Occupancy grid

Karel Zimmermann

Motivation

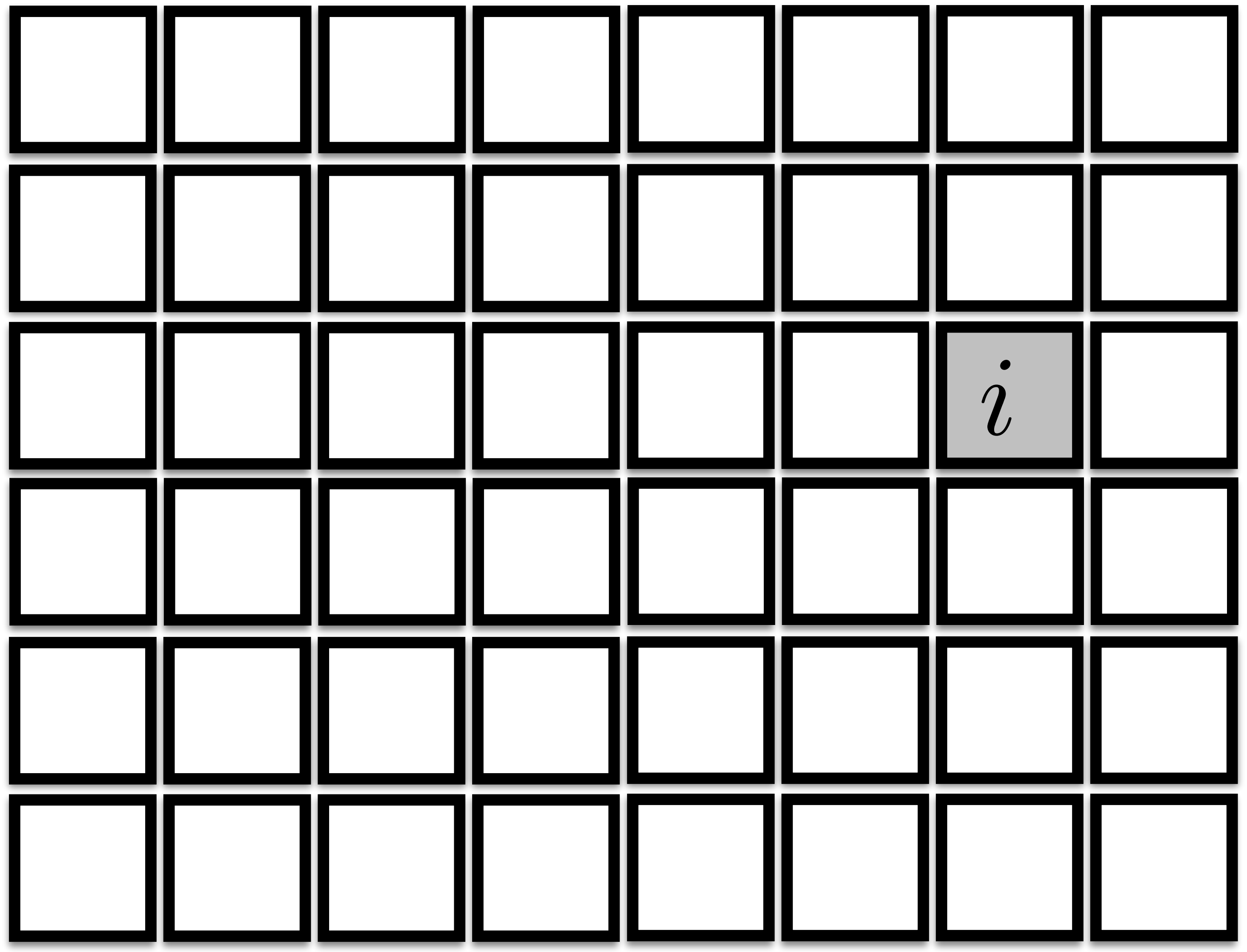
- planning a robot path typically requires to distinguish "unoccupied" (free) space from "unknown" space.
- simplest representation which allows to do this, is occupancy grid.

…. occupied (+1)

…. unknown (0)

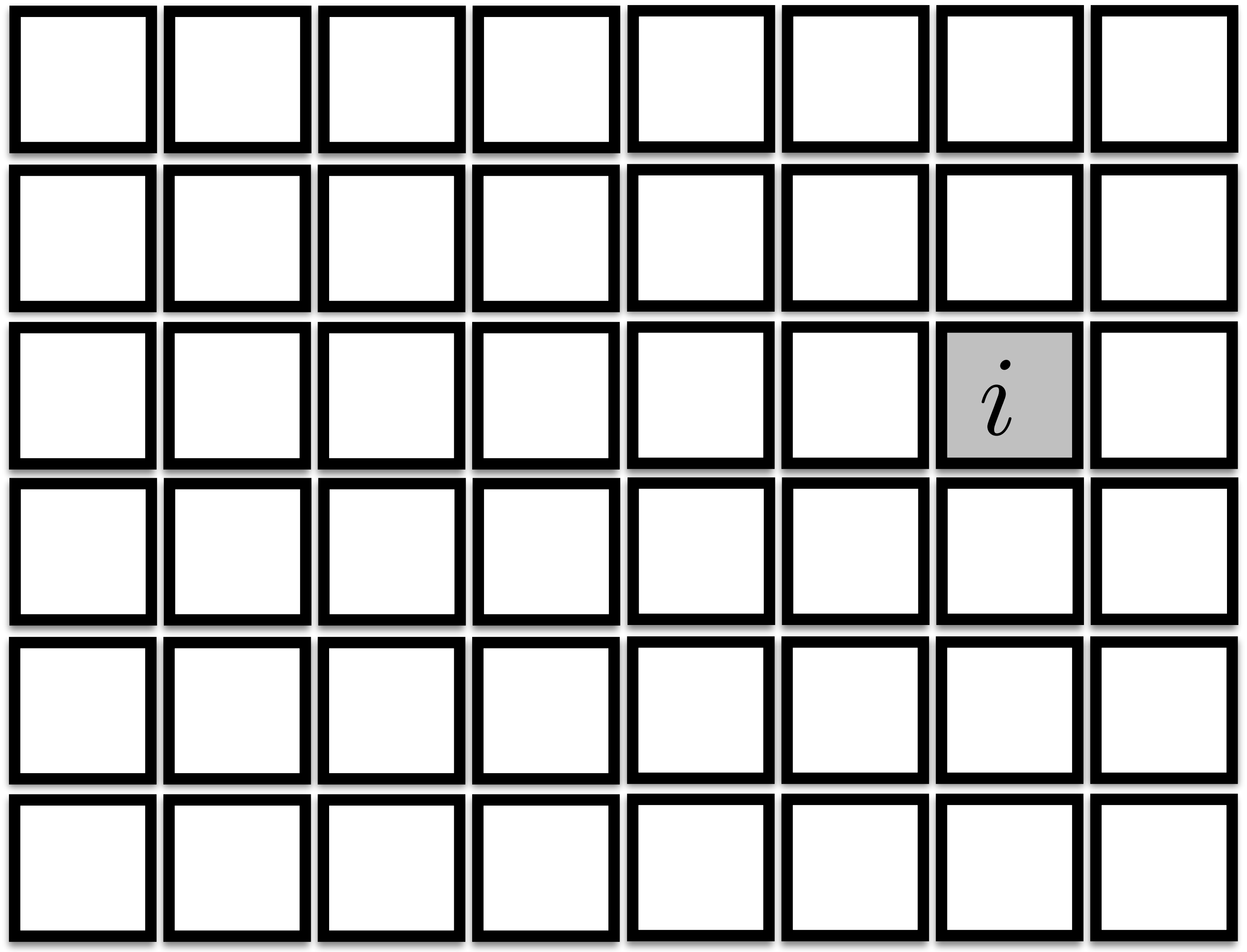
…. unoccupied (-1)

- Assumptions:
  - occupancy of a cell is binary random variable independent of other cells,
  - world is static
  - robot poses $\mathbf{x}_{1:t}$ are known.
- We model only the probability $p(o_i|\mathbf{z}_{1:t}, \cancel{\mathbf{x}_{0:t}, \mathbf{u}_{1:t}})$ that cell i is occupied given measurements $\mathbf{z}_{1:t}$



2D occupancy grid

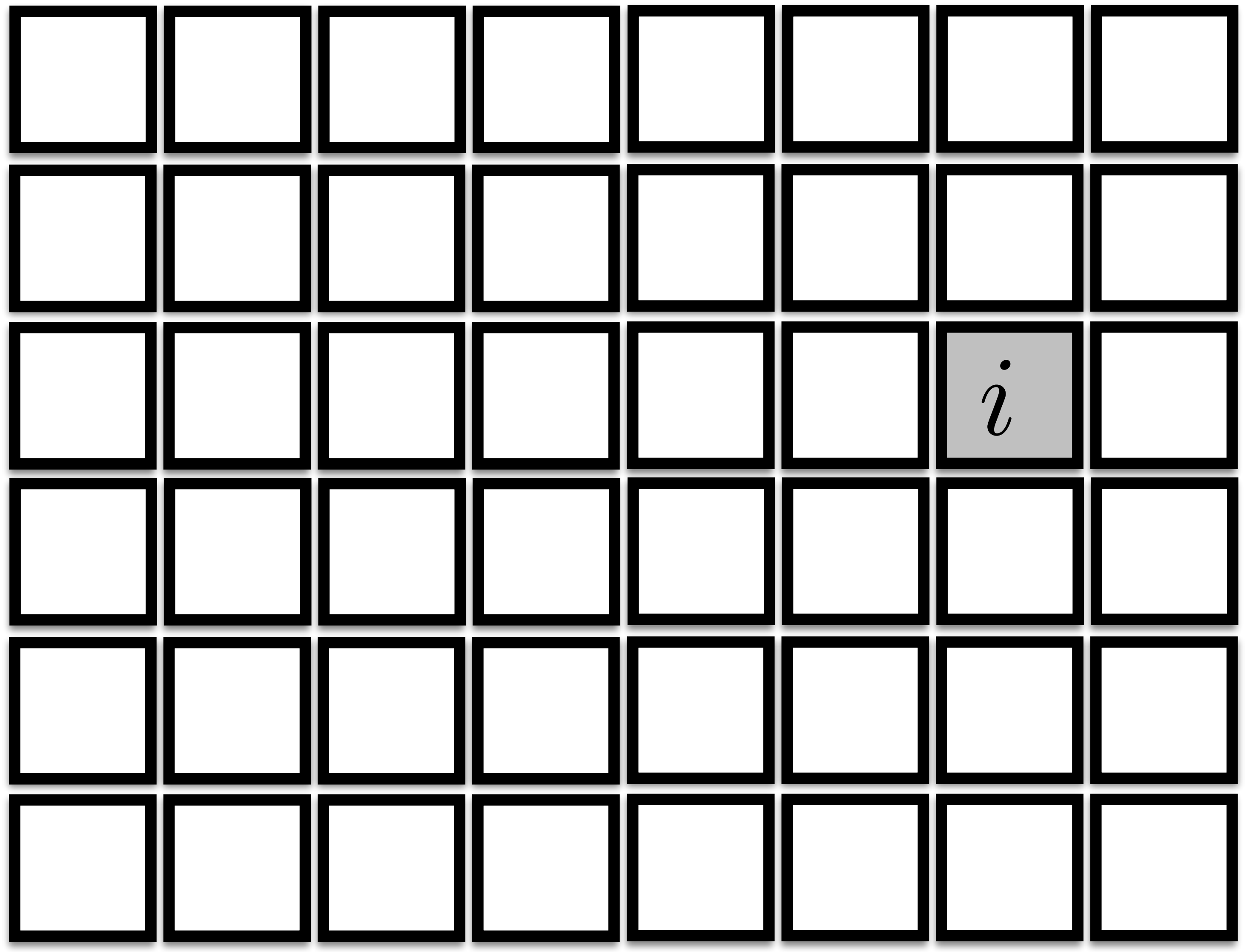
- Assumptions:
  - occupancy of a cell is binary random variable independent of other cells,
  - world is static
  - robot poses $\mathbf{x}_{1:t}$ are known.
- We model only the probability $p(o_i|\mathbf{z}_{1:t})$ that cell i is occupied given measurements $\mathbf{z}_{1:t}$



