Iterative Closest Point SLAM

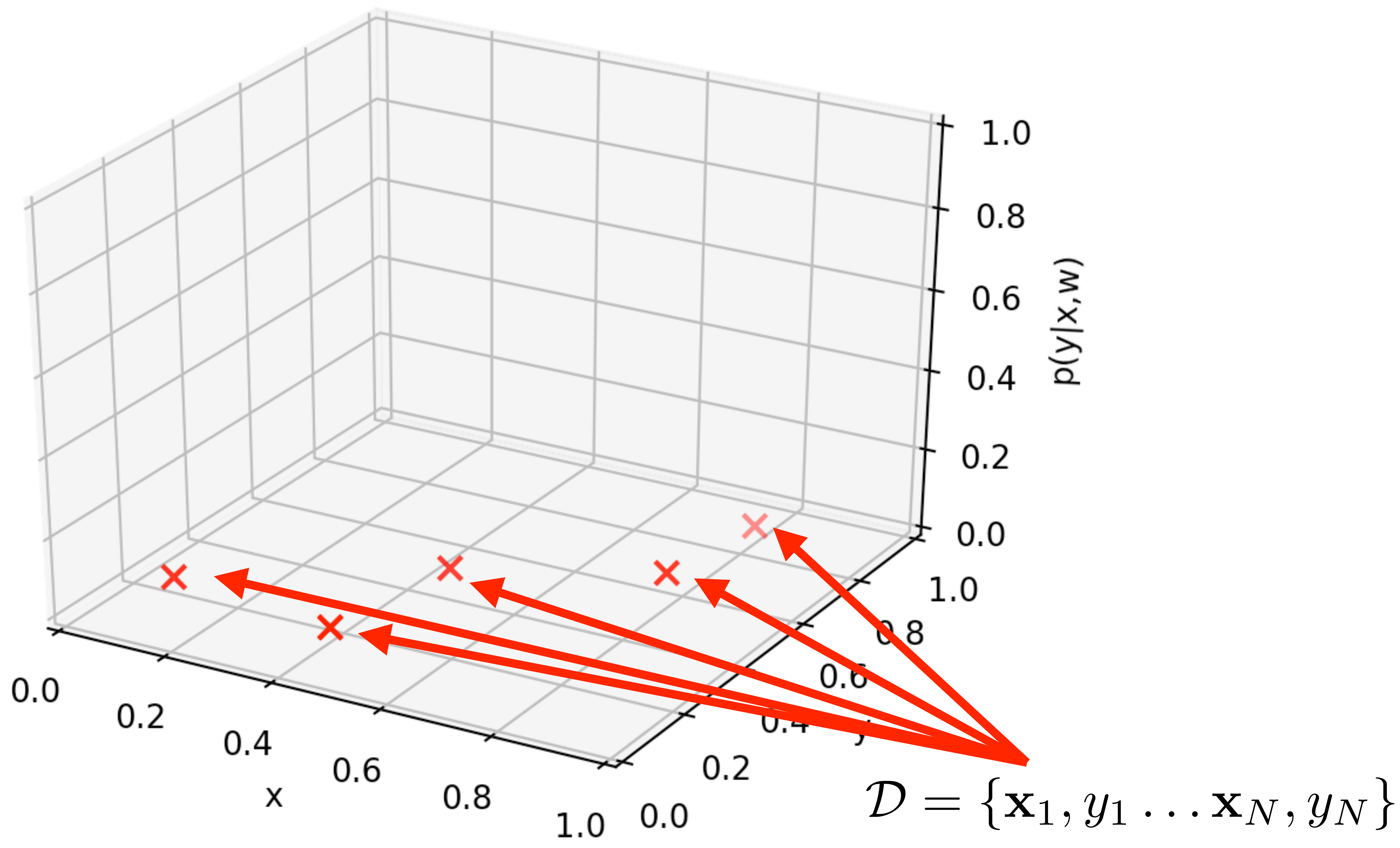
Karel Zimmermann

Prerequisities

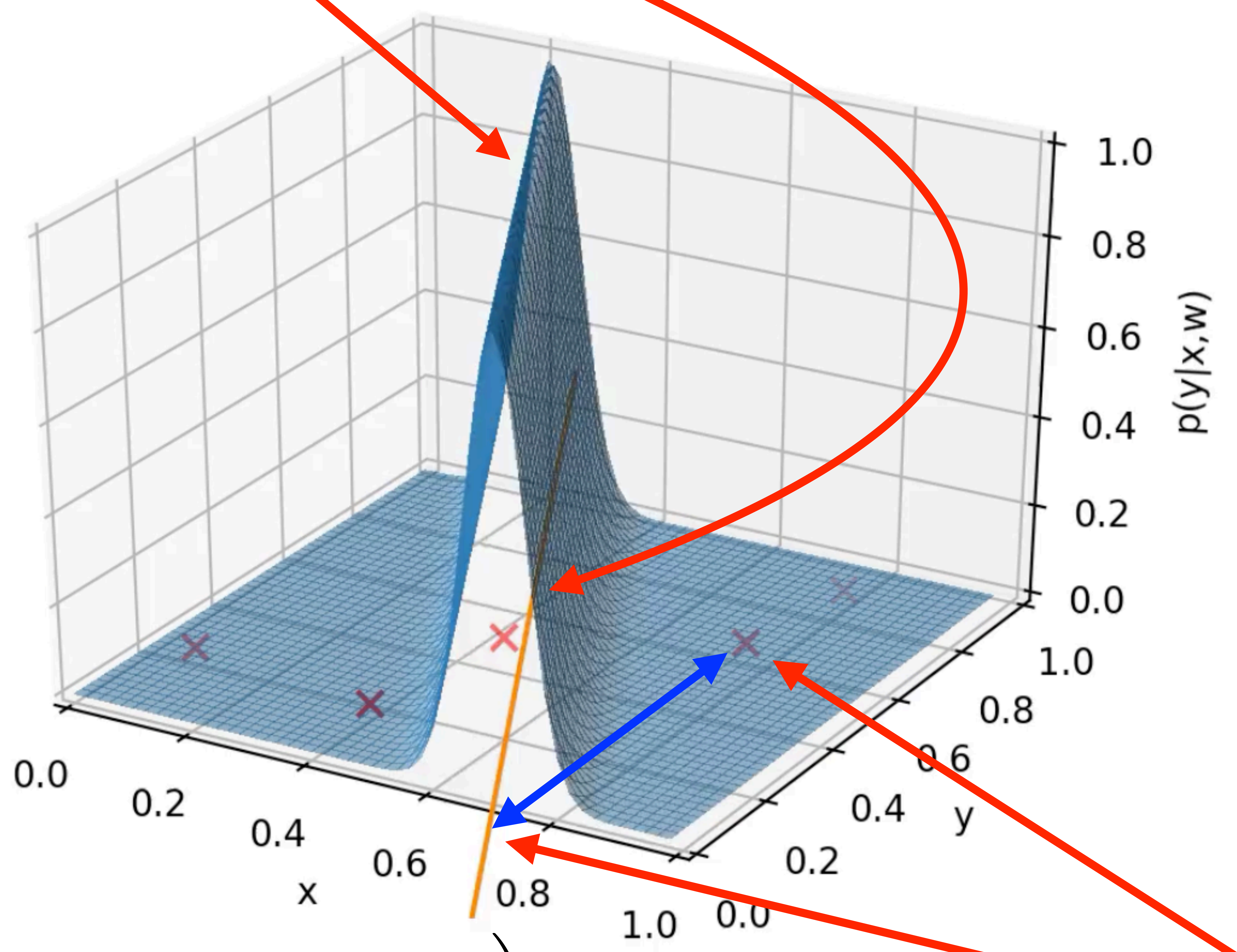
maximizing product of gaussians <=> minimizing the sum of L2 differences.

MLE LS

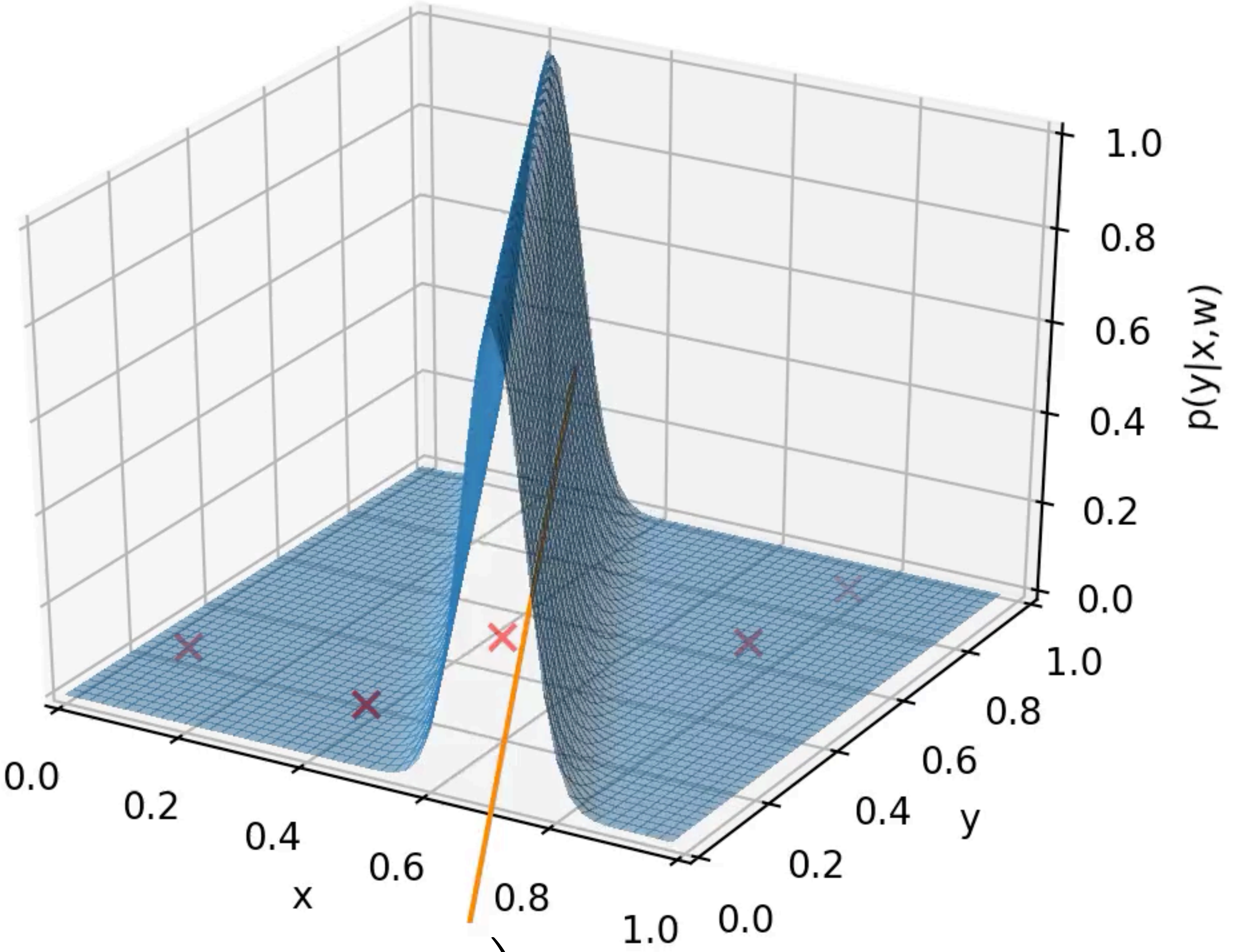
$$\mu^* = \arg\max_{\mu} \left(\prod_i \mathcal{N}_{y_i}(\mu, \sigma^2) \right) = \arg\min_{\mu} \sum_i \|y_i - \mu\|_2^2$$


p(y|x,w)
x
y
0.0
0.2
0.4
0.6
0.8
1.0
$\mathcal{D} = \{\mathbf{x}_1, y_1 \ldots \mathbf{x}_N, y_N\}$

$$\mathcal{N}_y(w_1 x + w_0,\ \sigma^2)$$



$$\mathbf{w}^* = \arg\max_{\mathbf{w}} \left(\prod_i \mathcal{N}_{y_i}(w_1 x_i + w_0, \sigma^2) \right) = \arg\min_{\mathbf{w}} \sum_i (w_1 x_i + w_0 - y_i)^2$$



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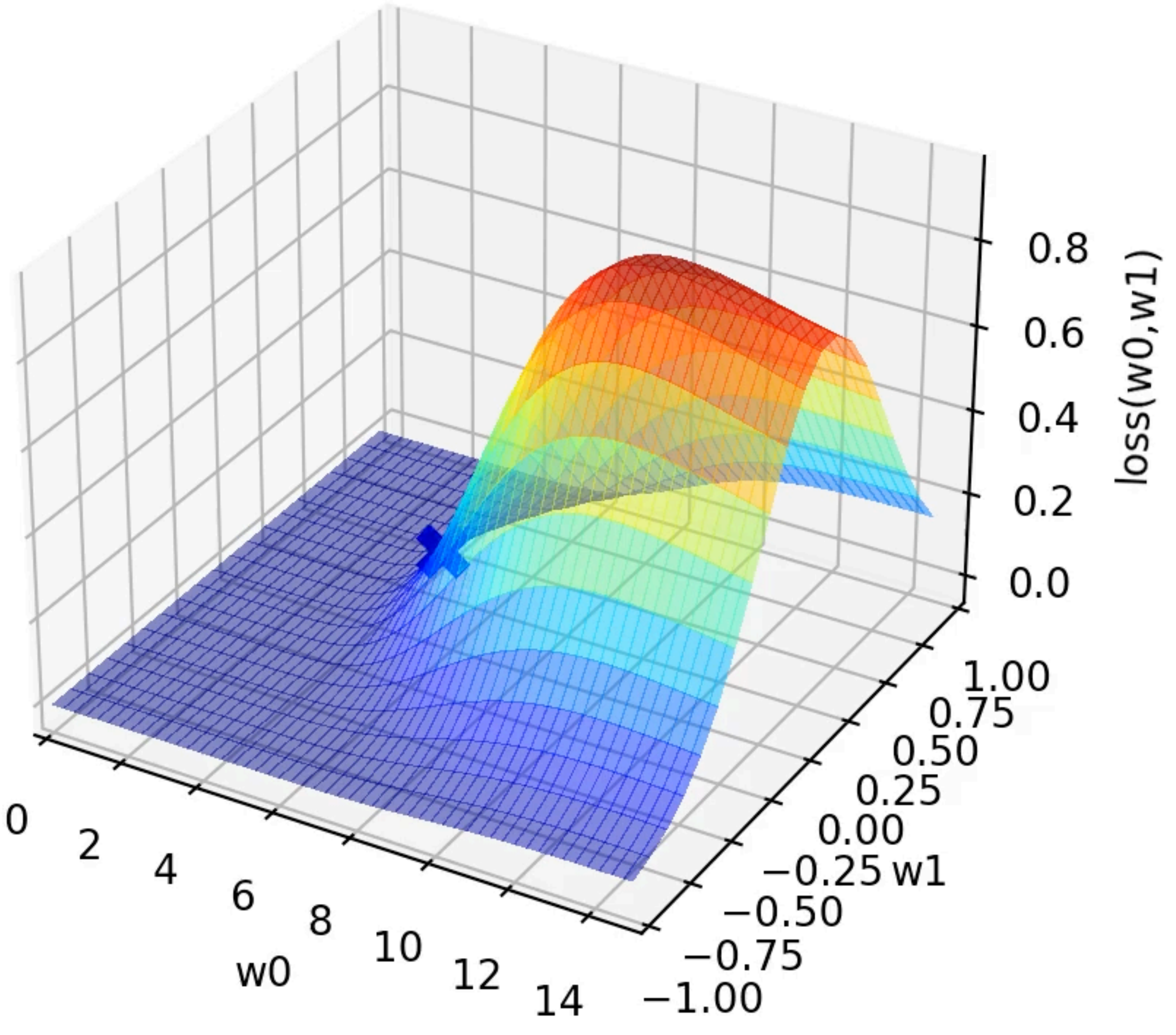
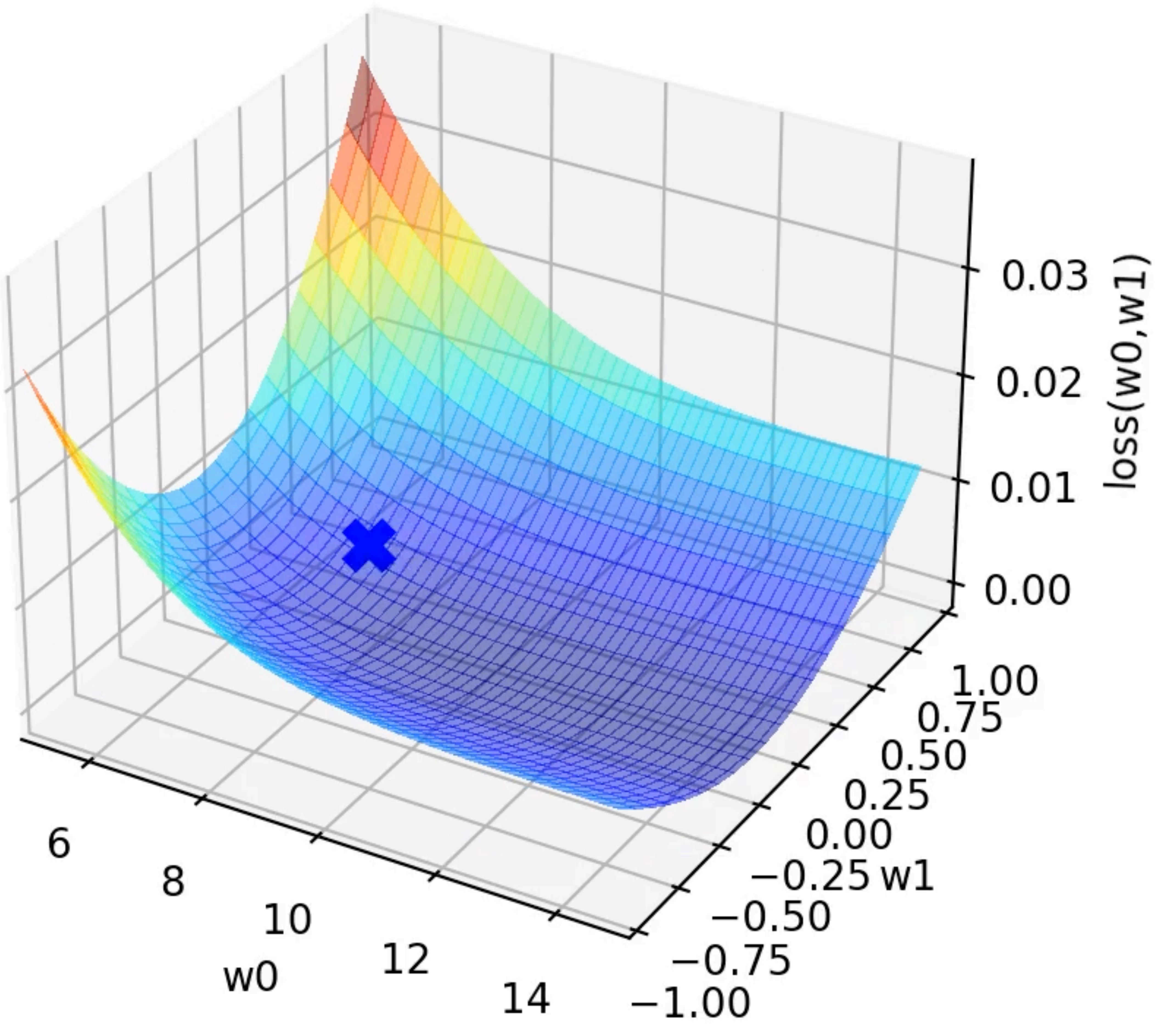


MLE



LS

$$\mathbf{w}^* = \arg\max_{\mathbf{w}} \left(\prod_i \mathcal{N}_{y_i}(w_1 x_i + w_0, \sigma^2) \right) = \arg\min_{\mathbf{w}} \sum_i (w_1 x_i + w_0 - y_i)^2$$

Example: Extended Kalman Filter

Turtlebot transition prob.

$$\underbrace{\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}}_{\mathbf{x}_t} = \underbrace{\begin{bmatrix} x_{t-1} + \frac{v_t}{\omega_t}\big(+ \sin(\theta_{t-1} + \omega_t \Delta t) - \sin(\theta_{t-1})\big) \\ y_{t-1} + \frac{v_t}{\omega_t}\big(- \cos(\theta_{t-1} + \omega_t \Delta t) + \cos(\theta_{t-1})\big) \\ \theta_{t-1} + \omega \Delta t \end{bmatrix}}_{g(\mathbf{u}_t, \mathbf{x}_{t-1})} + \mathcal{N}_{\mathbf{x}_t}(\mathbf{0}, \mathbf{R}_t)$$

WHEEL ENCODERS

/odom

IMU measurement probability

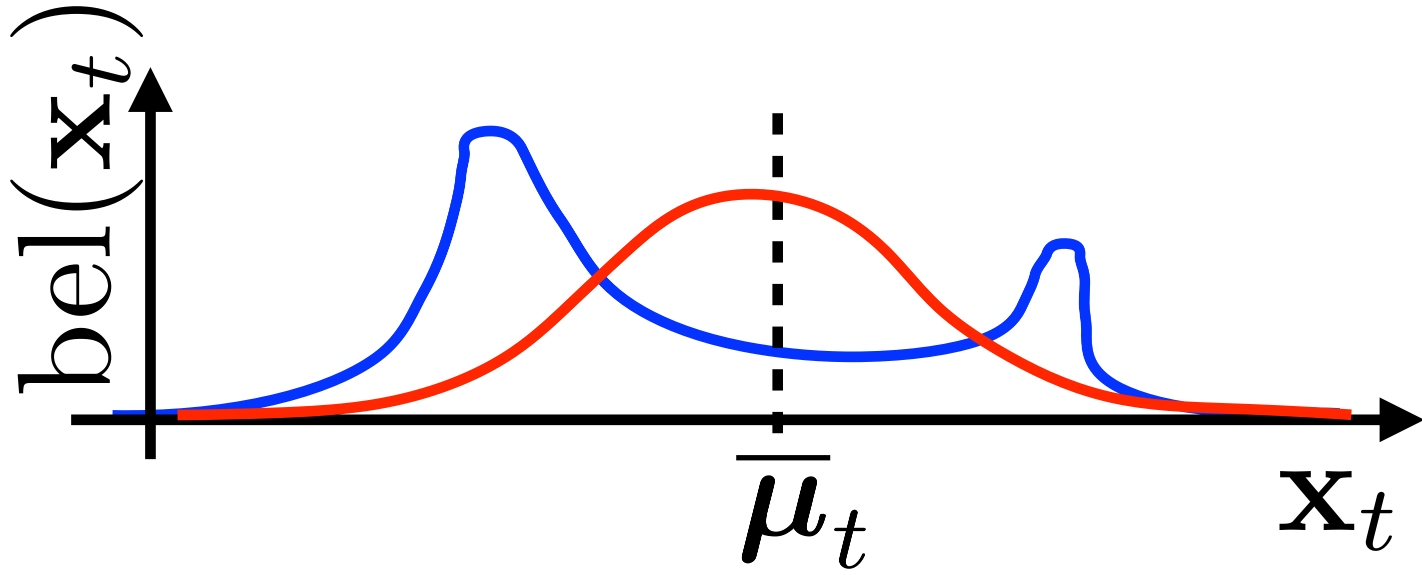
$$\underbrace{\begin{bmatrix} \theta_t^{\mathrm{IMU}} \end{bmatrix}}_{\mathbf{z}_t^{\mathrm{IMU}}} = \underbrace{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}}_{h^{\mathrm{IMU}}(\mathbf{x}_t)} + \mathcal{N}_{\mathbf{z}_t}(0, Q_t^{\mathrm{IMU}})$$

IMU

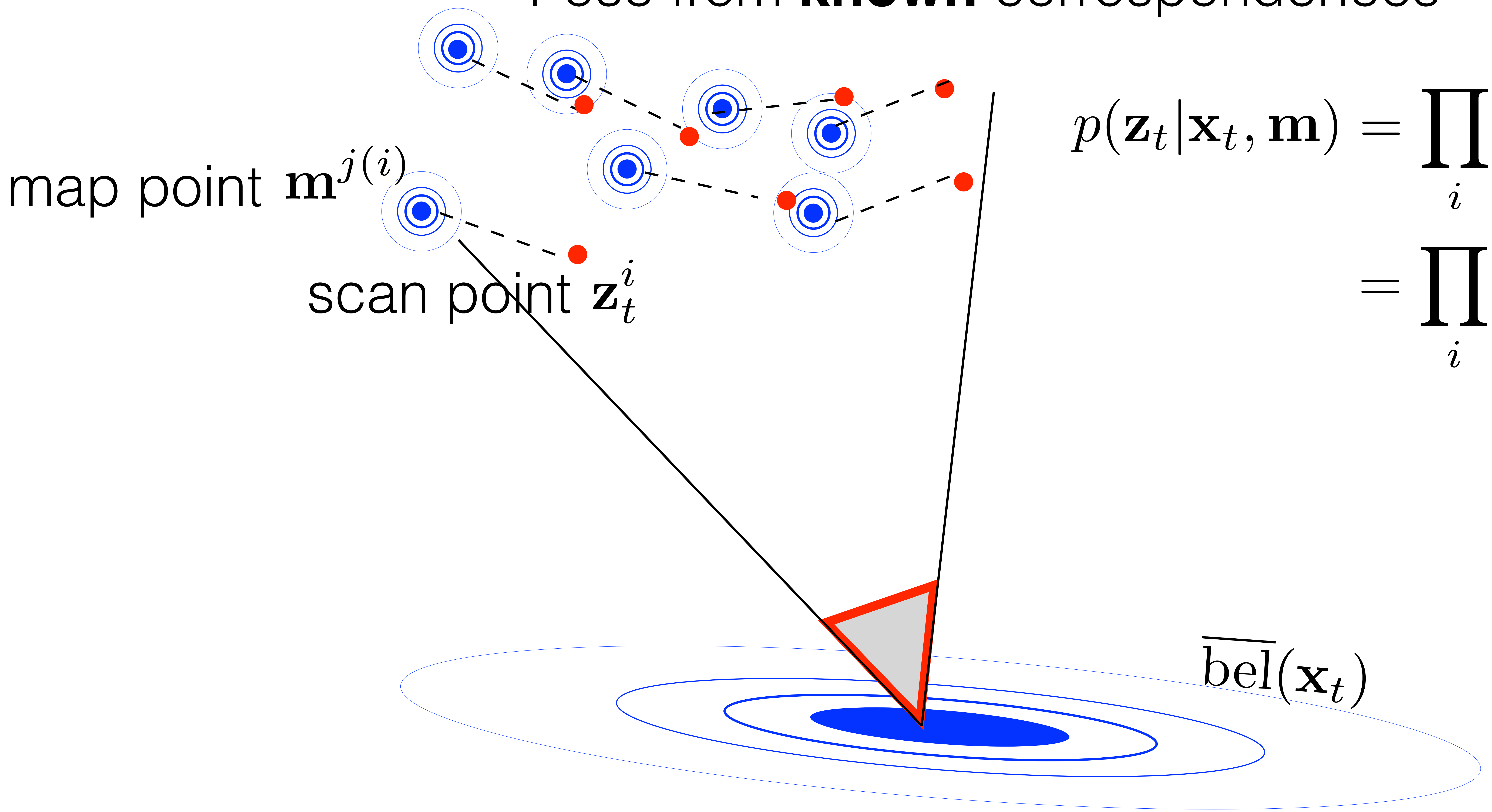
/imu_data

LIDAR measurement probability

$$p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{m})$$



Pose from **known** correspondences



$$p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{m}) = \prod_i p(\mathbf{z}_t^i|\mathbf{x}_t, \mathbf{m}^{j(i)})$$

$$= \prod_i \mathcal{N}_{T(\mathbf{z}_t^i, \mathbf{x}_t)}(\mathbf{m}^{j(i)}, \mathbf{Q}_t^i)$$

$$\mathbf{z}_t^{\text{LIDAR}} = \arg\max_{\mathbf{x}_t} \; p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{m}) \; = \arg\max_{\mathbf{x}_t} \; \prod_i \mathcal{N}_{T(\mathbf{z}_t^i, \mathbf{x}_t)}(\mathbf{m}^{j(i)}, \mathbf{I})$$

$$= \arg\min_{\mathbf{x}_t} \; \|T(\mathbf{z}_t^i, \mathbf{x}_t) - \mathbf{m}^{j(i)}\|_2^2$$ … absolute orientation problem

