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# A4M33MAS - Multiagent Systems
# Distributed Constraint Satisfaction

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OTEVŘENÁ INFORMATIKA

In parts based on Multi-agent Constraint Programming, Boi Faltings, Laboratoire d'Intelligence Artificielle, EPFL

# Constraint Satisfaction Problem

Given $< X, D, C >$ where:

- $X = \{x_1, .., x_n\}$ is a set of $n$ variables.
- $D = \{d_1, .., d_n\}$ is a set of $n$ domains.
- $C = \{c_1, .., c_m\}$ is a set of $m$ constraints.

Find solution $= (x_1 = v_1 \in d_1, ..., x_n = v_n \in d_n)$ such that for all constraints, value combinations are allowed by relations.

= Assignment

$C =$ represented as a list of Boolean predicate on 1 ... $n$ variables in $X$ and their values from $D$, so that $\mathcal{P}(\mathrm{X}, D) \rightarrow \{0,1\}$

# Multi-agent Constraint Satisfaction

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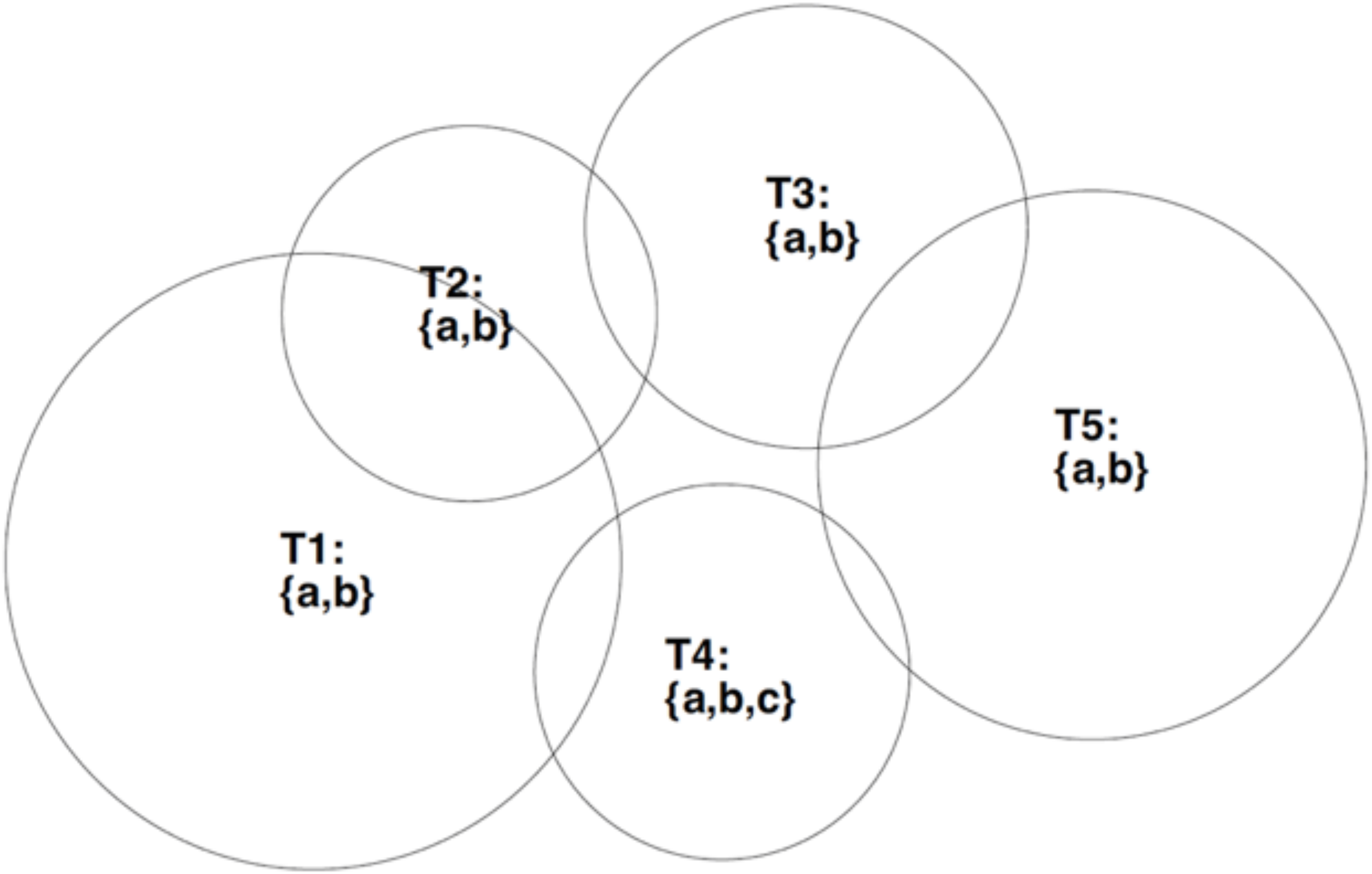
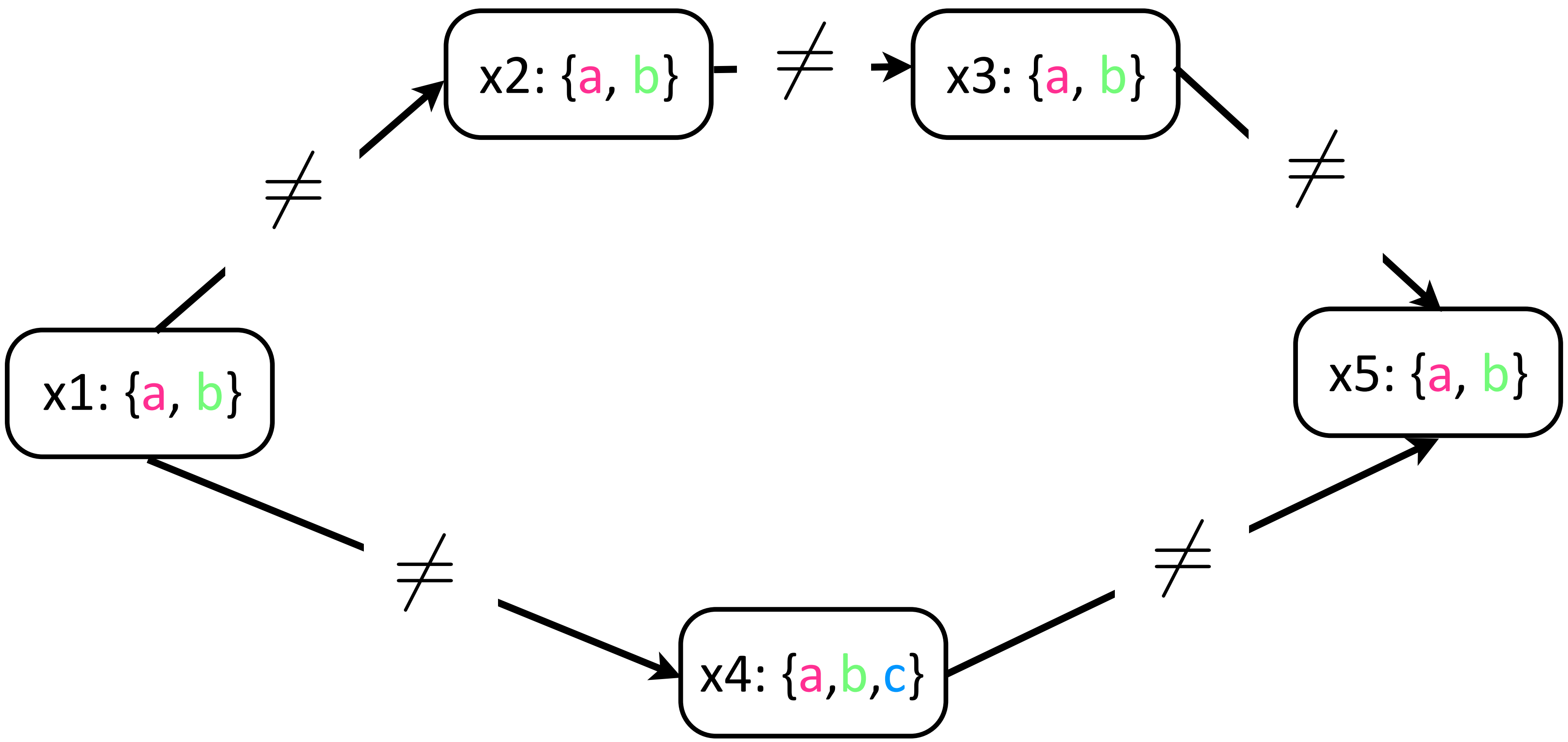
# Example



# Example

CSP model:

- Variables = choice of frequency
- Domains = frequency bands
- Constraints = inequalities between overlapping ranges
- Agents control transmitters



# Multi-agent Constraint Satisfaction

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# Multiagent Constraint Optimization

Given $< X, D, C, A >$ where:

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- $A = \{a_1, .., a_n\}$ is a set of $n$ agents, not necessarily all different.

