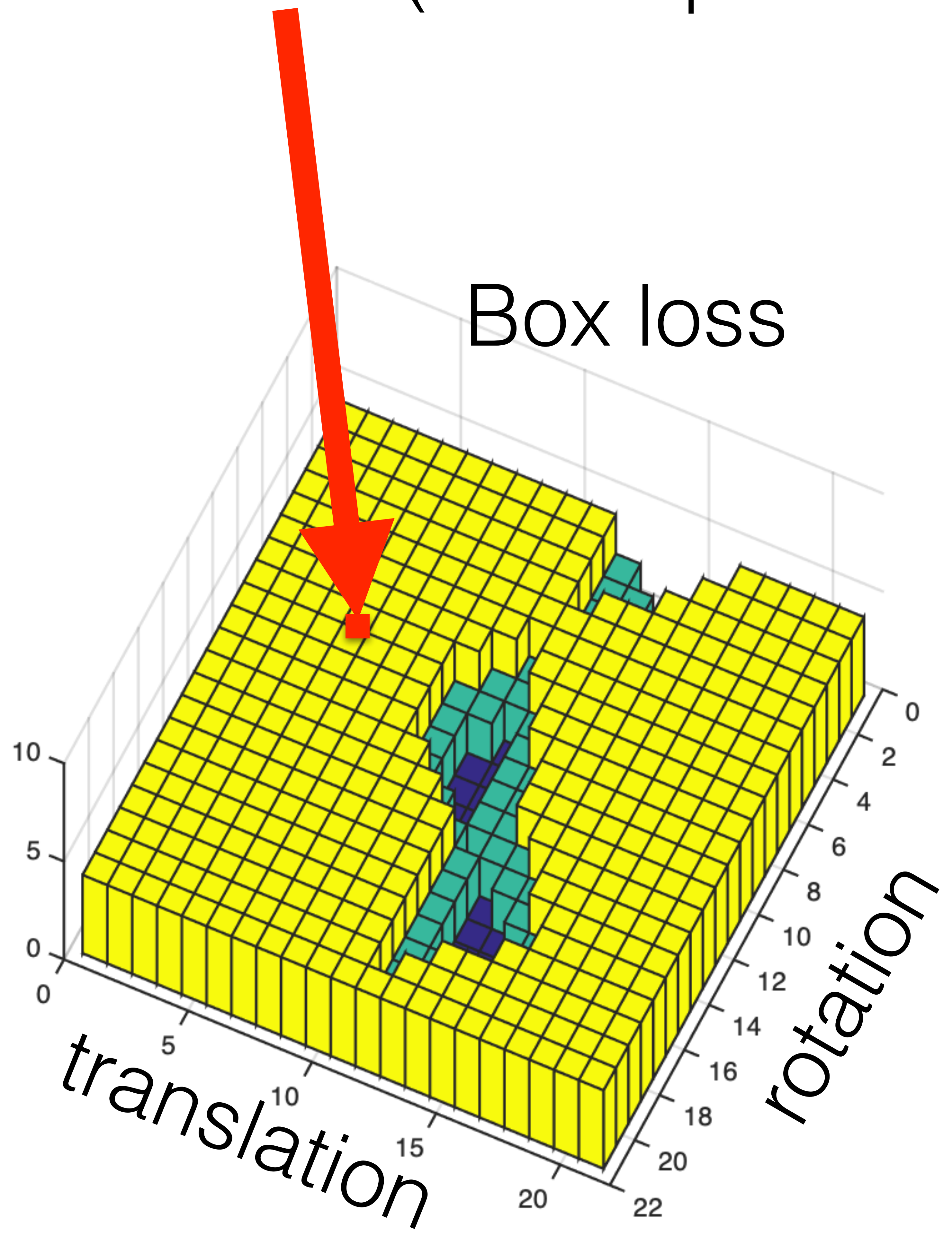


- Zero gradients
- Combinatorial optimization

Optimizing box-loss

Naive optimization algorithm:

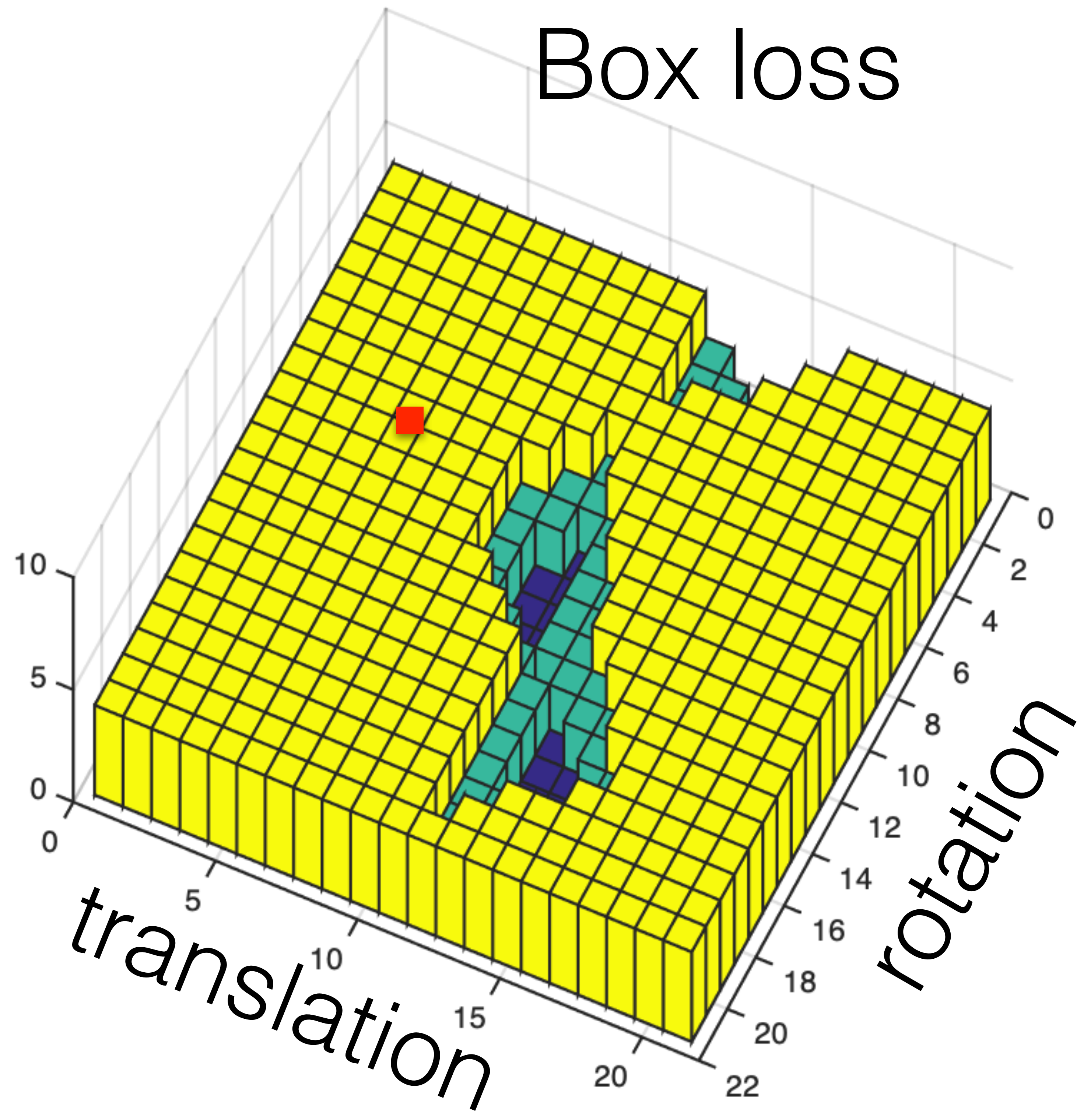
1. Sample hypothesis (R,t) at random
2. Evaluate value of the box-loss function (at this point R,t)



Optimizing box-loss

Naive optimization algorithm:

1. Sample hypothesis (R,t) at random
2. Evaluate value of the box-loss function (at this point R,t)
3. Remember the lowest value so far
4. repeat K times