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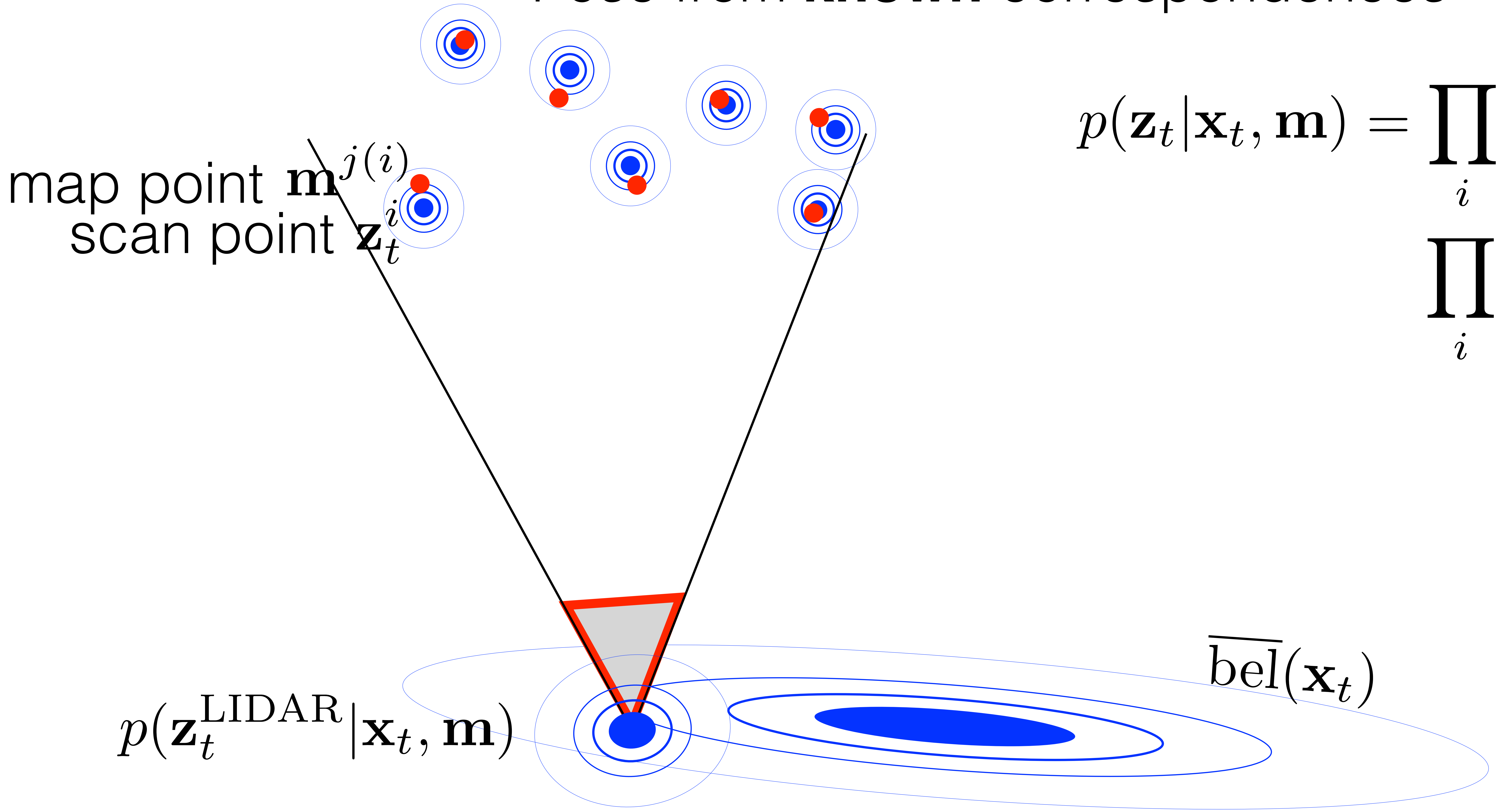

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… absolute orientation problem

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WHEEL ENCODERS

/odom

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IMU

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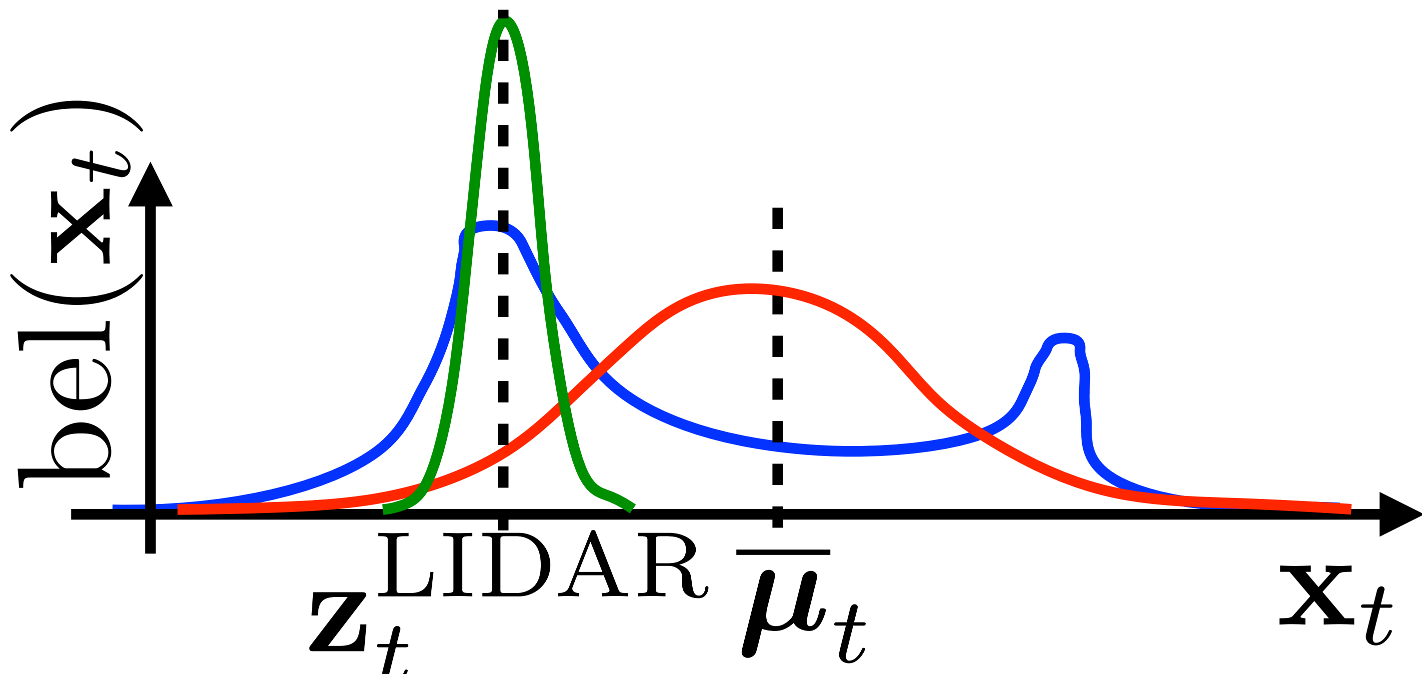
$$\mathbf{z}_t^{\mathrm{LIDAR}} = \arg\max_{\mathbf{x}_t} \; p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{m})$$

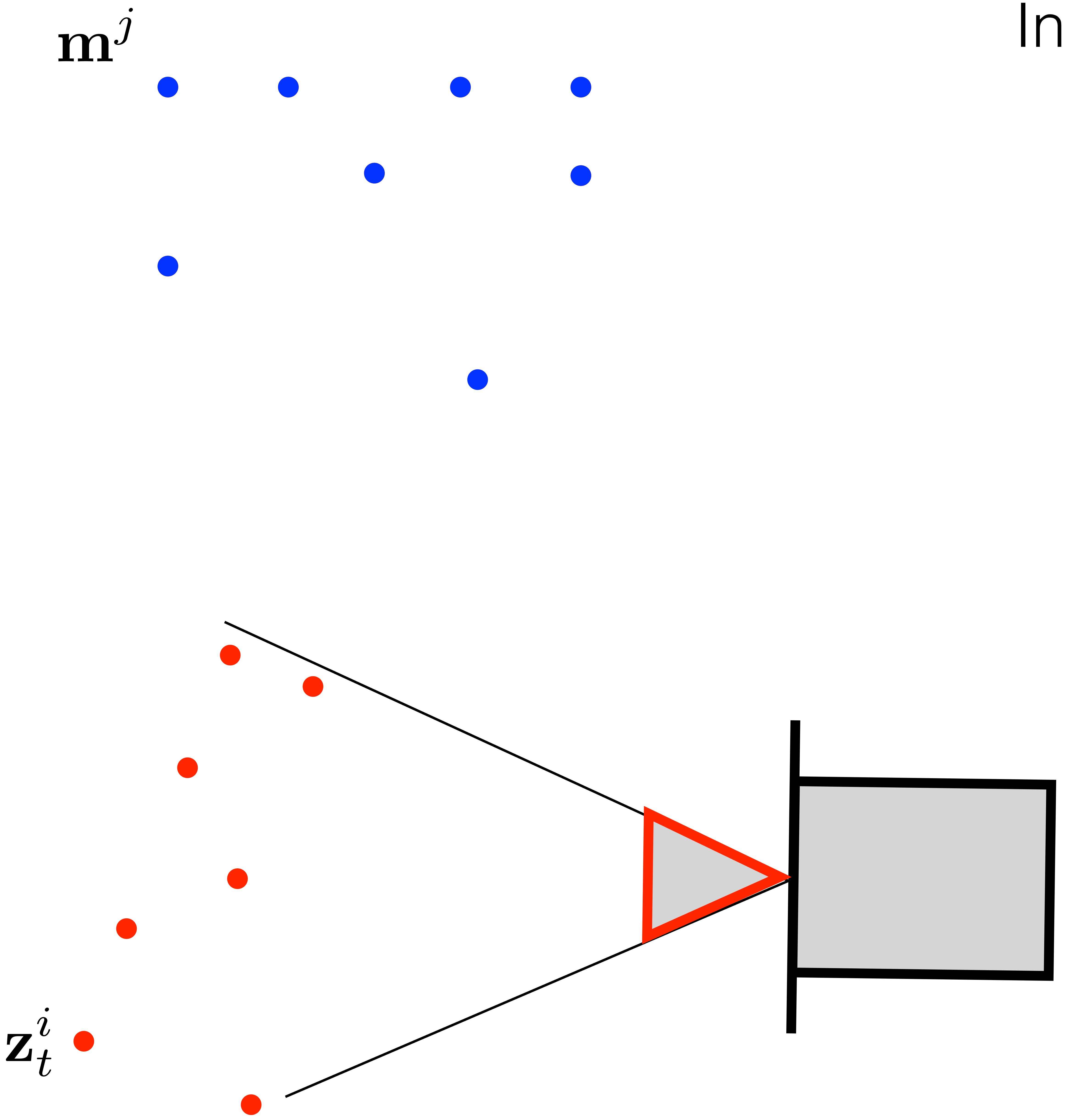
$$\underbrace{\begin{bmatrix} x_t^{\mathrm{LIDAR}} \\ y_t^{\mathrm{LIDAR}} \\ \theta_t^{\mathrm{LIDAR}} \end{bmatrix}}_{\mathbf{z}_t^{\mathrm{LIDAR}}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} + \mathcal{N}_{\mathbf{z}_t}(\mathbf{0}, \mathbf{Q}_t^{\mathrm{LIDAR}})$$

generalized ICP localization / SLAM



Pose from **known** correspondences

Input: • map $\mathbf{m}^j$, scan $\mathbf{z}_t^i$



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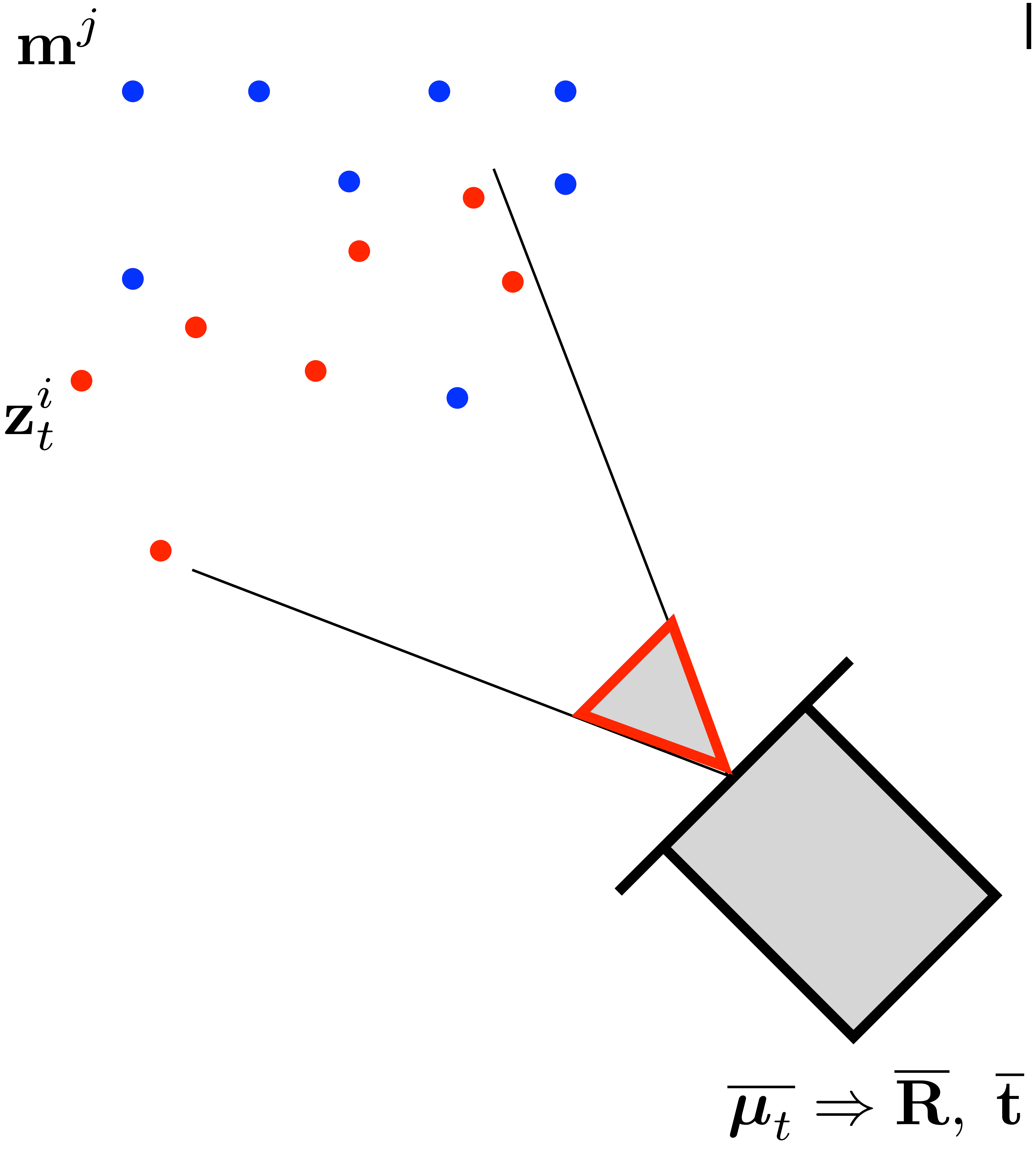
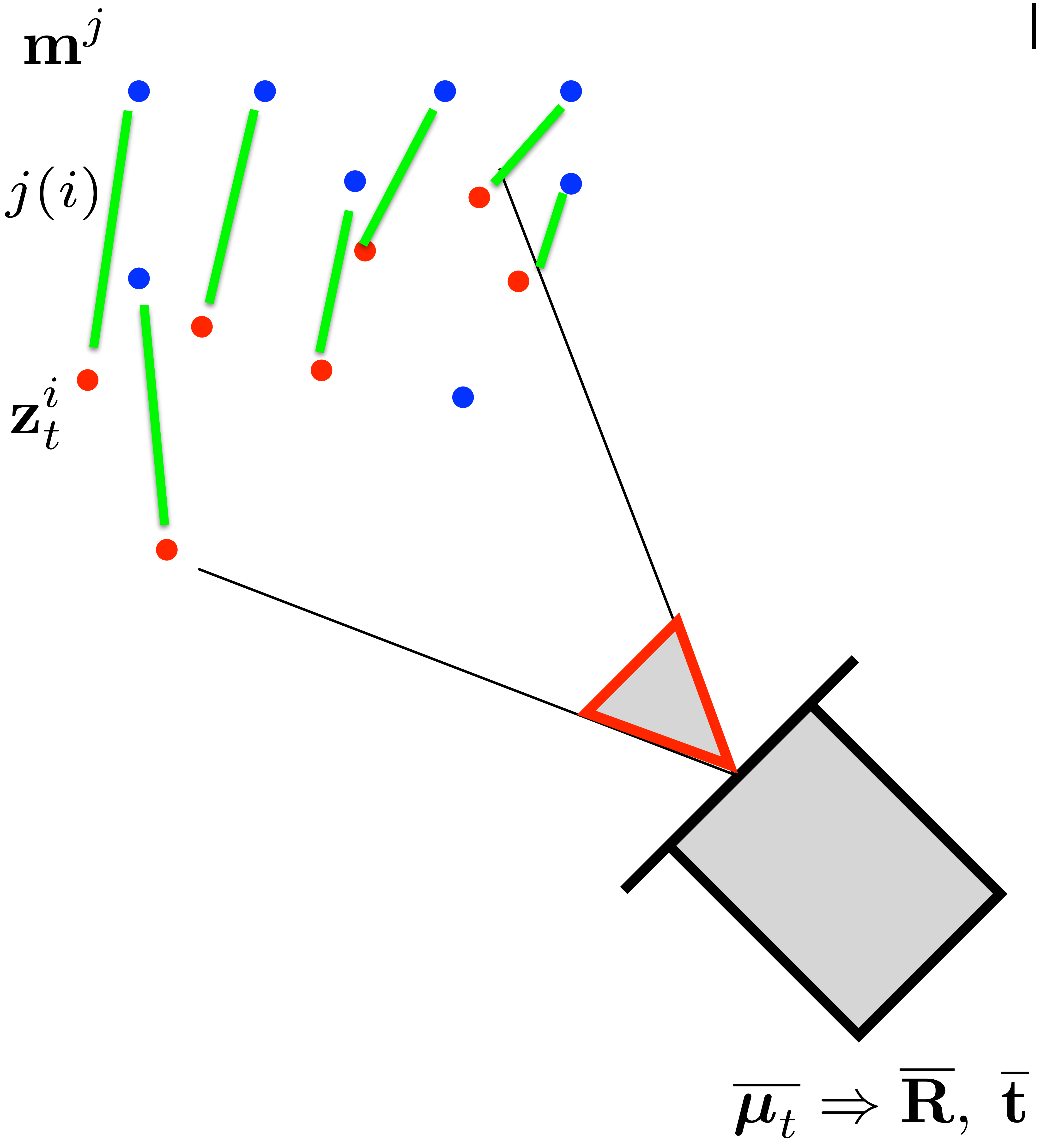


Input:
- map $\mathbf{m}^j$, scan $\mathbf{z}_t^i$
- mean $\overline{\boldsymbol{\mu}_t} \Rightarrow \overline{\mathbf{R}},\ \overline{\mathbf{t}}$

Pose from **known** correspondences



Input:
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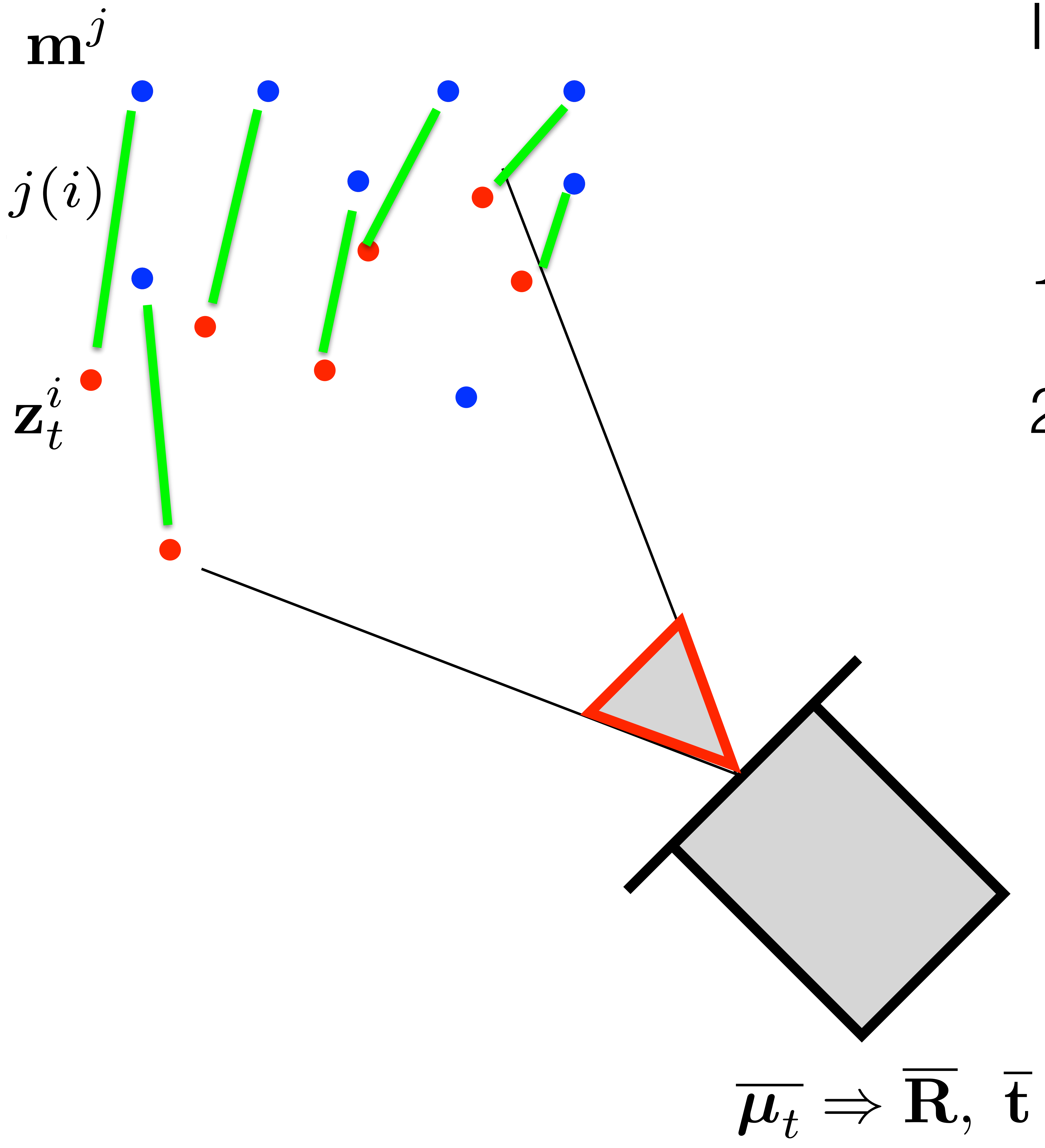
- correspondences $j(i)$

Pose from **known** correspondences



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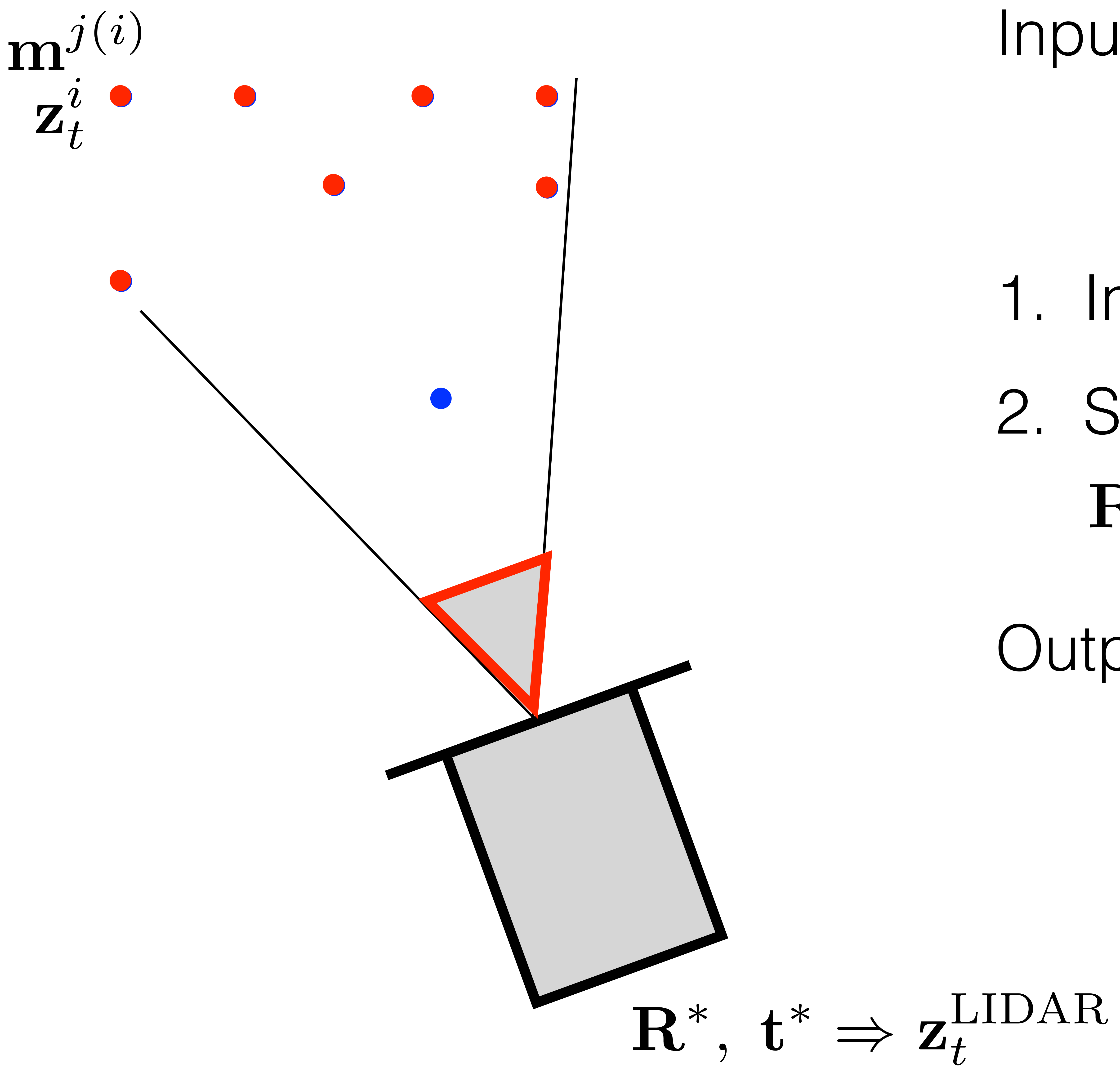
- correspondences $j(i)$