Find solution $= (x_1 = v_1 \in d_1, ..., x_n = v_n \in d_n)$ such that for all the overall cost of the assignment is minimized

$$\text{Cost}(\{v_1, .., v_n\}) = \sum_{\forall c_i \in C} c_i(\{v_1, .., v_n\})$$

$C$ = represented as a list of cost functions on 1 ... $n$ variables in $X$ and their values from $D$, so that $\mathcal{P}(X,D) \rightarrow \mathbf{R}$

# Solving CSP

- Importance of CSP: large theory and tools for computing solutions
- 2 common methods:
  - backtrack search: assign one variable at a time, backtrack when no assignment without satisfying constraints.
  - local search: start with random assignment, make local changes to reduce number of constraint violations.

# Multiagent/Distributed CSP & COP

- Problem is distributed in a network of *agents*.
- Each variable *belongs* to one agent who is responsible for setting its value (typically these are connected to complex local subproblems).
- Constraints are known to all agents with variables in it.
- Distributed $\neq$ parallel: distribution of variables to agents cannot be chosen to optimize performance.

- WHY?
  - Real world problems are distributed, no agreement on a common model.
  - Costly to formalize constraints and preferences for all possible cases.
  - No trusted third party, privacy concerns.
  - but generally not efficiency!

# Multiagent/Distributed CSP & COP

- Top-down approaches:
  - **Pruning algorithms:** used mainly as a preprocessing step
    - *Filtering, Hyper-resolution*
  - **Search algorithms:**
    - *Chronological (Synchronous) Backtracking,*
    - *Asynchronous Backtracking, ADOPT*
- Bottom-up approaches:
    - *Distributed breakout*

# Multiagent/Distributed CSP & COP

- Top-down approaches:
  - **Pruning algorithms:** used mainly as a preprocessing step
    - *Filtering, Hyper-resolution*
  - **Search algorithms:**
    - *Chronological (Synchronous) Backtracking,*
      - A few agents are active, most are waiting
      - Active agents take decisions with updated information
      - Low degree of concurrency / poor robustness
      - Algorithms: direct extensions of centralized ones
    - *Asynchronous Backtracking, ADOPT*
      - All agents are active simultaneously
      - Information is less updated, obsolescence appears
      - High degree of concurrency / robust approaches
      - Algorithms: new approaches

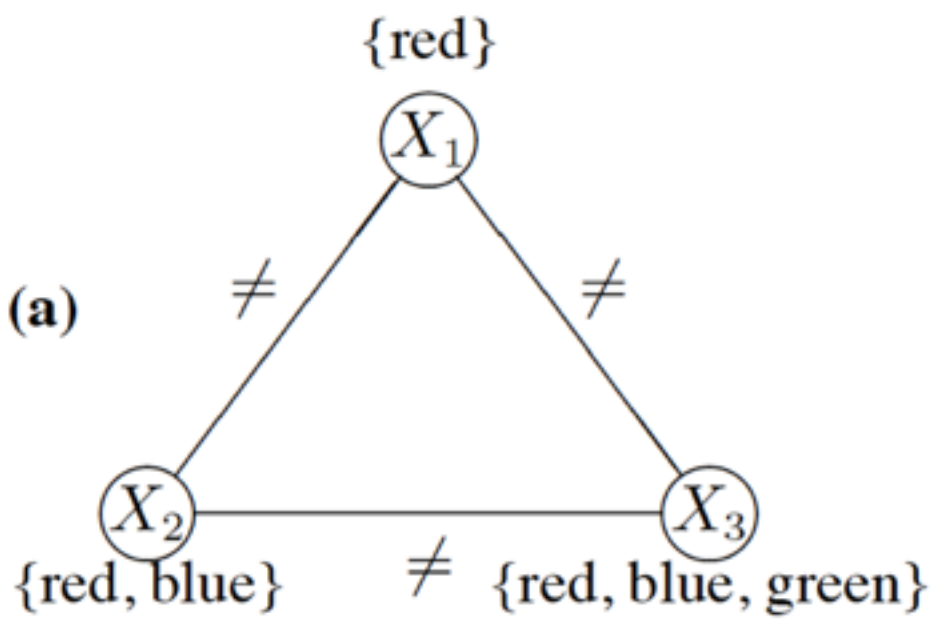
# Domain Pruning Algorithms

- Filtering algorithm:
  - For each node $x_i$ repeatadly execute **Revise**$(x_i, x_j)$ with each neighbour $x_j$.

**procedure** Revise$(x_i, x_j)$
**forall** $v_i \in D_i$ **do**
  **if** *there is no value* $v_j \in D_j$ *such that* $v_i$ *is consistent with* $v_j$ **then**
    delete $v_i$ from $D_i$

- Filtering terminates when no further elimination happens:
  - The solution is found if there is one value for each variable only
  - If there is an empty set assigned for one of the variables, -> no solution
  - If there is non-singleton set for one variable, the result is nonconlusive

# Domain Pruning Algorithms



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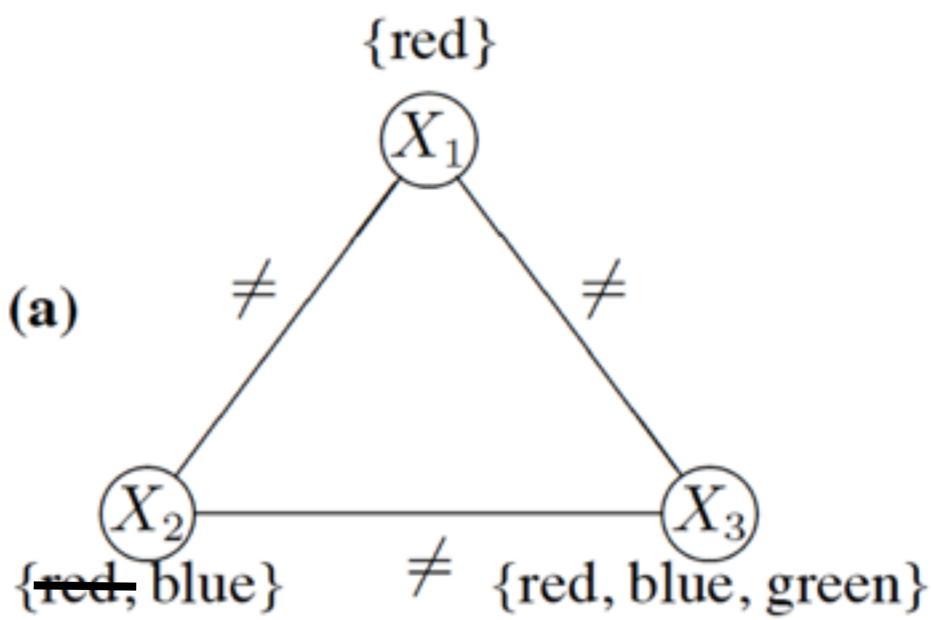

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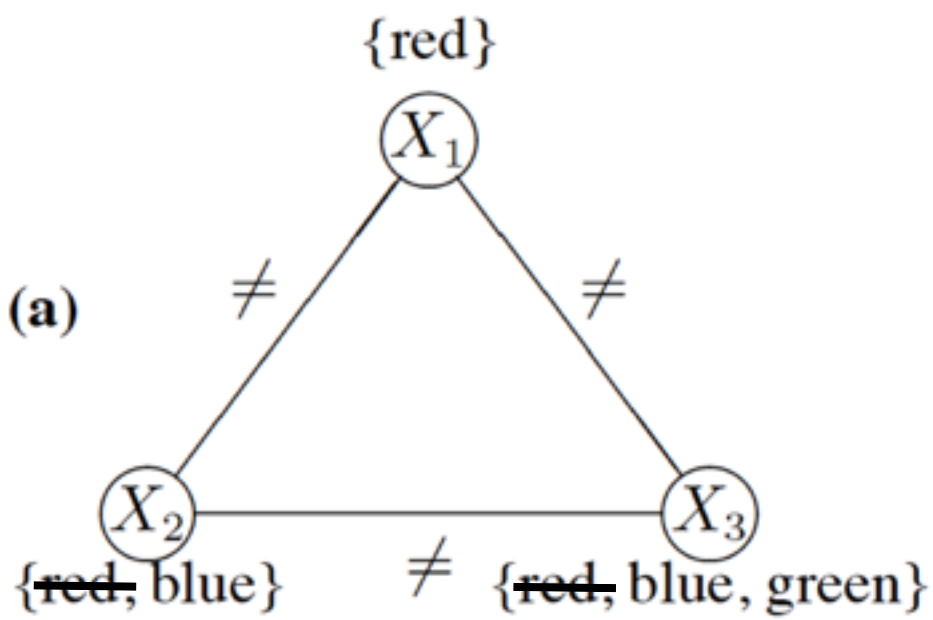