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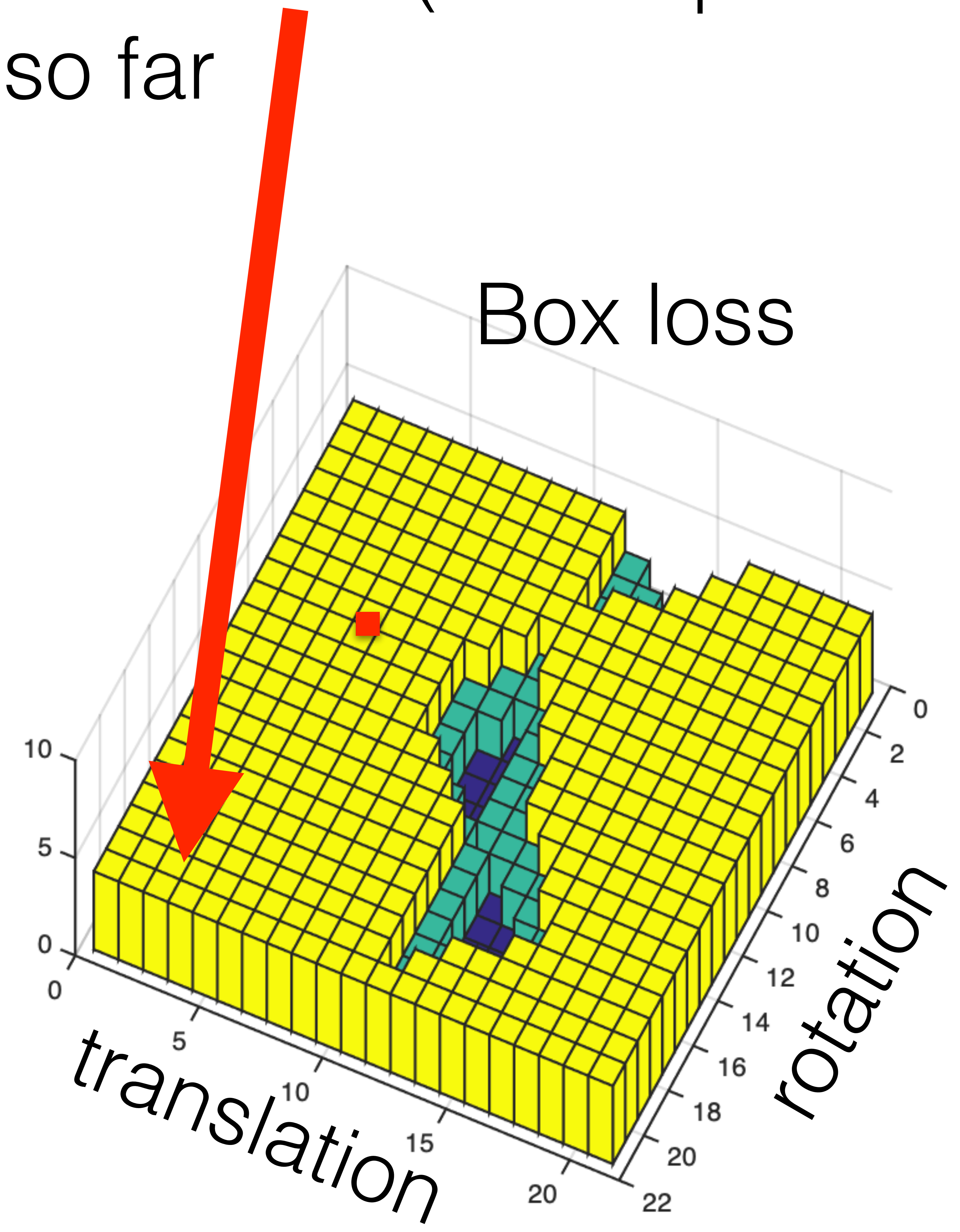

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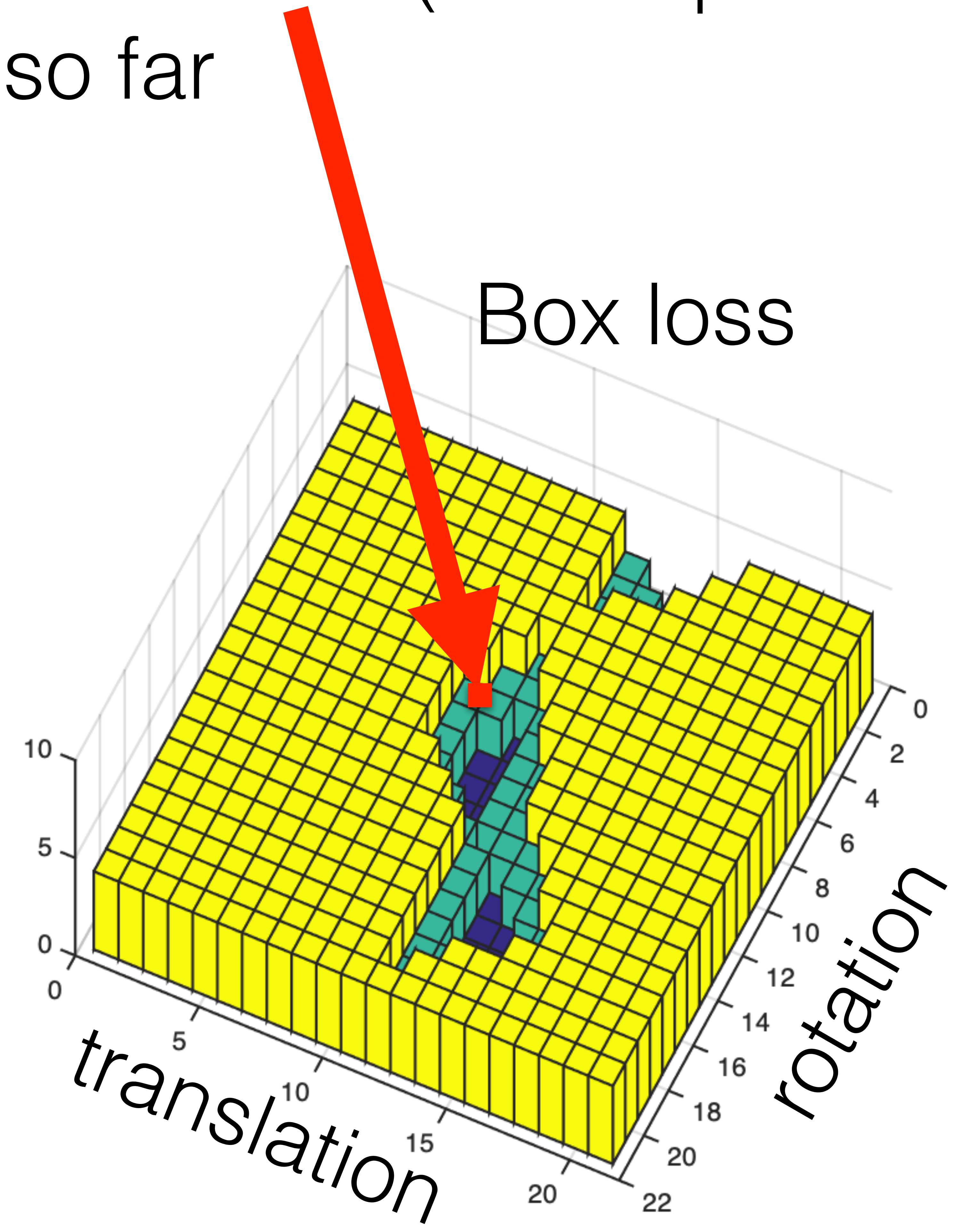