1. Initialize $\mathbf{R} = \overline{\mathbf{R}},\ \mathbf{t} = \overline{\mathbf{t}}$
2. Solve absolute orientation:

$$\mathbf{R}^*, \mathbf{t}^* = \arg\min_{\mathbf{R}\in SO(3), \mathbf{t}\in\mathcal{R}^3} \sum_i \|\mathbf{R}\mathbf{z}_t^i + \mathbf{t} - \mathbf{m}^{j(i)}\|_2^2$$

Pose from **known** correspondences



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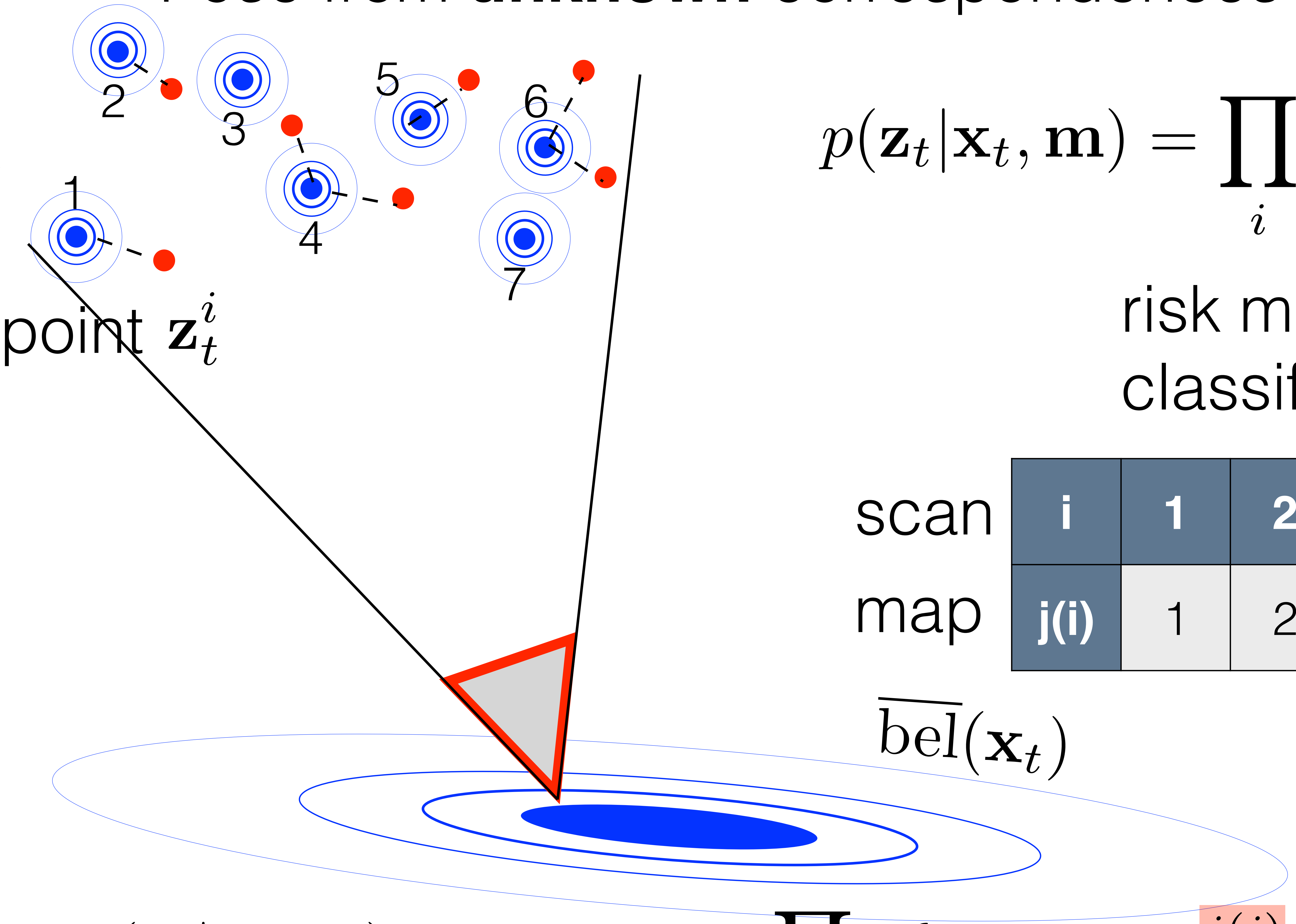
Output:
- posterior mean $\mathbf{R}^*,\ \mathbf{t}^* \Rightarrow \mathbf{z}_t^{\mathrm{LIDAR}}$

Pose from **unknown** correspondences



$$p(\mathbf{z}_t|\mathbf{x}_t,\mathbf{m}) = \prod_i p(\mathbf{z}_t^i|\mathbf{x}_t,\mathbf{m}^{j(i)})$$

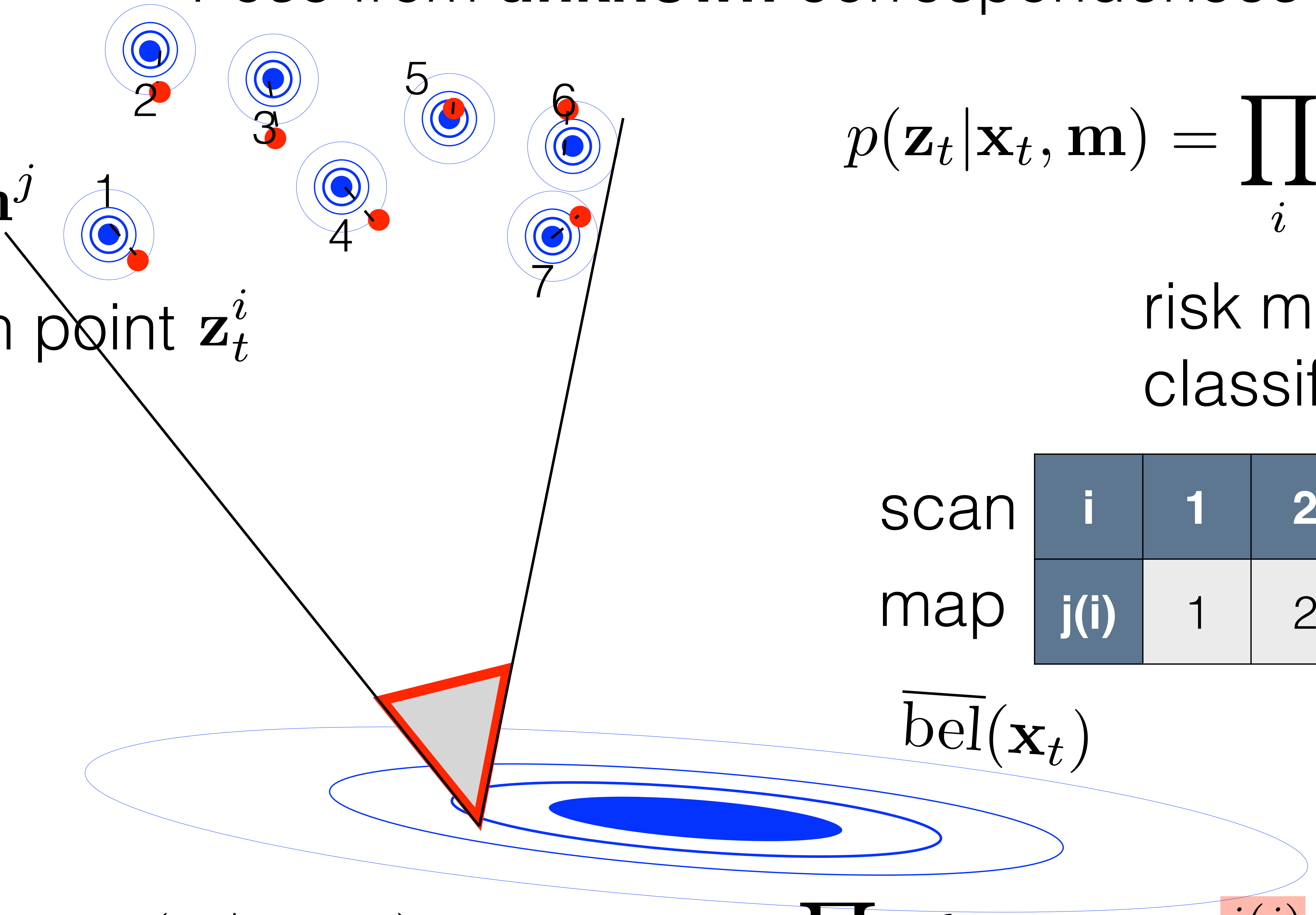
risk minimizing 7-class classification problem

scan	**i**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
map	**j(i)**	1	2	4	4	5	6	7

$$\mathbf{z}_t^{\mathrm{LIDAR}} = \arg\max_{\mathbf{x}_t} p(\mathbf{z}_t|\mathbf{x}_t,\mathbf{m}) = \arg\max_{\mathbf{x}_t, j(i)} \prod_i \mathcal{N}_{T(\mathbf{z}_t^i,\mathbf{x}_t)}(\mathbf{m}^{j(i)},\mathbf{I})$$

1. $\arg\min_{j(i)} \|T(\mathbf{z}_t^i,\mathbf{x}_t) - \mathbf{m}^{j(i)}\|_2^2$ … Nearest neighbour problem
2. $\arg\min_{\mathbf{x}_t} \|T(\mathbf{z}_t^i,\mathbf{x}_t) - \mathbf{m}^{j(i)}\|_2^2$ … Known absolute orientation problem

Pose from **unknown** correspondences



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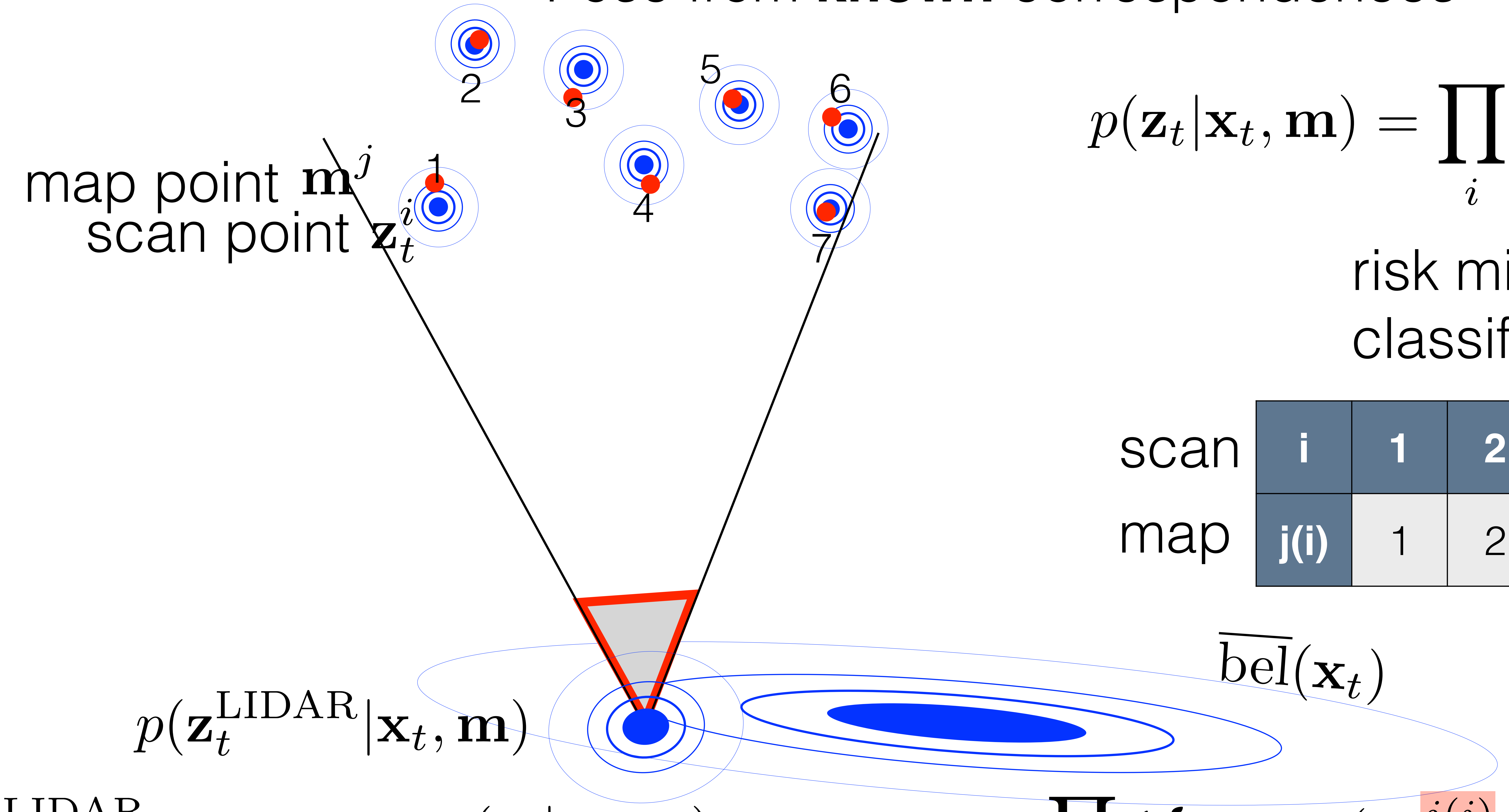
Pose from **known** correspondences



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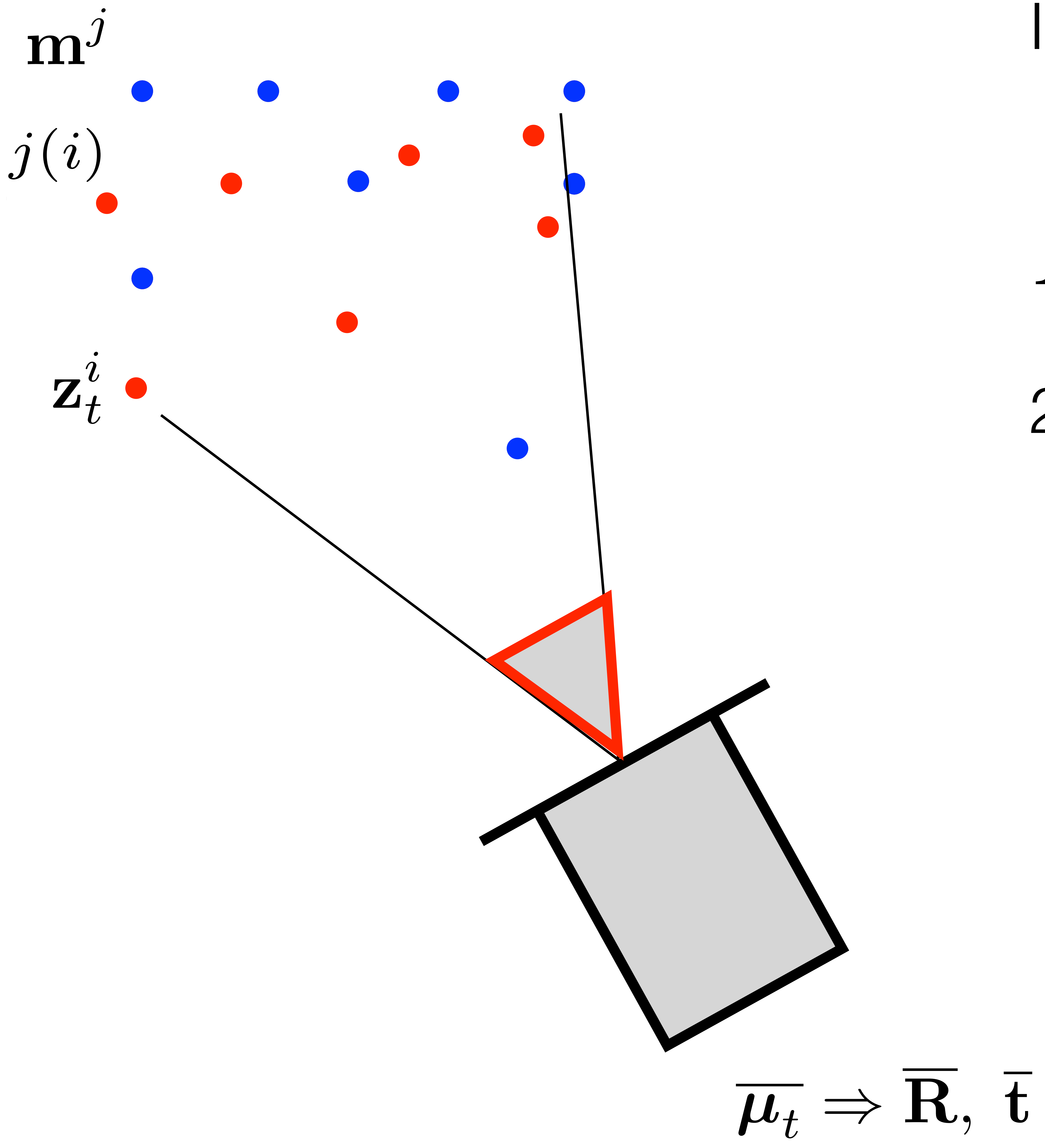
Pose from **unknown** correspondences



Input:
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- ~~correspondences $j(i)$~~

1. Initialize $\mathbf{R}^* = \overline{\mathbf{R}},\ \mathbf{t}^* = \overline{\mathbf{t}}$
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$$j(i)^* = \underset{j(i) \in J}{\arg\min} \sum_i \|\mathbf{R}^* \mathbf{z}_t^i + \mathbf{t}^* - \mathbf{m}^{j(i)}\|_2^2$$

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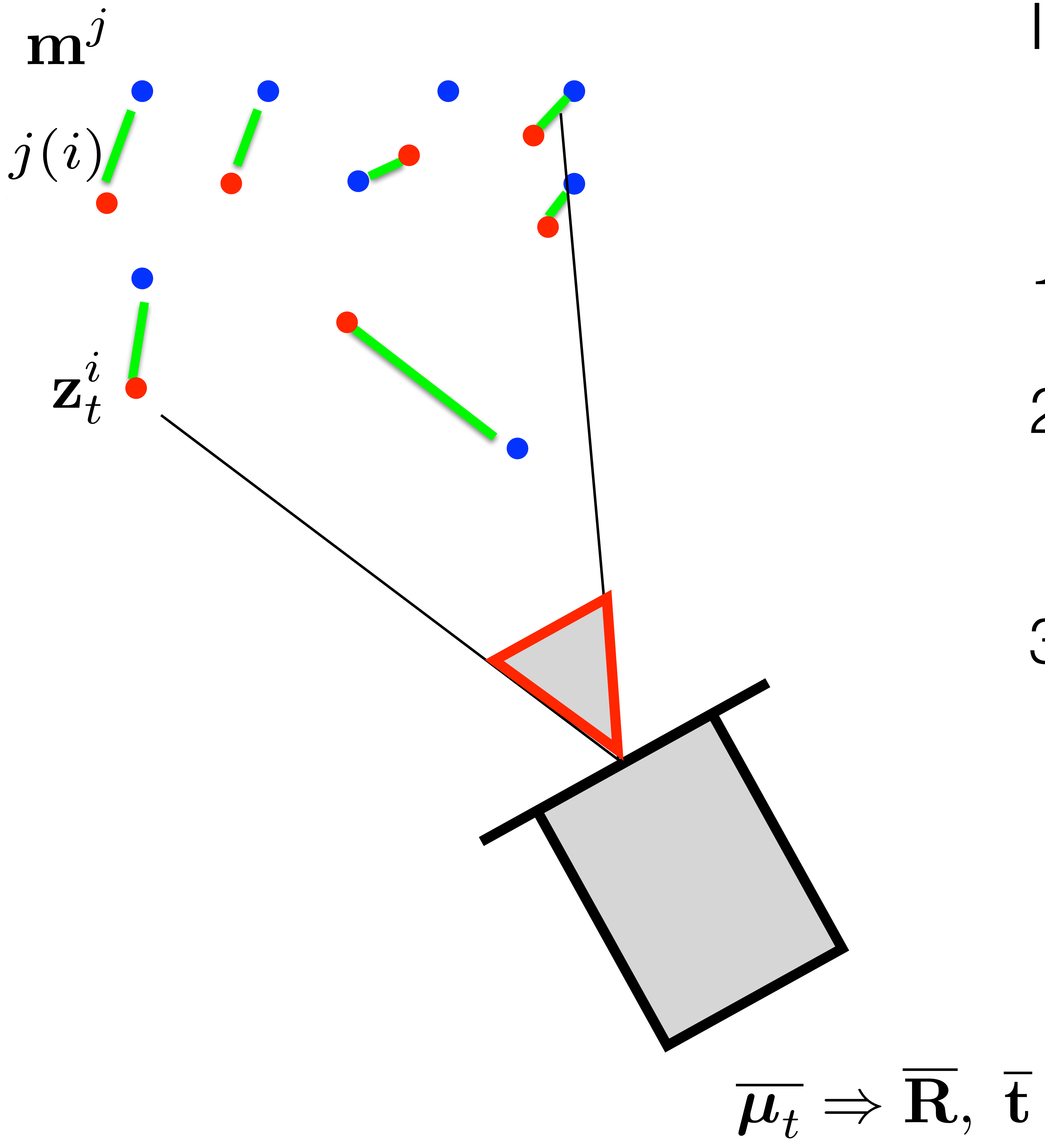

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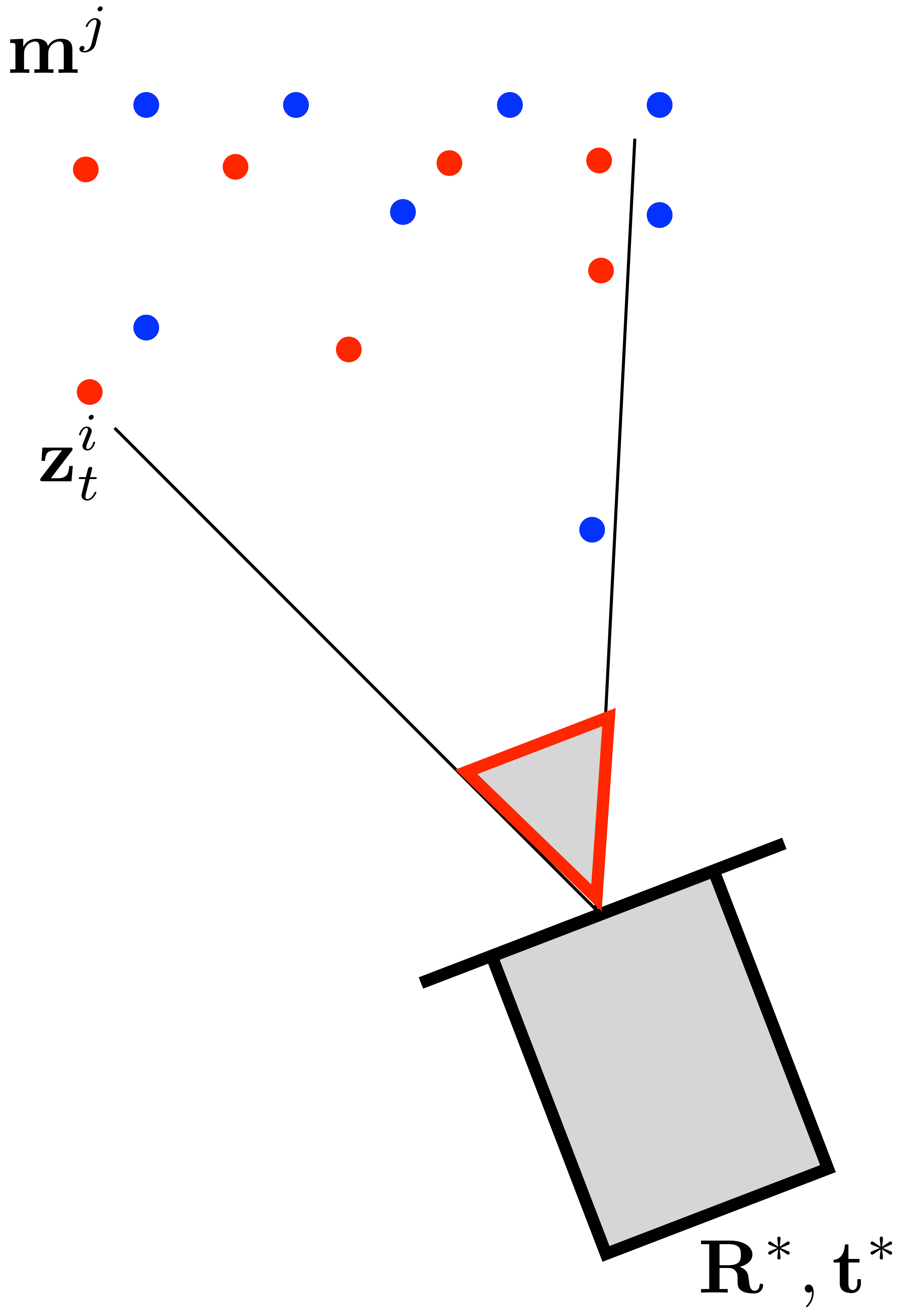
$$\mathbf{R}^*, \mathbf{t}^* = \underset{\mathbf{R} \in SO(3), \mathbf{t} \in \mathcal{R}^3}{\arg\min} \sum_i \|\mathbf{R}\mathbf{z}_t^i + \mathbf{t} - \mathbf{m}^{j(i)^*}\|_2^2$$

