Problem 1. Let $X_1,\ldots,X_n \sim \text{Poisson}(\lambda)$. Derive the maximum likelihood estimator of $\lambda$.

Problem 2. Suppose the lifetime of a Skoda car lights are exponentially distributed with unknown $\lambda$. We test 5 light bulbs and they have lifetimes of 2, 3, 1, 3, 4 years. What is the maximum likelihood estimate for $\lambda$? Given your estimate for $\lambda$, what is the probability that a randomly chosen light bulb will last more than 5 years?

Problem 3. A friend comes to you with a strange coin. You flip it and you get the following sequences: HHTHH, TTHHH, HTHH, HTH. What is the probability of getting a head with this coin? Here you should verify your intuition by finding the maximum likelihood estimate of $p$ of the binomial distribution.

Problem 4. Consider one-dimensional observations $\{-3, -2, 0, 1, 3, 2\}$ independently selected from an unknown distribution. What is the estimate $\hat{p}(x = 0.5)$ using parzen window estimation with uniform density kernel of radius 1.25 units?

Problem 5. In the Maximum likelihood lab, you evaluated the quality of the MLE estimate for $\hat{\mu}, \hat{\sigma}$ by plotting their variance as a function of the size of the training set. Why is this a good metric? Are there other possibilities?

Problem 6. In the Maximum likelihood lab, you assumed that the measured feature was a normally distributed. Is this a valid assumption? Why or why not?