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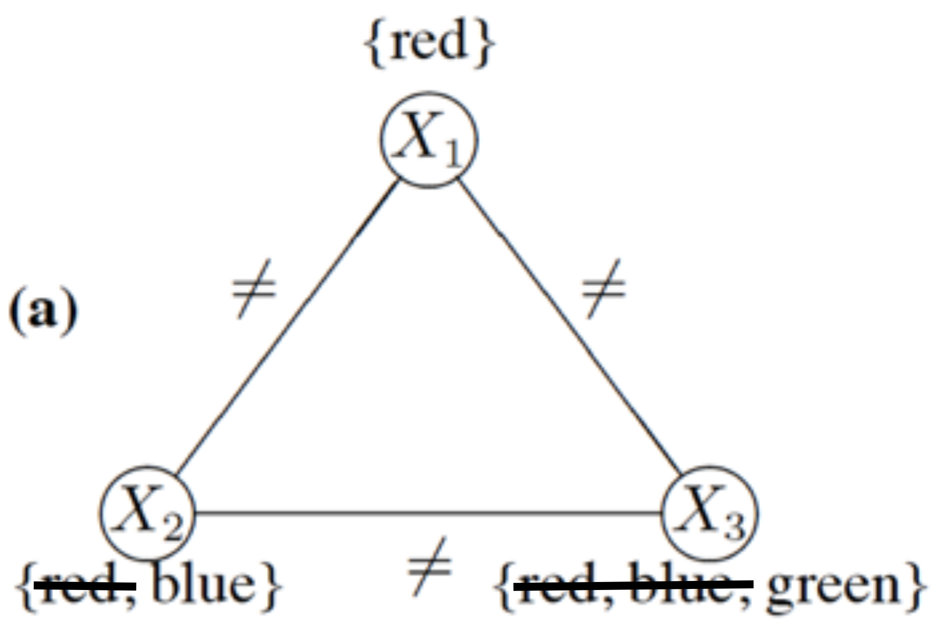

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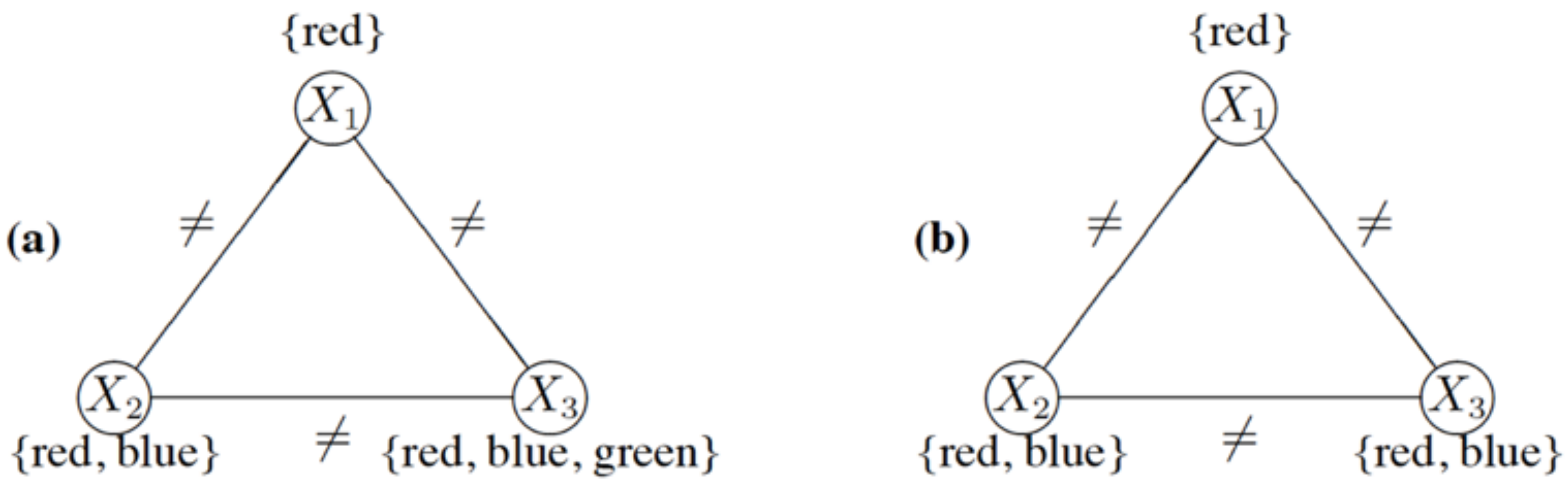

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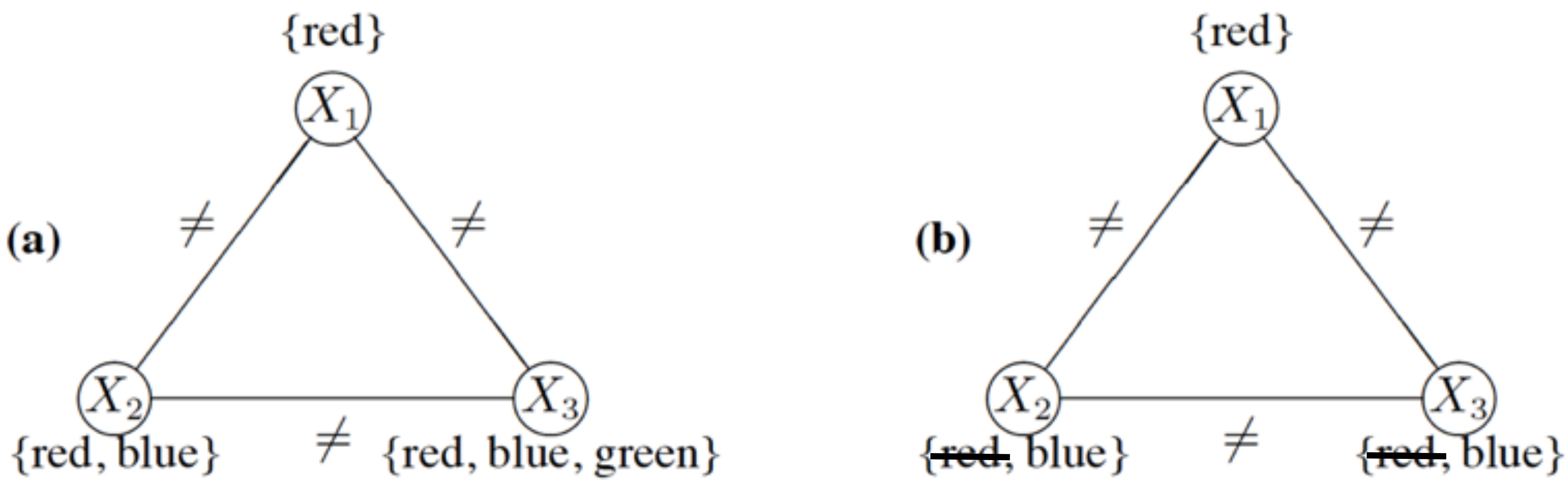