Pose from **unknown** correspondences



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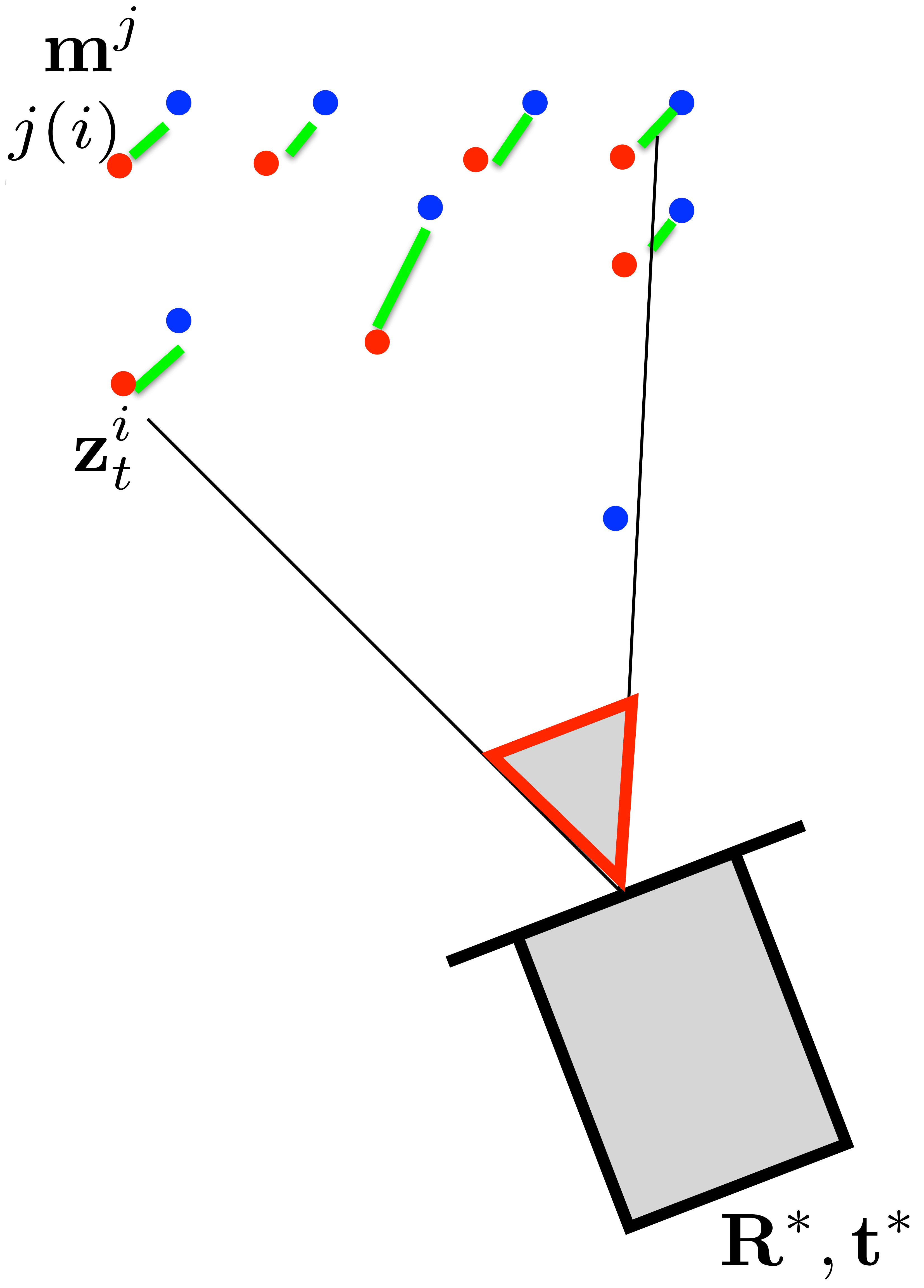

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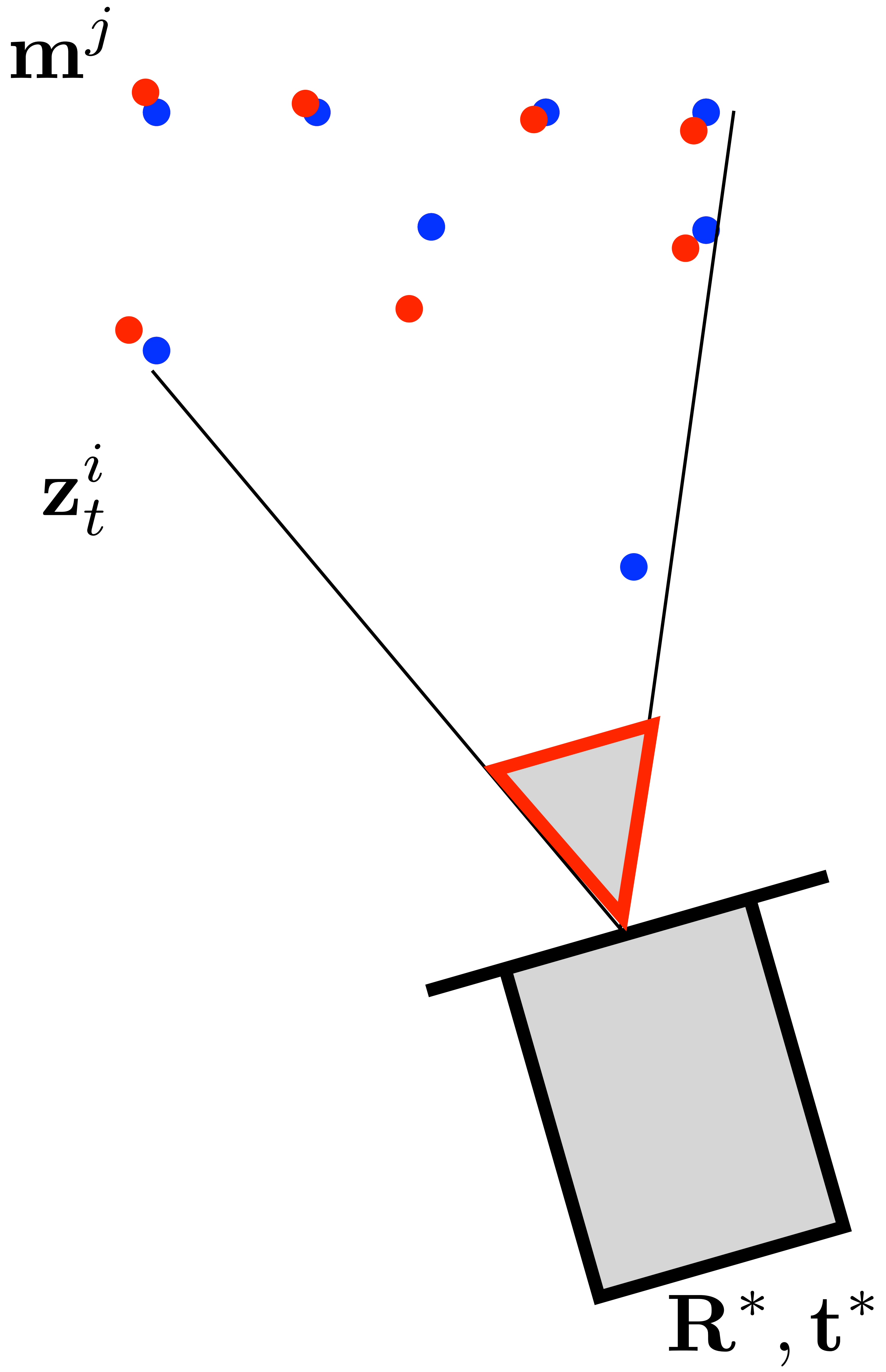

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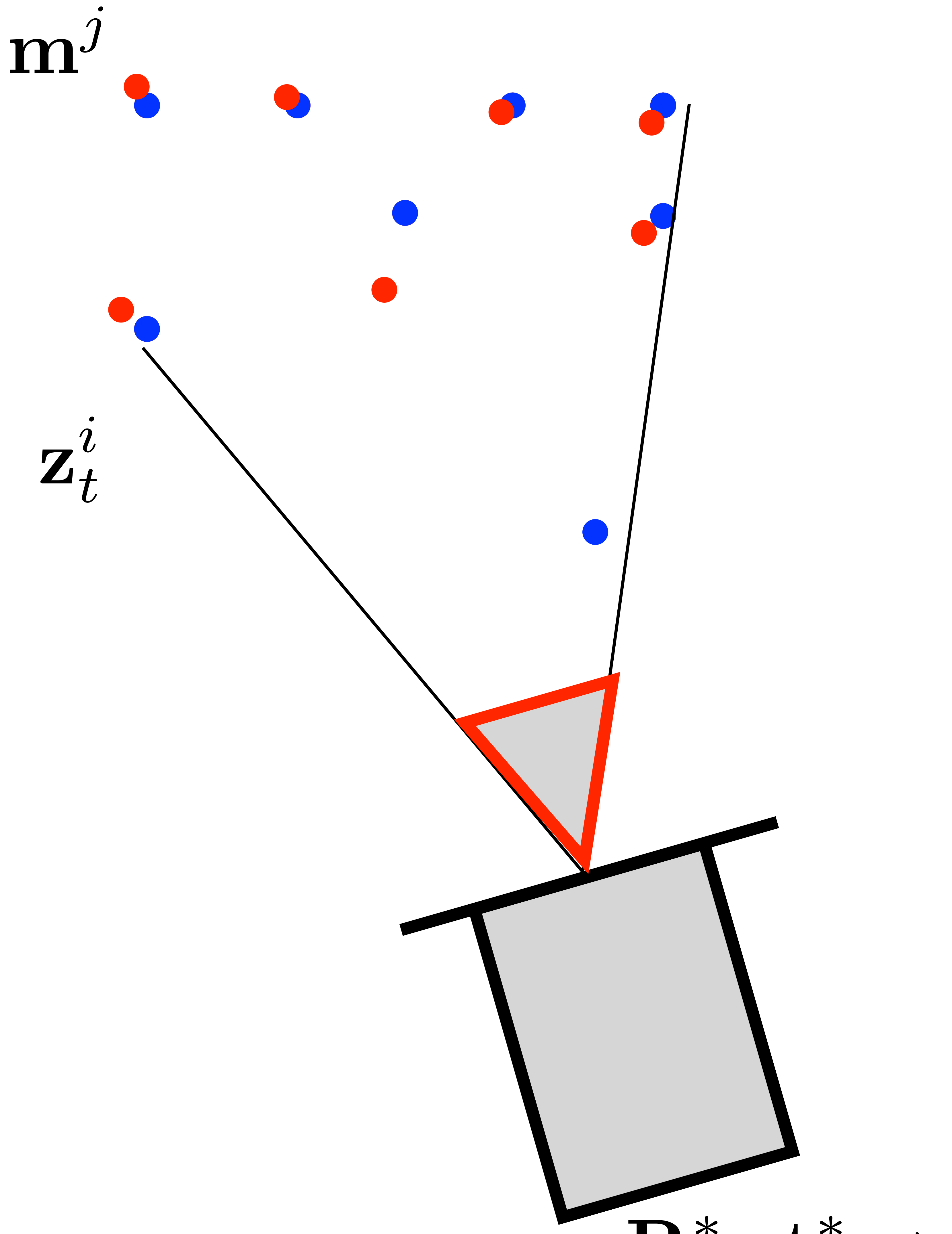
iterate

Pose from **unknown** correspondences



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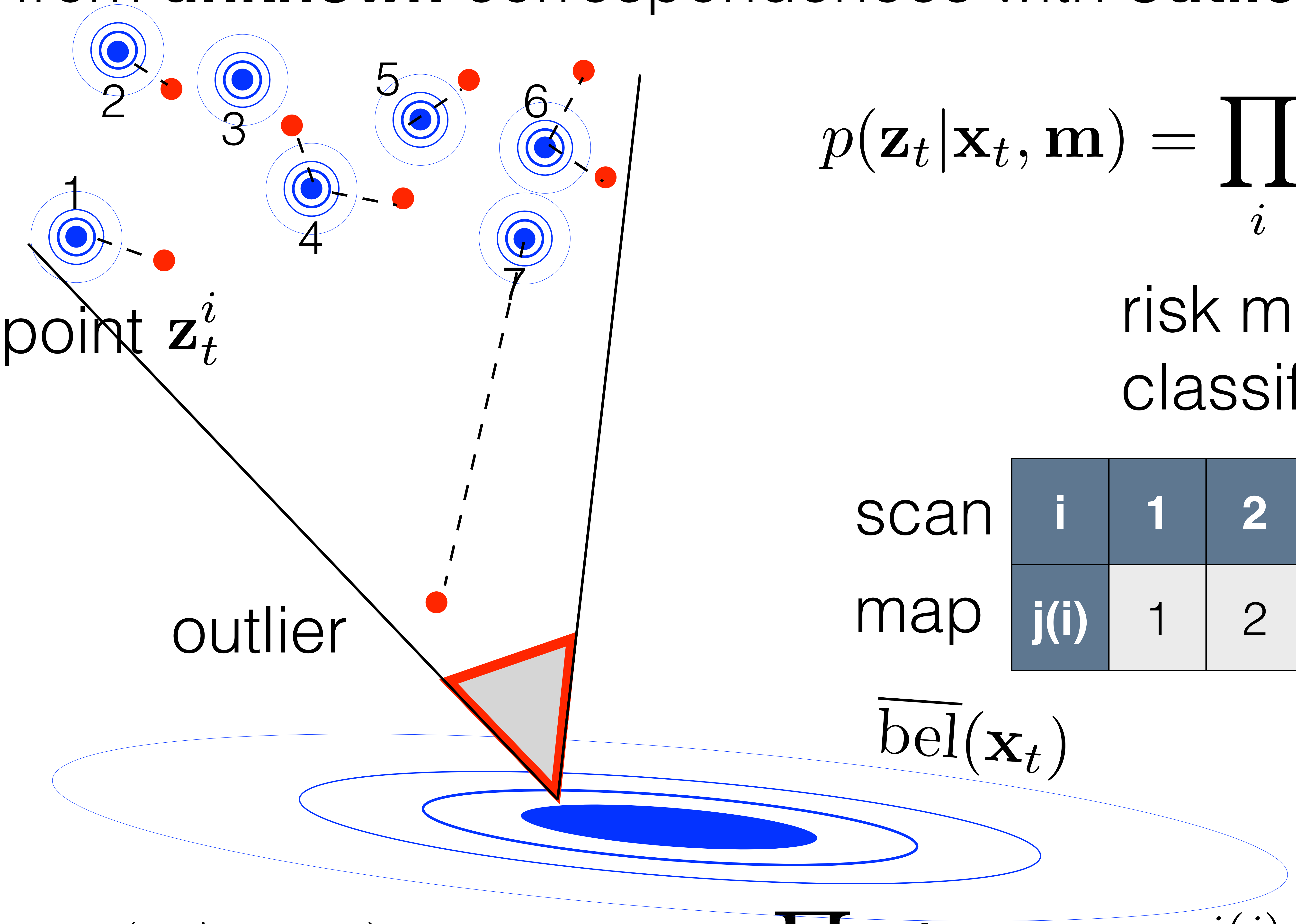
Output:
- posterior mean $\mathbf{R}^*,\ \mathbf{t}^* \Rightarrow \mathbf{z}_t^{\text{LIDAR}}$

Pose from **unknown** correspondences with **outlier** rejection



$$p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{m}) = \prod_i p(\mathbf{z}_t^i|\mathbf{x}_t, \mathbf{m}^{j(i)})$$

risk minimizing 7-class classification problem

scan	i	1	2	3	4	5	6	7	8
map	j(i)	1	2	4	4	5	6	6	7

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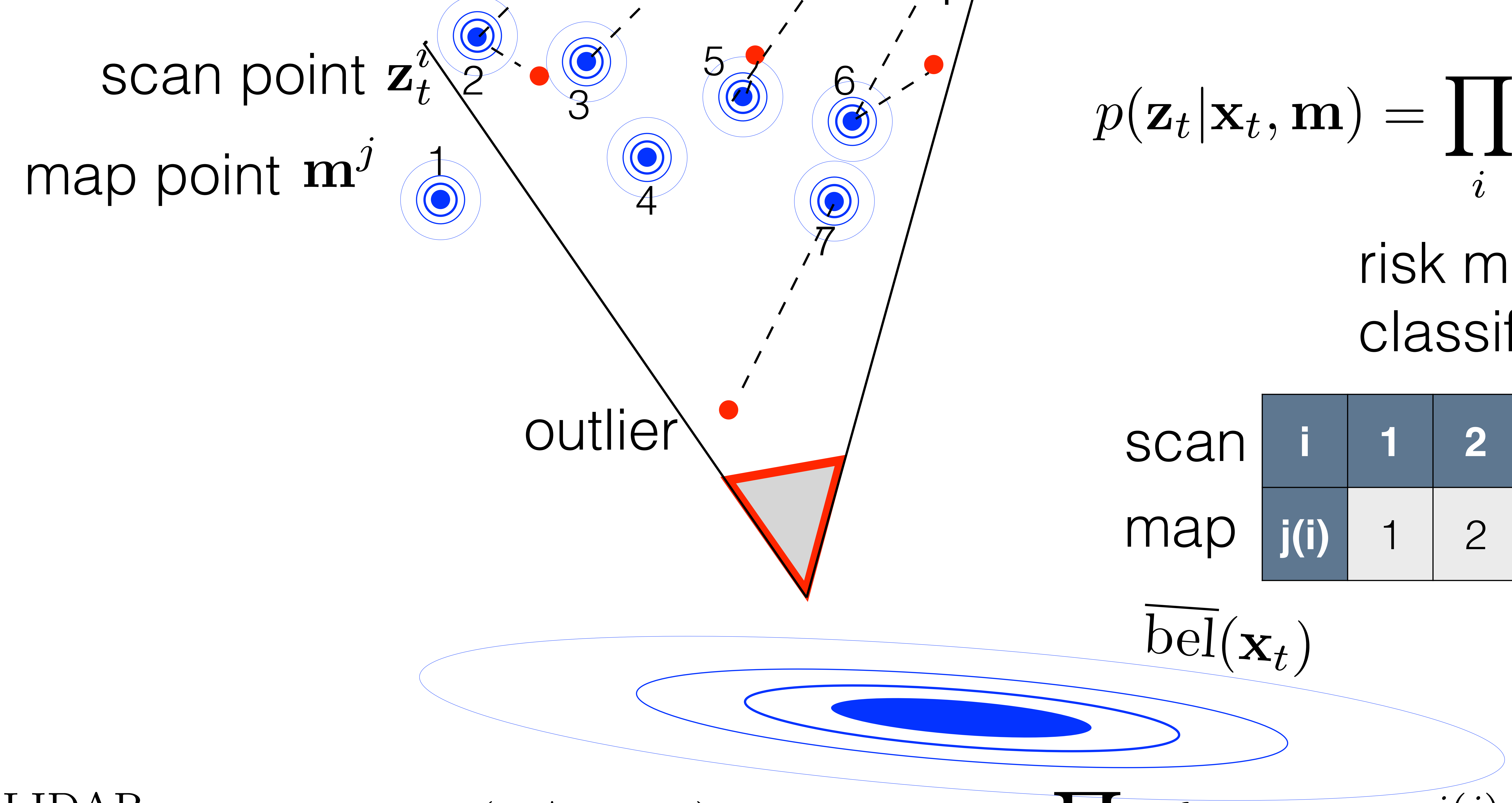


$$p(\mathbf{z}_t|\mathbf{x}_t,\mathbf{m}) = \prod_i p(\mathbf{z}_t^i|\mathbf{x}_t,\mathbf{m}^{j(i)})$$

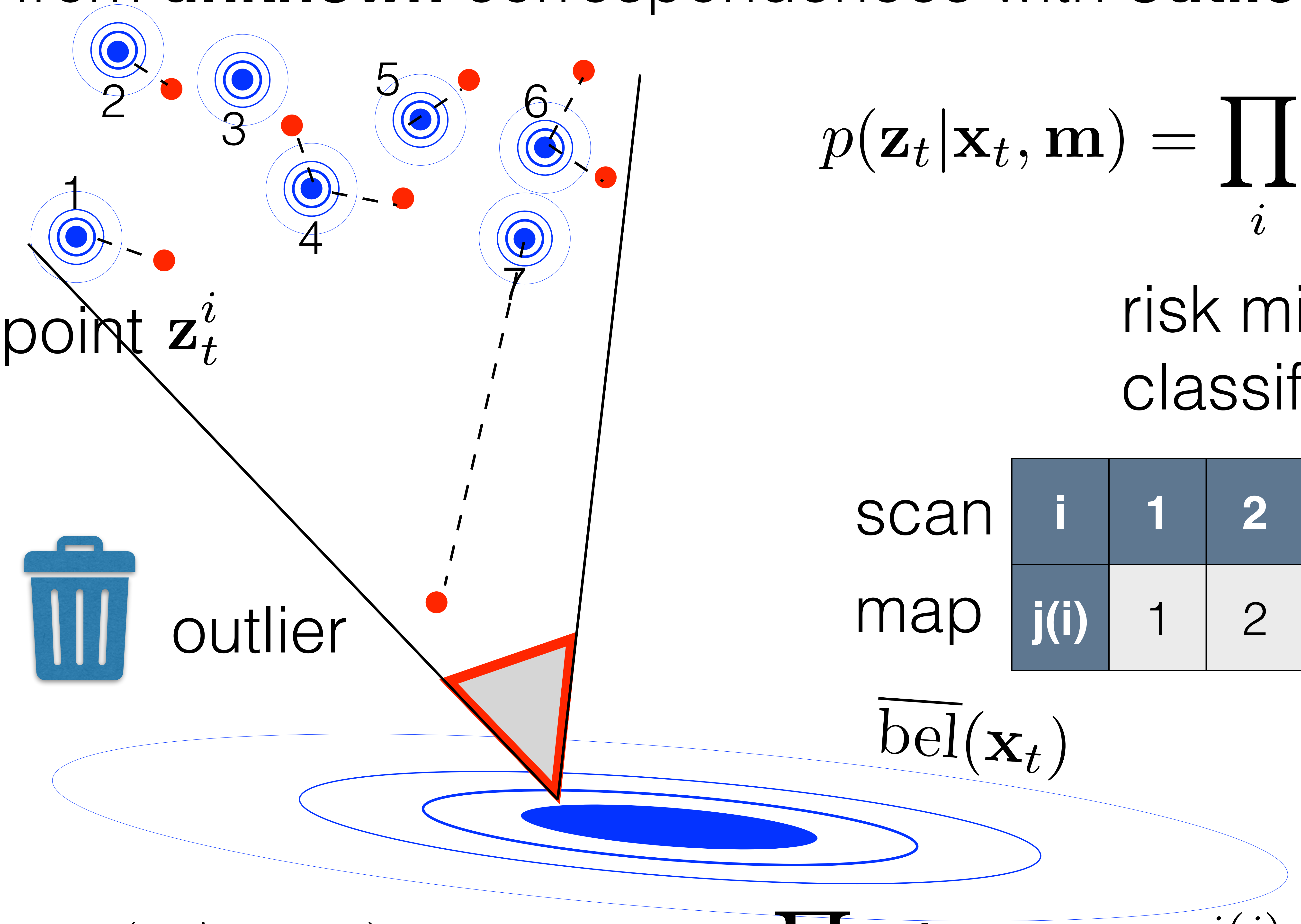
risk minimizing 7-class classification problem

scan	**i**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
map	**j(i)**	1	2	4	4	5	6	6	7

$$\mathbf{z}_t^{\mathrm{LIDAR}} = \arg\max_{\mathbf{x}_t} p(\mathbf{z}_t|\mathbf{x}_t,\mathbf{m}) = \arg\max_{\mathbf{x}_t, j(i)} \prod_i \mathcal{N}_{T(\mathbf{z}_t^i,\mathbf{x}_t)}(\mathbf{m}^{j(i)},\mathbf{I})$$

1. $\arg\min_{j(i)} \|T(\mathbf{z}_t^i,\mathbf{x}_t) - \mathbf{m}^{j(i)}\|_2^2$ … Nearest neighbour problem
2. $\arg\min_{\mathbf{x}_t} \|T(\mathbf{z}_t^i,\mathbf{x}_t) - \mathbf{m}^{j(i)}\|_2^2$ … Known absolute orientation problem

Pose from **unknown** correspondences with **outlier** rejection



$$p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{m}) = \prod_i p(\mathbf{z}_t^i|\mathbf{x}_t, \mathbf{m}^{j(i)})$$

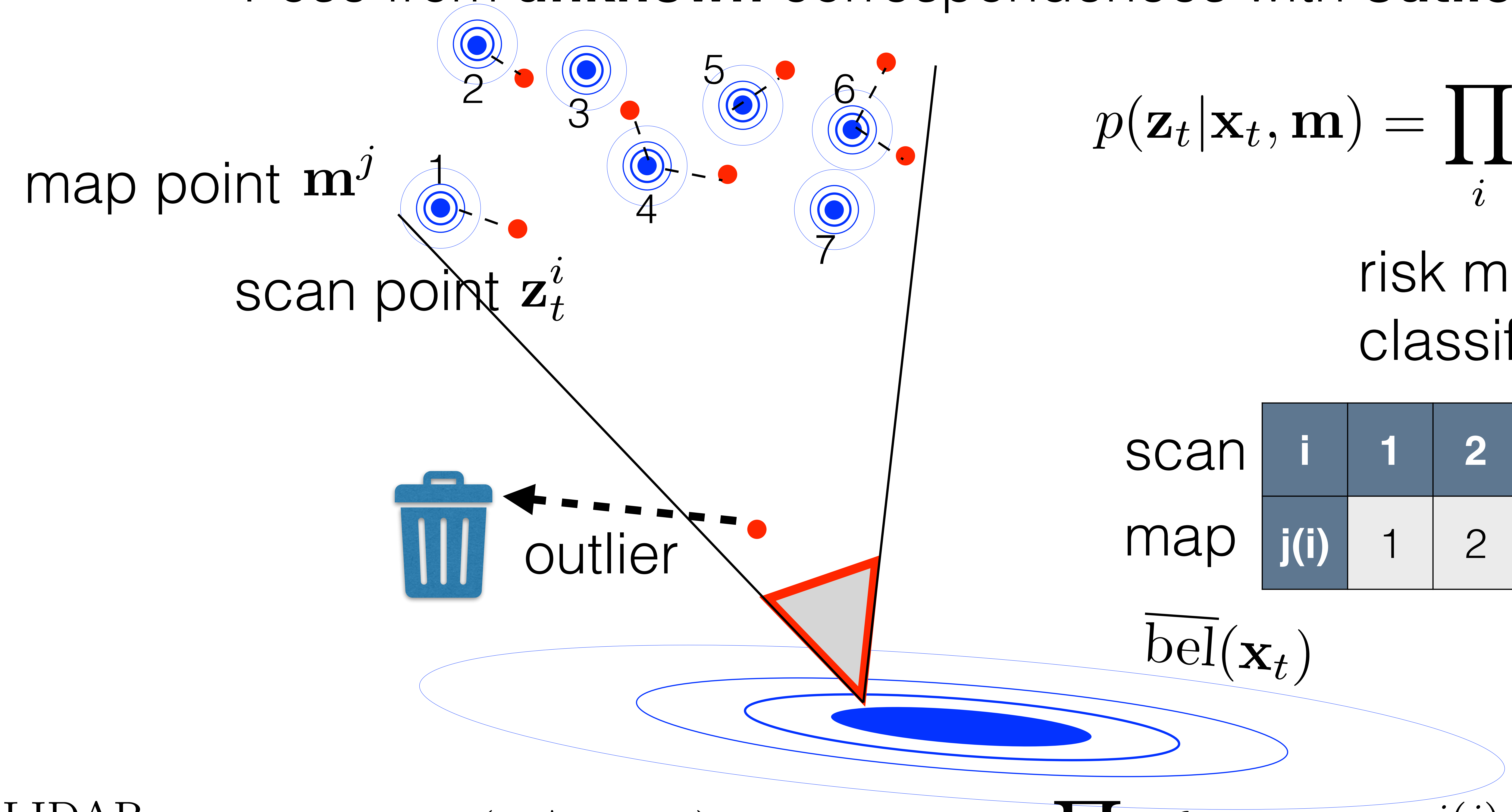
risk minimizing **8**-class classification problem

scan	**i**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
map	**j(i)**	1	2	4	4	5	6	6	7

$$\mathbf{z}_t^{\text{LIDAR}} = \arg\max_{\mathbf{x}_t} p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{m}) = \arg\max_{\mathbf{x}_t, j(i)} \prod_i \mathcal{N}_{T(\mathbf{z}_t^i, \mathbf{x}_t)}(\mathbf{m}^{j(i)}, \mathbf{I})$$