2D occupancy grid

Occupancy grid

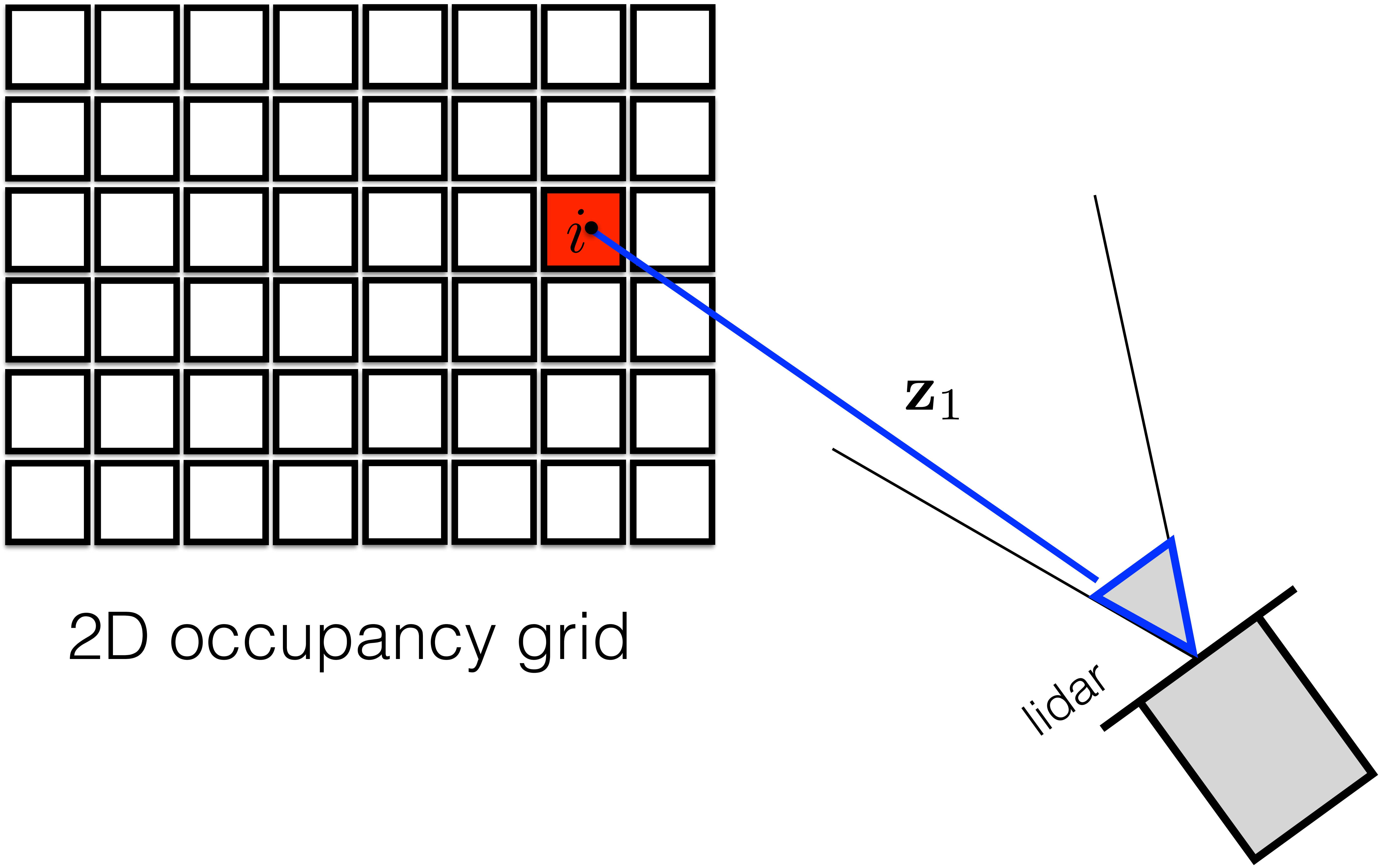
- We model only the probability $p(o_i|\mathbf{z}_{1:t})$ that cell i is occupied given measurements $\mathbf{z}_{1:t}$



2D occupancy grid

Occupancy grid

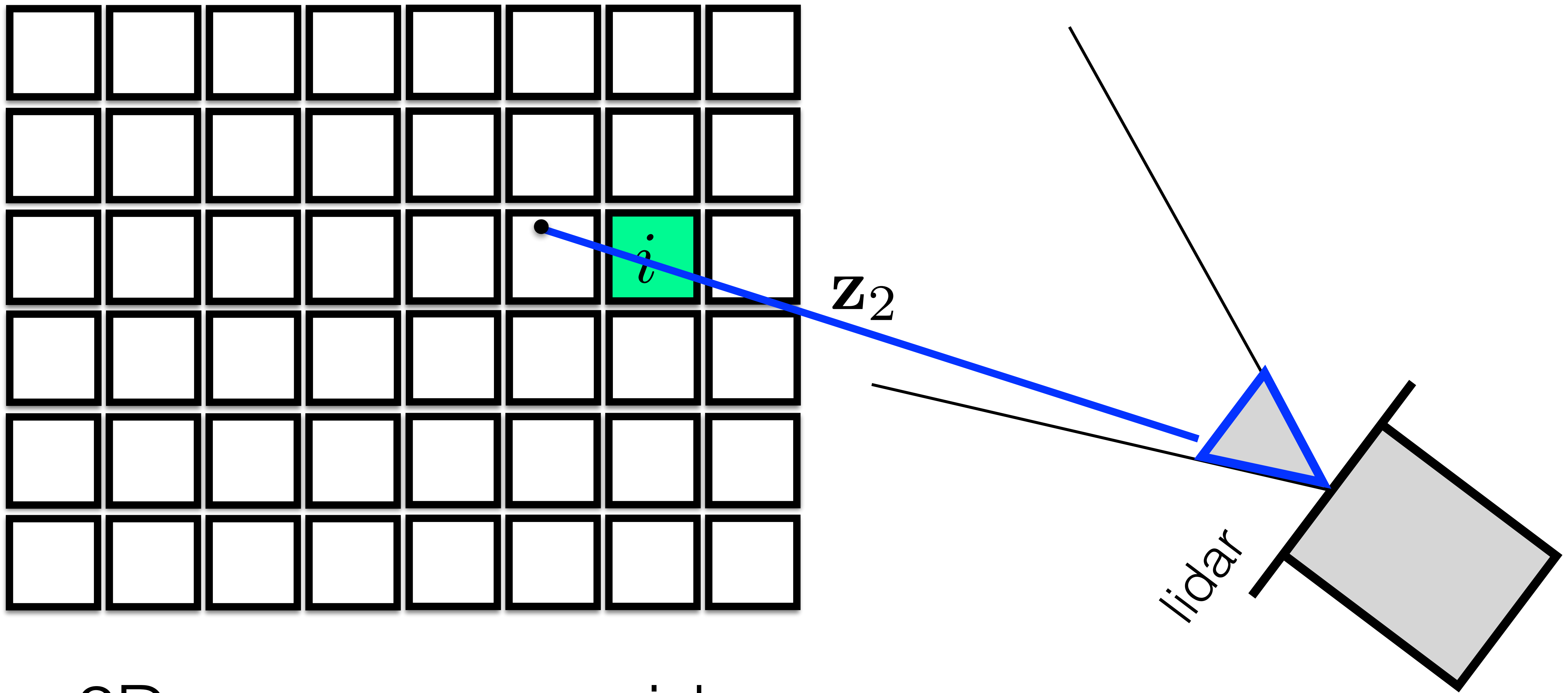
- We model only the probability $p(o_i|\mathbf{z}_{1:t})$ that cell i is occupied given measurements $\mathbf{z}_{1:t}$



2D occupancy grid

Occupancy grid

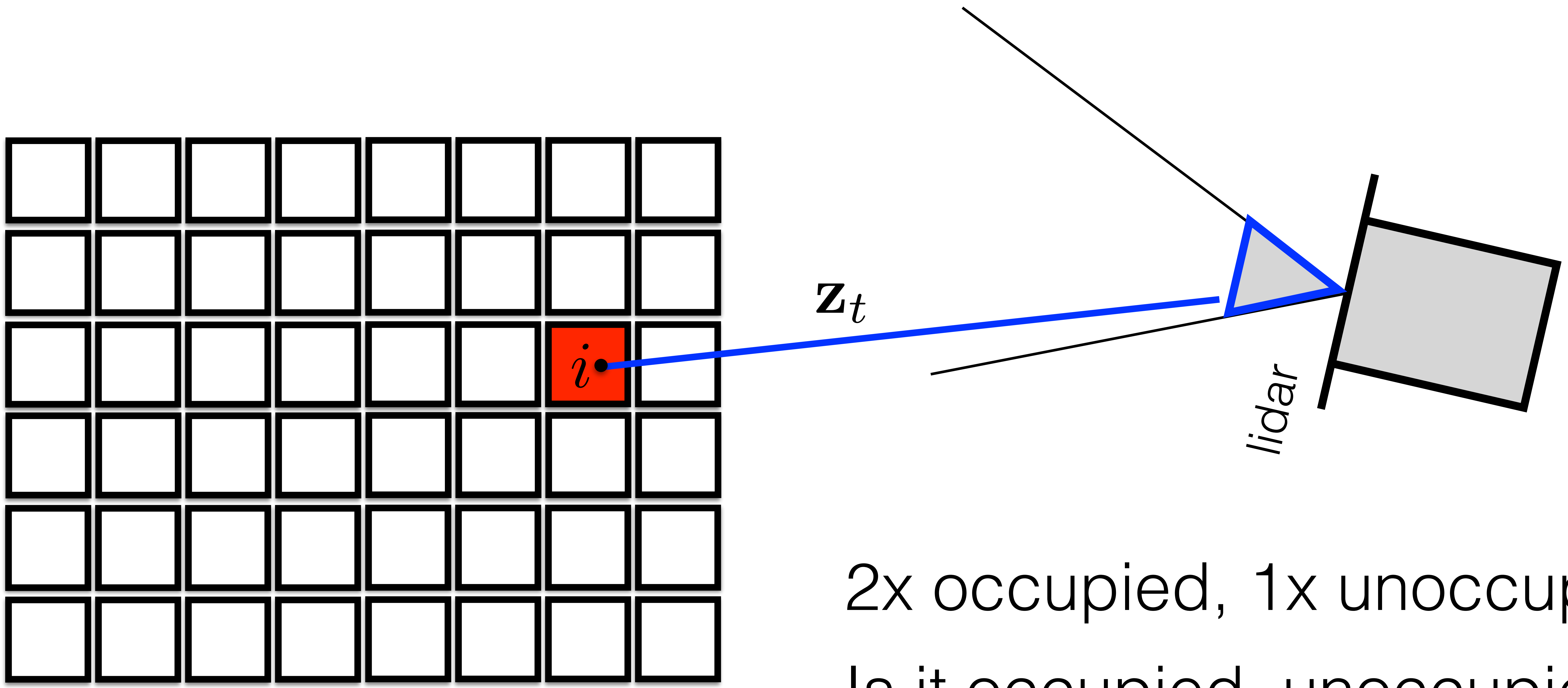
- We model only the probability $p(o_i|\mathbf{z}_{1:t})$ that cell i is occupied given measurements $\mathbf{z}_{1:t}$



2D occupancy grid

Occupancy grid

- We model only the probability $p(o_i|\mathbf{z}_{1:t})$ that cell i is occupied given measurements $\mathbf{z}_{1:t}$



2D occupancy grid

2x occupied, 1x unoccupied

Is it occupied, unoccupied or unknown?

Simple hack: $b = 2 - 1$

$b \leq \theta_L$ …. unoccupied

$\theta_L < b < \theta_H$ …. unknown

$b \geq \theta_H$ …. occupied

We will search for probabilistic justification of this hack!