Optimizing box-loss

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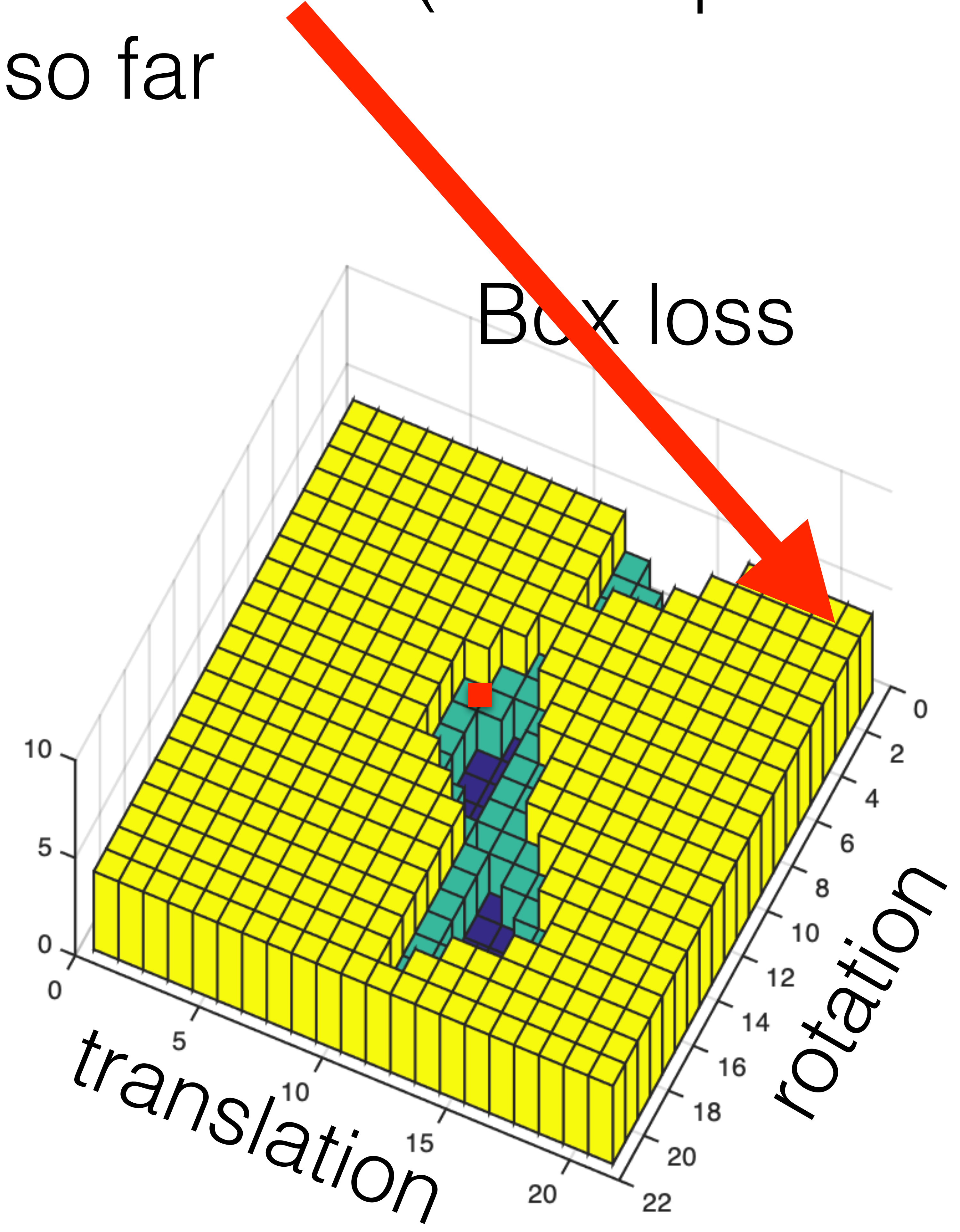

Optimizing box-loss

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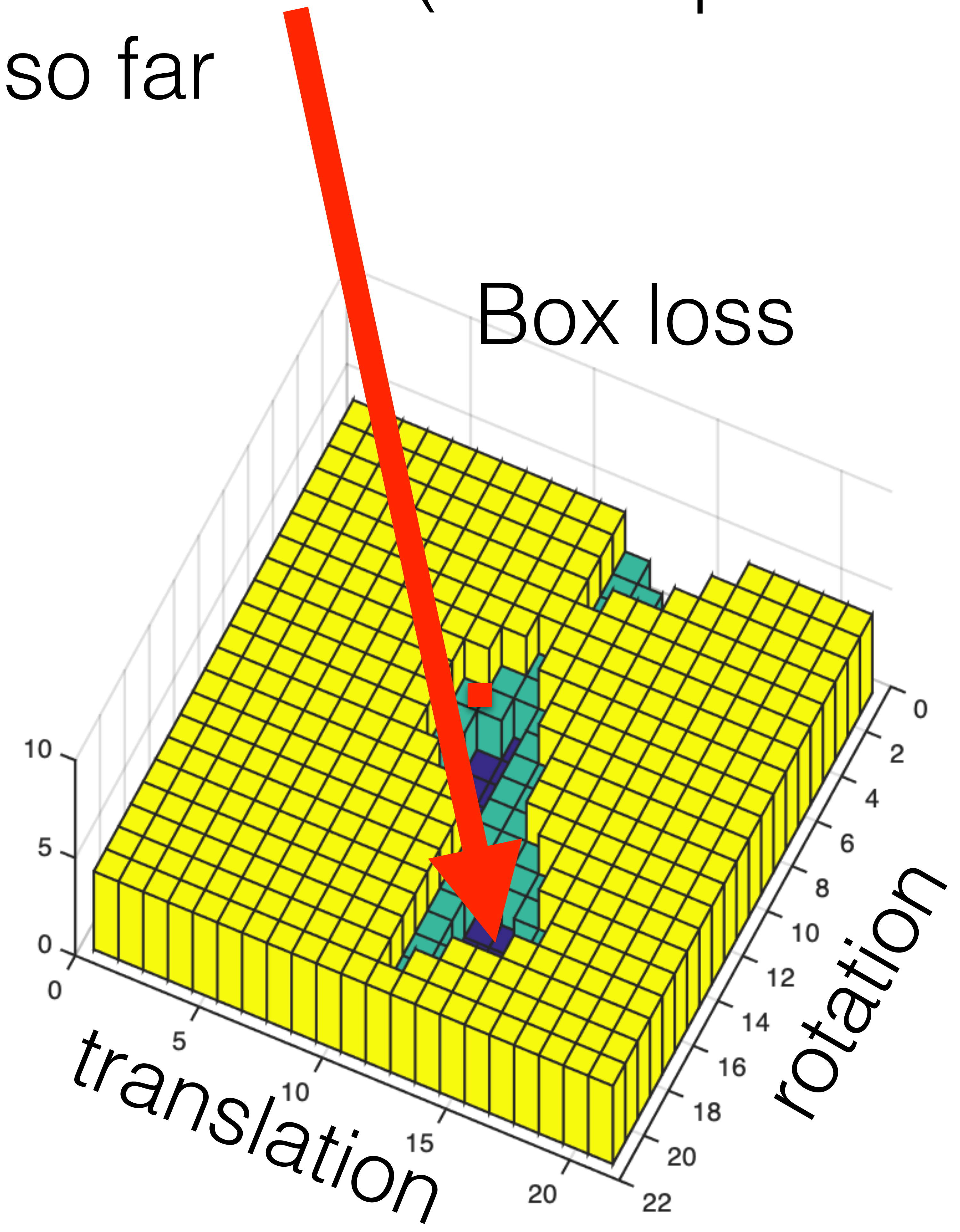

Optimizing box-loss

Naive optimization algorithm:

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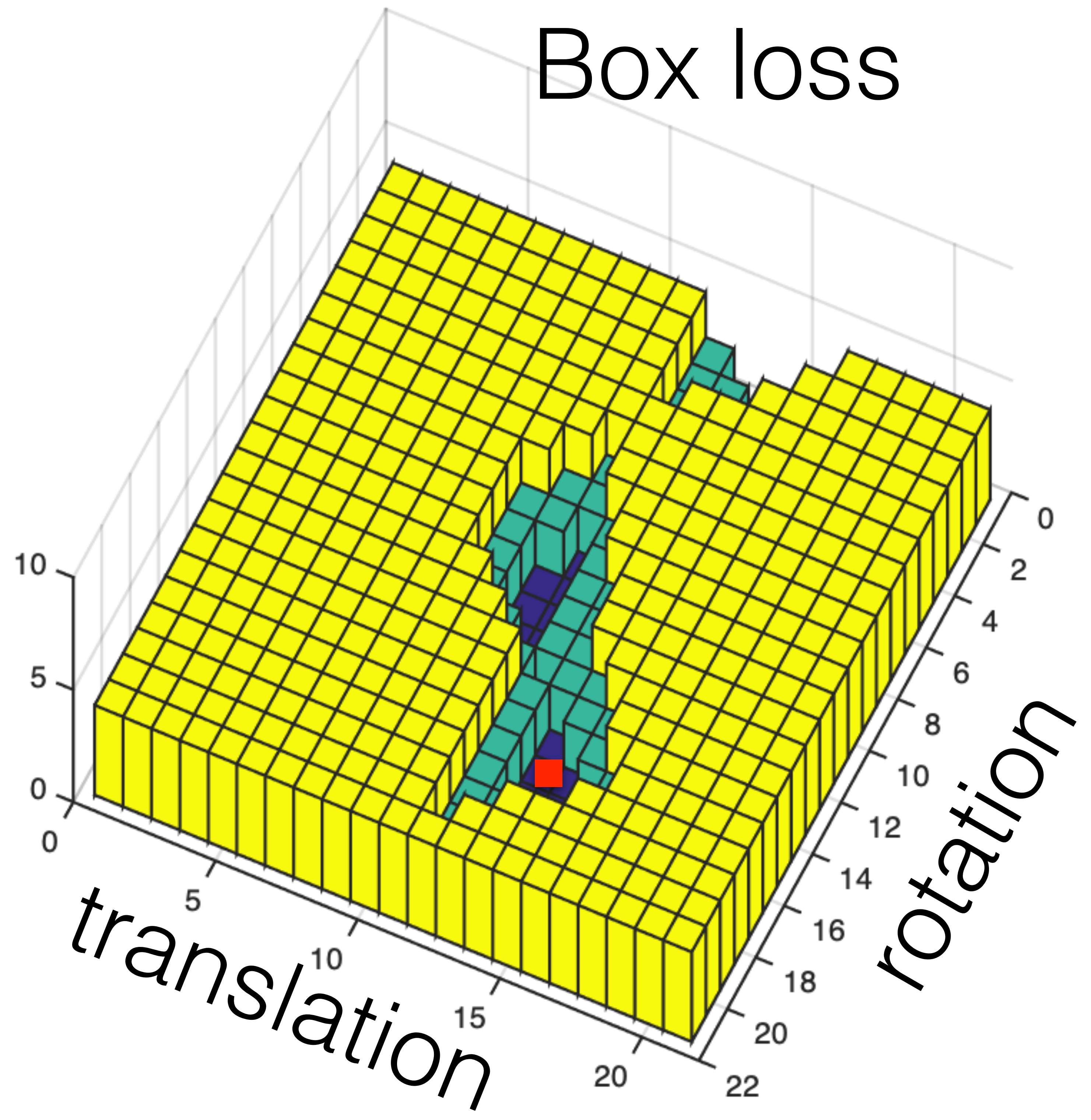
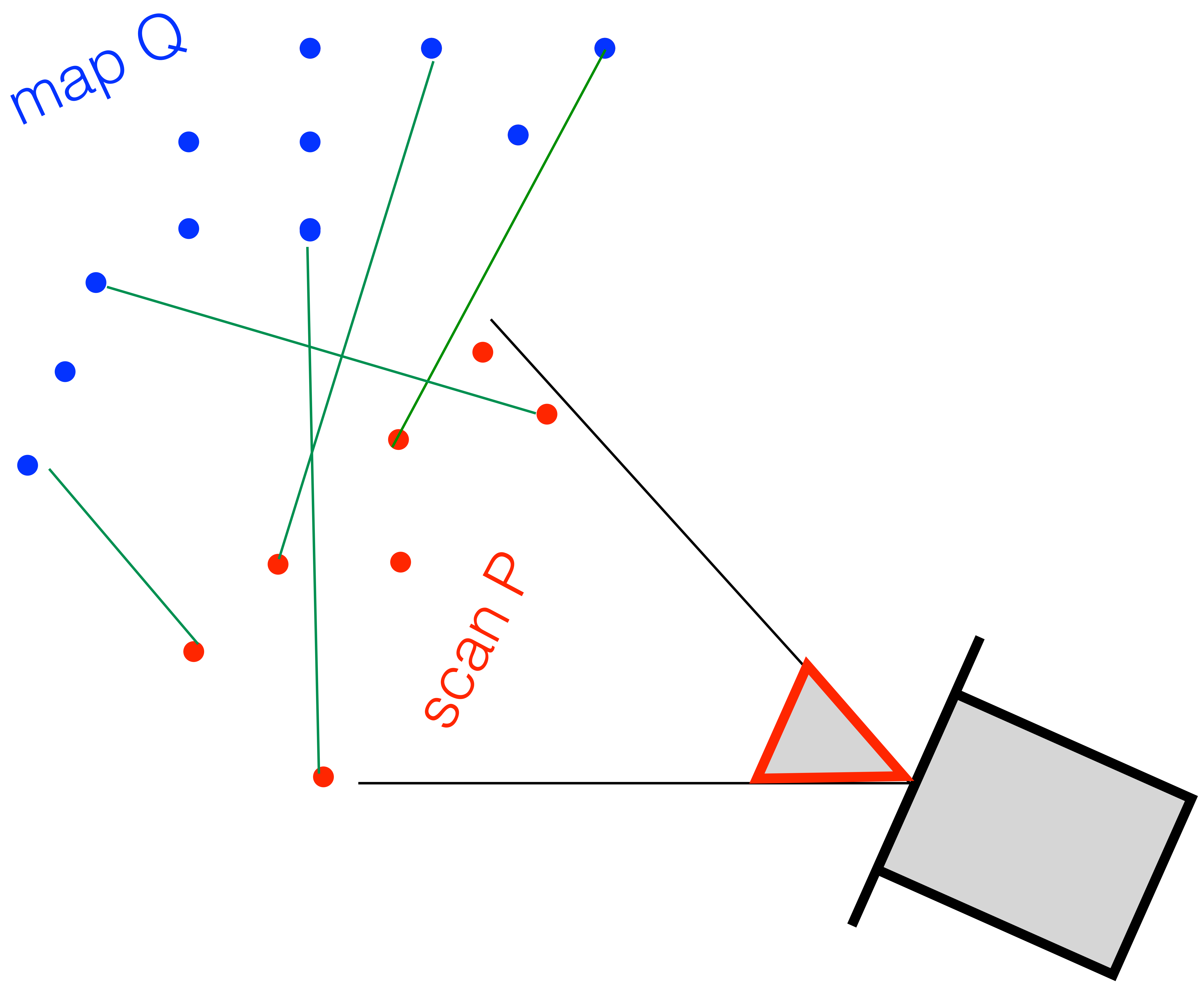
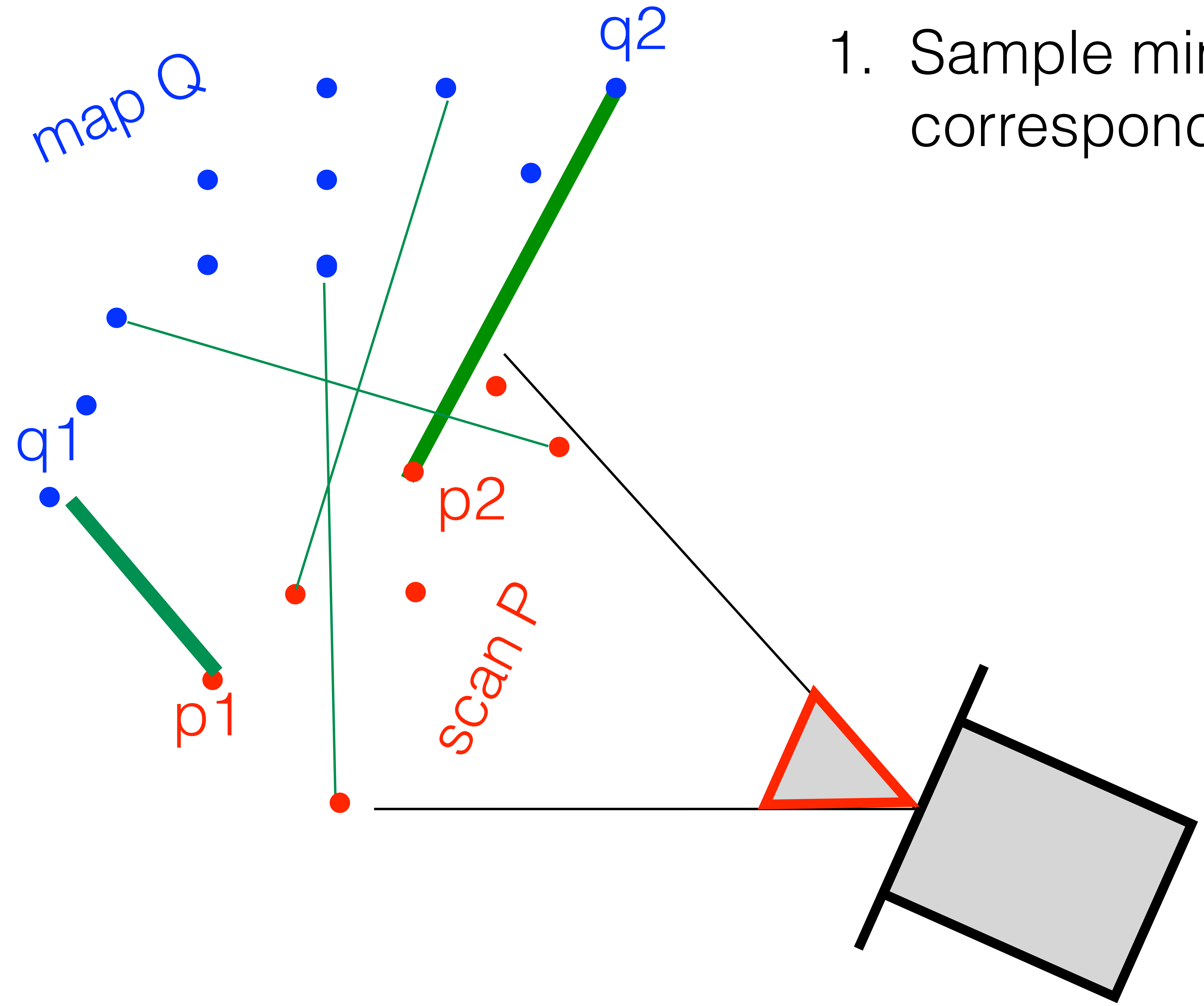
if K is huge and you are lucky