1. $\arg\min_{j(i)} \|T(\mathbf{z}_t^i, \mathbf{x}_t) - \mathbf{m}^{j(i)}\|_2^2$ … Nearest neighbour problem
2. $\arg\min_{\mathbf{x}_t} \|T(\mathbf{z}_t^i, \mathbf{x}_t) - \mathbf{m}^{j(i)}\|_2^2$ … Known absolute orientation problem

Pose from **unknown** correspondences with **outlier** rejection



$$p(\mathbf{z}_t|\mathbf{x}_t,\mathbf{m}) = \prod_i p(\mathbf{z}_t^i|\mathbf{x}_t,\mathbf{m}^{j(i)})$$

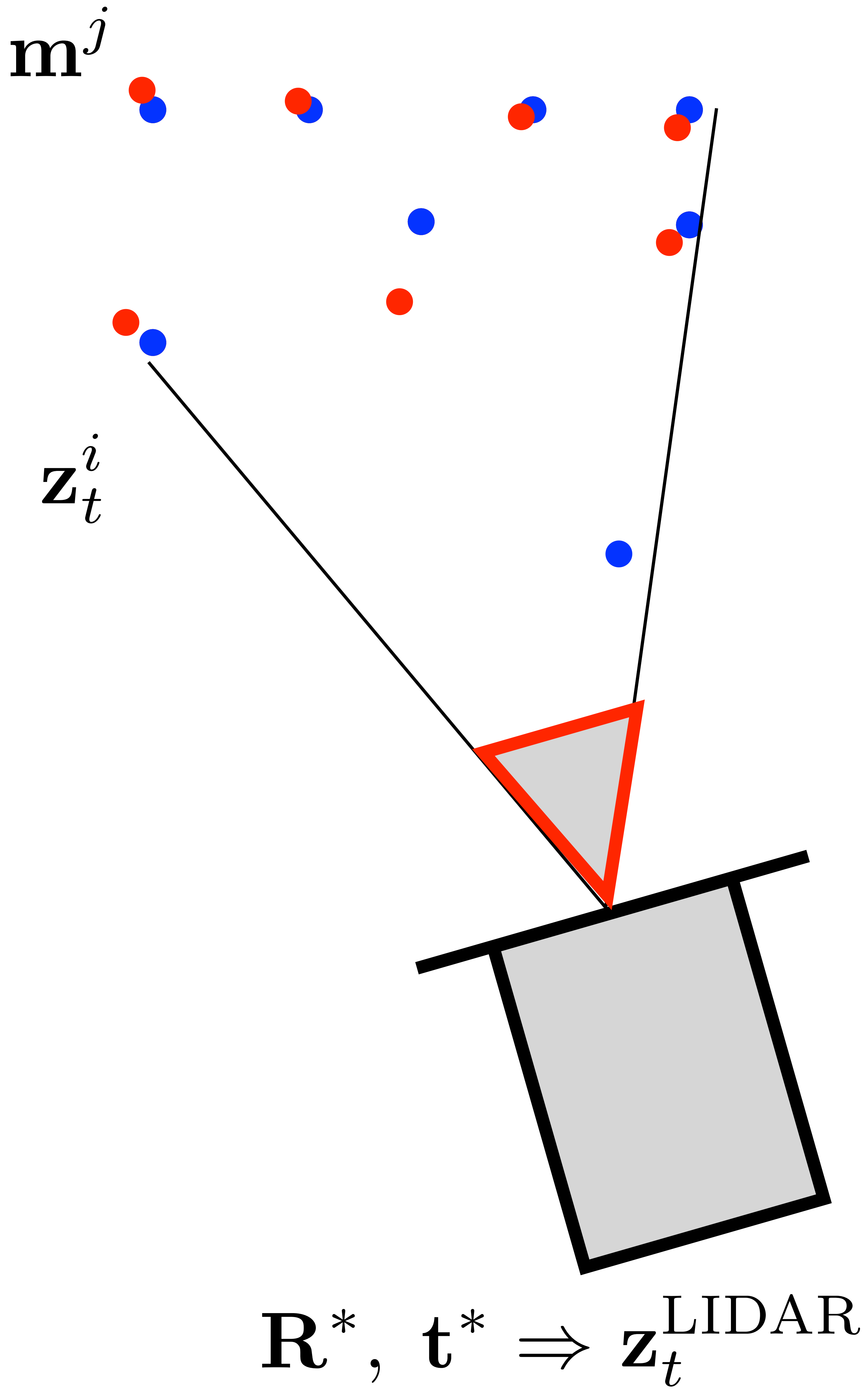
risk minimizing **8**-class classification problem

scan	i	1	2	3	4	5	6	7	8
map	j(i)	1	2	4	4	5	6	7	(trash)

$$\mathbf{z}_t^{\text{LIDAR}} = \arg\max_{\mathbf{x}_t} p(\mathbf{z}_t|\mathbf{x}_t,\mathbf{m}) = \arg\max_{\mathbf{x}_t, j(i)} \prod_i \mathcal{N}_{T(\mathbf{z}_t^i,\mathbf{x}_t)}(\mathbf{m}^{j(i)},\mathbf{I})$$

1. $\arg\min_{j(i)} \|T(\mathbf{z}_t^i,\mathbf{x}_t) - \mathbf{m}^{j(i)}\|_2^2$ … Nearest neighbour problem
2. $\arg\min_{\mathbf{x}_t} \|T(\mathbf{z}_t^i,\mathbf{x}_t) - \mathbf{m}^{j(i)}\|_2^2$ … Known absolute orientation problem

Pose from **unknown** correspondences with **outlier** rejection



Input:
- map $\mathbf{m}^j$, scan $\mathbf{z}_t^i$
- mean $\overline{\boldsymbol{\mu}_t} \Rightarrow \overline{\mathbf{R}},\ \overline{\mathbf{t}}$
- ~~correspondences $j(i)$~~

1. Initialize $\mathbf{R}^* = \overline{\mathbf{R}},\ \mathbf{t}^* = \overline{\mathbf{t}}$
2. Solve nearest neighbour:

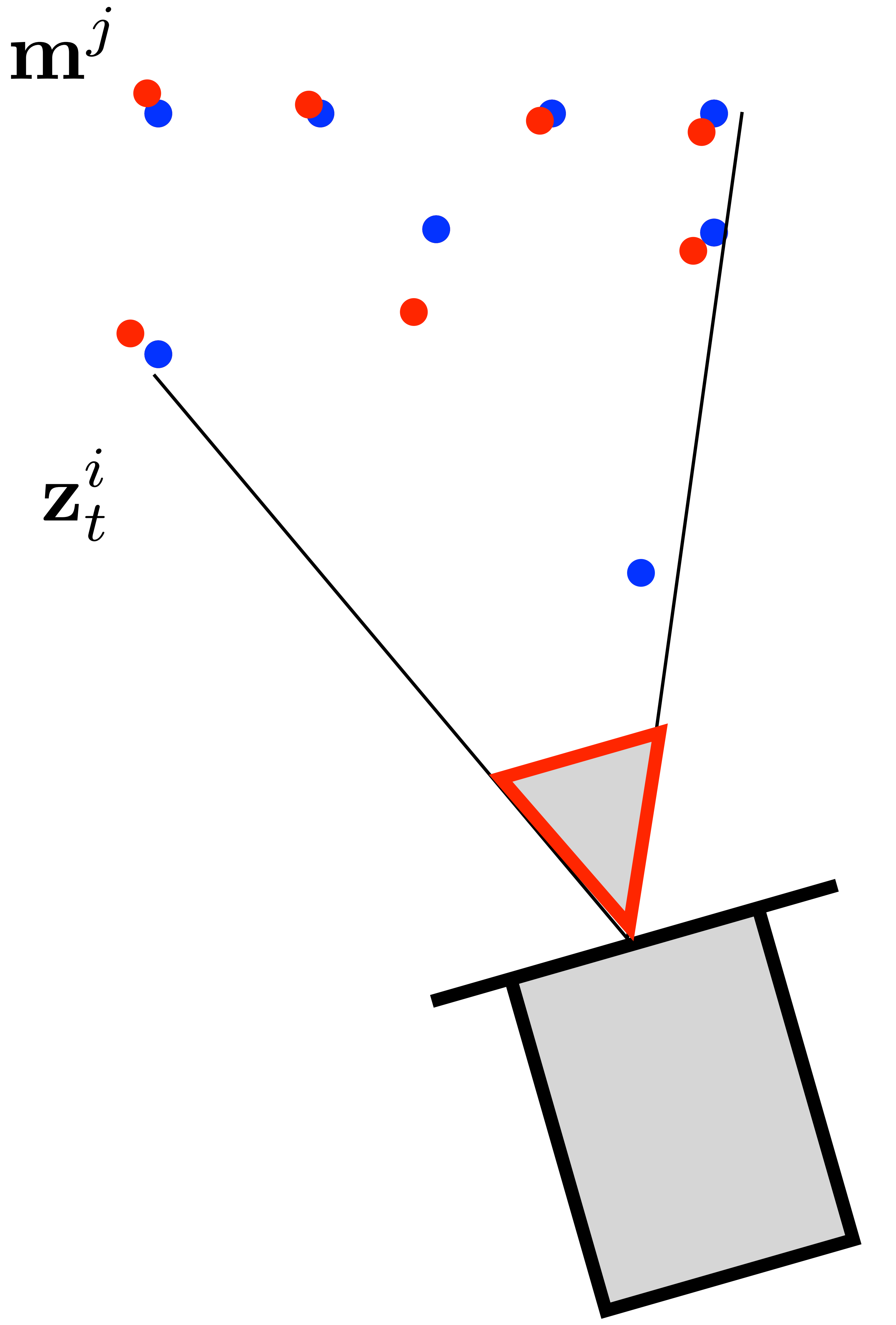
$$j(i)^* = \underset{j(i)\in J}{\arg\min} \sum_i \|\mathbf{R}^*\mathbf{z}_t^i + \mathbf{t}^* - \mathbf{m}^{j(i)}\|_2^2$$

3. Solve absolute orientation:

$$\mathbf{R}^*, \mathbf{t}^* = \underset{\mathbf{R}\in SO(3),\mathbf{t}\in\mathcal{R}^3}{\arg\min} \sum_i \|\mathbf{R}\mathbf{z}_t^i + \mathbf{t} - \mathbf{m}^{j(i)^*}\|_2^2$$

Output:
- posterior mean $\mathbf{R}^*,\ \mathbf{t}^* \Rightarrow \mathbf{z}_t^{\mathrm{LIDAR}}$

Pose from **unknown** correspondences with **outlier** rejection



$\mathbf{R}^*,\ \mathbf{t}^* \Rightarrow \mathbf{z}_t^{\text{LIDAR}}$

Input:
- map $\mathbf{m}^j$, scan $\mathbf{z}_t^i$
- mean $\overline{\boldsymbol{\mu}_t} \Rightarrow \overline{\mathbf{R}},\ \overline{\mathbf{t}}$
- ~~correspondences $j(i)$~~

1. Initialize $\mathbf{R}^* = \overline{\mathbf{R}},\ \mathbf{t}^* = \overline{\mathbf{t}}$

2. Solve nearest neighbour:

$$j(i)^* = \underset{j(i)\in J}{\arg\min} \sum_i \|\mathbf{R}^*\mathbf{z}_t^i + \mathbf{t}^* - \mathbf{m}^{j(i)}\|_2^2$$

3. Outlier rejection by median thresholding:

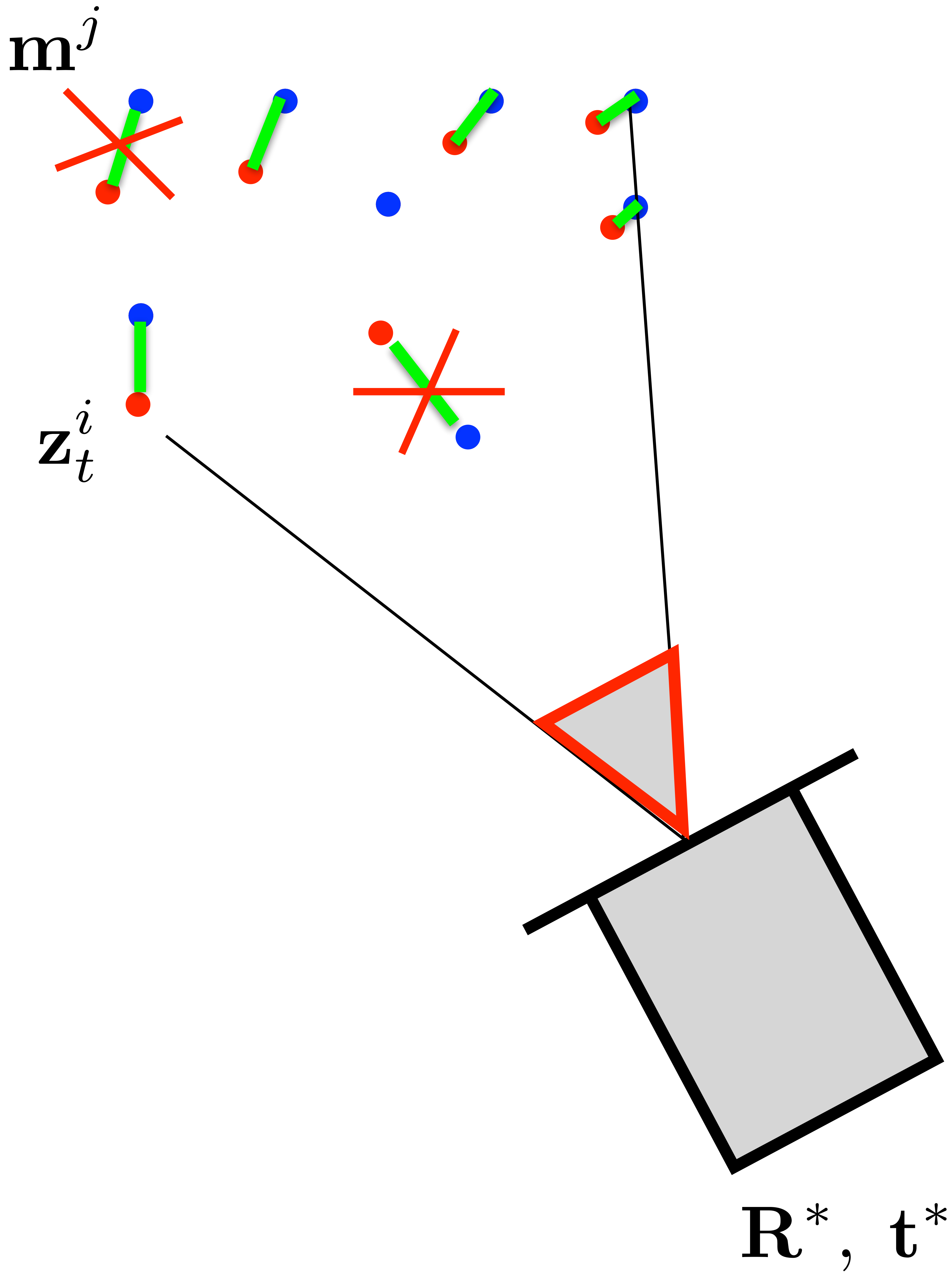
$$\texttt{if}\ \|\mathbf{R}^*\mathbf{z}_t^i + \mathbf{t}^* - \mathbf{m}^{j(i)^*}\|_2^2 \geq \theta\ \texttt{then}\ j(i)^* = 🗑$$

4. Solve absolute orientation:

$$\mathbf{R}^*, \mathbf{t}^* = \underset{\mathbf{R}\in SO(3), \mathbf{t}\in\mathcal{R}^3}{\arg\min} \sum_i \|\mathbf{R}\mathbf{z}_t^i + \mathbf{t} - \mathbf{m}^{j(i)^*}\|_2^2$$

Output:
- posterior mean $\mathbf{R}^*,\ \mathbf{t}^* \Rightarrow \mathbf{z}_t^{\text{LIDAR}}$

Pose from **unknown** correspondences with **outlier** rejection



Input:
- map $\mathbf{m}^j$, scan $\mathbf{z}_t^i$
- mean $\overline{\boldsymbol{\mu}_t} \Rightarrow \overline{\mathbf{R}},\ \overline{\mathbf{t}}$
- ~~correspondences $j(i)$~~

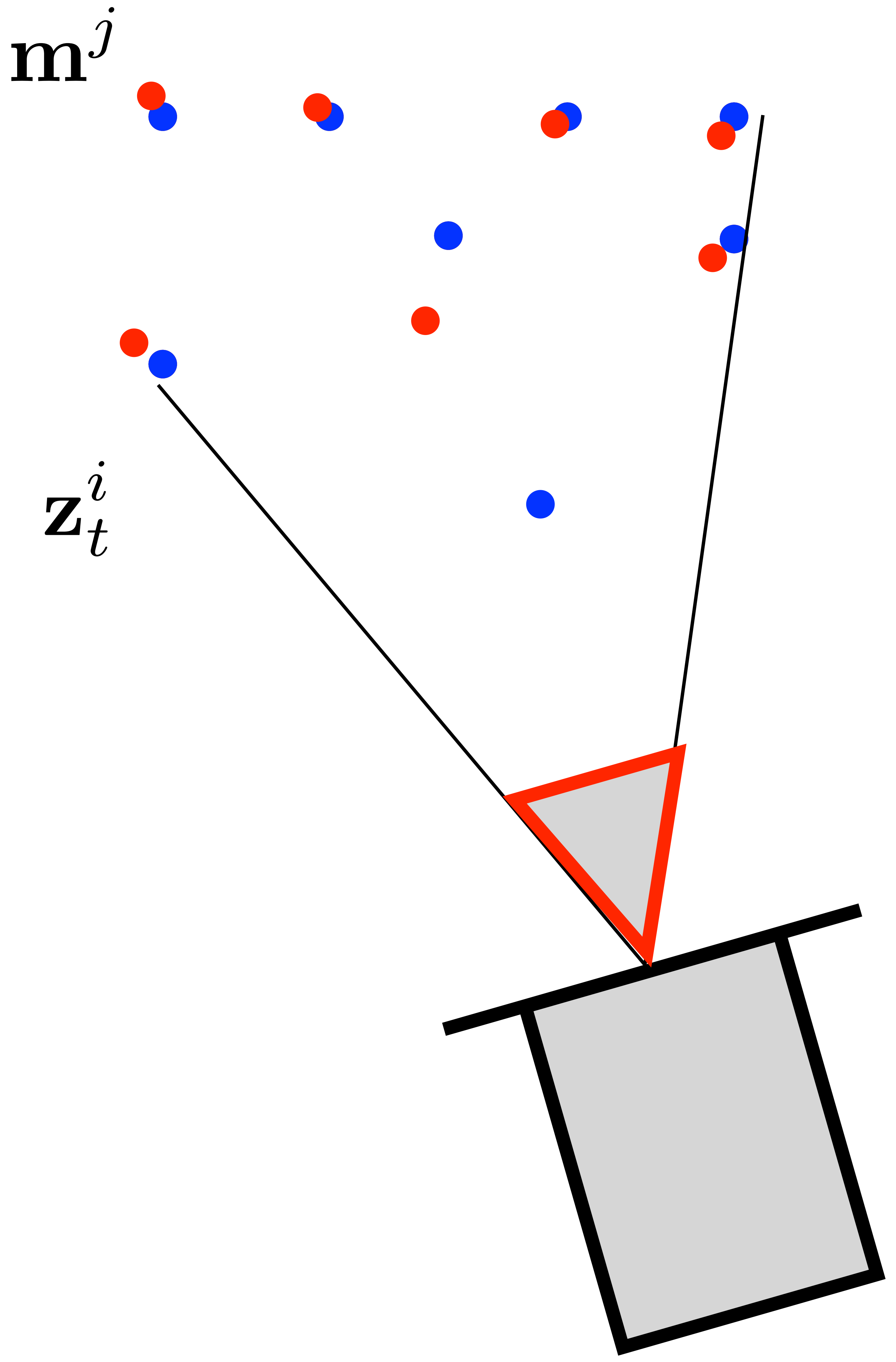
1. Initialize $\mathbf{R}^* = \overline{\mathbf{R}},\ \mathbf{t}^* = \overline{\mathbf{t}}$
2. Solve nearest neighbour:

$$j(i)^* = \operatorname*{arg\,min}_{j(i)\in J} \sum_i \|\mathbf{R}^*\mathbf{z}_t^i + \mathbf{t}^* - \mathbf{m}^{j(i)}\|_2^2$$

3. Outlier rejection by median thresholding:
   `if` $\|\mathbf{R}^*\mathbf{z}_t^i + \mathbf{t}^* - \mathbf{m}^{j(i)^*}\|_2^2 \geq \theta$ `then` $j(i)^* =$ 🗑
4. Solve absolute orientation:

$$\mathbf{R}^*, \mathbf{t}^* = \operatorname*{arg\,min}_{\mathbf{R}\in SO(3), \mathbf{t}\in\mathcal{R}^3} \sum_i \|\mathbf{R}\mathbf{z}_t^i + \mathbf{t} - \mathbf{m}^{j(i)^*}\|_2^2$$

Pose from **unknown** correspondences with **outlier** rejection



$\mathbf{R}^*,\ \mathbf{t}^* \Rightarrow \mathbf{z}_t^{\text{LIDAR}}$

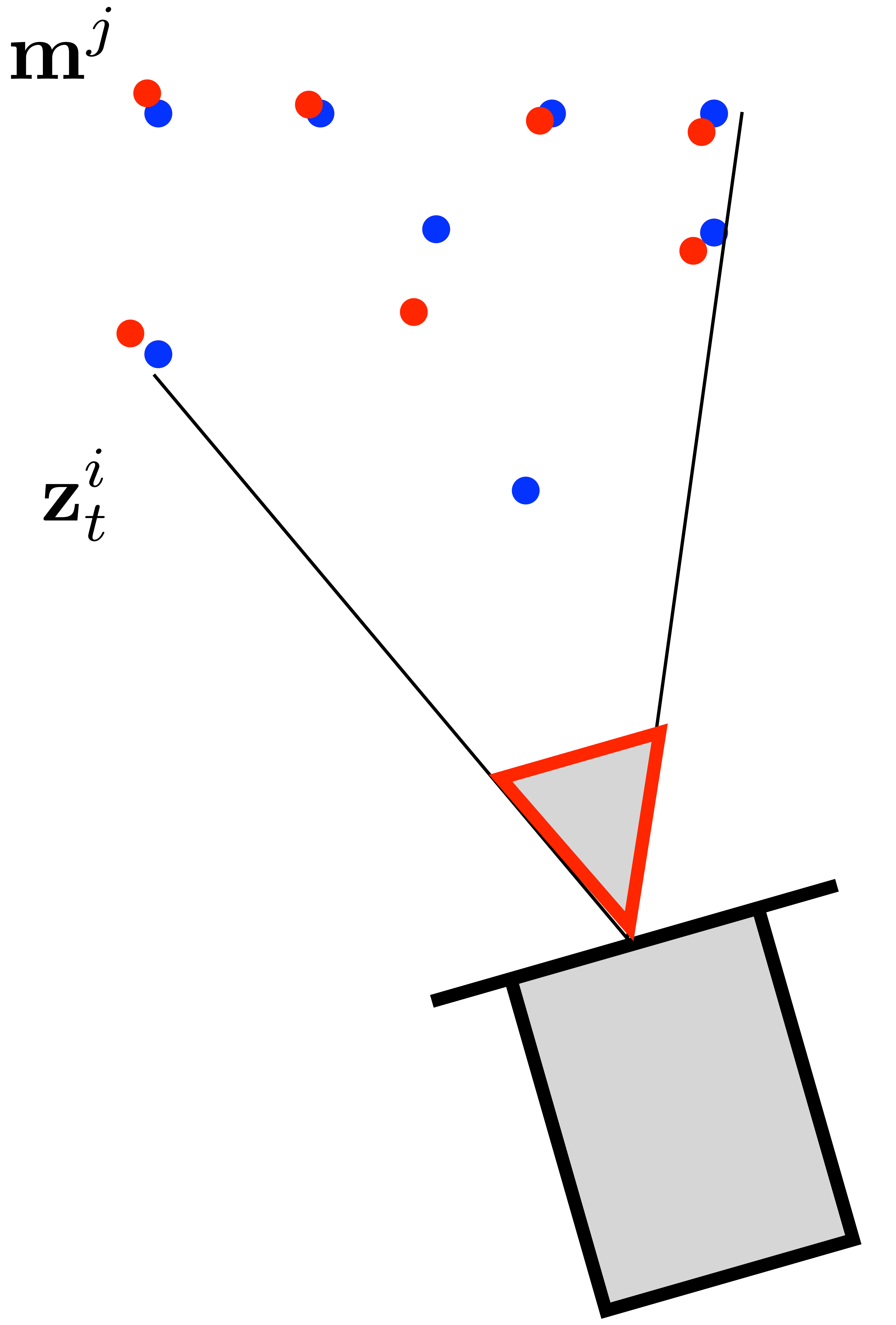
Input:
- map $\mathbf{m}^j$, scan $\mathbf{z}_t^i$
- mean $\overline{\boldsymbol{\mu}_t} \Rightarrow \overline{\mathbf{R}},\ \overline{\mathbf{t}}$
- ~~correspondences $j(i)$~~

iterate

1. Initialize $\mathbf{R}^* = \overline{\mathbf{R}},\ \mathbf{t}^* = \overline{\mathbf{t}}$
2. Solve nearest neighbour:
$$j(i)^* = \underset{j(i)\in J}{\arg\min} \sum_i \|\mathbf{R}^*\mathbf{z}_t^i + \mathbf{t}^* - \mathbf{m}^{j(i)}\|_2^2$$
3. Outlier rejection by median thresholding:
$\texttt{if}\ \|\mathbf{R}^*\mathbf{z}_t^i + \mathbf{t}^* - \mathbf{m}^{j(i)^*}\|_2^2 \geq \theta\ \texttt{then}\ j(i)^* =$ 🗑
4. Solve absolute orientation:
$$\mathbf{R}^*, \mathbf{t}^* = \underset{\mathbf{R}\in SO(3),\mathbf{t}\in\mathcal{R}^3}{\arg\min} \sum_i \|\mathbf{R}\mathbf{z}_t^i + \mathbf{t} - \mathbf{m}^{j(i)^*}\|_2^2$$