Occupancy grid

- We model only the probability $p(o_i|\mathbf{z}_{1:t})$ that cell i is occupied given measurements $\mathbf{z}_{1:t}$

$$p(o_i|\mathbf{z}_{1:t}) \overset{\text{BR}}{=} \frac{p(\mathbf{z}_t|\mathbf{z}_{1:t-1}, o_i)\ p(o_i|\mathbf{z}_{1:t-1})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})} \overset{\text{CI}}{=} \frac{p(\mathbf{z}_t|o_i)\ p(o_i|\mathbf{z}_{1:t-1})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}$$

$$\overset{\text{BR}}{=} \frac{p(o_i|\mathbf{z}_t)p(\mathbf{z}_t)}{p(o_i)} \frac{p(o_i|\mathbf{z}_{1:t-1})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}$$

Conditional independence: $p(\mathbf{z}_t|\mathbf{z}_{1:t-1}, o_i) = p(\mathbf{z}_t|o_i)$

Occupancy grid

- We model only the probability $p(o_i|\mathbf{z}_{1:t})$ that cell i is occupied given measurements $\mathbf{z}_{1:t}$

$$p(o_i|\mathbf{z}_{1:t}) \overset{\text{BR}}{=} \frac{p(\mathbf{z}_t|\mathbf{z}_{1:t-1}, o_i) \;\; p(o_i|\mathbf{z}_{1:t-1})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})} \overset{\text{CI}}{=} \frac{p(\mathbf{z}_t|o_i) \;\; p(o_i|\mathbf{z}_{1:t-1})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}$$

$$\overset{\text{BR}}{=} \frac{p(o_i|\mathbf{z}_t)p(\mathbf{z}_t)}{p(o_i)} \frac{p(o_i|\mathbf{z}_{1:t-1})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}$$

Conditional independence: $p(\mathbf{z}_t|\mathbf{z}_{1:t-1}, o_i) = p(\mathbf{z}_t|o_i)$

$$p(\neg o_i|\mathbf{z}_{1:t}) = \frac{p(\neg o_i|\mathbf{z}_t)p(\mathbf{z}_t)}{p(\neg o_i)} \frac{p(\neg o_i|\mathbf{z}_{1:t-1})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}$$

Occupancy grid

$$\frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{\dfrac{p(o_i|\mathbf{z}_t)\cancel{p(\mathbf{z}_t)} \; p(o_i|\mathbf{z}_{1:t-1})}{p(o_i) \; \cancel{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}}}{\dfrac{p(\neg o_i|\mathbf{z}_t)\cancel{p(\mathbf{z}_t)} \; p(\neg o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i) \; \cancel{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}}}$$

Occupancy grid

$$\frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{\dfrac{p(o_i|\mathbf{z}_t)\cancel{p(\mathbf{z}_t)}}{p(o_i)} \dfrac{p(o_i|\mathbf{z}_{1:t-1})}{\cancel{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}}}{\dfrac{p(\neg o_i|\mathbf{z}_t)\cancel{p(\mathbf{z}_t)}}{p(\neg o_i)} \dfrac{p(\neg o_i|\mathbf{z}_{1:t-1})}{\cancel{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}}}$$

$$= \frac{p(o_i|\mathbf{z}_t) \; p(o_i|\mathbf{z}_{1:t-1})}{p(o_i)}$$

Occupancy grid

$$\frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{\dfrac{p(o_i|\mathbf{z}_t)p(\mathbf{z}_t)}{p(o_i)} \dfrac{p(o_i|\mathbf{z}_{1:t-1})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}}{\dfrac{p(\neg o_i|\mathbf{z}_t)p(\mathbf{z}_t)}{p(\neg o_i)} \dfrac{p(\neg o_i|\mathbf{z}_{1:t-1})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}}$$

$$= \frac{p(o_i|\mathbf{z}_t) \; p(o_i|\mathbf{z}_{1:t-1}) \qquad p(\neg o_i)}{p(o_i) \qquad p(\neg o_i|\mathbf{z}_t)p(\neg o_i|\mathbf{z}_{1:t-1})}$$

Occupancy grid

$$\frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{\dfrac{p(o_i|\mathbf{z}_t)\cancel{p(\mathbf{z}_t)}}{p(o_i)} \dfrac{p(o_i|\mathbf{z}_{1:t-1})}{\cancel{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}}}{\dfrac{p(\neg o_i|\mathbf{z}_t)\cancel{p(\mathbf{z}_t)}}{p(\neg o_i)} \dfrac{p(\neg o_i|\mathbf{z}_{1:t-1})}{\cancel{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}}}$$

$$= \frac{p(o_i|\mathbf{z}_t) \; p(o_i|\mathbf{z}_{1:t-1})}{p(o_i)} \frac{p(\neg o_i)}{p(\neg o_i|\mathbf{z}_t)p(\neg o_i|\mathbf{z}_{1:t-1})}$$

$$= \frac{\qquad}{\qquad}$$

Occupancy grid

$$\frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{\dfrac{p(o_i|\mathbf{z}_t)\cancel{p(\mathbf{z}_t)}}{p(o_i)} \dfrac{p(o_i|\mathbf{z}_{1:t-1})}{\cancel{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}}}{\dfrac{p(\neg o_i|\mathbf{z}_t)\cancel{p(\mathbf{z}_t)}}{p(\neg o_i)} \dfrac{p(\neg o_i|\mathbf{z}_{1:t-1})}{\cancel{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}}}$$

$$= \frac{p(o_i|\mathbf{z}_t)\ p(o_i|\mathbf{z}_{1:t-1})}{p(o_i)} \frac{p(\neg o_i)}{p(\neg o_i|\mathbf{z}_t)p(\neg o_i|\mathbf{z}_{1:t-1})}$$

$$= \frac{p(o_i|\mathbf{z}_t)}{p(\neg o_i|\mathbf{z}_t)}$$

Occupancy grid

$$\frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{\dfrac{p(o_i|\mathbf{z}_t)\cancel{p(\mathbf{z}_t)}\ \ p(o_i|\mathbf{z}_{1:t-1})}{p(o_i)\ \ \cancel{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}}}{\dfrac{p(\neg o_i|\mathbf{z}_t)\cancel{p(\mathbf{z}_t)}\ \ p(\neg o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i)\ \ \cancel{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}}}$$

$$= \frac{p(o_i|\mathbf{z}_t)\ \ p(o_i|\mathbf{z}_{1:t-1})}{p(o_i)} \frac{p(\neg o_i)}{p(\neg o_i|\mathbf{z}_t)p(\neg o_i|\mathbf{z}_{1:t-1})}$$

$$= \frac{p(o_i|\mathbf{z}_t)\ \ \ \ p(o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i|\mathbf{z}_t)\ \ \ \ p(\neg o_i|\mathbf{z}_{1:t-1})}$$

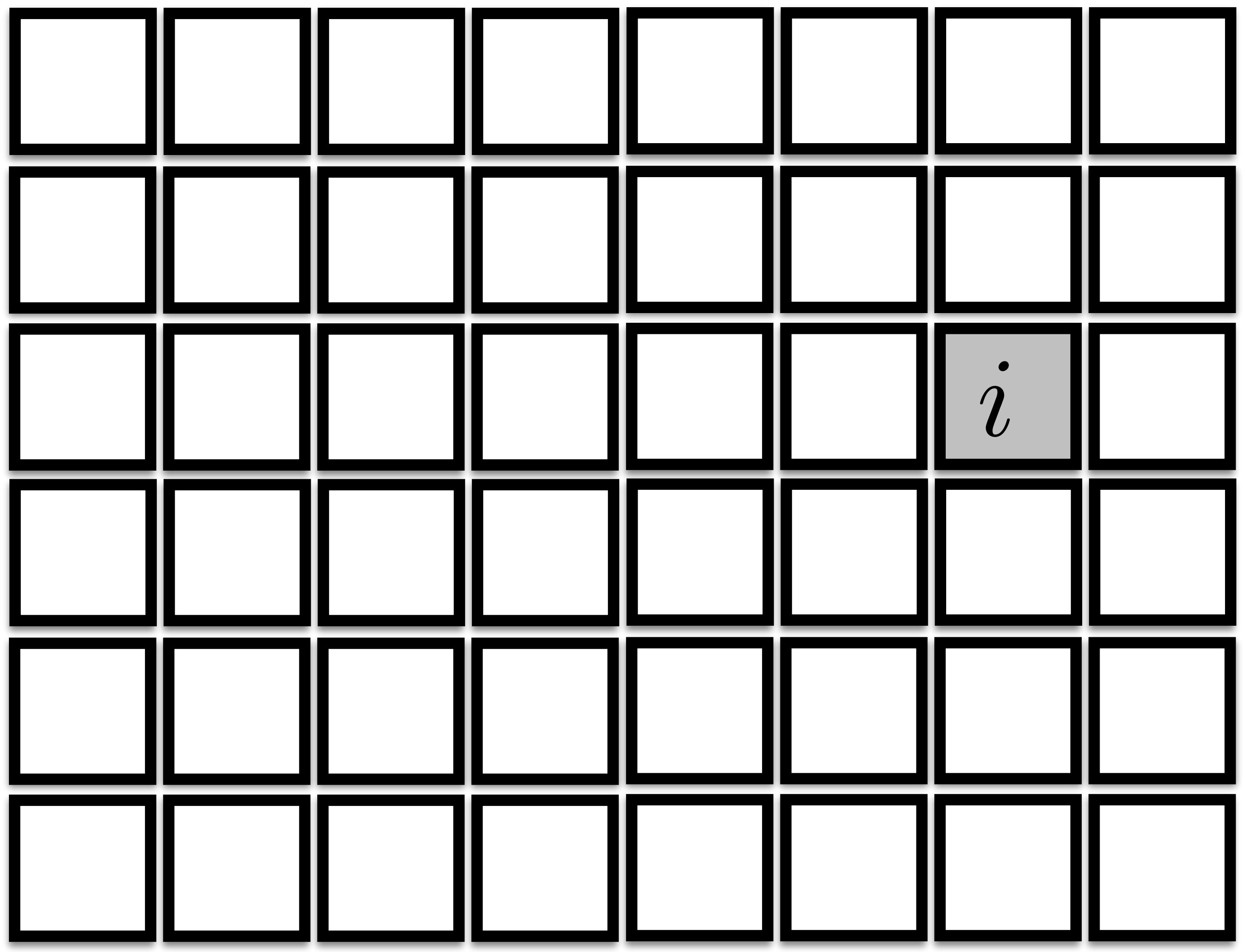
Occupancy grid

$$\frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{\dfrac{p(o_i|\mathbf{z}_t)\cancel{p(\mathbf{z}_t)} \quad p(o_i|\mathbf{z}_{1:t-1})}{p(o_i) \qquad \cancel{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}}}{\dfrac{p(\neg o_i|\mathbf{z}_t)\cancel{p(\mathbf{z}_t)} \quad p(\neg o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i) \qquad \cancel{p(\mathbf{z}_t|\mathbf{z}_{1:t-1})}}}$$

$$= \frac{p(o_i|\mathbf{z}_t) \quad p(o_i|\mathbf{z}_{1:t-1})}{p(o_i)} \frac{p(\neg o_i)}{p(\neg o_i|\mathbf{z}_t)p(\neg o_i|\mathbf{z}_{1:t-1})}$$

$$= \frac{p(o_i|\mathbf{z}_t) \qquad p(o_i|\mathbf{z}_{1:t-1}) \qquad p(\neg o_i)}{p(\neg o_i|\mathbf{z}_t) \qquad p(\neg o_i|\mathbf{z}_{1:t-1}) \qquad p(o_i)}$$

$$r = \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{p(o_i|\mathbf{z}_t)\quad p(o_i|\mathbf{z}_{1:t-1})\quad p(\neg o_i)}{p(\neg o_i|\mathbf{z}_t)\quad p(\neg o_i|\mathbf{z}_{1:t-1})\quad p(o_i)}$$



2D occupancy grid

Optimal (risk-minimizing) strategy for two-class classification problem with rejection option (unknown), is achieved by thresholding the likelihood ratio.

$r<0.5$ …. unoccupied

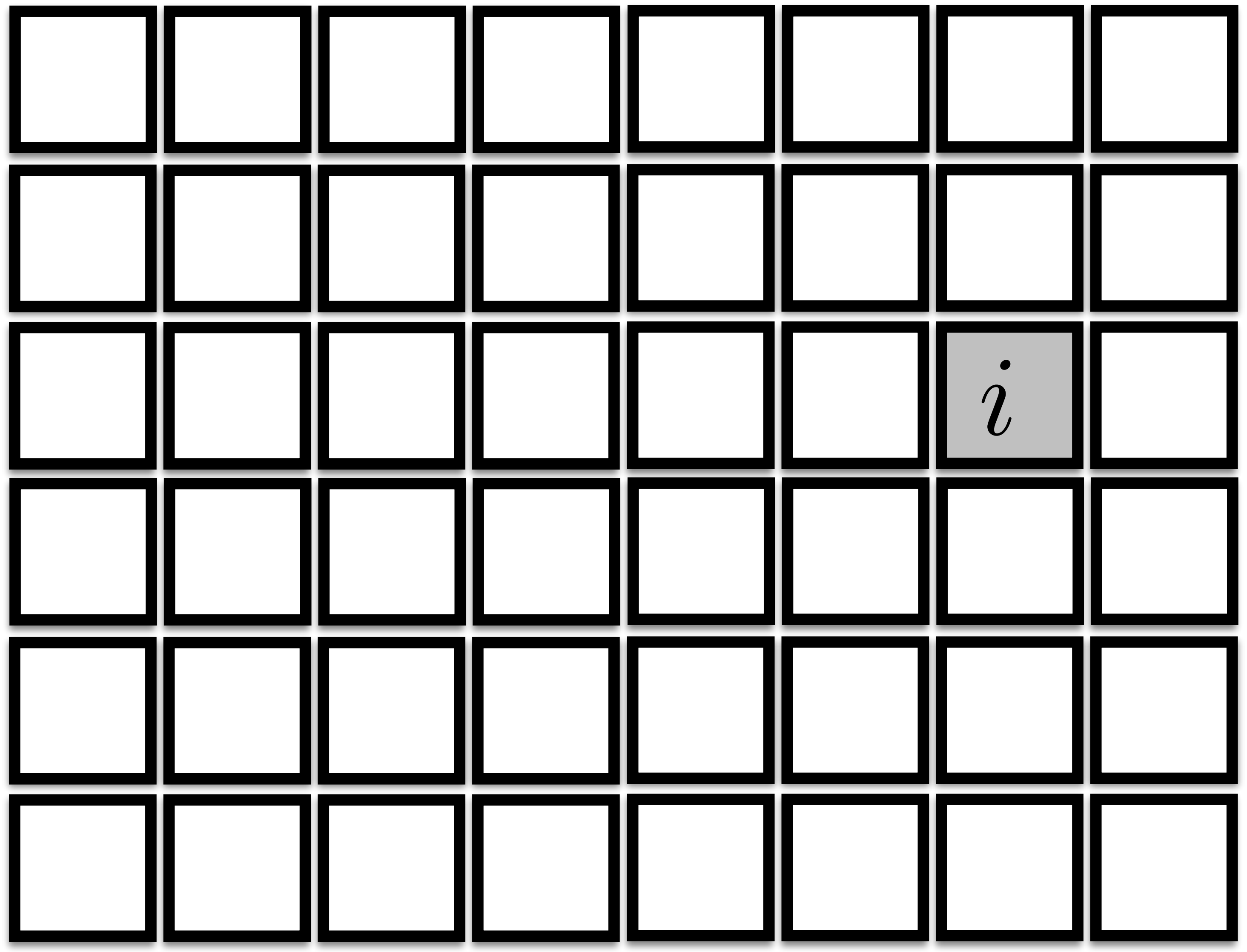
$0.5<r<2$ …. unknown

$r>2$ …. occupied

$$r = \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{p(o_i|\mathbf{z}_t) \quad p(o_i|\mathbf{z}_{1:t-1}) \quad p(\neg o_i)}{p(\neg o_i|\mathbf{z}_t) \quad p(\neg o_i|\mathbf{z}_{1:t-1}) \quad p(o_i)}$$