RANSAC (RAndom SAmple Consensus)

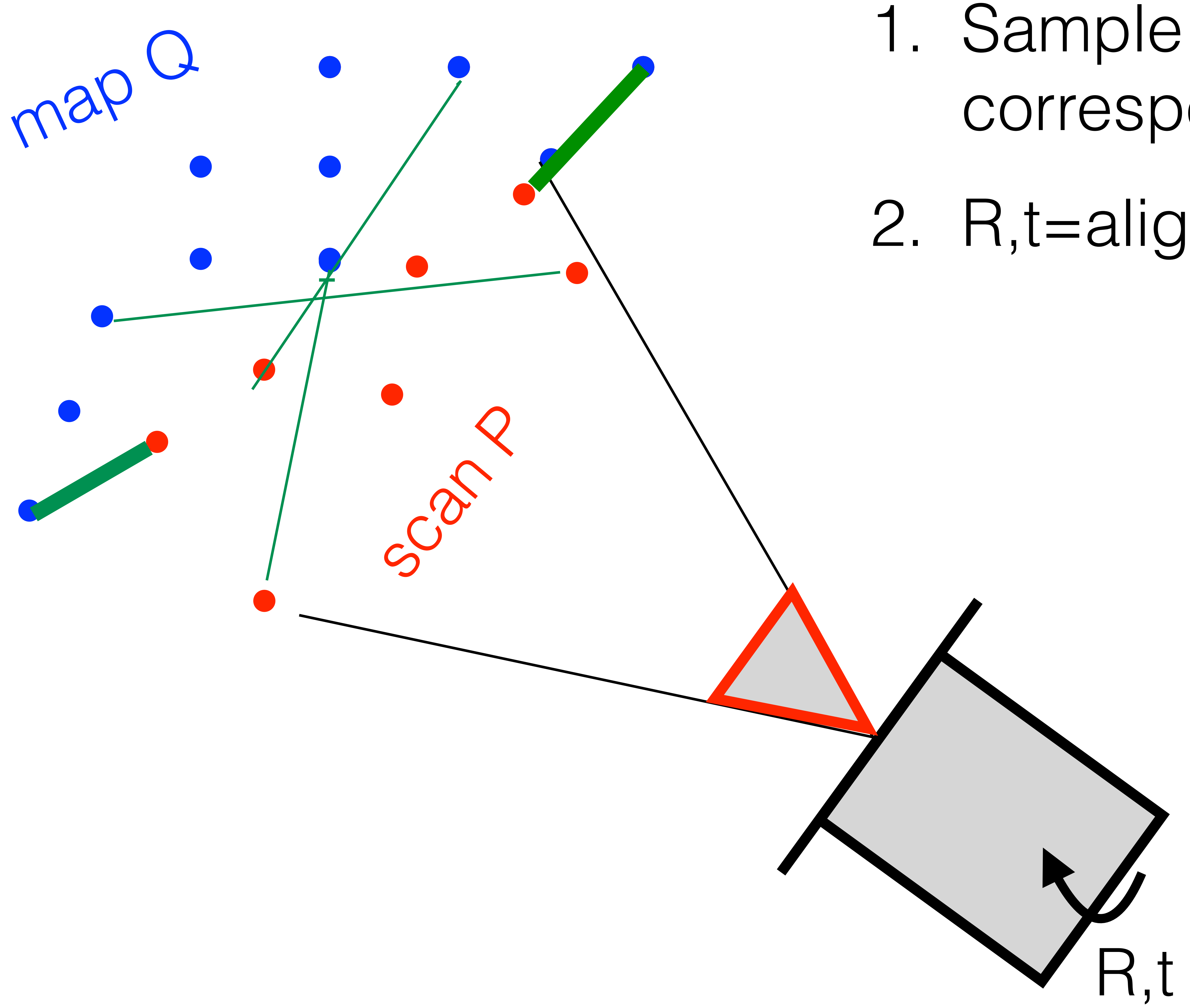


RANSAC (RAndom SAmple Consensus)



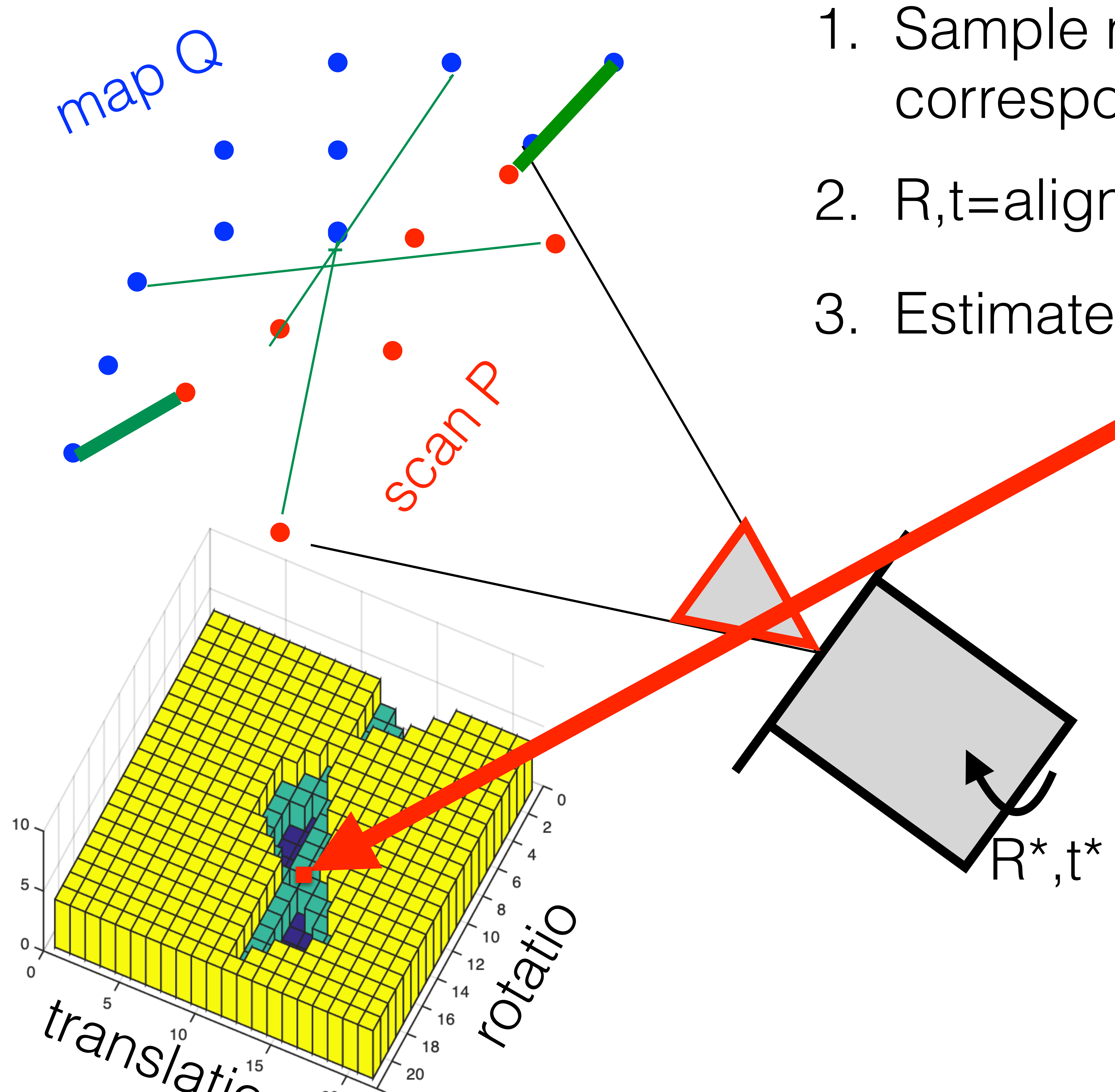
1. Sample minimal subset of correspondences (p, q).