Output:
- posterior mean $\mathbf{R}^*,\ \mathbf{t}^* \Rightarrow \mathbf{z}_t^{\text{LIDAR}}$

Pose from **unknown** correspondences with **outlier** rejection



Input:
- map $\mathbf{m}^j$, scan $\mathbf{z}_t^i$
- mean $\overline{\boldsymbol{\mu}_t} \Rightarrow \overline{\mathbf{R}},\ \overline{\mathbf{t}}$
- ~~correspondences $j(i)$~~

iterate

1. Initialize $\mathbf{R}^* = \overline{\mathbf{R}},\ \mathbf{t}^* = \overline{\mathbf{t}}$
2. Solve nearest neighbour:

$$j(i)^* = \underset{j(i)\in J}{\arg\min} \sum_i \|\mathbf{R}^*\mathbf{z}_t^i + \mathbf{t}^* - \mathbf{m}^{j(i)}\|_2^2$$

3. Outlier rejection by median thresholding:

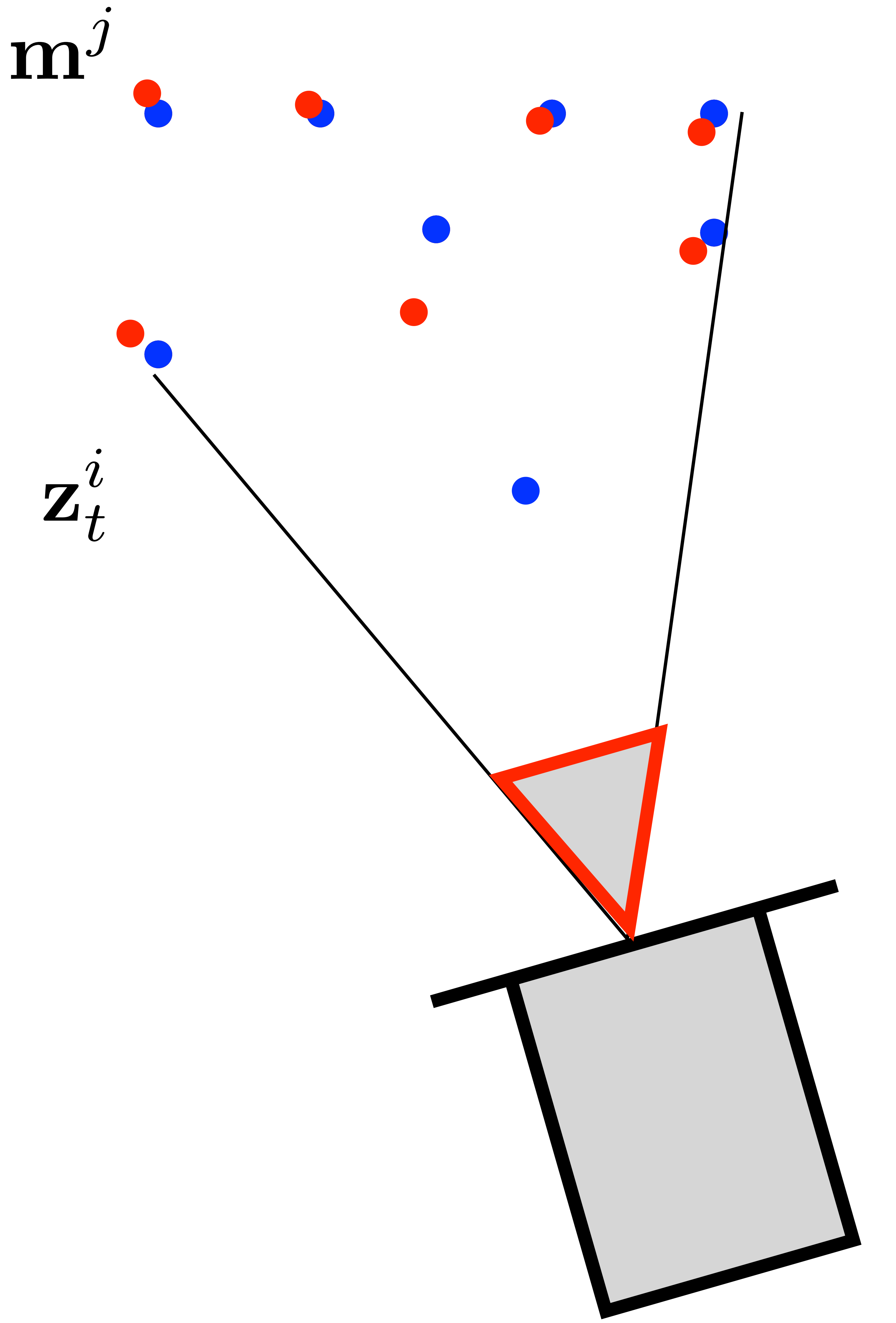
$\texttt{if}\ \|\mathbf{R}^*\mathbf{z}_t^i + \mathbf{t}^* - \mathbf{m}^{j(i)^*}\|_2^2 \geq \theta\ \texttt{then}\ j(i)^* =$ 🗑

4. Solve absolute orientation:

$$\mathbf{R}^*, \mathbf{t}^* = \underset{\mathbf{R}\in SO(3), \mathbf{t}\in\mathcal{R}^3}{\arg\min} \sum_i \|\mathbf{R}\mathbf{z}_t^i + \mathbf{t} - \mathbf{m}^{j(i)^*}\|_2^2$$

Output:
- posterior mean $\mathbf{R}^*,\ \mathbf{t}^* \Rightarrow \mathbf{z}_t^{\mathrm{LIDAR}}$

Pose from **unknown** correspondences with **outlier** rejection



$\mathbf{R}^*,\ \mathbf{t}^* \Rightarrow \mathbf{z}_t^{\mathrm{LIDAR}}$

Input:
- map $\mathbf{m}^j$, scan $\mathbf{z}_t^i$
- mean $\overline{\boldsymbol{\mu}_t} \Rightarrow \overline{\mathbf{R}},\ \overline{\mathbf{t}}$
- ~~correspondences $j(i)$~~

$\mathbf{R}^*,\ \mathbf{t}^*$ = `align(` $\mathbf{z}_t^i$ `,` $\mathbf{m}^j$`,` $\overline{\mathbf{R}},\ \overline{\mathbf{t}}$ `)`

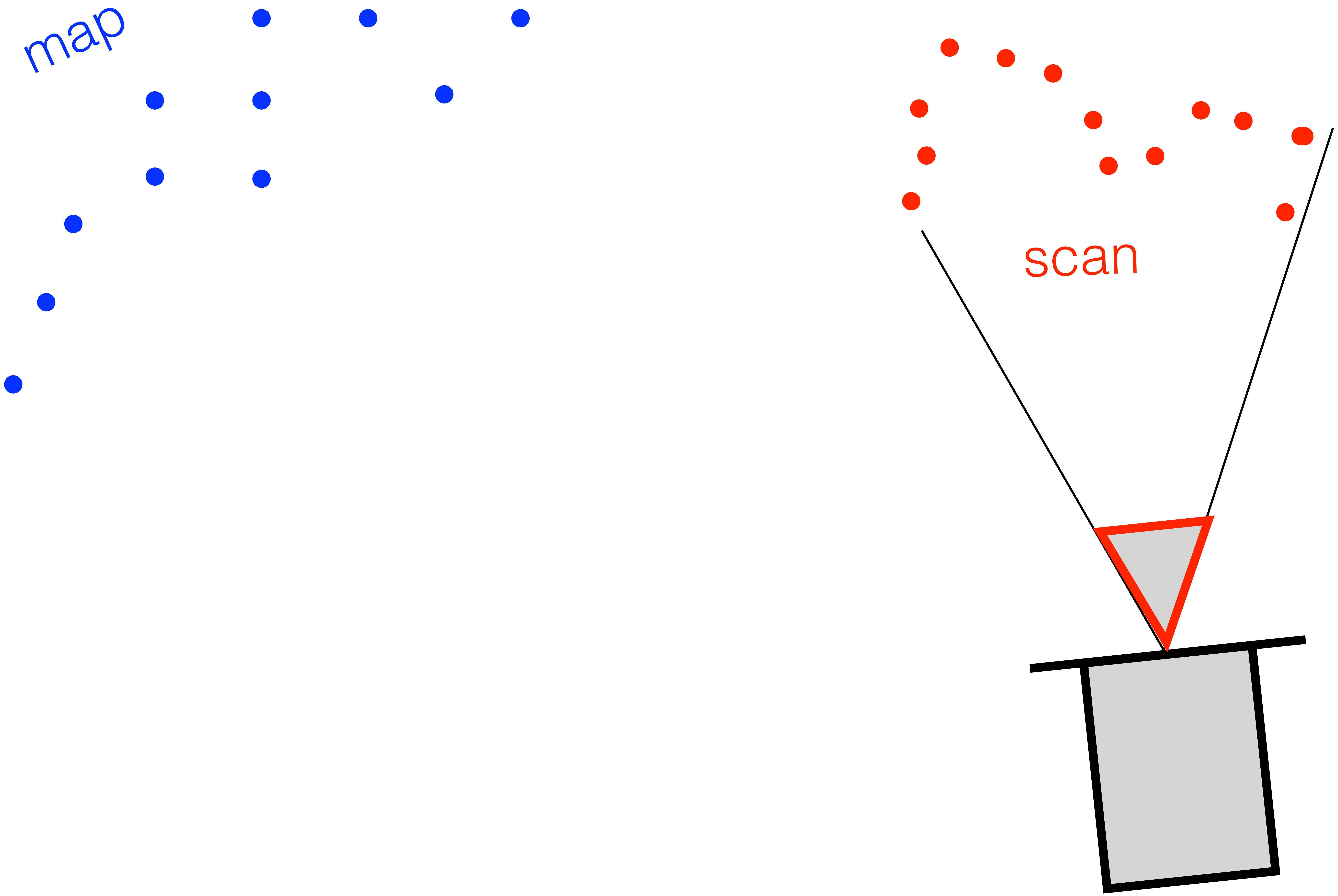
Output:
- posterior mean $\mathbf{R}^*,\ \mathbf{t}^* \Rightarrow \mathbf{z}_t^{\mathrm{LIDAR}}$

Iterative Closest Point (ICP) [Besl and McKay 92]

Successive localization based on the previous lidar scan only is usually very inaccurate => some kind of map is typically needed

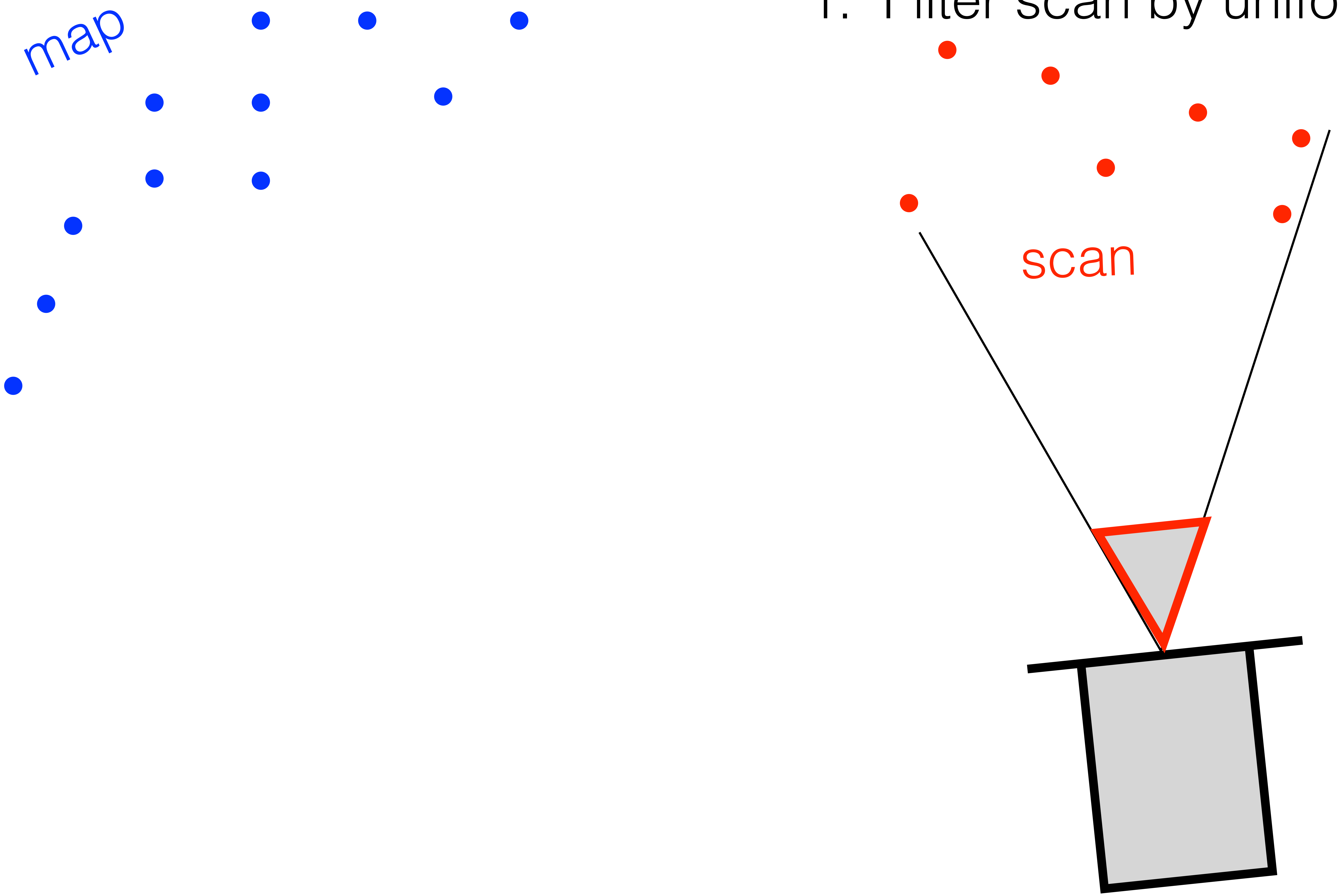
- 3D pointcloud map
- Occupancy grid
- Surfel map
- 2.5D hightmap

Iterative Closest Point (ICP) [Besl and McKay 92]

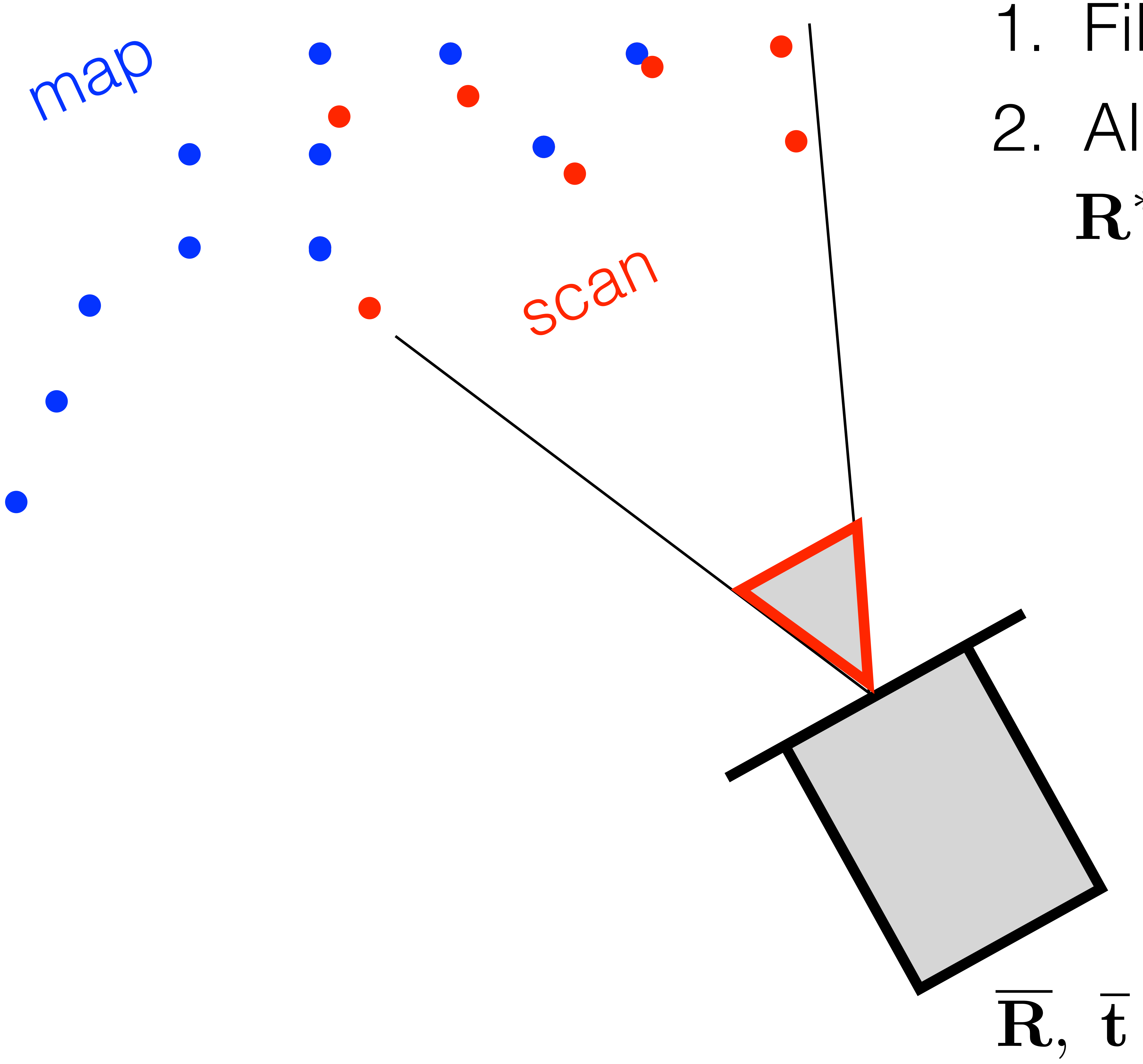


Iterative Closest Point (ICP) [Besl and McKay 92]

1. Filter scan by uniform sampling



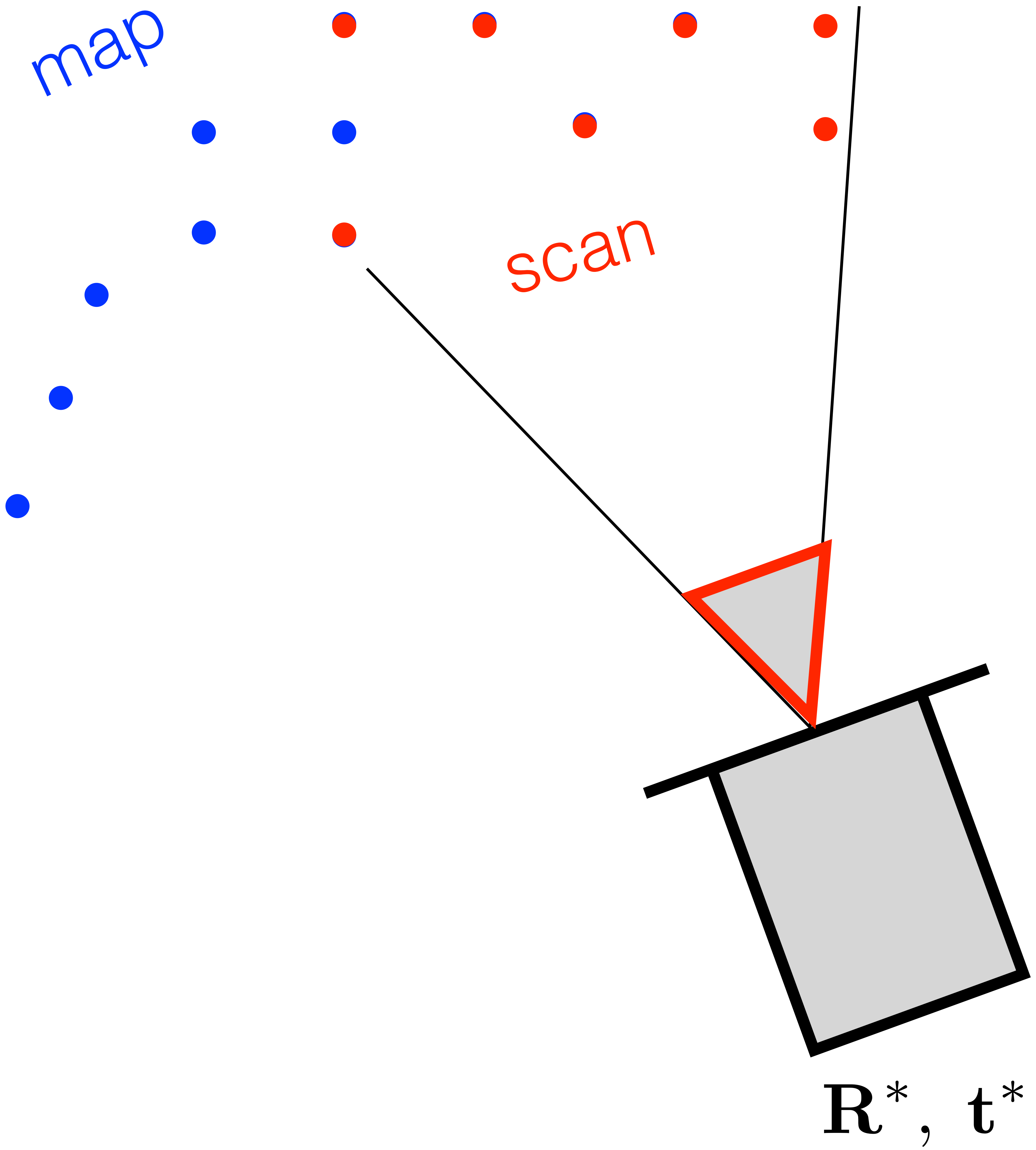
Iterative Closest Point (ICP) [Besl and McKay 92]



1. Filter scan by uniform sampling
2. Align:

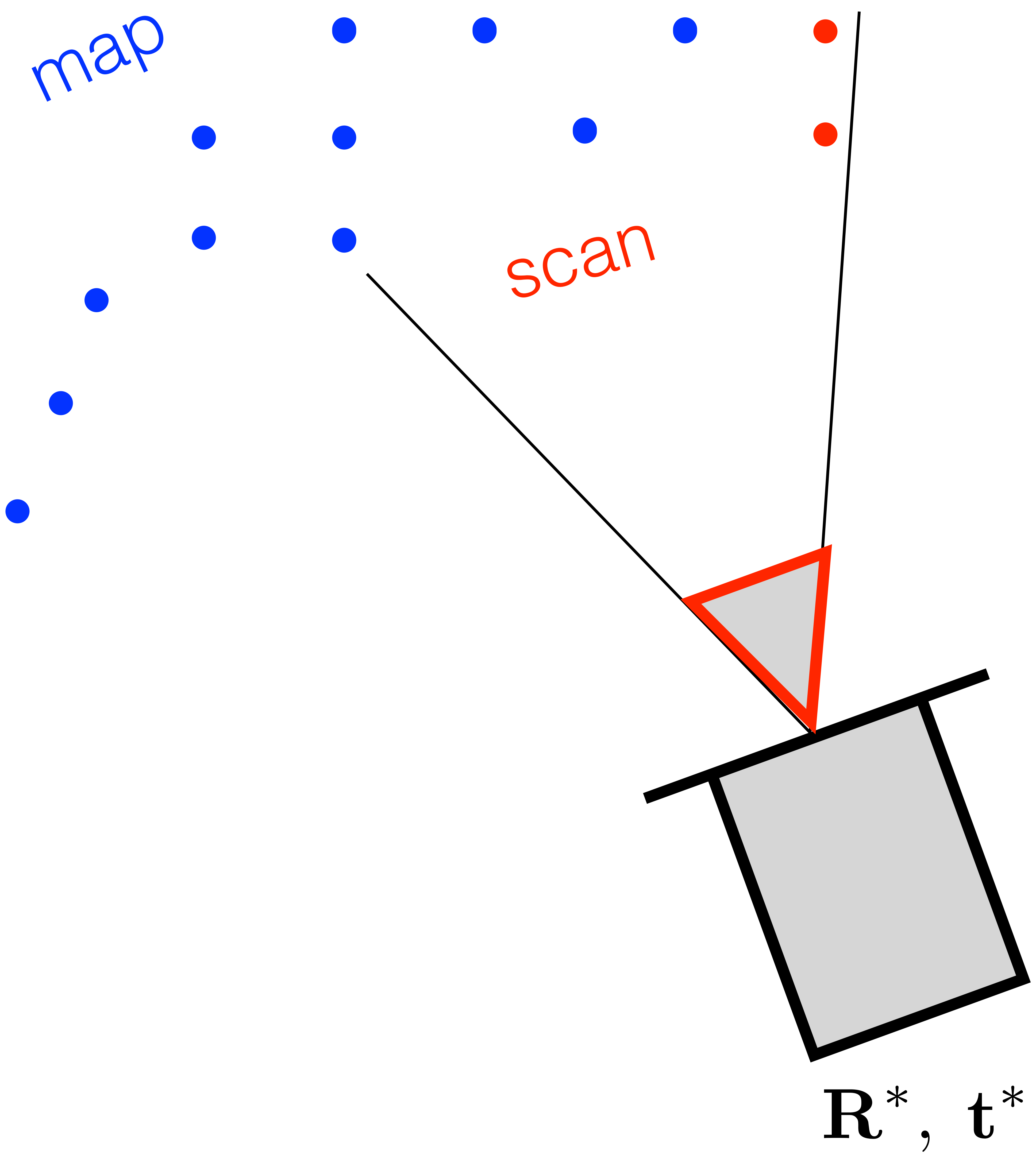
$$\mathbf{R}^*, \mathbf{t}^* = \texttt{align}(\mathbf{z}_t^i, \mathbf{m}^j, \overline{\mathbf{R}}, \bar{\mathbf{t}})$$

Iterative Closest Point (ICP) [Besl and McKay 92]



1. Filter scan by uniform sampling
2. Align:
   $\mathbf{R}^*,\ \mathbf{t}^* = \mathtt{align}(\mathbf{z}_t^i, \mathbf{m}^j, \overline{\mathbf{R}}, \bar{\mathbf{t}})$
3. Update map