inverse measurement model	previous belief	prior

$p(o_i)$ =0.5



2D occupancy grid

r<0.5 …. unoccupied

0.5<r<2 …. unknown

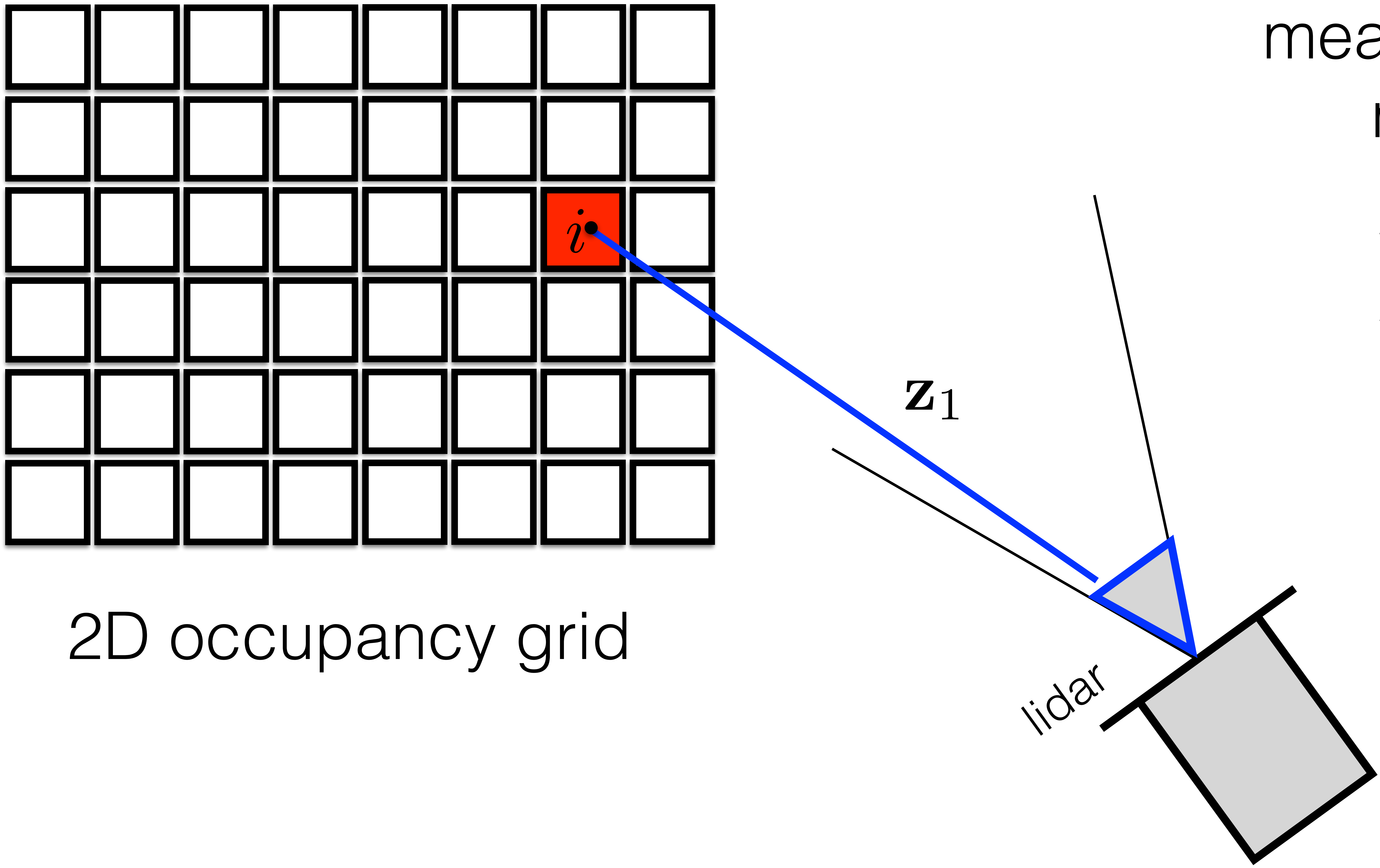
r>2 …. occupied

$$r = \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{p(o_i|\mathbf{z}_t)}{p(\neg o_i|\mathbf{z}_t)} \frac{p(o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i|\mathbf{z}_{1:t-1})} \frac{p(\neg o_i)}{p(o_i)}$$

inverse measurement model — previous belief — prior

$p(o_i) = 0.5$

$p(o_i|\mathbf{z}_t) = 0.9$ if point in cell i



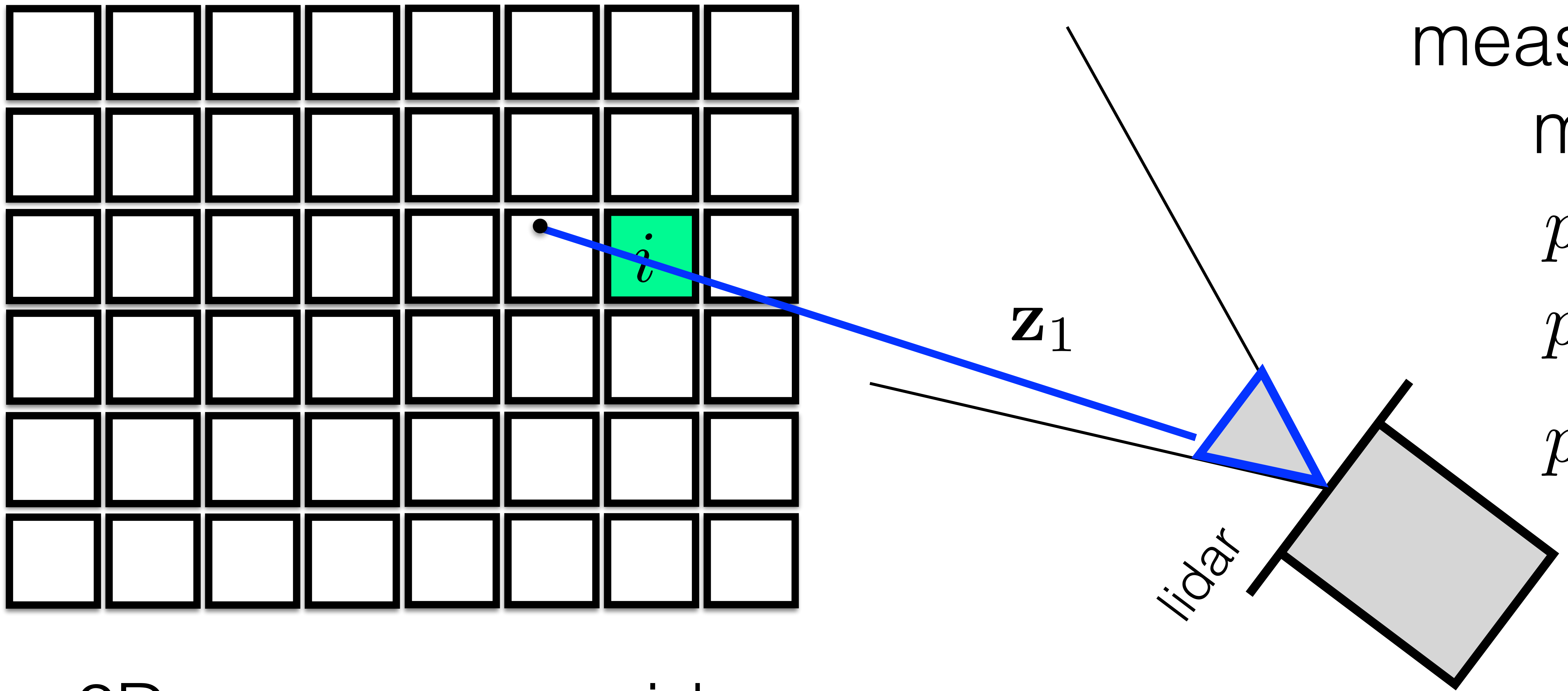
r<0.5 …. unoccupied

0.5<r<2 …. unknown

r>2 …. occupied

$$r = \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{p(o_i|\mathbf{z}_t)}{p(\neg o_i|\mathbf{z}_t)} \frac{p(o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i|\mathbf{z}_{1:t-1})} \frac{p(\neg o_i)}{p(o_i)}$$

inverse measurement model | previous belief | prior



2D occupancy grid

$p(o_i)$ =0.5

$p(o_i|\mathbf{z}_t)$ =0.9 if point in cell i

$p(o_i|\mathbf{z}_t)$ =0.1 if ray intersects cell i

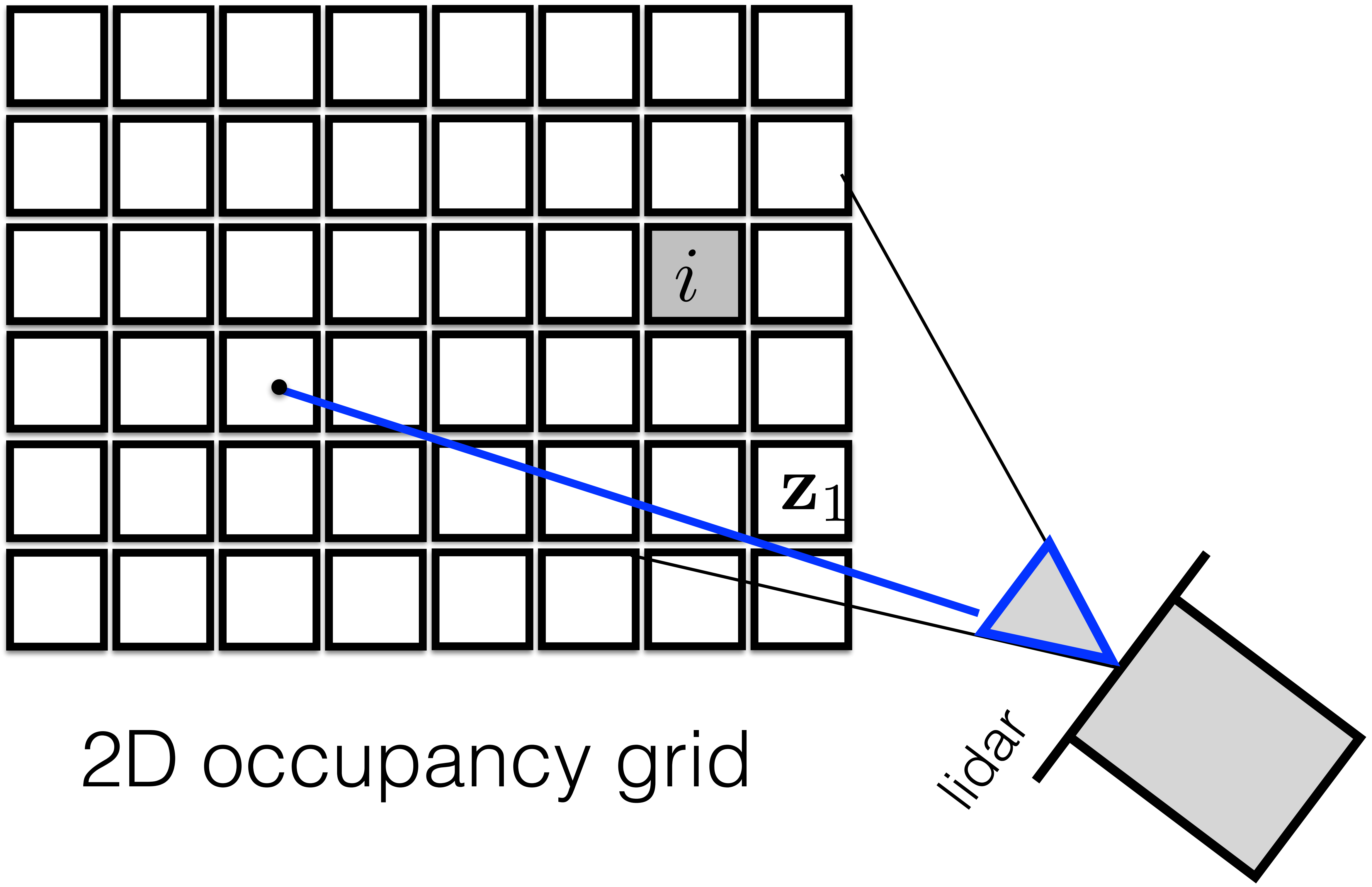
r<0.5 …. unoccupied

0.5<r<2 …. unknown

r>2 …. occupied

$$r = \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{p(o_i|\mathbf{z}_t) \quad p(o_i|\mathbf{z}_{1:t-1}) \quad p(\neg o_i)}{p(\neg o_i|\mathbf{z}_t) \quad p(\neg o_i|\mathbf{z}_{1:t-1}) \quad p(o_i)}$$