RANSAC (RAndom SAmple Consensus)



1. Sample minimal subset of correspondences (p, q).
2. R,t=align_L2(p, q)

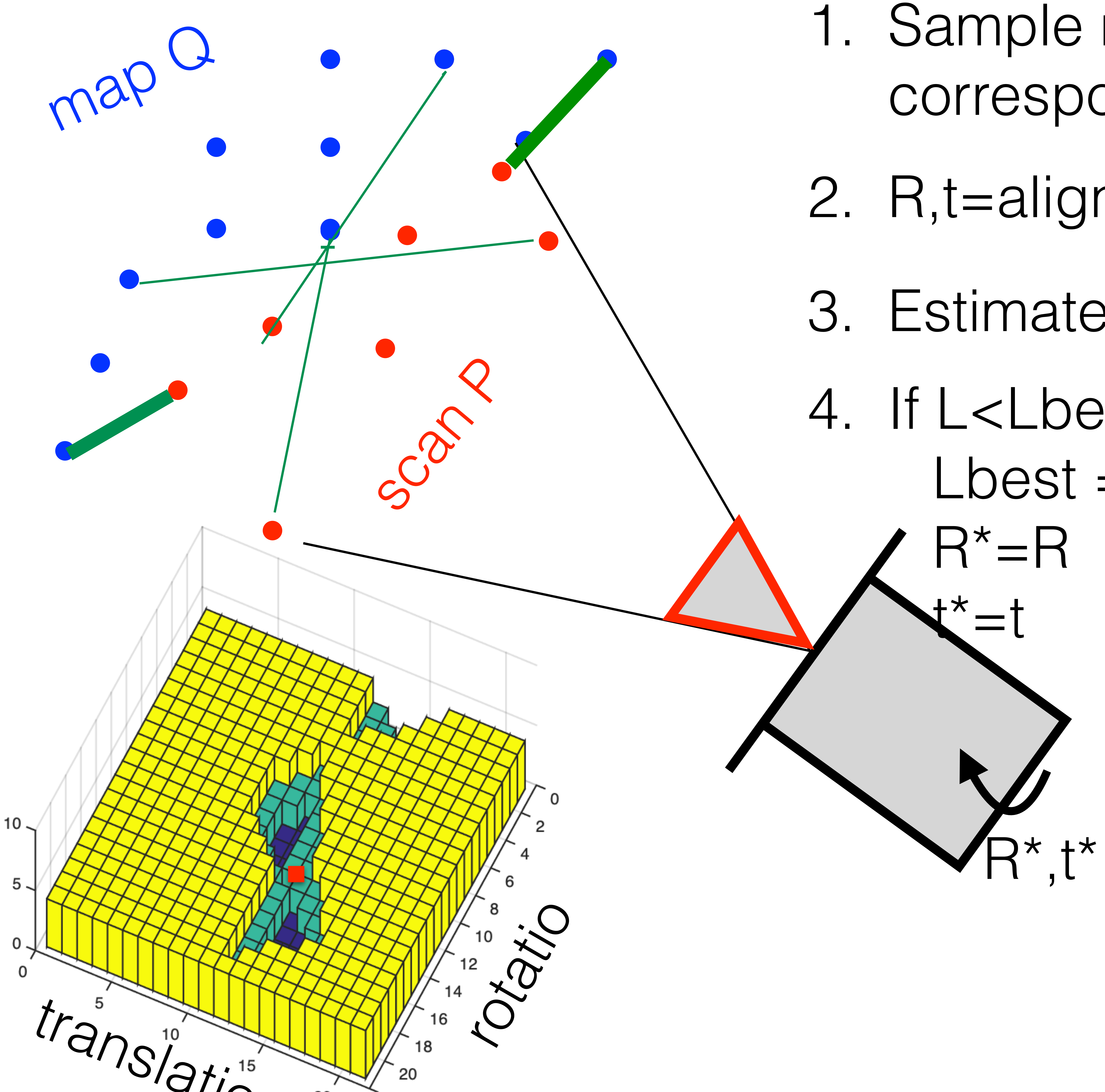
RANSAC (RAndom SAmple Consensus)



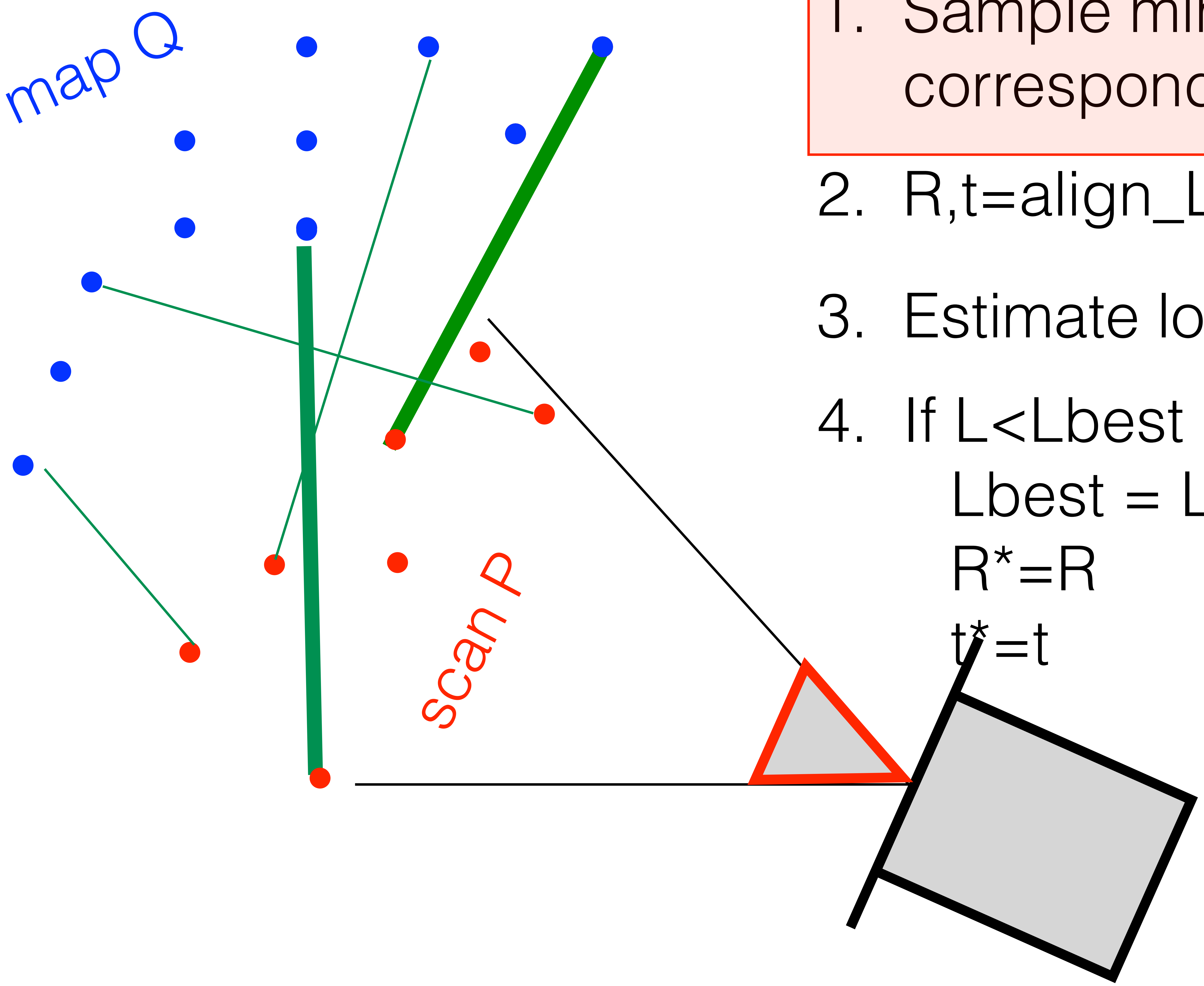
1. Sample minimal subset of correspondences (p, q).
2. R,t=align_L2(p, q)
3. Estimate loss **L=3**

RANSAC (RAndom SAmple Consensus)

1. Sample minimal subset of correspondences (p, q).
2. R,t=align_L2(p, q)
3. Estimate loss **L=3**
4. If L<Lbest
   Lbest = L
   R*=R
   t*=t