Iterative Closest Point (ICP) [Besl and McKay 92]

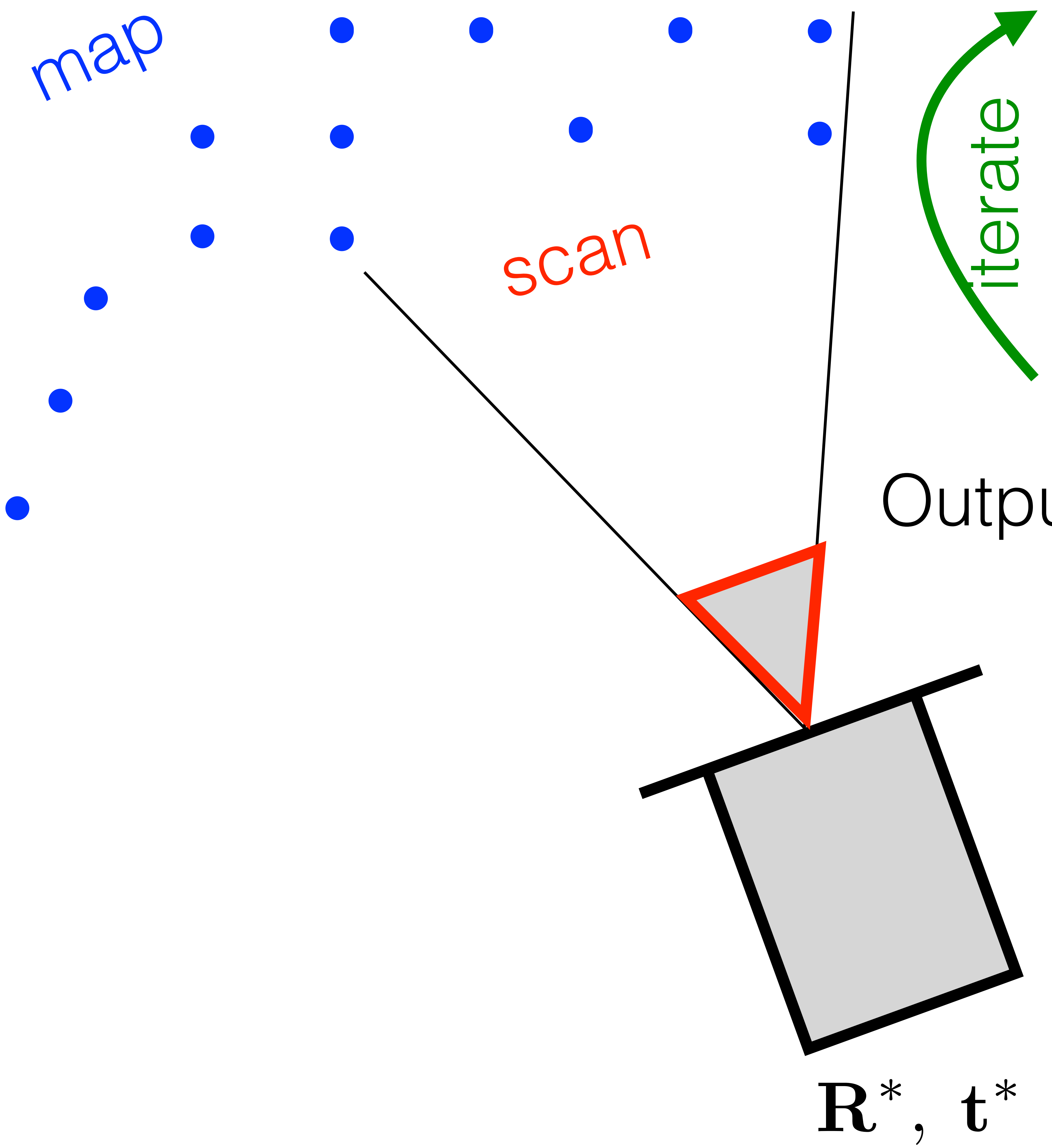


1. Filter scan by uniform sampling
2. Align:

   $$\mathbf{R}^*, \mathbf{t}^* = \texttt{align}(\mathbf{z}_t^i, \mathbf{m}^j, \overline{\mathbf{R}}, \overline{\mathbf{t}})$$

3. Update map from detected outliers

ICP with 3D pointcloud map



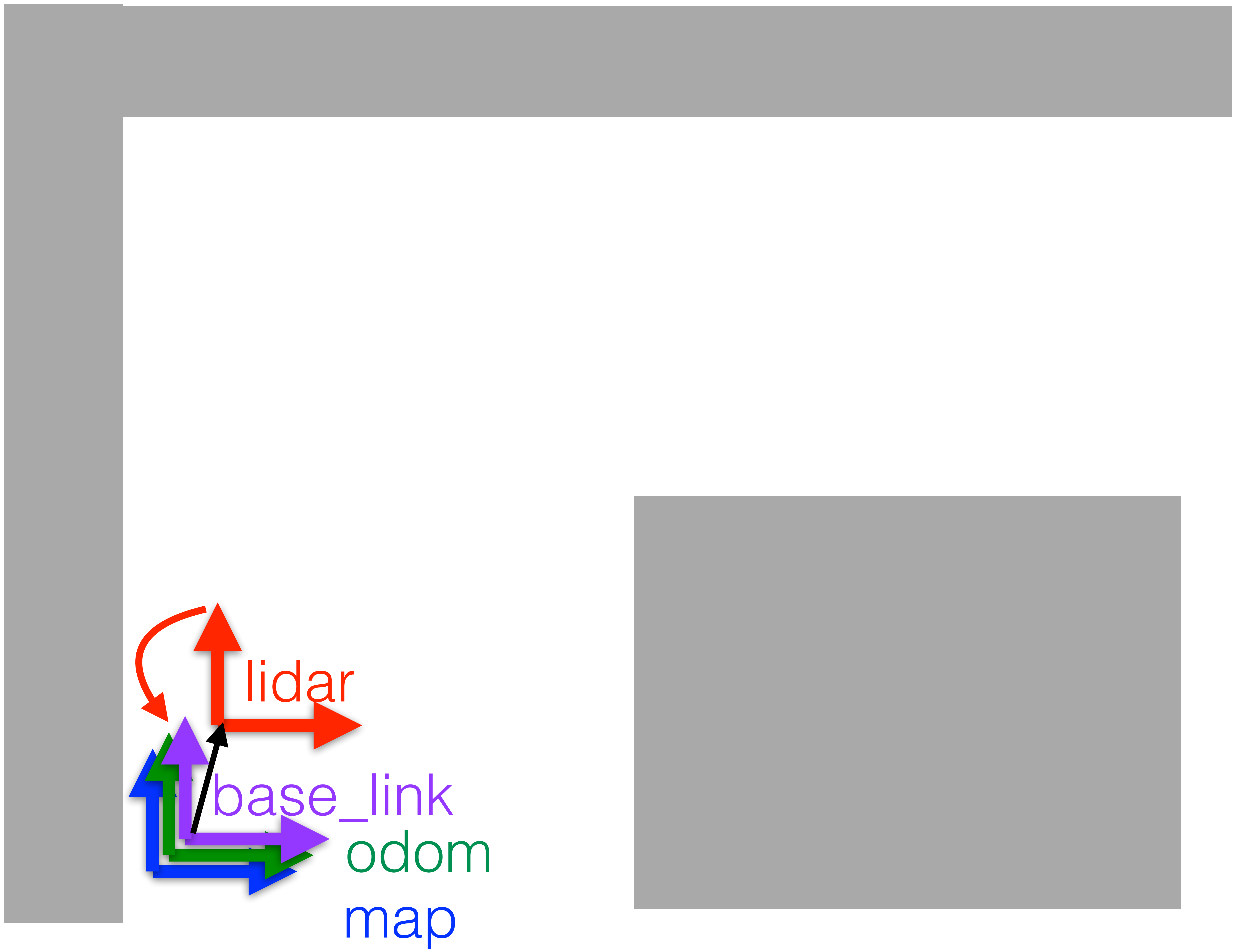
1. Filter scan by uniform sampling
2. Align:
   $\mathbf{R}^*,\ \mathbf{t}^* = \texttt{align}(\ \mathbf{z}_t^i\ ,\ \mathbf{m}^j,\ \overline{\mathbf{R}},\ \bar{\mathbf{t}}\ )$
3. Update map from detected outliers

Output: • posterior mean $\mathbf{R}^*,\ \mathbf{t}^* \Rightarrow \mathbf{z}_t^{\mathrm{LIDAR}}$

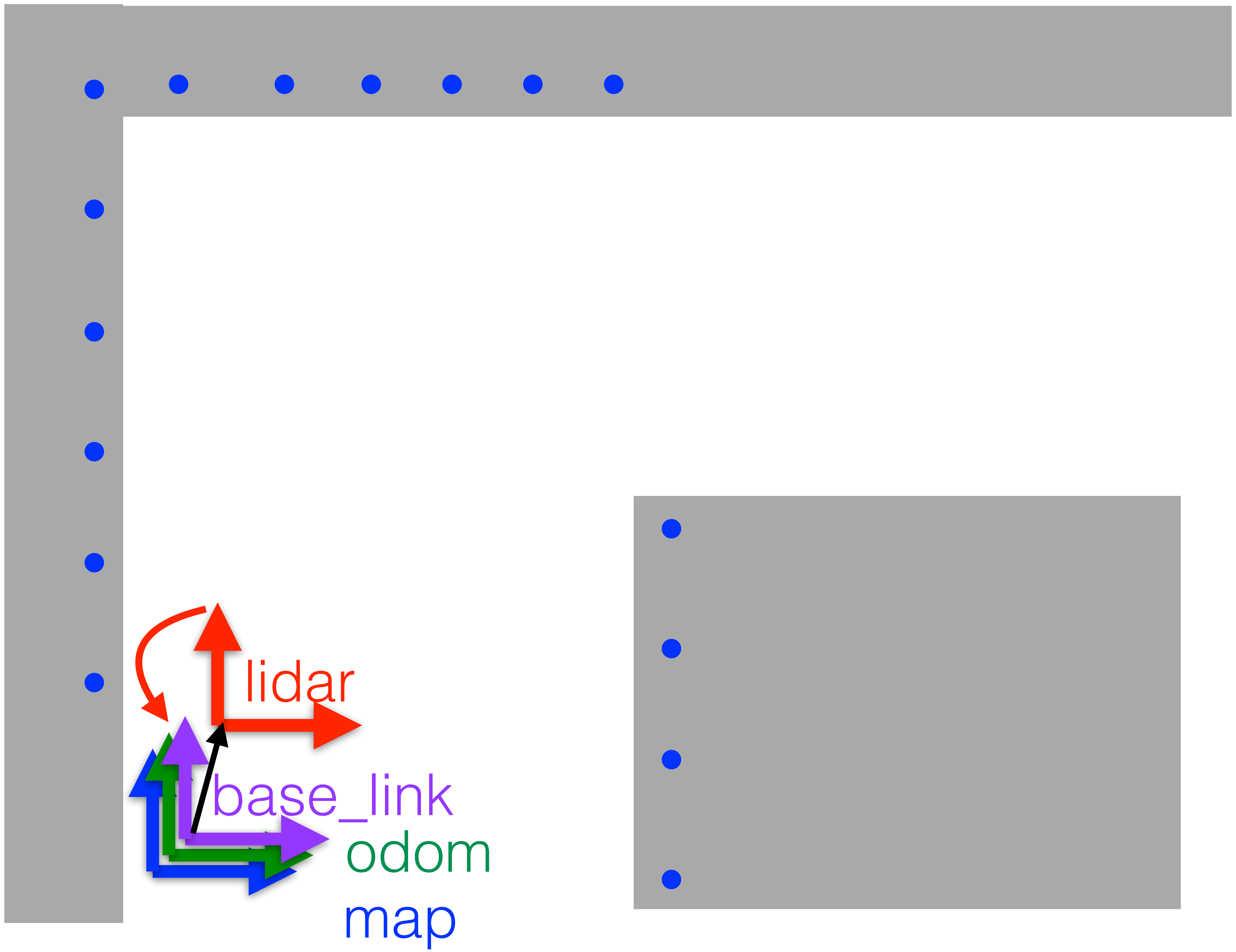
ICP SLAM - known issues

- Converges to a local minima (unknown region of convergence => sensitive to good initialization)
- Large map requires huge amount of memory => better representation (e.g. surfel map)
- Does not work in degenerate environments (e.g. forward motion in long hallways, or rotation in circular room)
- Sensitive to outliers due to L2 norm => RANSAC