inverse measurement model — previous belief — prior



2D occupancy grid

$p(o_i)$ =0.5

$p(o_i|\mathbf{z}_t)$ =0.9 if point in cell i

$p(o_i|\mathbf{z}_t)$ =0.1 if ray intersects cell i

$p(o_i|\mathbf{z}_t) = p(o_i)$ otherwise

r<0.5 …. unoccupied

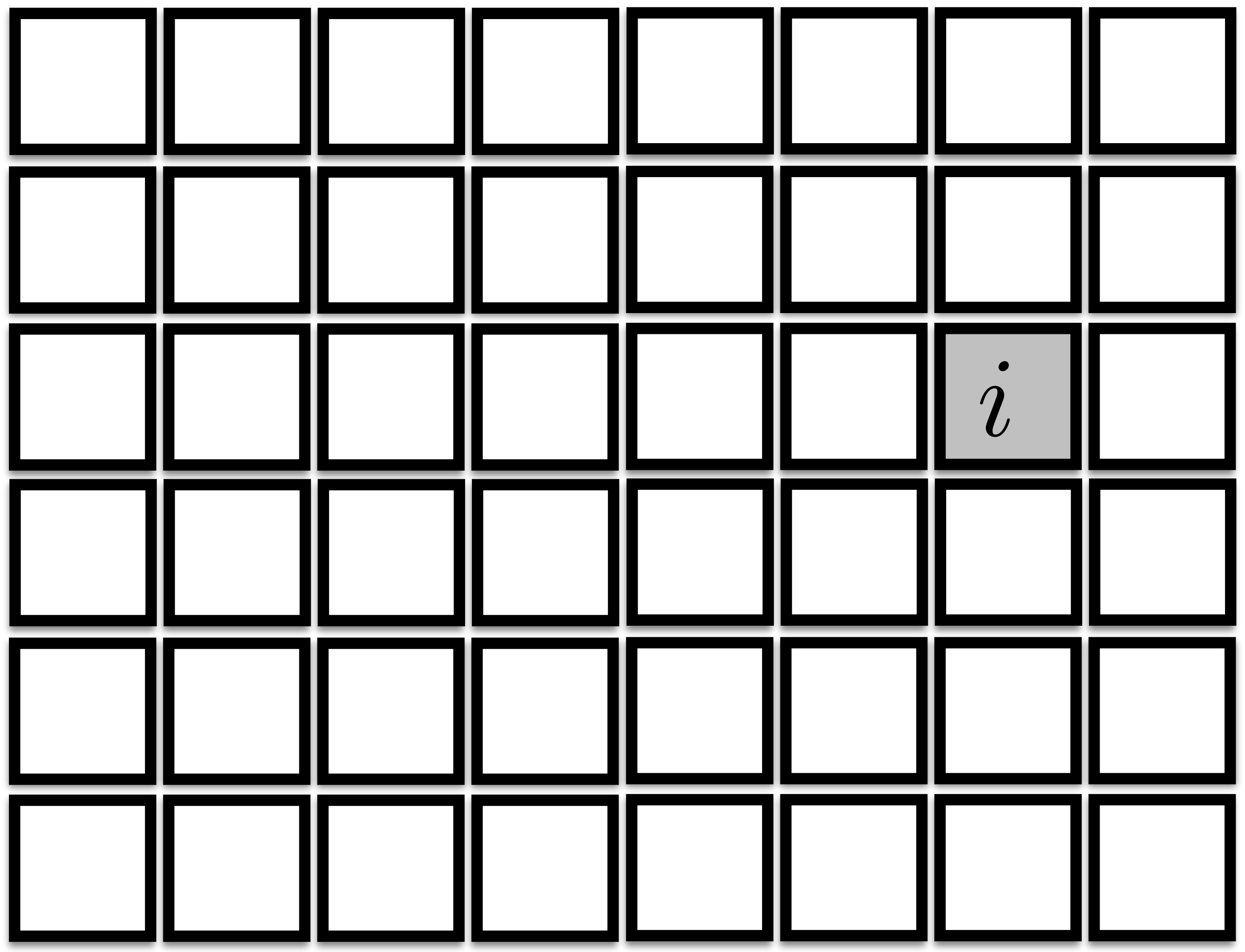
0.5<r<2 …. unknown

r>2 …. occupied

$$1 = \frac{0.5}{0.5}$$

$$r = \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{p(o_i|\mathbf{z}_t)}{p(\neg o_i|\mathbf{z}_t)} \; \frac{p(o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i|\mathbf{z}_{1:t-1})} \; \frac{p(\neg o_i)}{p(o_i)}$$

inverse measurement model — previous belief — prior



2D occupancy grid

$p(o_i)$ =0.5

$p(o_i|\mathbf{z}_t)$ =0.9 if point in cell i

$p(o_i|\mathbf{z}_t)$ =0.1 if ray intersects cell i

$p(o_i|\mathbf{z}_t) = p(o_i)$ otherwise

r<0.5 …. unoccupied

0.5<r<2 …. unknown

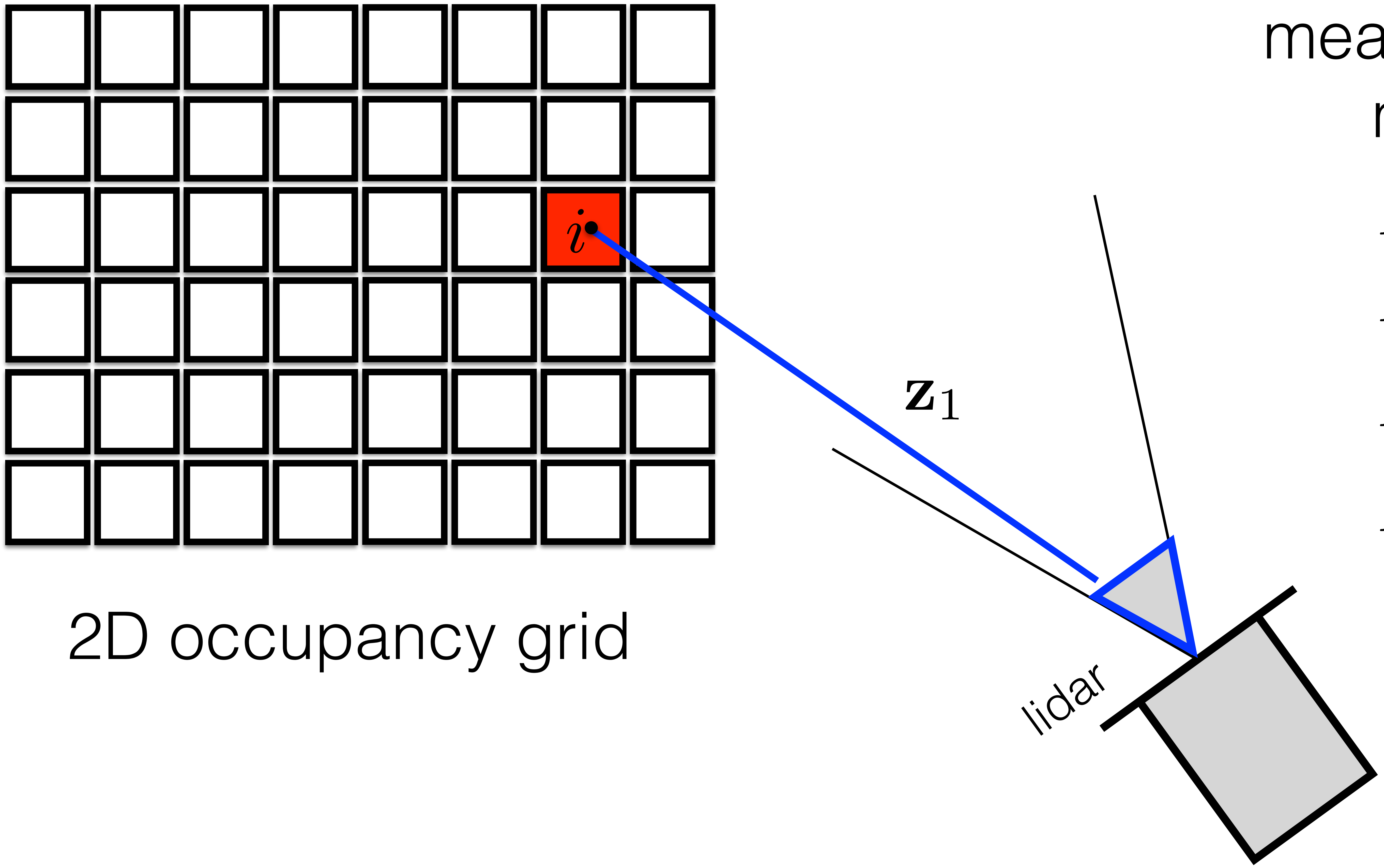
r>2 …. occupied

$$9 \quad = \quad \frac{0.9}{0.1} \qquad\qquad \frac{0.5}{0.5}$$

$$r = \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{p(o_i|\mathbf{z}_t)}{p(\neg o_i|\mathbf{z}_t)} \frac{p(o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i|\mathbf{z}_{1:t-1})} \frac{p(\neg o_i)}{p(o_i)}$$