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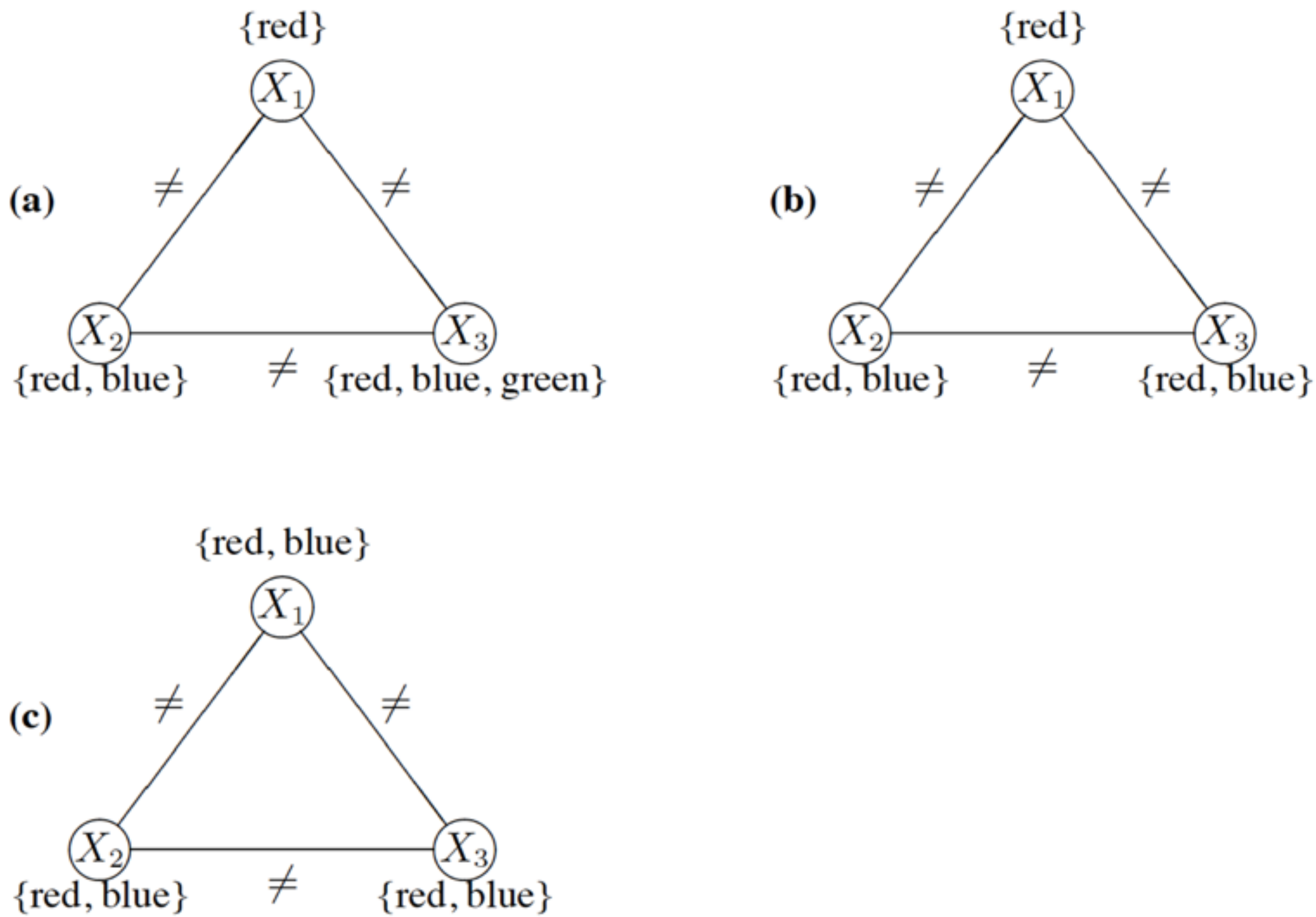

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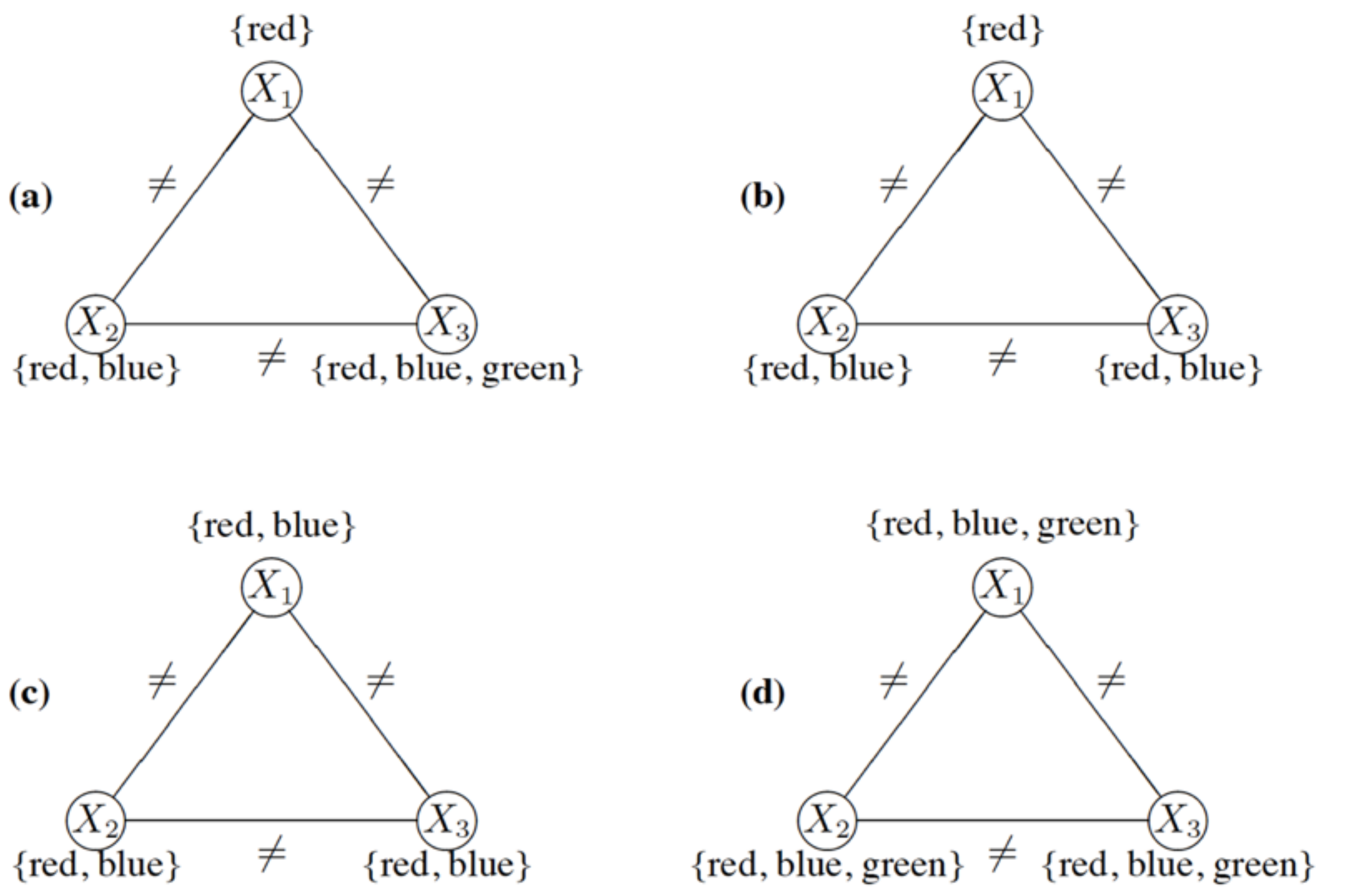
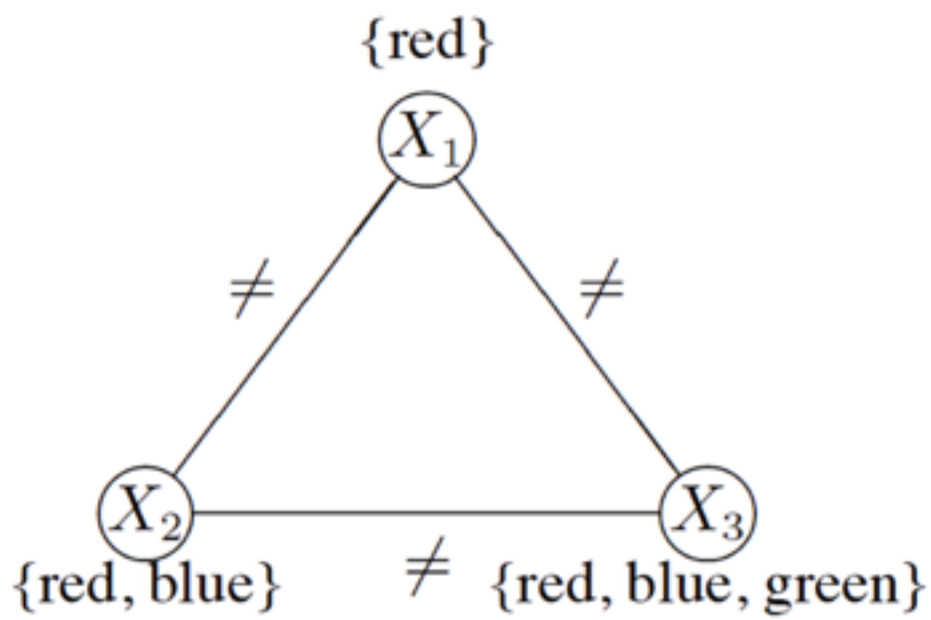


# Filtering based on hyper-resolution

- Works with the concept of forbidden combinations: NOGOOD (NG)
  - example: $\neg(x_1 = red \wedge x_2 = red)$
- Unit resolution:

$$\frac{\begin{array}{l} x_1 = red \\ \neg(x_1 = red \wedge x_2 = red) \end{array}}{\neg(x_2 = red)}$$