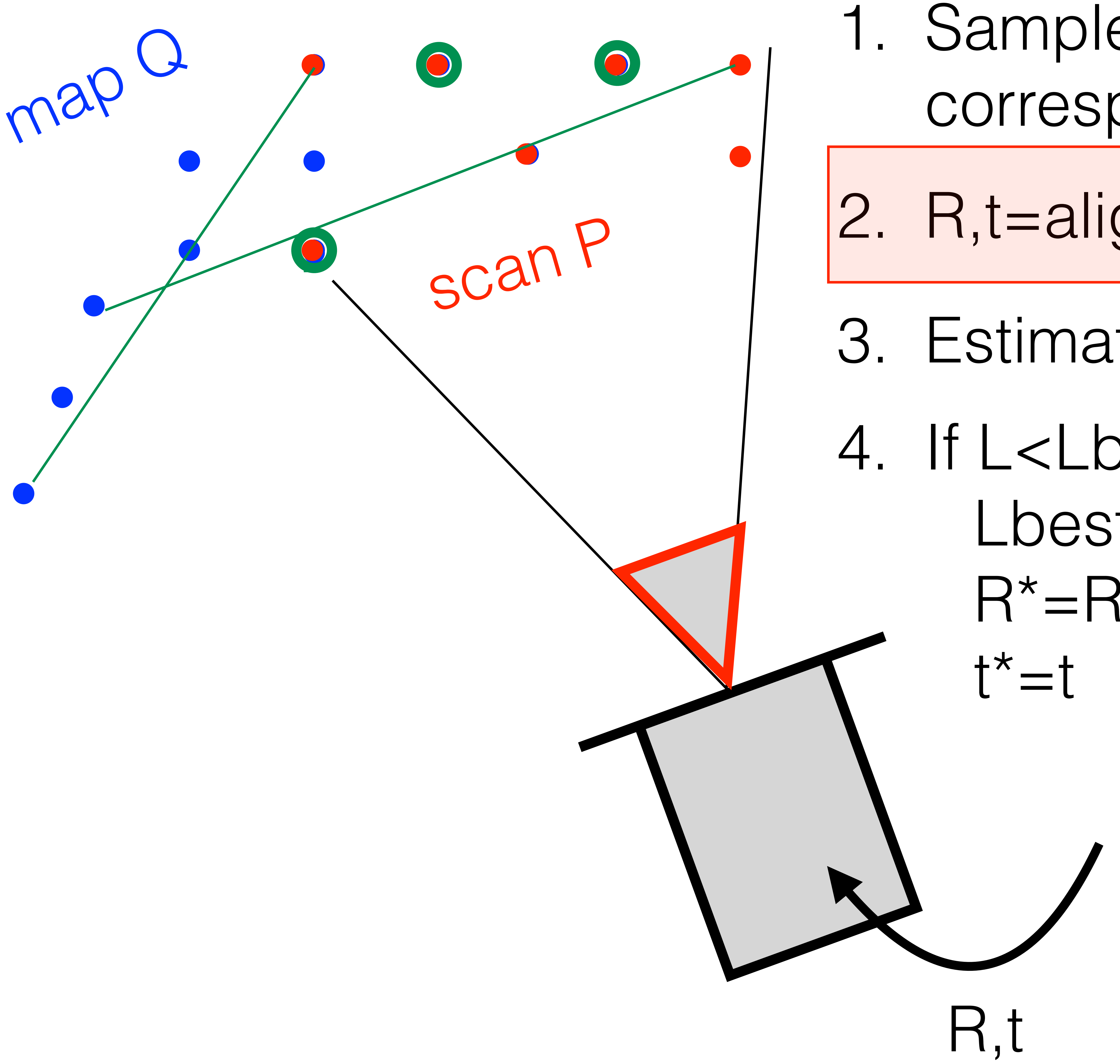


RANSAC



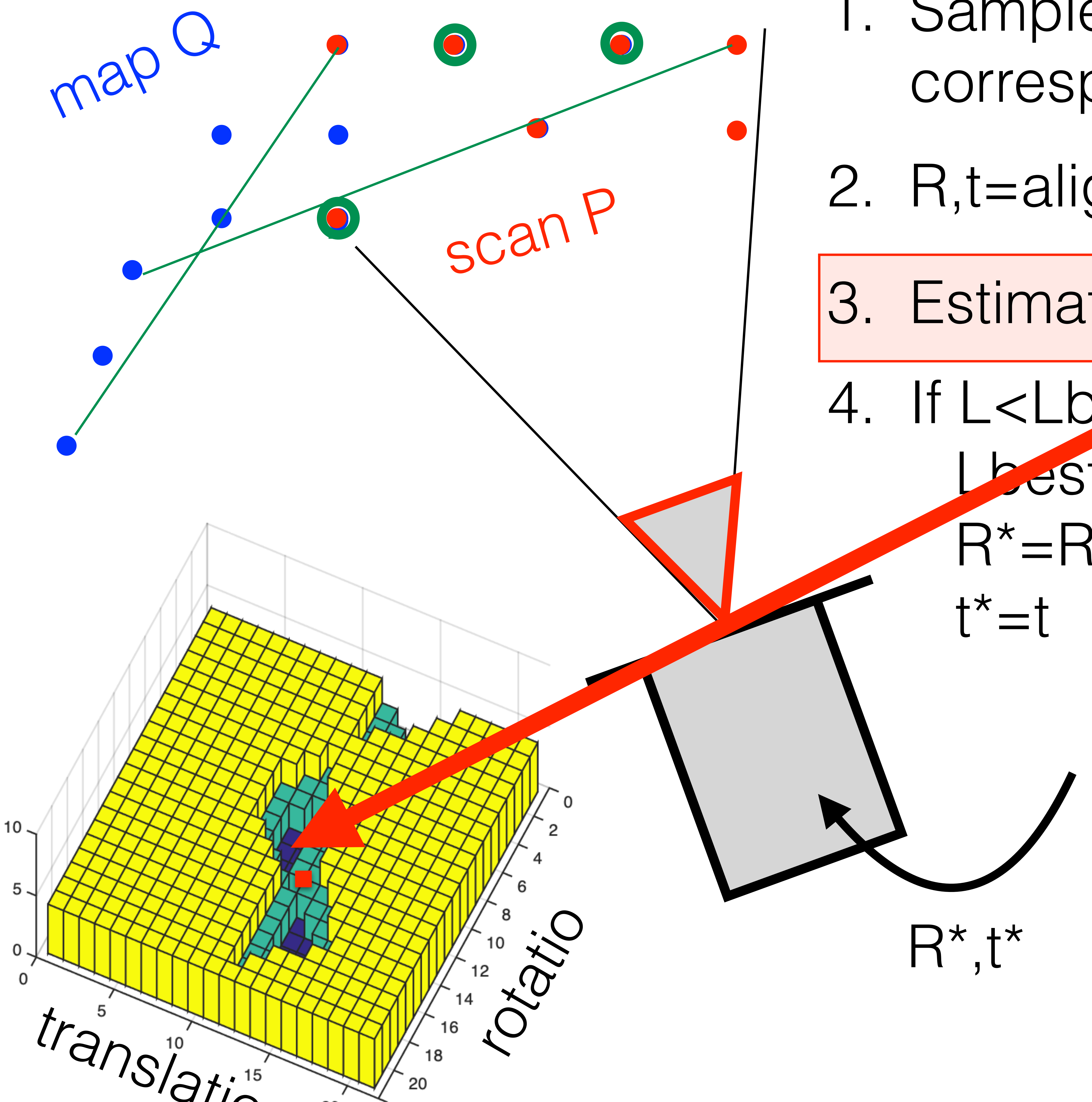
1. Sample minimal subset of correspondences (p, q).
2. R,t=align_L2(p, q)
3. Estimate loss
4. If L<Lbest
   Lbest = L
   R*=R
   t*=t

RANSAC (RAndom SAmple Consensus)