inverse measurement model — previous belief — prior

$t = 1$



2D occupancy grid

$p(o_i) = 0.5$

$p(o_i|\mathbf{z}_t) = 0.9$ if point in cell i

$p(o_i|\mathbf{z}_t) = 0.1$ if ray intersects cell i

$p(o_i|\mathbf{z}_t) = p(o_i)$ otherwise

r<0.5 …. unoccupied

0.5<r<2 …. unknown

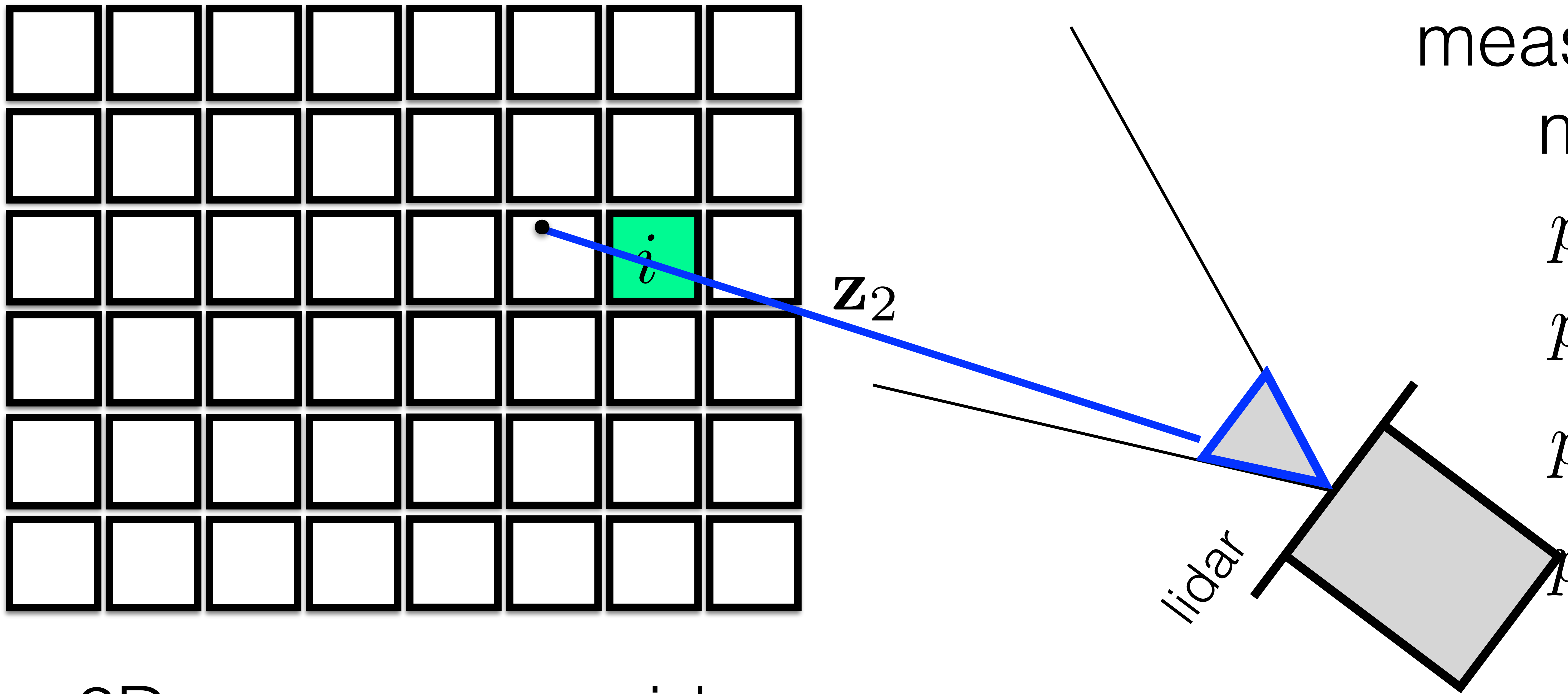
r>2 …. occupied

$$1 = \frac{0.1}{0.9} \quad 9 \quad \frac{0.5}{0.5}$$

$$r = \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{p(o_i|\mathbf{z}_t) \quad p(o_i|\mathbf{z}_{1:t-1}) \quad p(\neg o_i)}{p(\neg o_i|\mathbf{z}_t) \quad p(\neg o_i|\mathbf{z}_{1:t-1}) \quad p(o_i)}$$

inverse measurement model | previous belief | prior

$t = 2$



2D occupancy grid

$p(o_i) = 0.5$

$p(o_i|\mathbf{z}_t) = 0.9$ if point in cell i

$p(o_i|\mathbf{z}_t) = 0.1$ if ray intersects cell i

$p(o_i|\mathbf{z}_t) = p(o_i)$ otherwise

r<0.5 …. unoccupied

0.5<r<2 …. unknown

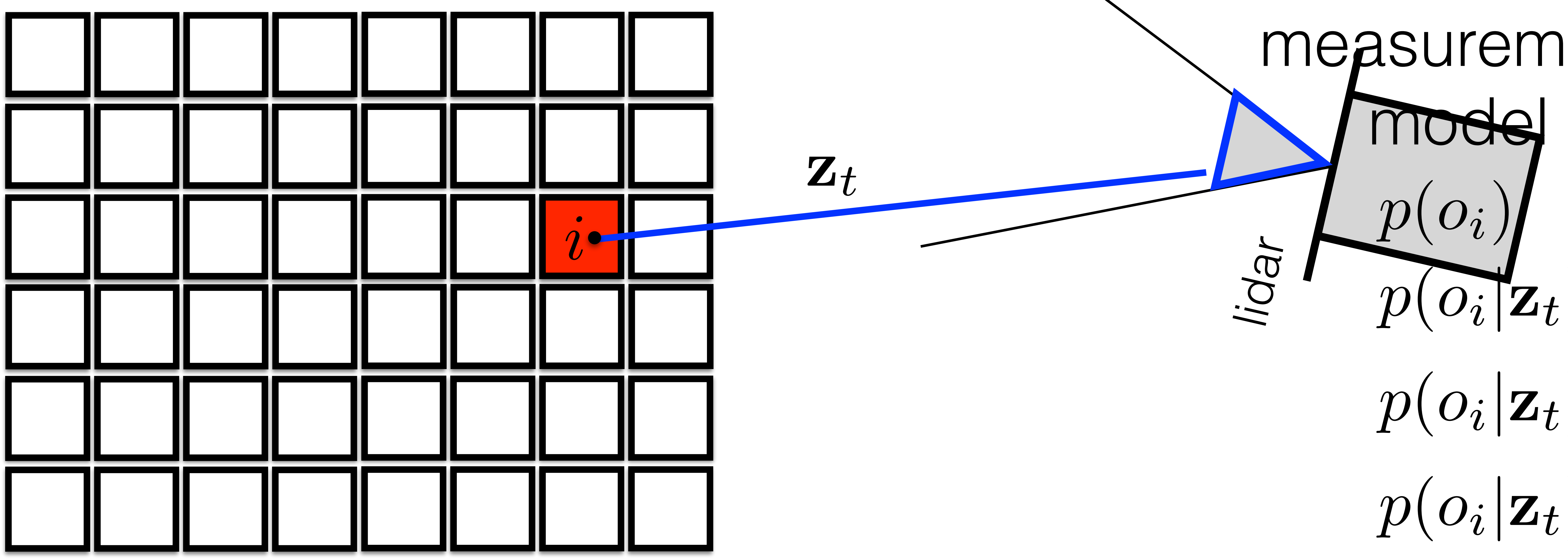
r>2 …. occupied

$$9 = \frac{0.9}{0.1} \quad 1 \quad \frac{0.5}{0.5}$$

$$r = \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{p(o_i|\mathbf{z}_t) \quad p(o_i|\mathbf{z}_{1:t-1}) \quad p(\neg o_i)}{p(\neg o_i|\mathbf{z}_t) \quad p(\neg o_i|\mathbf{z}_{1:t-1}) \quad p(o_i)}$$

inverse measurement model | previous belief | prior

$t = 3$



2D occupancy grid

$p(o_i)$ =0.5

$p(o_i|\mathbf{z}_t)$ =0.9 if point in cell i

$p(o_i|\mathbf{z}_t)$ =0.1 if ray intersects cell i

$p(o_i|\mathbf{z}_t) = p(o_i)$ otherwise

r<0.5 …. unoccupied

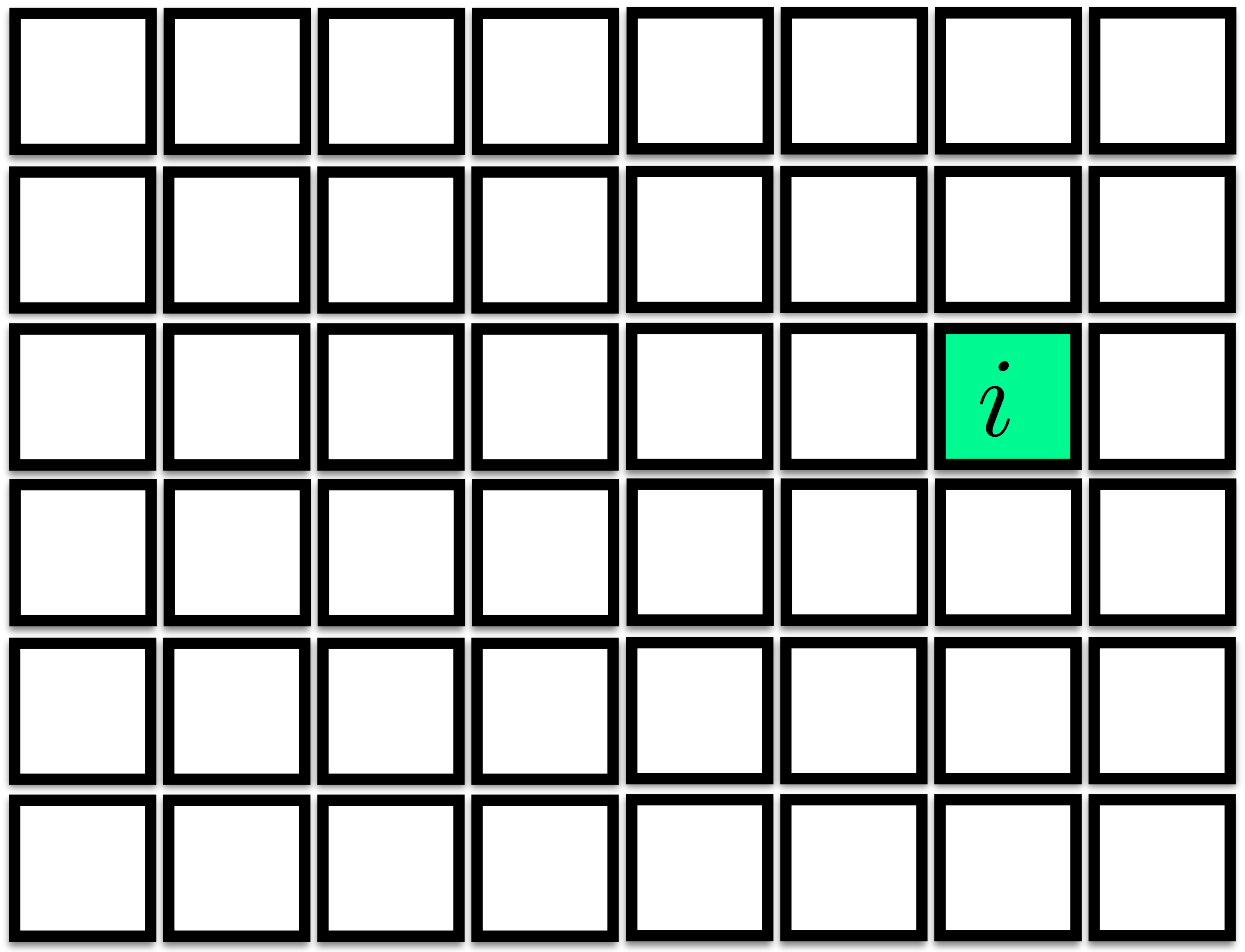
0.5<r<2 …. unknown

r>2 …. occupied

$$0{,}000001771561 = 0.11 * 0.11 * 0.11 * \ldots * 0.11 \frac{0.5}{0.5}$$

$$r = \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{p(o_i|\mathbf{z}_t) \quad p(o_i|\mathbf{z}_{1:t-1}) \quad p(\neg o_i)}{p(\neg o_i|\mathbf{z}_t) \quad p(\neg o_i|\mathbf{z}_{1:t-1}) \quad p(o_i)}$$

$t = 3$



2D occupancy grid

Multiplying floating-point-numbers:

- suffers from rounding errors.
- is more computationally demanding adding.