- Hyper-resolution:

$$\frac{\begin{array}{l} A_1 \vee A_2 \vee \cdots \vee A_m \\ \neg(A_1 \wedge A_{1,1} \wedge A_{1,2} \wedge \cdots) \\ \neg(A_2 \wedge A_{2,1} \wedge A_{2,2} \wedge \cdots) \\ \vdots \\ \neg(A_m \wedge A_{m,1} \wedge A_{m,2} \wedge \cdots) \end{array}}{\neg(A_{1,1} \wedge \cdots \wedge A_{2,1} \wedge \cdots \wedge A_{m,1} \wedge \cdots)}$$

# Filtering based on hyper-resolution

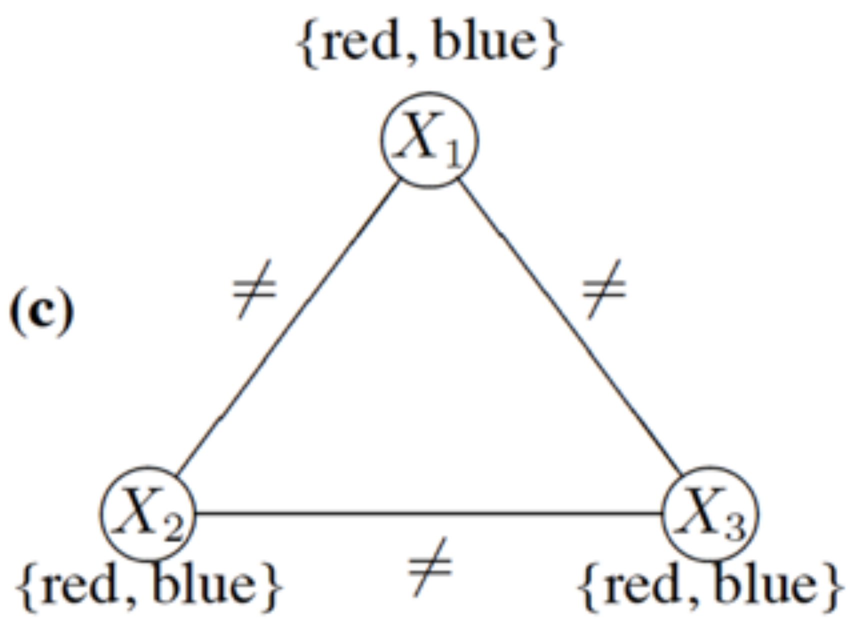
- Each agent repeatedly generates new constraints for his neighbors, notifies them of these new constraints, and prunes his own domain based on new constraints passed to him by his neighbors.

**procedure ReviseHR**($NG_i, NG_j^*$)
**repeat**
    $NG_i \leftarrow NG_i \bigcup NG_j^*$
    let $NG_i^*$ denote the set of new Nogoods that $i$ can derive from $NG_i$ and his domain using hyper-resolution
    **if** $NG_i^*$ *is nonempty* **then**
        $NG_i \leftarrow NG_i \bigcup NG_i^*$
        send the Nogoods $NG_i^*$ to all neighbors of $i$
        **if** $\{\} \in NG_i^*$ **then**
            stop
**until** *there is no change in $i$'s set of Nogoods $NG_i$*

# Filtering based on hyper-resolution

- As hyper-resolution is sound and complete for propositional logic it gave a rise to an efficient while complete distributed CSP algorithm.
- The algorithm is guaranteed to converge in the sense that after sending and receiving a finite number of messages, each agent will stop sending messages and generating Nogoods.

# Filtering based on hyper-resolution




NOGOODS:

$\{x_1 = red, x_2 = red\}, \{x_1 = red, x_3 = red\}$
$\{x_1 = blue, x_2 = blue\}, \{x_1 = blue, x_3 = blue\}$

$$x_1 = red \lor x_1 = blue$$
$$\neg(x_1 = red \land x_2 = red)$$
$$\neg(x_1 = blue \land x_3 = blue)$$

---

$$\neg(x_2 = red \land x_3 = blue)$$

NOGOODS:

$\{x_2 = red, x_3 = blue\}$ $\{x_2 = blue, x_3 = red\}$.

$$x_2 = red \lor x_2 = blue$$
$$\neg(x_2 = red \land x_3 = blue)$$
$$\neg(x_2 = blue \land x_3 = blue)$$

---

$$\neg(x_3 = blue)$$

similarly:

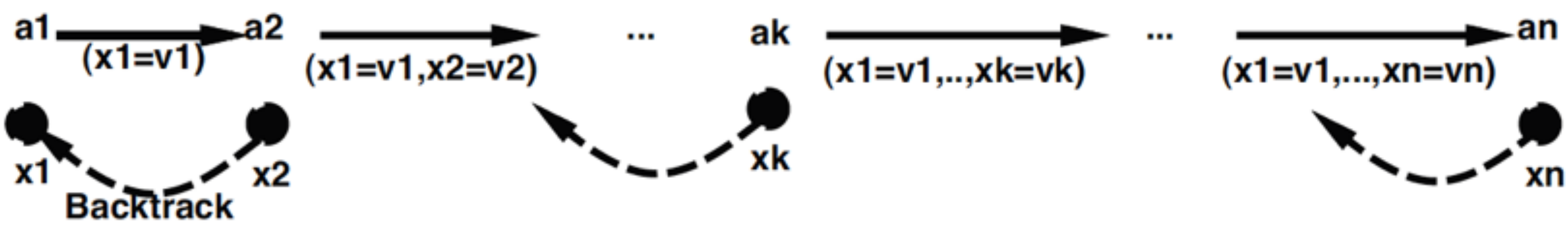
$$\neg(x_3 = red)$$

# Chronological Backtracking



Agents agree on an variable order and repeat:

1. send partial solution up to $x_{k-1}$ to k-th agent.
2. k-th agent generates the next extension to this partial solution.
3. if solution cannot be extended consistently, $k \leftarrow k - 1$.
4. if solution can be extended consistently, $k \leftarrow k + 1$.
5. if $k < 1$, stop: unsolvable.
6. if $k > n$, assigment $=$ solution.



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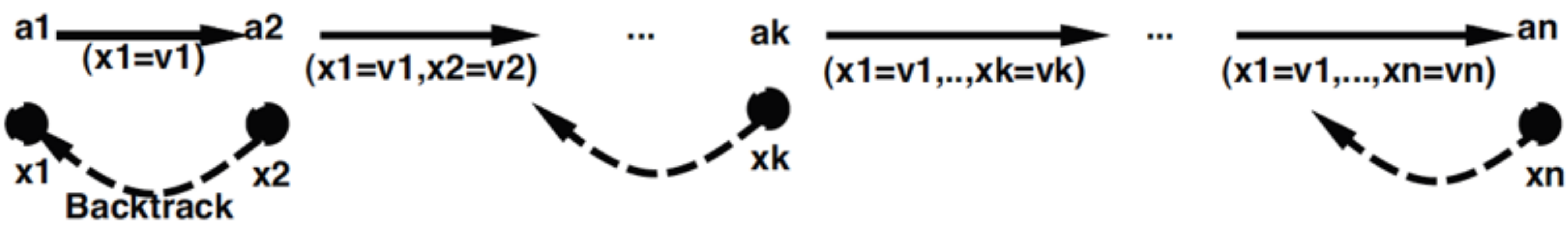


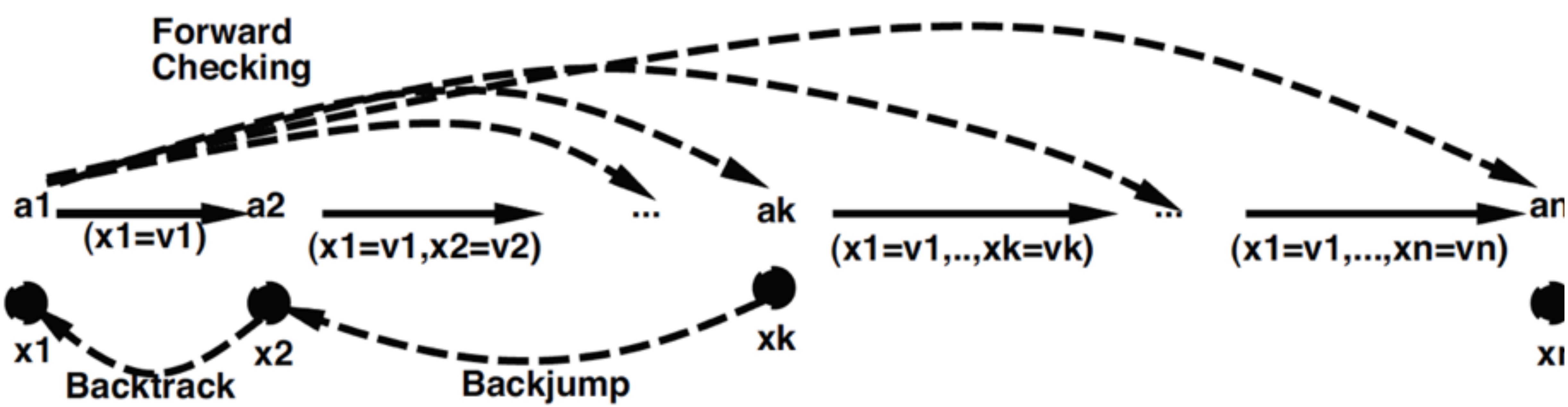
# Towards optimization: Synchronous branch-bounds

- Extend synchronous backtracking to optimization
  - every constraint contributes a cost.
  - upper bound = lowest cost of full assignment found so far.
  - partial assignment extended while cost < upper bound.
  - result = solution with lowest cost

# Improvements

Synchronous backtracking allows common CSP heuristics:

- forward checking: send partial solution to all higher agents.
- dynamic variable ordering: select next variable according to domain size.
- backjumping: reduce $k$ to last variable involved in conflict.