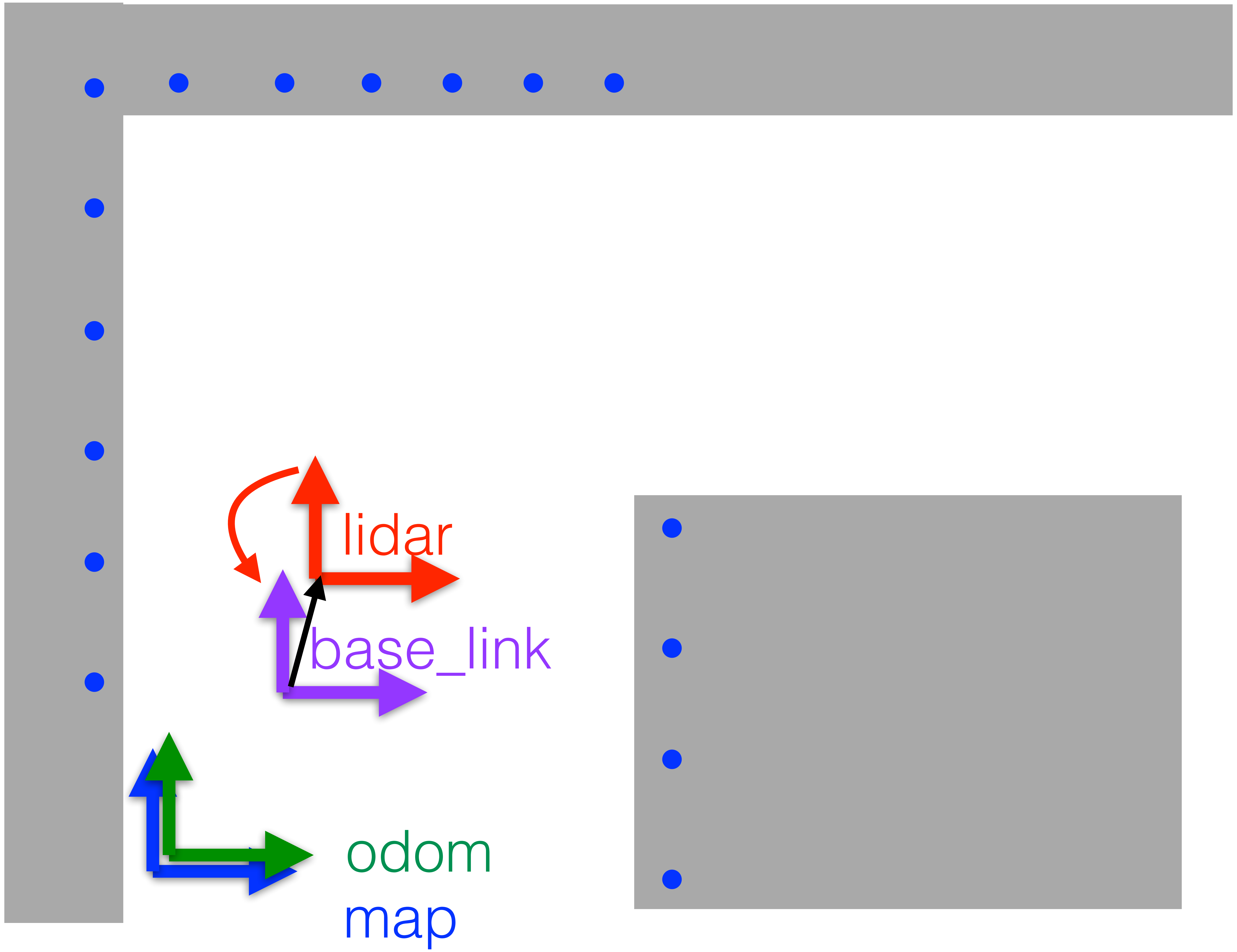
Initialization from wheel odometry



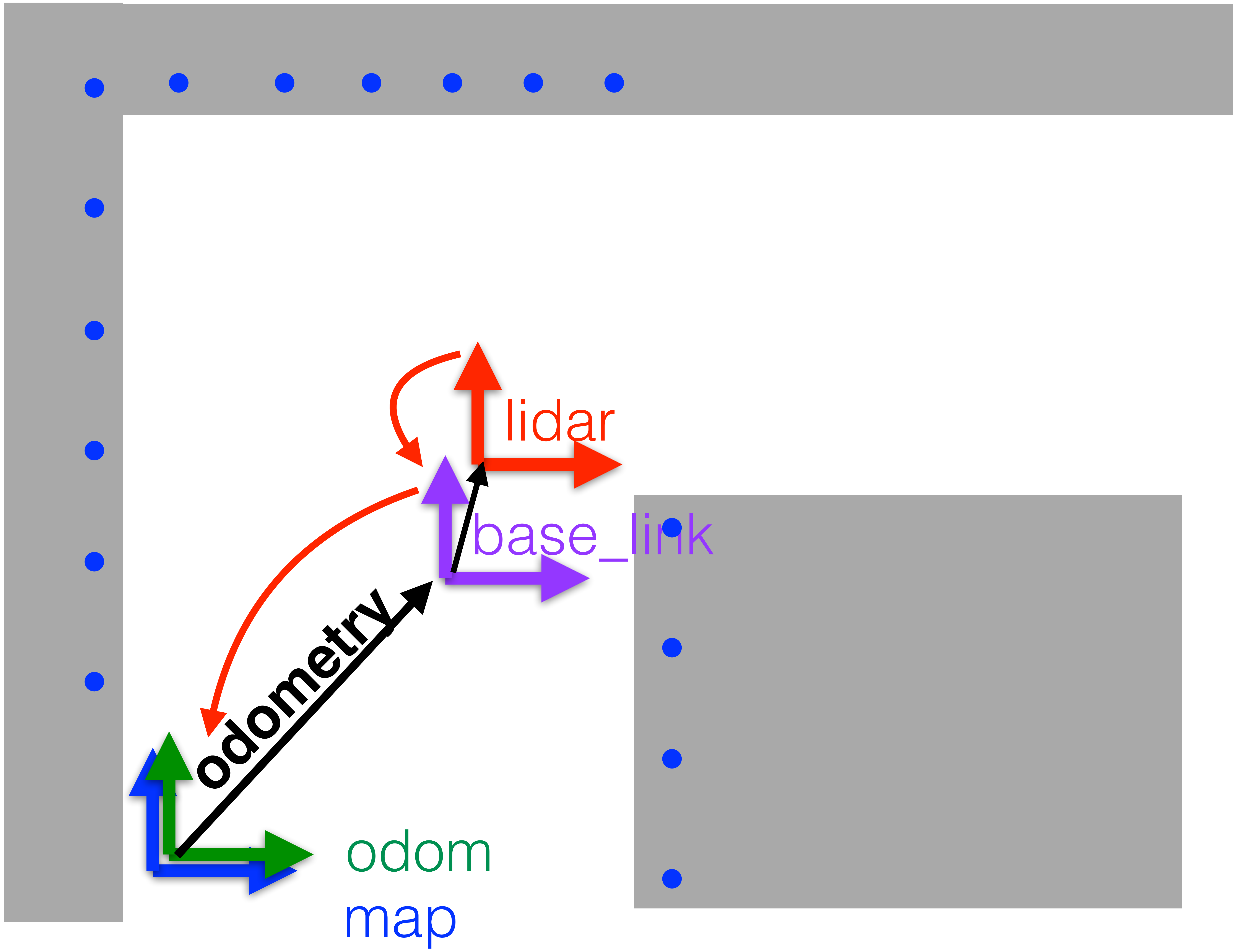
Initialization from wheel odometry



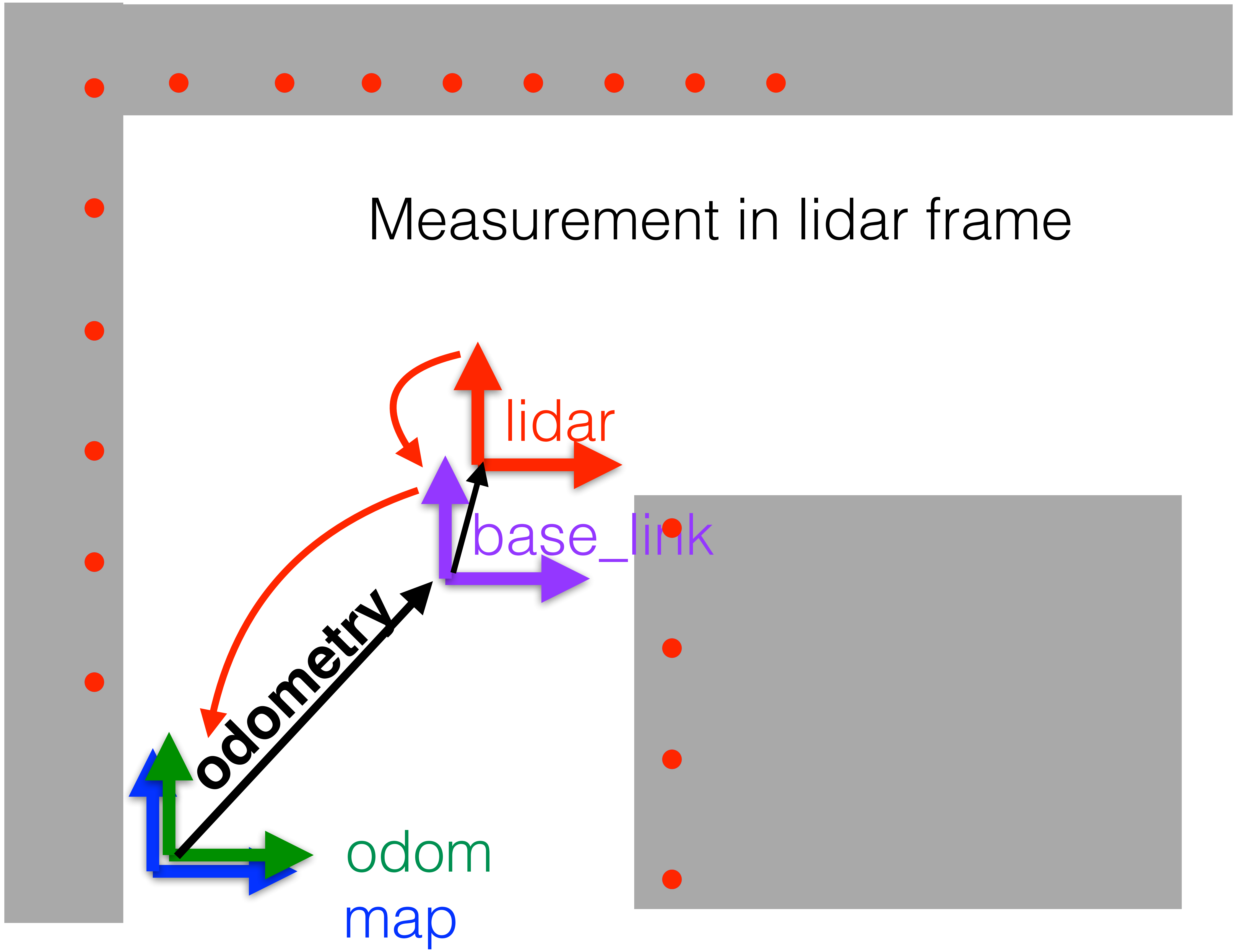
Initialization from wheel odometry



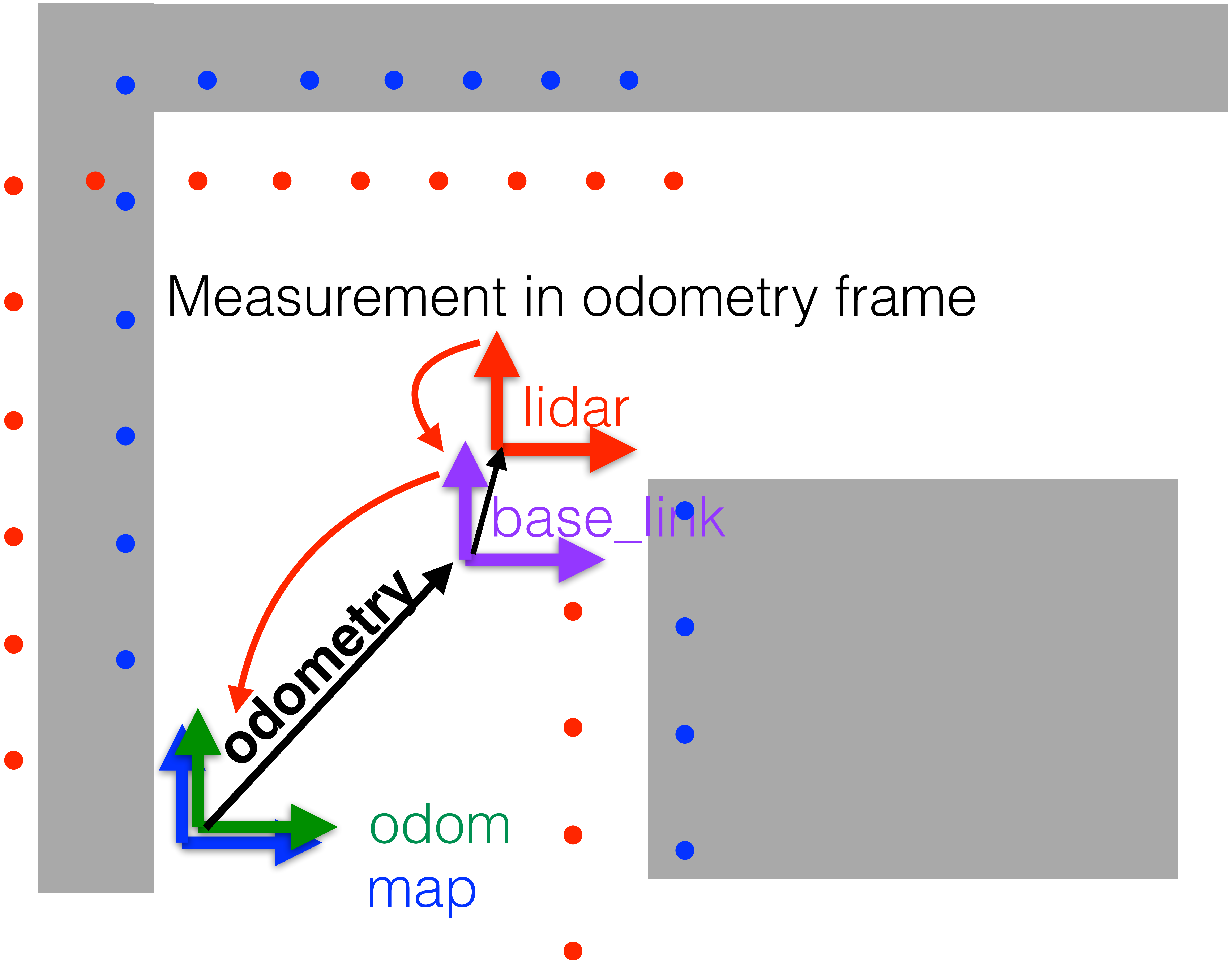
Initialization from wheel odometry



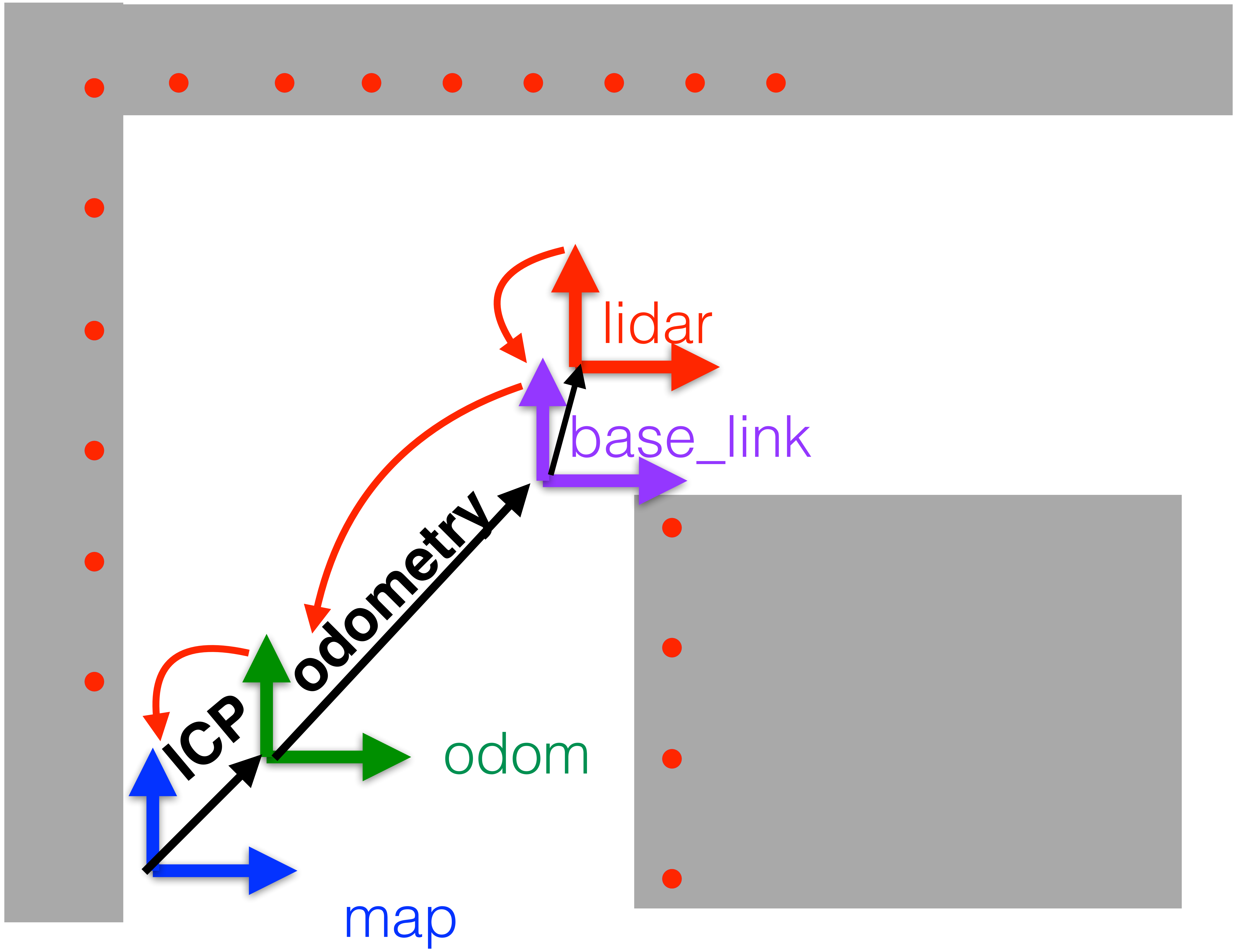
Initialization from wheel odometry



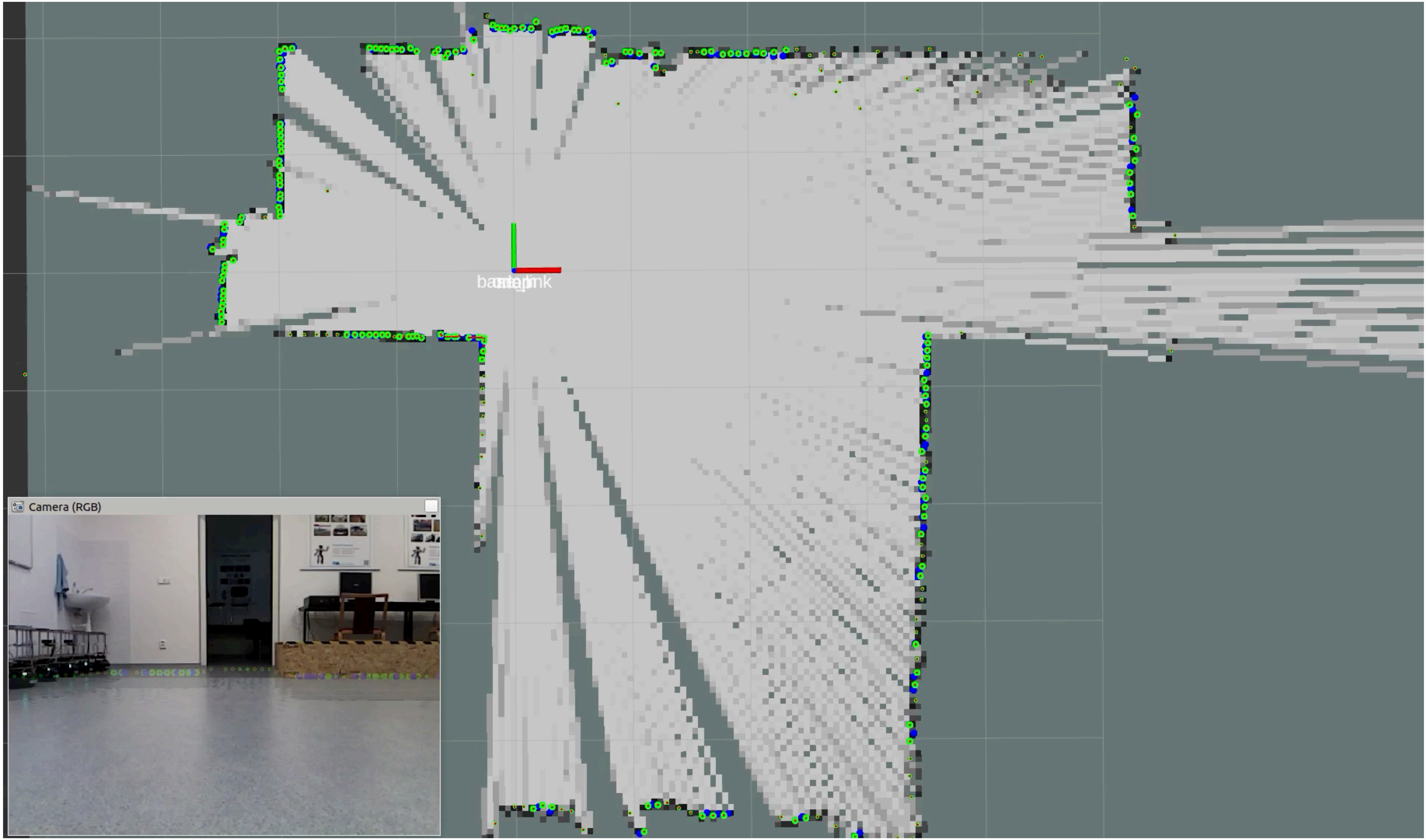
Initialization from wheel odometry



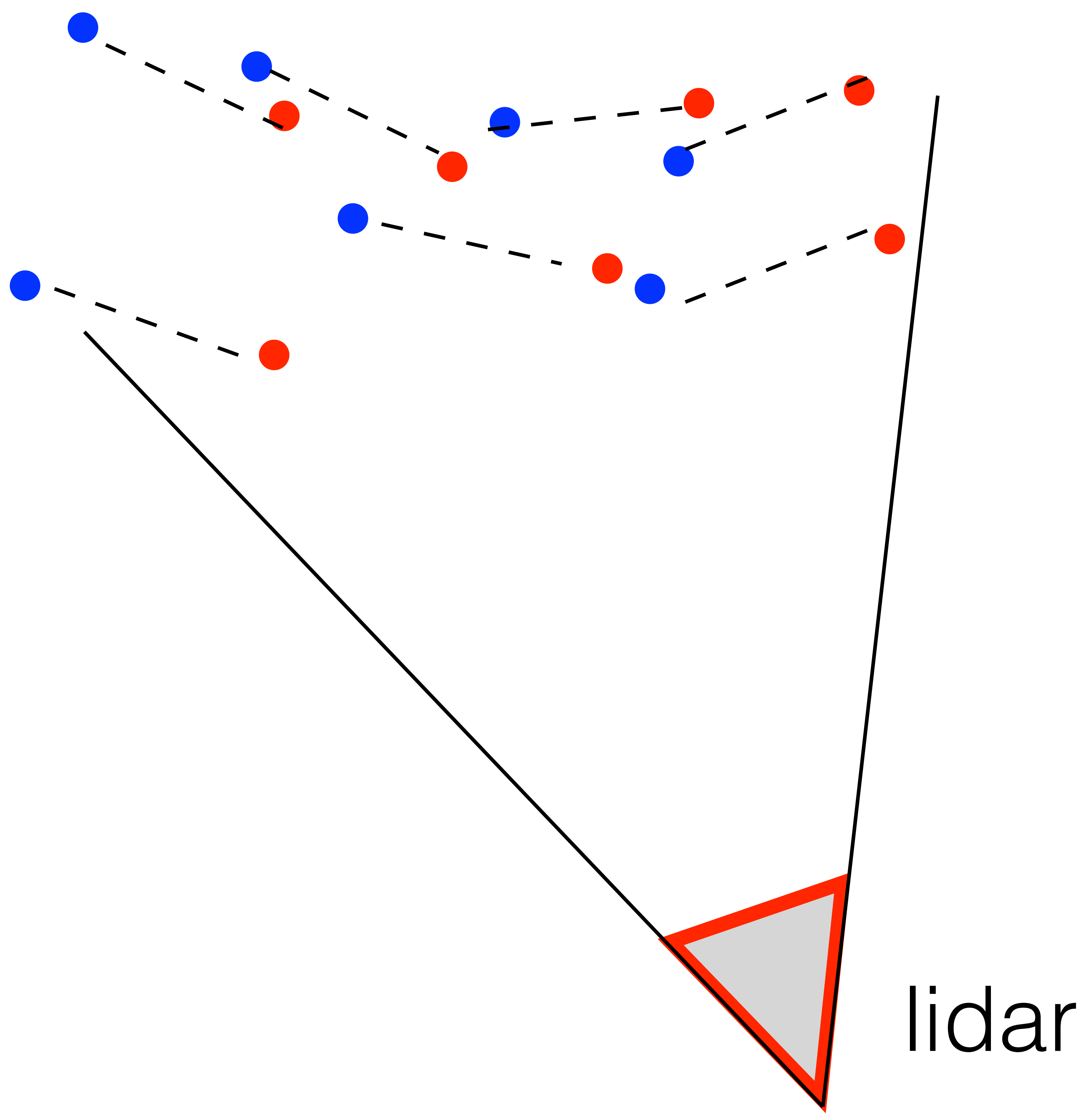
Initialization from wheel odometry



ICP correction

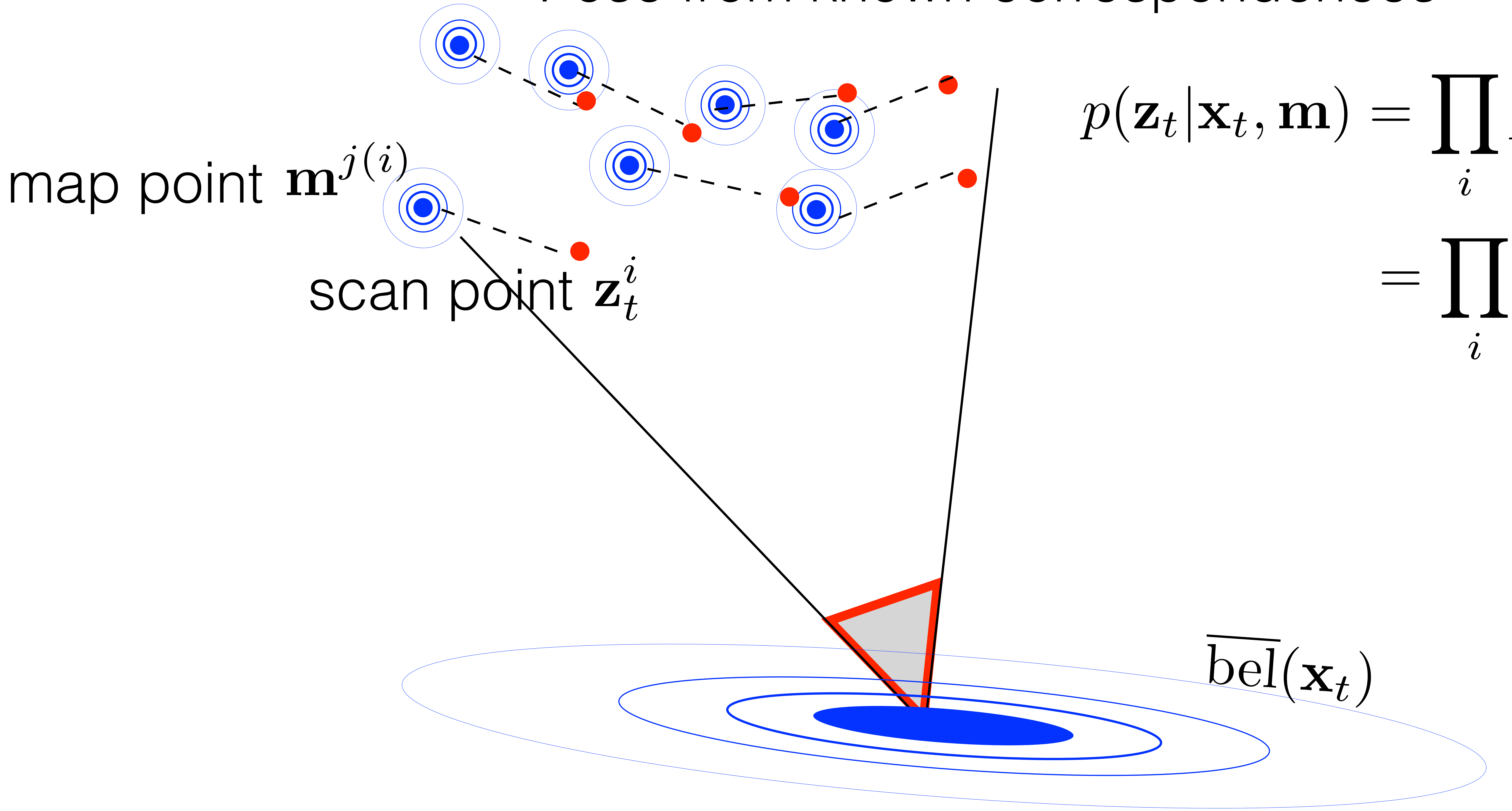


Alignment from known correspondences



$$\text{bel}(\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{m})\overline{\text{bel}}(\mathbf{x}_t)$$

Pose from known correspondences



$$p(\mathbf{z}_t|\mathbf{x}_t,\mathbf{m}) = \prod_i p(\mathbf{z}_t^i|\mathbf{x}_t,\mathbf{m}^{j(i)})$$

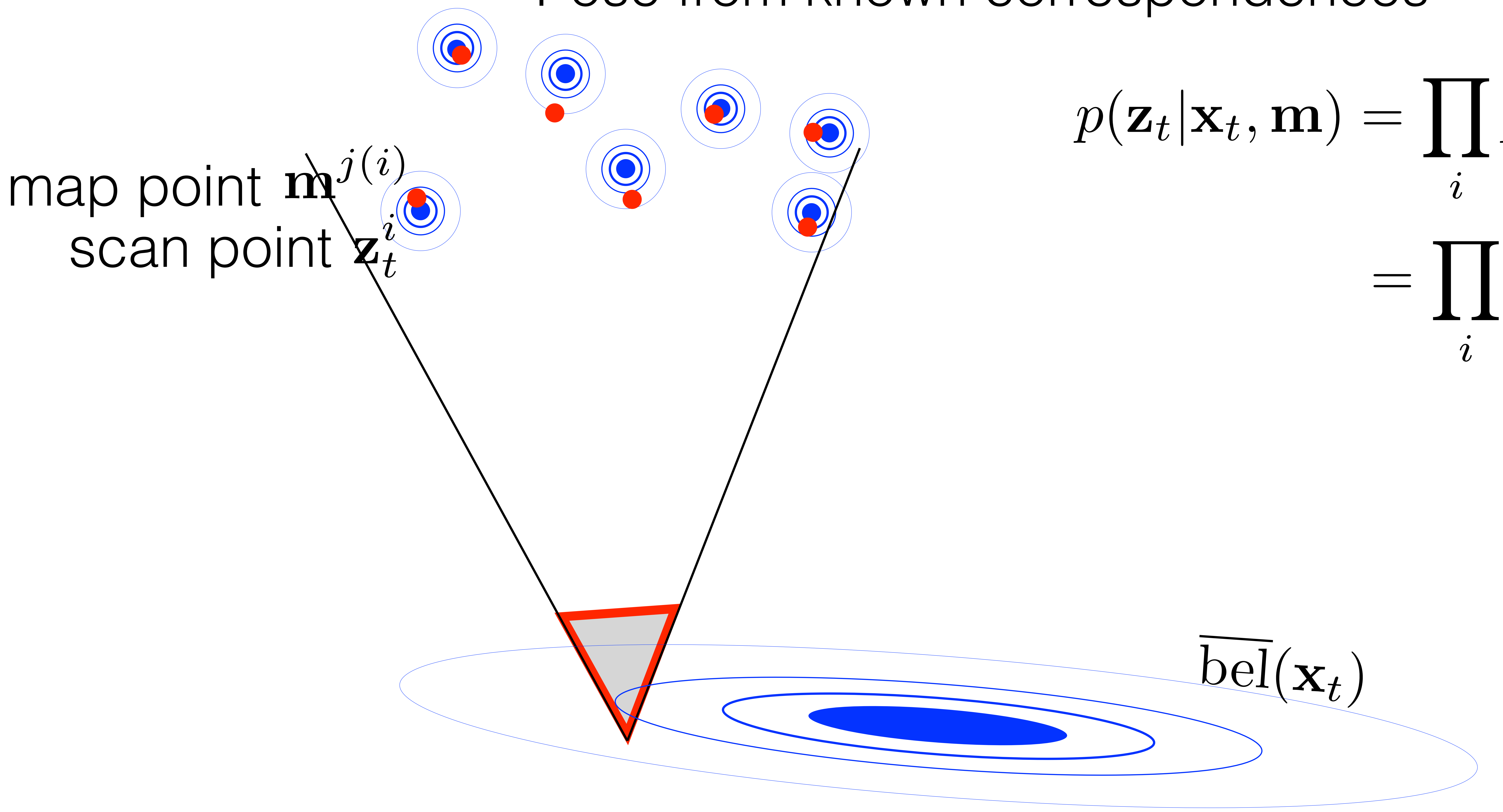
$$= \prod_i \mathcal{N}_{T(\mathbf{z}_t^i,\mathbf{x}_t,)}(\mathbf{m}^{j(i)},\mathbf{Q}_t)$$

$$\arg\max_{\mathbf{x}_t} \; p(\mathbf{z}_t|\mathbf{x}_t,\mathbf{m})\cdot\overline{\text{bel}}(\mathbf{x}_t) = \arg\max_{\mathbf{x}_t} \prod_i \mathcal{N}_{T(\mathbf{z}_t^i,\mathbf{x}_t)}(\mathbf{m}^{j(i)},\mathbf{I})\cdot\mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t,\overline{\boldsymbol{\Sigma}}_t)$$

$$= \arg\min_{\mathbf{x}_t} \; \|T(\mathbf{z}_t^i,\mathbf{x}_t)-\mathbf{m}^{j(i)}\|_2^2 + \sout{(\mathbf{x}_t-\overline{\boldsymbol{\mu}}_t)^\top\overline{\boldsymbol{\Sigma}}_t^{-1}(\mathbf{x}_t-\overline{\boldsymbol{\mu}}_t)}$$ … Levenberg-Marquardt

$$\approx \arg\min_{\mathbf{x}_t} \|T(\mathbf{z}_t^i,\mathbf{x}_t)-\mathbf{m}^{j(i)}\|_2^2$$ … Closed-form solution

Pose from known correspondences



$$p(\mathbf{z}_t|\mathbf{x}_t,\mathbf{m}) = \prod_i p(\mathbf{z}_t^i|\mathbf{x}_t,\mathbf{m}^{j(i)})$$

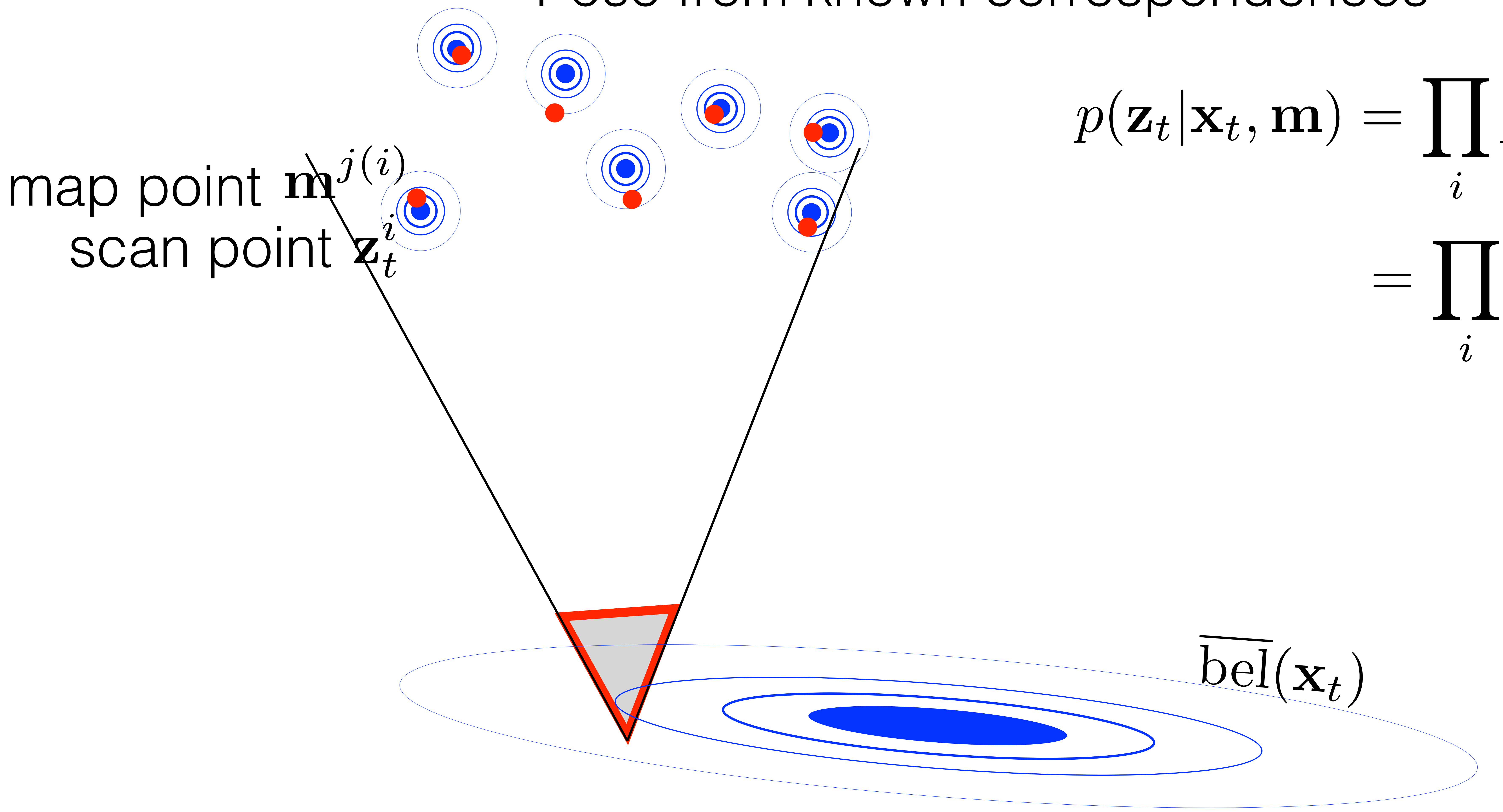
$$= \prod_i \mathcal{N}_{T(\mathbf{z}_t^i,\mathbf{x}_t,)}(\mathbf{m}^{j(i)},\mathbf{Q}_t)$$

$$\arg\max_{\mathbf{x}_t}\; p(\mathbf{z}_t|\mathbf{x}_t,\mathbf{m})\cdot\overline{\mathrm{bel}}(\mathbf{x}_t) = \arg\max_{\mathbf{x}_t}\; \prod_i \mathcal{N}_{T(\mathbf{z}_t^i,\mathbf{x}_t)}(\mathbf{m}^{j(i)},\mathbf{I})\cdot\mathcal{N}_{\mathbf{x}_t}(\bar{\boldsymbol{\mu}}_t,\bar{\boldsymbol{\Sigma}}_t)$$

$$= \arg\min_{\mathbf{x}_t}\; \|T(\mathbf{z}_t^i,\mathbf{x}_t)-\mathbf{m}^{j(i)}\|_2^2 + \xcancel{(\mathbf{x}_t-\bar{\boldsymbol{\mu}}_t)^\top\bar{\boldsymbol{\Sigma}}_t^{-1}(\mathbf{x}_t-\bar{\boldsymbol{\mu}}_t)}$$ ... Levenberg-Marquardt

$$\approx \arg\min_{\mathbf{x}_t} \|T(\mathbf{z}_t^i,\mathbf{x}_t)-\mathbf{m}^{j(i)}\|_2^2$$ ... Closed-form solution

Pose from known correspondences



$$p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{m}) = \prod_i p(\mathbf{z}_t^i|\mathbf{x}_t, \mathbf{m}^{j(i)})$$

$$= \prod_i \mathcal{N}_{T(\mathbf{z}_t^i, \mathbf{x}_t,)}(\mathbf{m}^{j(i)}, \mathbf{Q}_t)$$

Optimal pose is gaussian and hessian of the criterion is its covariance