$r<0.5$ …. unoccupied

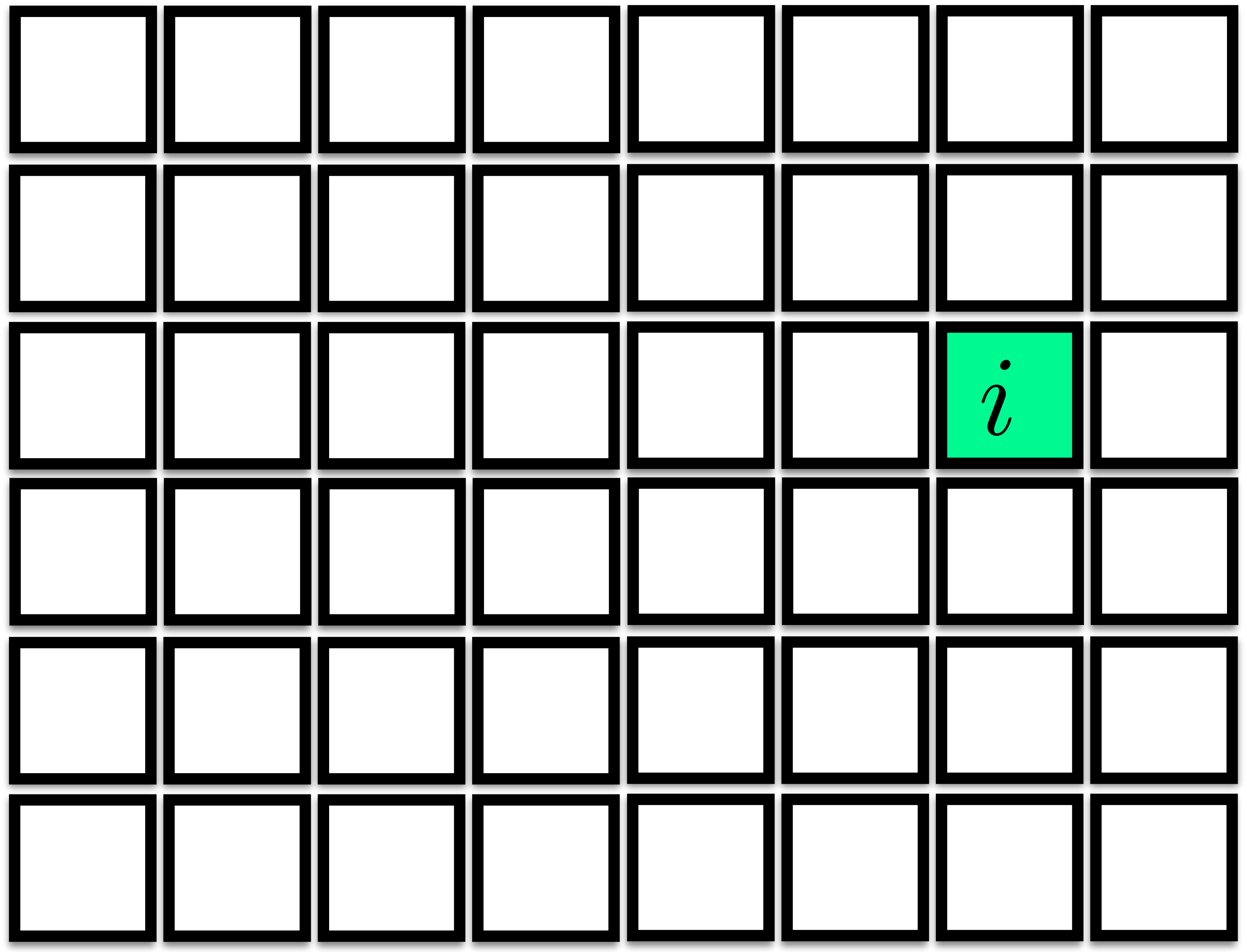
$0.5<r<2$ …. unknown

$r>2$ …. occupied

$$0,000001771561 = 0.11 * 0.11 * 0.11 * \ldots * 0.11 \frac{0.5}{0.5}$$

$$r = \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{p(o_i|\mathbf{z}_t) \quad p(o_i|\mathbf{z}_{1:t-1}) \quad p(\neg o_i)}{p(\neg o_i|\mathbf{z}_t) \quad p(\neg o_i|\mathbf{z}_{1:t-1}) \quad p(o_i)}$$

$t = 3$



2D occupancy grid

Multiplying floating-point-numbers:

- suffers from rounding errors.
- is more computationally demanding adding.

$$\log \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \log \frac{p(o_i|\mathbf{z}_t)}{p(\neg o_i|\mathbf{z}_t)} + \log \frac{p(o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i|\mathbf{z}_{1:t-1})} + \log \frac{p(\neg o_i)}{p(o_i)}$$

$r<0.5$ …. unoccupied

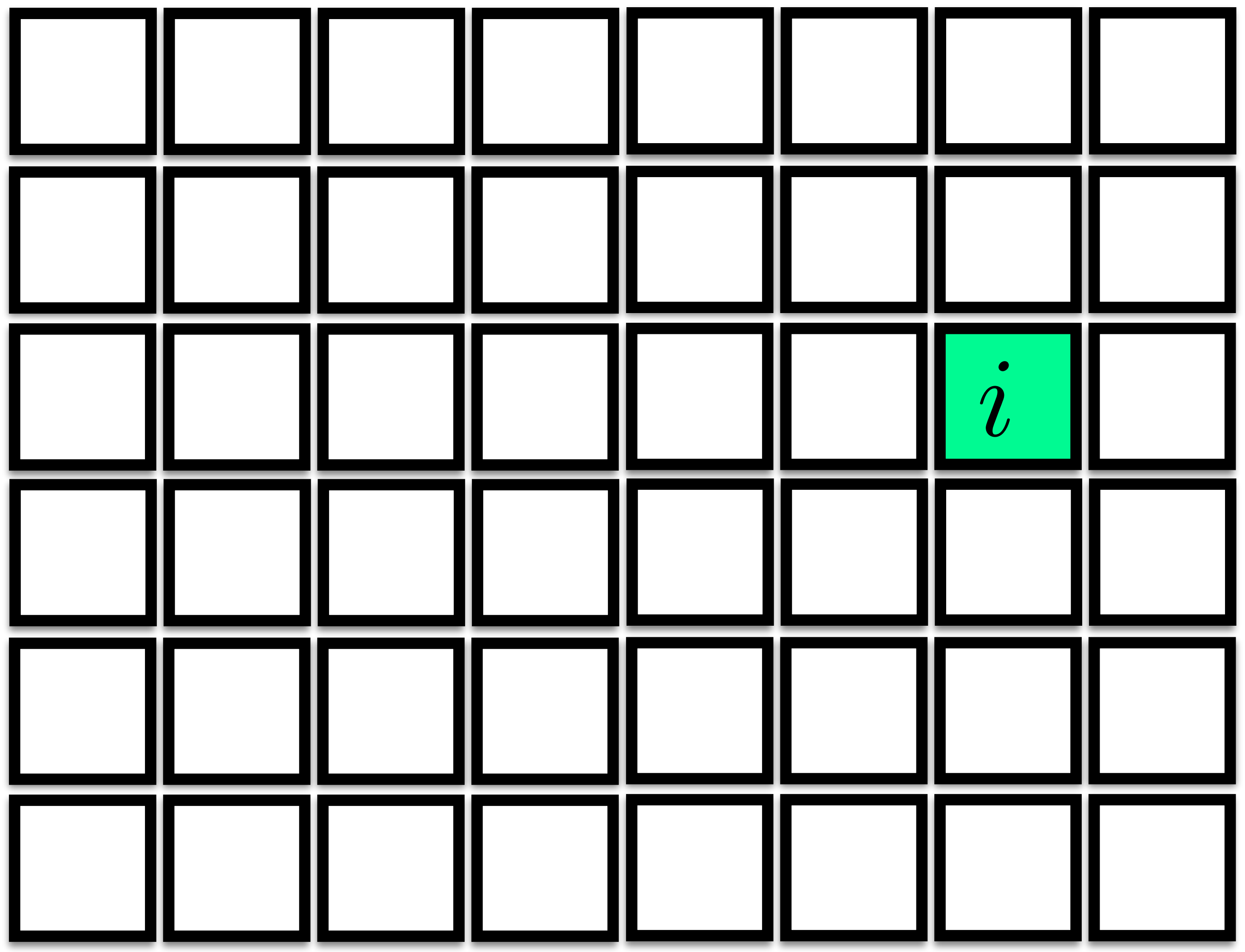
$0.5<r<2$ …. unknown

$r>2$ …. occupied

$$0{,}000001771561 = 0.11 * 0.11 * 0.11 * \ldots * 0.11 \frac{0.5}{0.5}$$

$$r = \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{p(o_i|\mathbf{z}_t)}{p(\neg o_i|\mathbf{z}_t)} \frac{p(o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i|\mathbf{z}_{1:t-1})} \frac{p(\neg o_i)}{p(o_i)}$$

$t = 3$



2D occupancy grid

Multiplying floating-point-numbers:

- suffers from rounding errors.
- is more computationally demanding adding.

$$\underbrace{\log \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})}}_{b} = \log \frac{p(o_i|\mathbf{z}_t)}{p(\neg o_i|\mathbf{z}_t)} + \log \frac{p(o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i|\mathbf{z}_{1:t-1})} + \log \frac{p(\neg o_i)}{p(o_i)}$$

log-odds ratio

can be always converted to prob

r<0.5 …. unoccupied

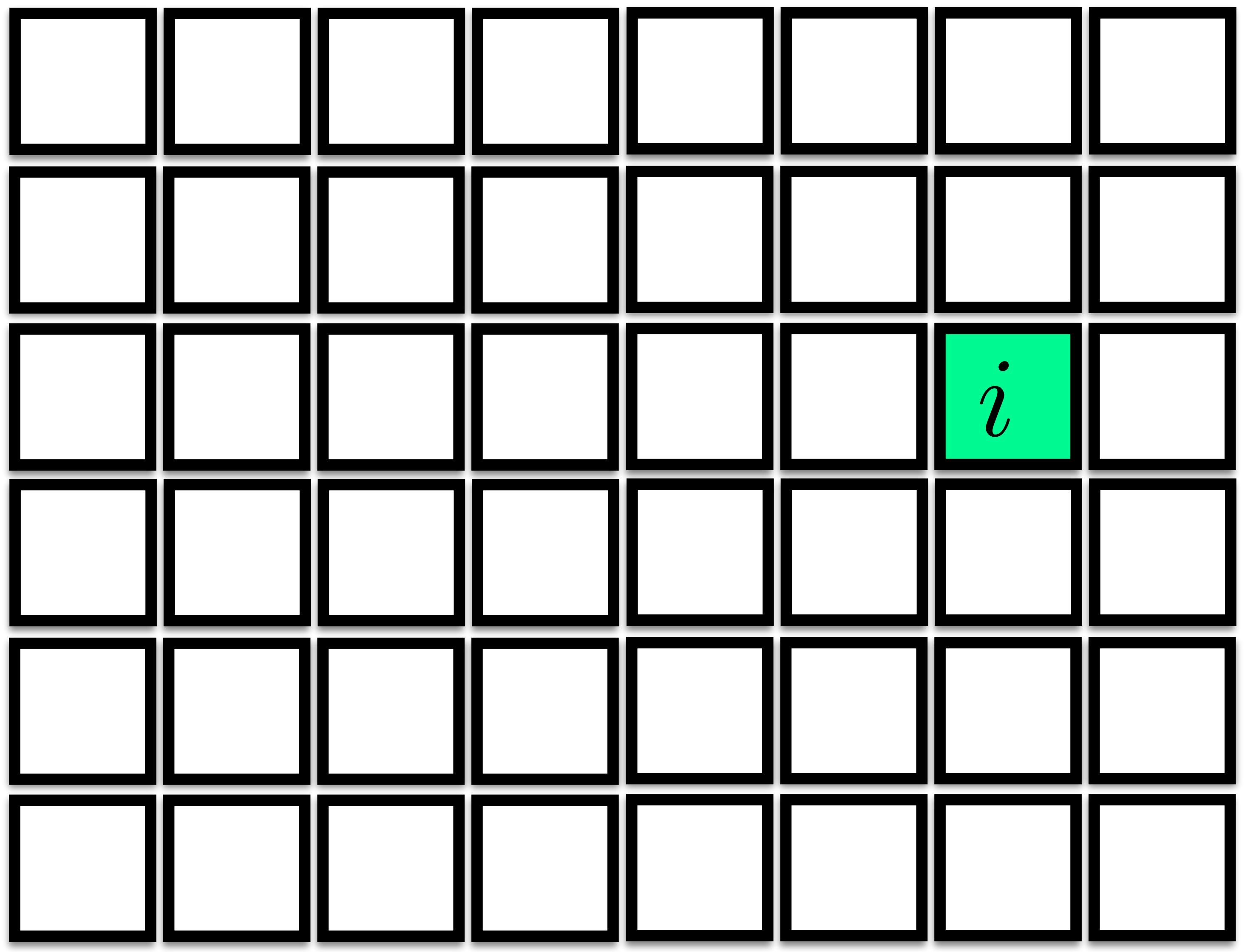
0.5<r<2 …. unknown

r>2 …. occupied

$$0{,}000001771561 = 0.11 * 0.11 * 0.11 * \ldots * 0.11$$

$$r = \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \frac{p(o_i|\mathbf{z}_t)}{p(\neg o_i|\mathbf{z}_t)} \frac{p(o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i|\mathbf{z}_{1:t-1})} \frac{p(\neg o_i)}{p(o_i)}$$

$t = 3$



2D occupancy grid

Multiplying floating-point-numbers:

- suffers from rounding errors.
- is more computationally demanding adding.