# Improvements

Distributed forward checking:

- $A(x_k)$ sends $(x_1 = v_1, .., x_k = v_k)$ to all $A(x_j)$, $j > k$
- $A(x_j)$ removes inconsistent values and initiates backtrack at $x_k$ whenever domain becomes empty

Can be done aynchronously (asynchronous forward checking)

Dynamic variable ordering:

- $A(x_j)$ sends back size of remaining domain for $x_j$
- $A(x_k)$ chooses smallest one to be $x_{k+1}$

Backjumping:

reduce $k$ to last variable involved in current conflict.

# Performance metrics

- non-concurrent constraint checks (NCCC): longest chain of constraint checks with serial dependency (ignores message delivery time).
- concurrent time: (simulated) time taken in parallel execution.
- wall clock time (time taken by the simulator).
- number of messages (ignores computation time and size of messages).
- amount of information exchanged (ignores computation time).

# Asynchronous backtracking (ABT)

- Assumptions:
  - Agents communicate by sending messages, agent send messages to others, iff it knows their identifiers (directed communication/no broadcasting)
  - The delay transmitting a message is finite but random, for any pair of agents, messages are delivered in the order they were sent
  - Agents know only the constraints in which they are involved
  - Each agent owns a single variable, constraints are binary
  - Asynchronous algorithm: Agents work in parallel without synchronization.
    - *all agents active, take a value and inform*
    - *no agent has to wait for other agents*
  - Global priority ordering among variables, and agents (to avoid cycles)
  - Constraints are directed: from higher-priority to lower-priority agents
- ABT plays in asynchronous distributed context the same role as backtracking in centralized

# ABT: Core principles

- Higher priority agent (j) informs the lower one (k) of its assignment
- Lower priority agent (k) evaluates the constraint with its own assignment
  - If permitted: no action
  - else: look for a value consistent with j
    - *If it exists k takes that value*
    - *else the agent view of k becomes a* NOGOOD (constraint) & *backtrack*

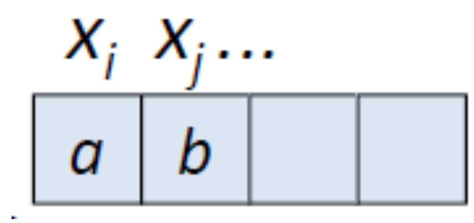
- NOGOOD: conjunction of (variable, value) pairs of higher priority agents, which removes a value of the current one
  - are required to ensure systematic traversal of search space in asynchronous, distributed context

# How ABT operates?

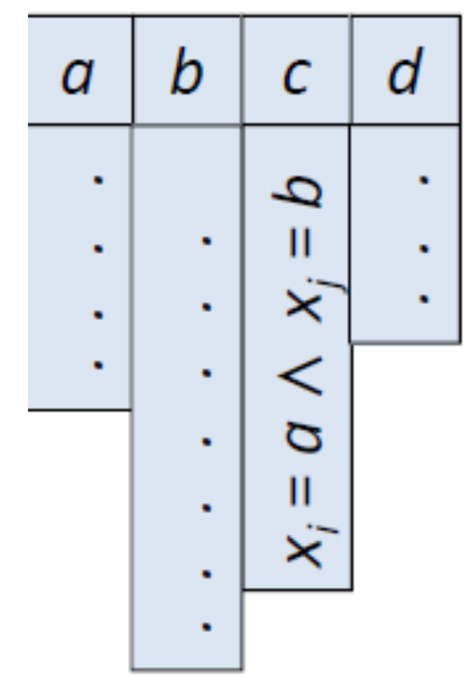
- ABT agents: asynchronous action; spontaneous assignment
- **Assignment**: *j* takes value *a*, *j* informs lower priority agents
- **Backtrack**: *k* has no consistent values with higher-priority agents, *k* resolves nogoods and sends a backtrack message
- **New links**: *j* receives a nogood mentioning *i*, unconnected with *j*; *j* asks *i* to set up a link
- **Stop:** "no solution" detected by an agent, stop
- **Solution:** when agents are silent for a while (quiescence), every constraint is satisfied => solution; detected by specialized algorithms

# ABT Data Structures

- **AgentView** (current assignment context):
  - values of higher-priority constrained agents




- **NOGOOD store**: each removed value has justifying NOGOOD
  - stored NOGOOD must be active wrt to AgentView



# ABT message passing

01

**when** *received* (***Ok?***, $(A_j, d_j)$) **do**
- add $(A_j, d_j)$ to *agent_view*
- **check_agent_view**

**when** *received* (***Nogood***, nogood) **do**
- add *nogood* to Nogood list
- **forall** $(A_k, d_k) \in$ nogood, *if* $A_k$ *is not a neighbor of* $A_i$ **do**
  - add $(A_k, d_k)$ to *agent_view*
  - request $A_k$ to add $A_i$ as a neighbor
- **check_agent_view**

**procedure** check_agent_view
**when** agent_view *and* current_value *are inconsistent* **do**
- **if** *no value in* $D_i$ *is consistent with* agent_view **then**
  - **backtrack**
- **else**
  - select $d \in D_i$ consistent with *agent_view*
  - *current_value* $\leftarrow d$
  - send (**ok?**, $(A_i, d)$) to lower-priority neighbors

# ABT message passing

01

**procedure** backtrack
*nogood* $\leftarrow$ some inconsistent set, using hyper-resolution or similar procedure
**if** nogood *is the empty set* **then**
    broadcast to other agents that there is no solution
    terminate this algorithm
**else**
    select $(A_j, d_j) \in$ *nogood* where $A_j$ has the lowest priority in *nogood*
    send (**Nogood**, *nogood*) to $A_j$
    remove $(A_j, d_j)$ from *agent_view*
    **check_agent_view**

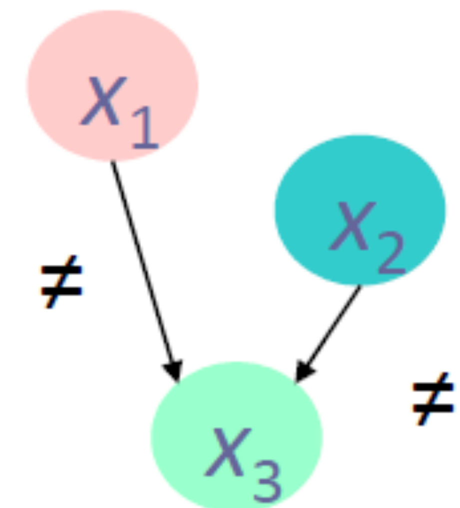
# Example

Variables $x_1, x_2, x_3$; $D_1 = \{b, a\}$, $D_2 = \{a\}$, $D_3 = \{a, b\}$

3 agents, lex ordered:


2 difference constraints: $c_{13}$ and $c_{23}$

Constraint graph:

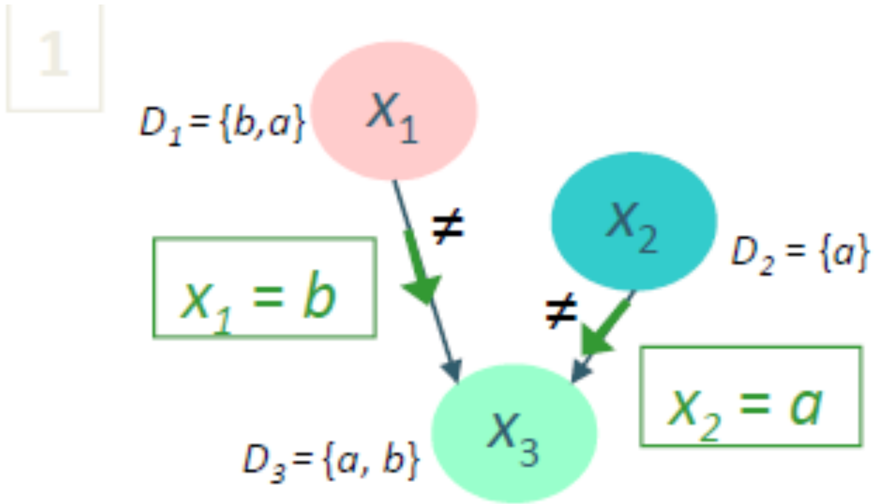



Value-sending agents: $x_1$ and $x_2$

Constraint-evaluating agent: $x_3$

Each agent *checks* constraints of incoming links: $Agent_1$ and $Agent_2$ check nothing, $Agent_3$ checks $c_{13}$ and $c_{23}$

# Example



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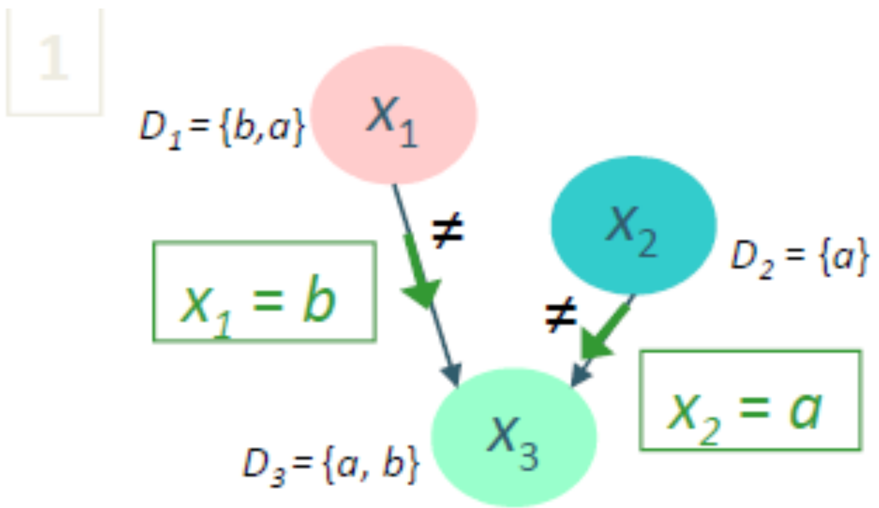
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$D_2 = \{a\}$
$x_1 = b$
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$x_2 = a$
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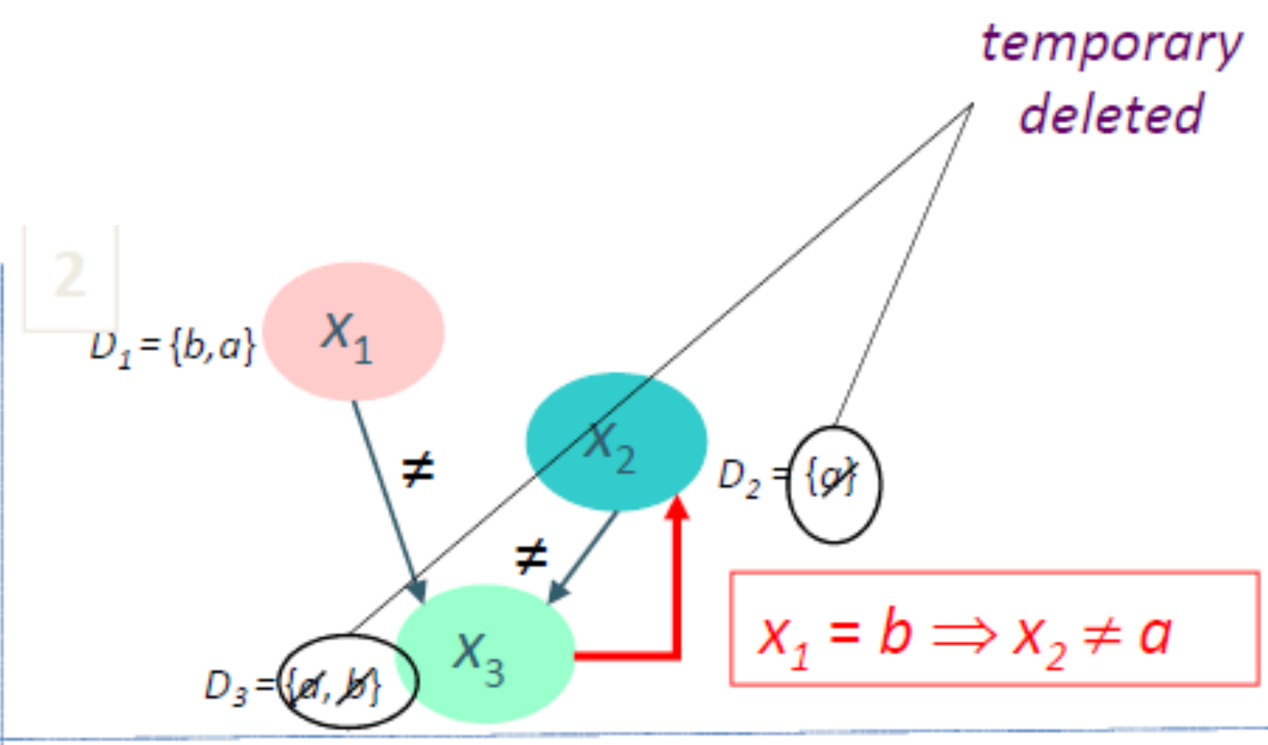
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2
$D_1 = \{b,a\}$
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$X_2$
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$x_1 = b \Rightarrow x_2 \neq a$
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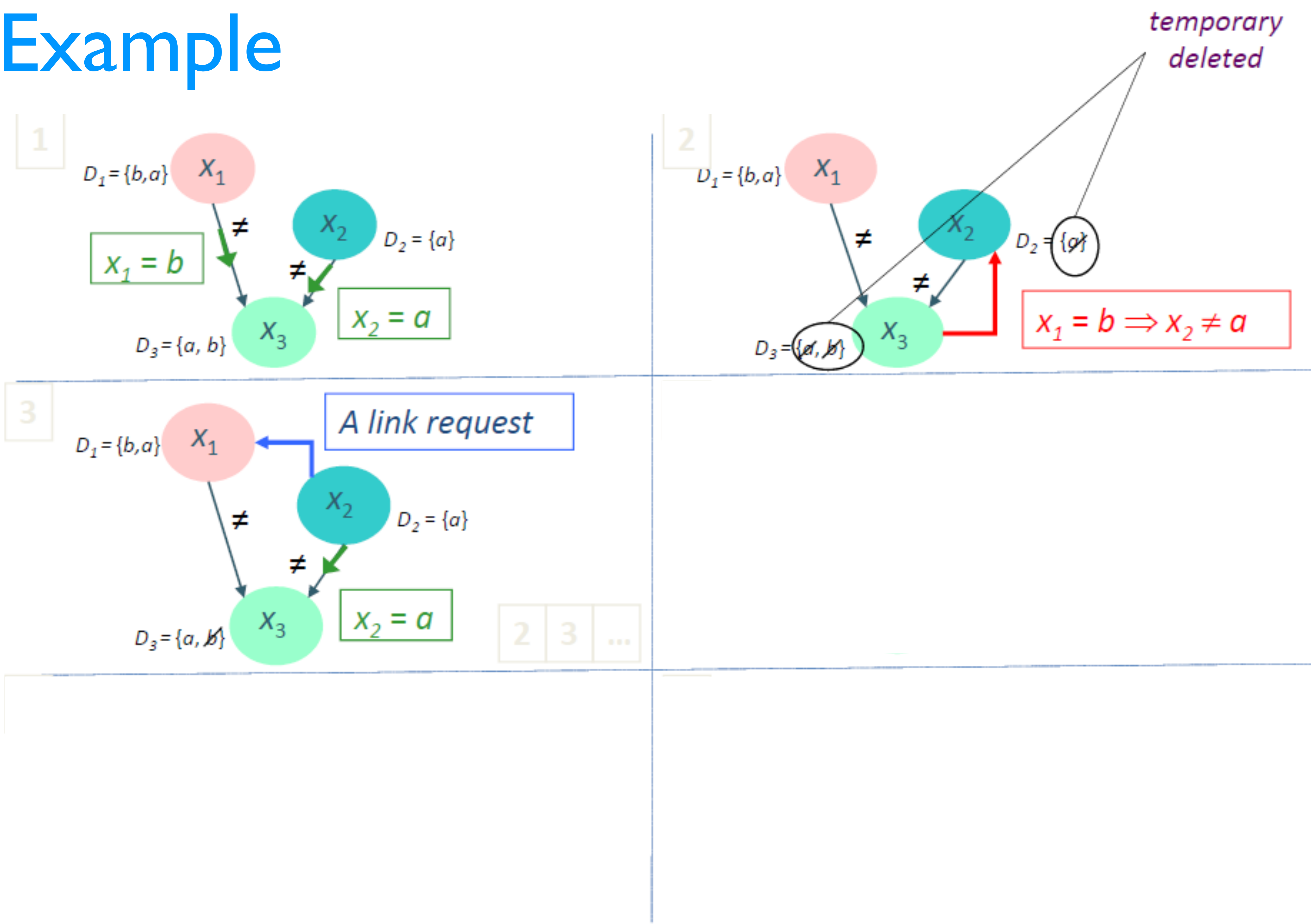
# Example