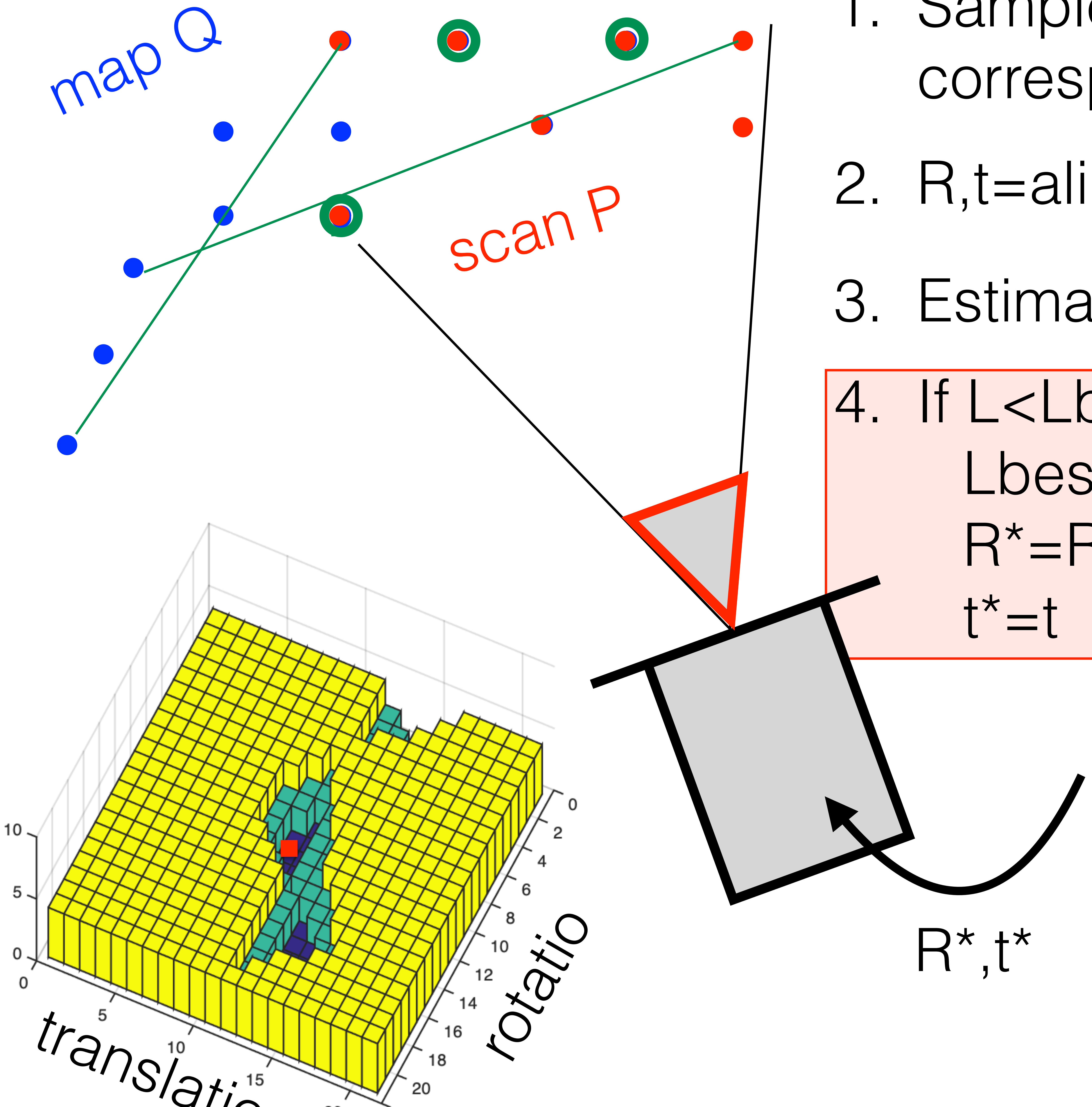
1. Sample minimal subset of correspondences (p, q).
2. R,t=align_L2(p, q)
3. Estimate loss
4. If L<Lbest
   Lbest = L
   R*=R
   t*=t

RANSAC (RAndom SAmple Consensus)



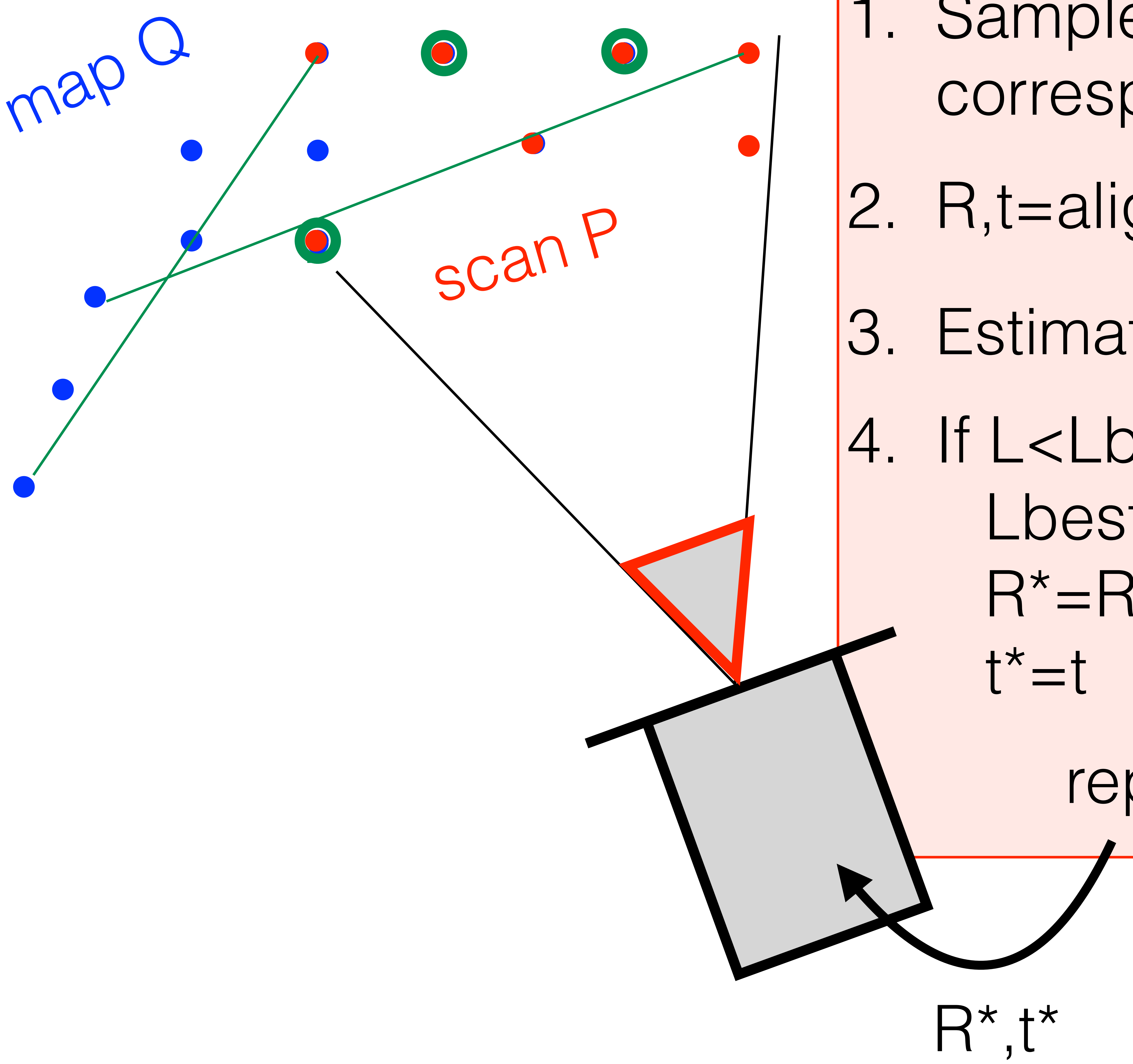
1. Sample minimal subset of correspondences (p, q).
2. R,t=align_L2(p, q)
3. Estimate loss **L=2**
4. If L<Lbest
   Lbest = L
   R*=R
   t*=t

RANSAC (RAndom SAmple Consensus)



1. Sample minimal subset of correspondences (p, q).
2. R,t=align_L2(p, q)
3. Estimate loss **L=2**
4. If L<Lbest
   Lbest = L
   R*=R
   t*=t

RANSAC (RAndom SAmple Consensus)



1. Sample minimal subset of correspondences (p, q).
2. R,t=align_L2(p, q)
3. Estimate loss **L=2**
4. If L<Lbest
   Lbest = L
   R*=R
   t*=t

repeat K-times

RANSAC (RAndom SAmple Consensus)

K … number of trials/iterations

p … probability, that we have selected a clean sample at least once out of K trials.

N … total number of correspondences (N=5)

w … fraction of inliers (w = 3/5 = 0.6)

s … size of |S| (s=2)

$$K = \frac{\log(1-p)}{\log(1-w^s)}$$

Summary

- Minimizing **L2-loss** on unclean correspondences (with **outliers**) yields **biased pose** estimate (and pointcloud alignment).
- Minimizing **robust norms** (Welsch) yields **complicated optimization** due to large plateaus with almost zero gradients.
- When **motion** between successive frames is sufficiently **small** (self-driving cars), odom-initialized **gradient minimization** of a robust loss is quite **OK**.
- When **motion is large** and **correspondences unclean** inlier detection method **RANSAC**, which randomly sample reasonable hypothesis (R,t).
- RANSAC is often used for 2D-2D correspondences and large motions (e.g. reconstruction of 3D world from collection of unordered RGB images).
- **Takehome message:** When designing the loss function always think about:
  A. Underlying probability distribution
  B. Optimization of the resulting landscape

Useful references

- SLAM implementations:
  - Nvidia Issac SLAM: https://github.com/NVIDIA-ISAAC-ROS/isaac_ros_visual_slam
  - ORB SLAM (RGBD SLAM): https://github.com/alsora/ros2-ORB_SLAM2
  - GTSAM (modular factorgraph SLAM implementation in C++) https://gtsam.org/
  - PyPose (modular factorgraph SLAM implementation in Python/Pytorch) https://pypose.org/
- Datasets, benchmarks and challenges:
  - Waymo https://waymo.com/intl/en_us/dataset-download-terms/
  - Kitti http://www.cvlibs.net/datasets/kitti/