$$\underbrace{\log \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})}}_{b} = \log \frac{p(o_i|\mathbf{z}_t)}{p(\neg o_i|\mathbf{z}_t)} + \log \frac{p(o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i|\mathbf{z}_{1:t-1})} + \log \frac{p(\neg o_i)}{p(o_i)}$$

log-odds ratio

can be always converted to prob

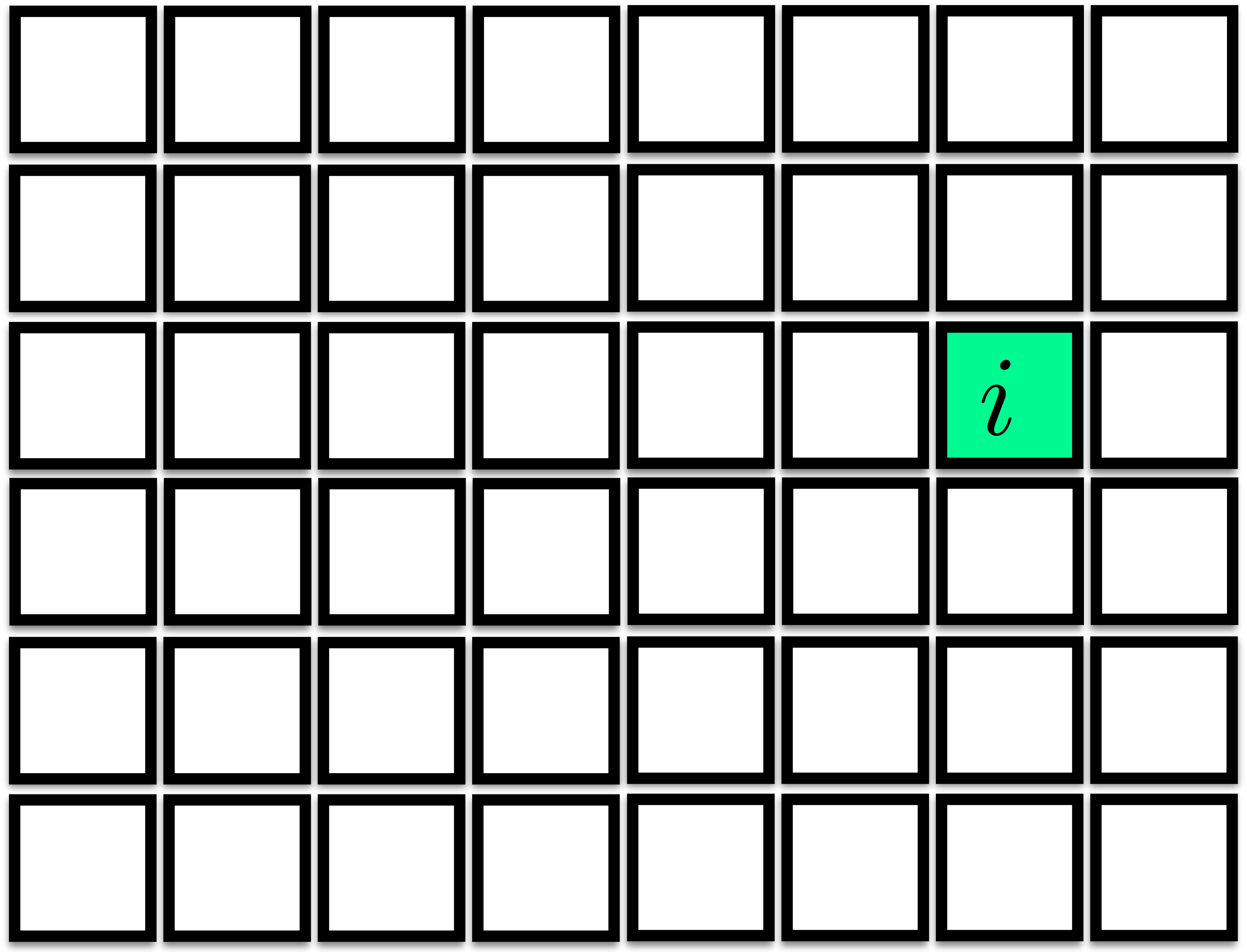
$b \leq \theta_L$ …. unoccupied

$\theta_L < b < \theta_H$ …. unknown

$b \geq \theta_H$ …. occupied

$$b = \log \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \log \frac{p(o_i|\mathbf{z}_t)}{p(\neg o_i|\mathbf{z}_t)} + \log \frac{p(o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i|\mathbf{z}_{1:t-1})} + \log \frac{p(\neg o_i)}{p(o_i)}$$

$t = 3$



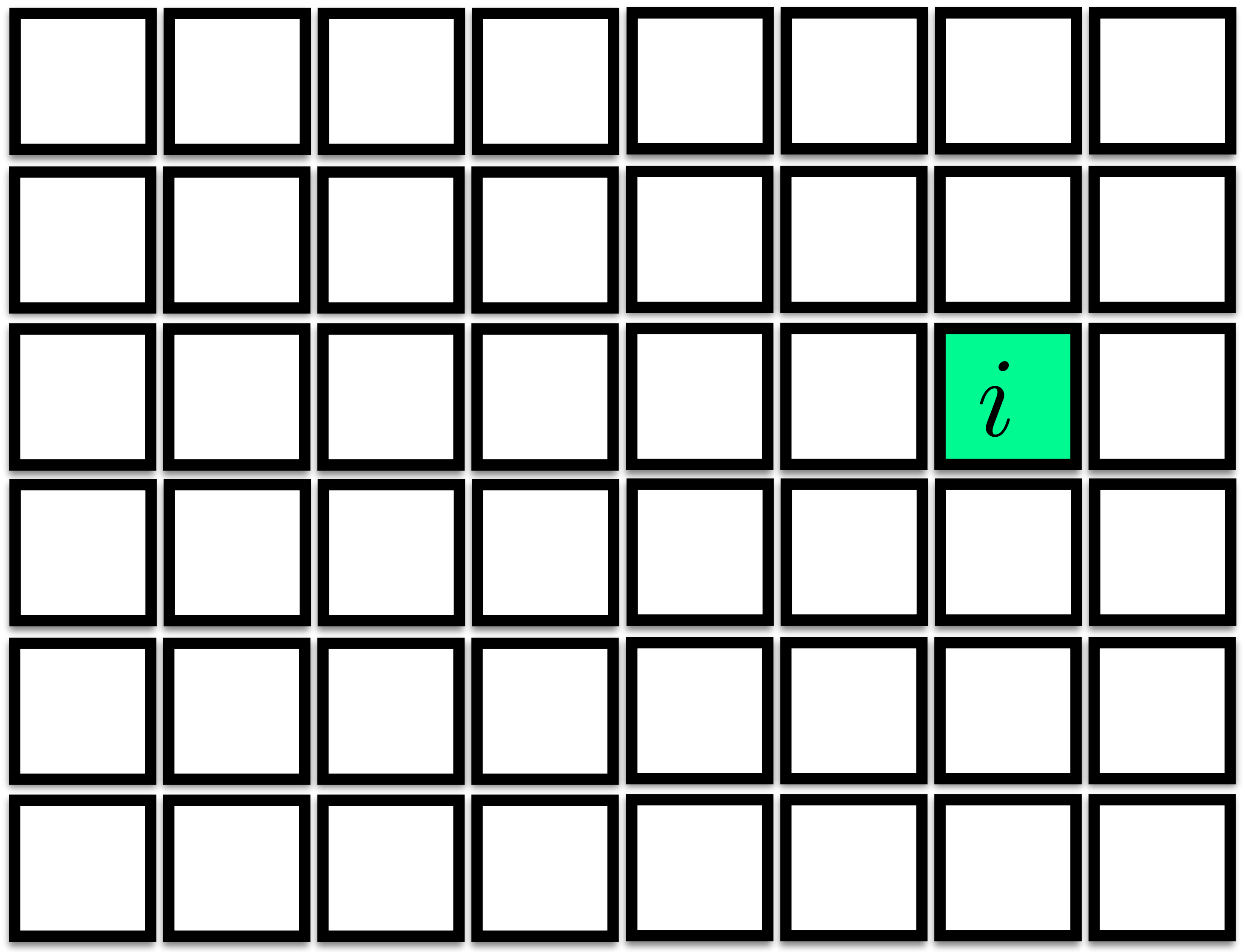
2D occupancy grid

$$-6 \quad = \quad (-1) + (-1) + \ldots + (-1) \quad +0$$

$$b = \log \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \log \frac{p(o_i|\mathbf{z}_t)}{p(\neg o_i|\mathbf{z}_t)} + \log \frac{p(o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i|\mathbf{z}_{1:t-1})} - \log \frac{p(o_i)}{p(\neg o_i)}$$

current belief | inverse measurement model | previous belief | prior belief

$t = 3$



2D occupancy grid

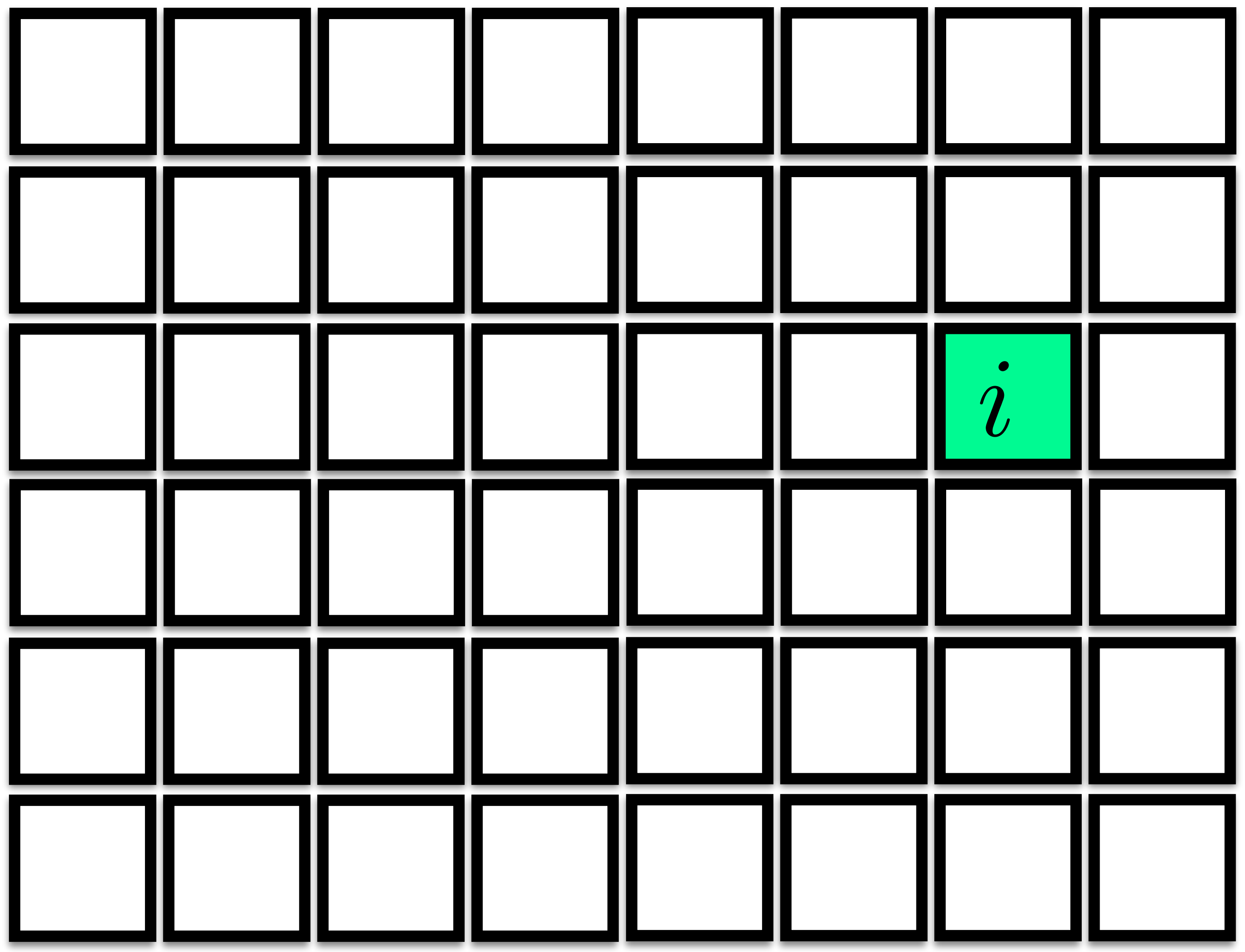
$$\log \frac{p(o_i)}{p(\neg o_i)} = 0$$

$$-6 \quad = \quad (-1) + (-1) + \ldots + (-1) \quad +0$$

$$b = \log \frac{p(o_i|\mathbf{z}_{1:t})}{p(\neg o_i|\mathbf{z}_{1:t})} = \log \frac{p(o_i|\mathbf{z}_t)}{p(\neg o_i|\mathbf{z}_t)} + \log \frac{p(o_i|\mathbf{z}_{1:t-1})}{p(\neg o_i|\mathbf{z}_{1:t-1})} - \log \frac{p(o_i)}{p(\neg o_i)}$$

current belief — inverse measurement model — previous belief — prior belief

$t = 3$



2D occupancy grid

$$\log \frac{p(o_i)}{p(\neg o_i)} = 0$$

$$\log \frac{p(o_i|\mathbf{z}_t)}{p(\neg o_i|\mathbf{z}_t)}$$

- $= +1$ if point in cell i
- $= -1$ if ray intersects cell i
- $= \log \frac{p(o_i)}{p(\neg o_i)}$ otherwise

Summary

- Occupancy grid is 2D (or 3D) array, which contains log-odds values of occupancy probabilities.
- Given the 3 assumptions (independence, static world, known poses), the log-odds values can be recurrently updated via Bayes filter.
- Resulting map is more suitable for exploration planning.
- Occupancy grid can be also used for localization (for 3D it might be too computationally demanding)
- Next: lets add RGB and RGBD sensors