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3
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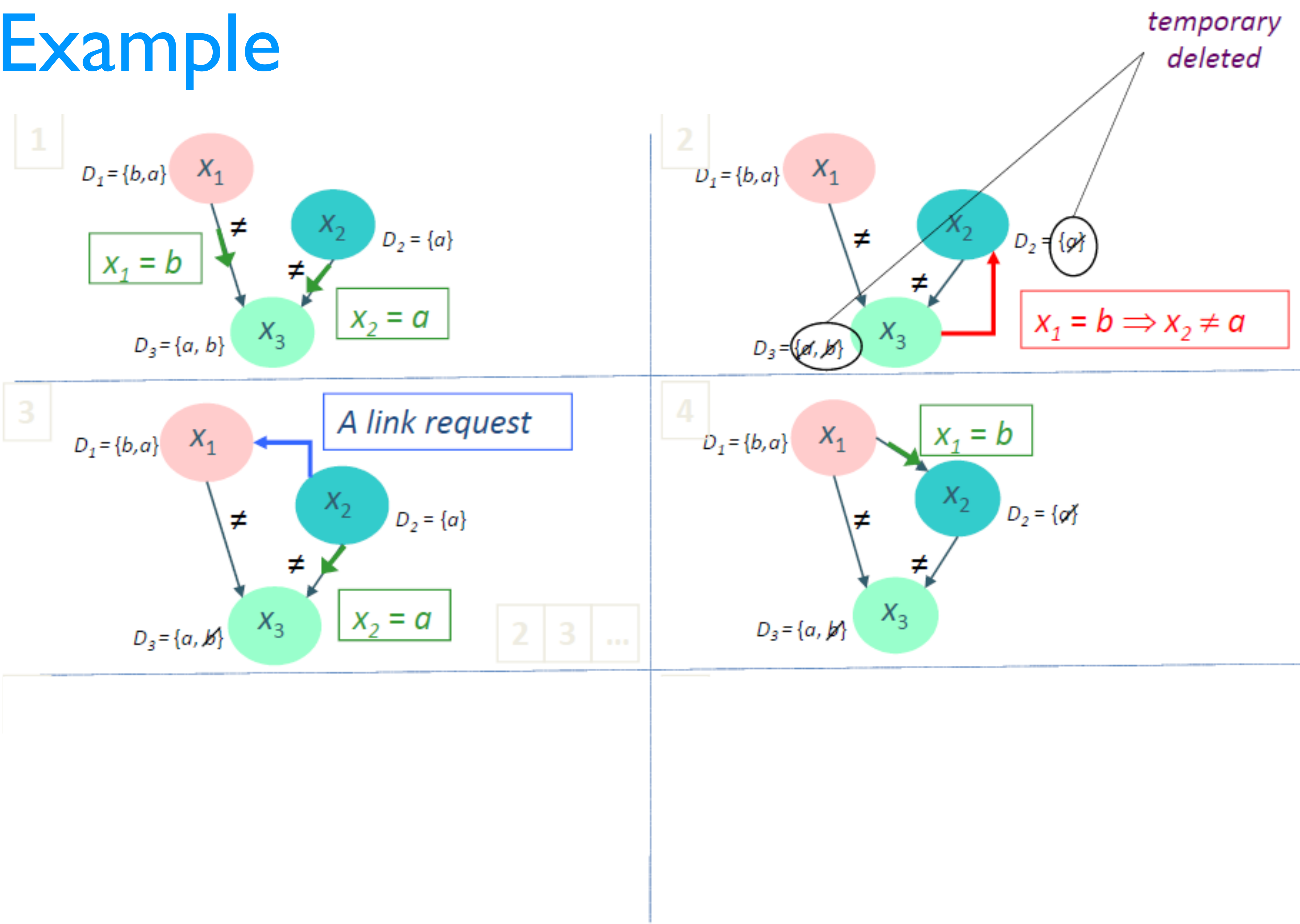
# Example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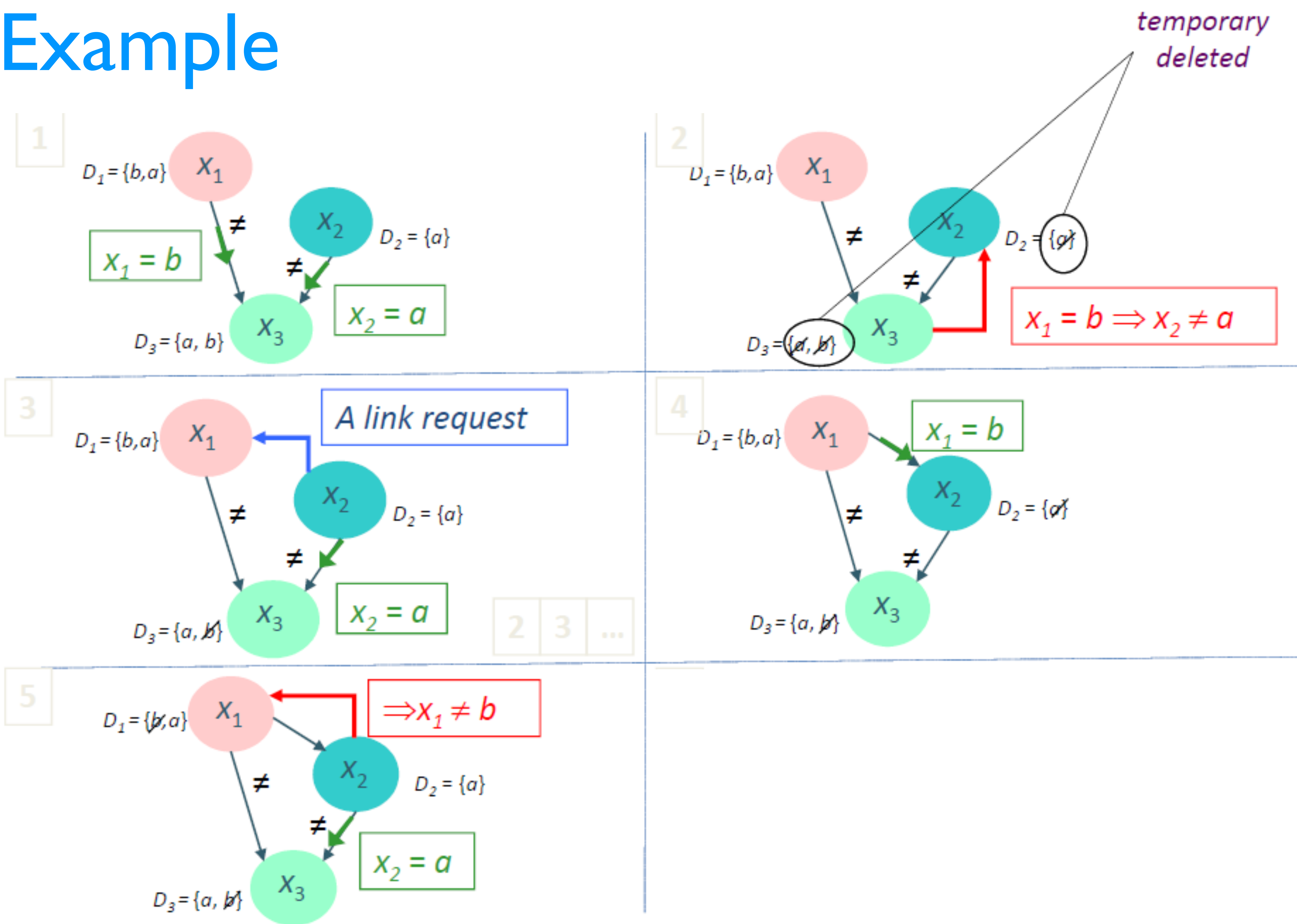

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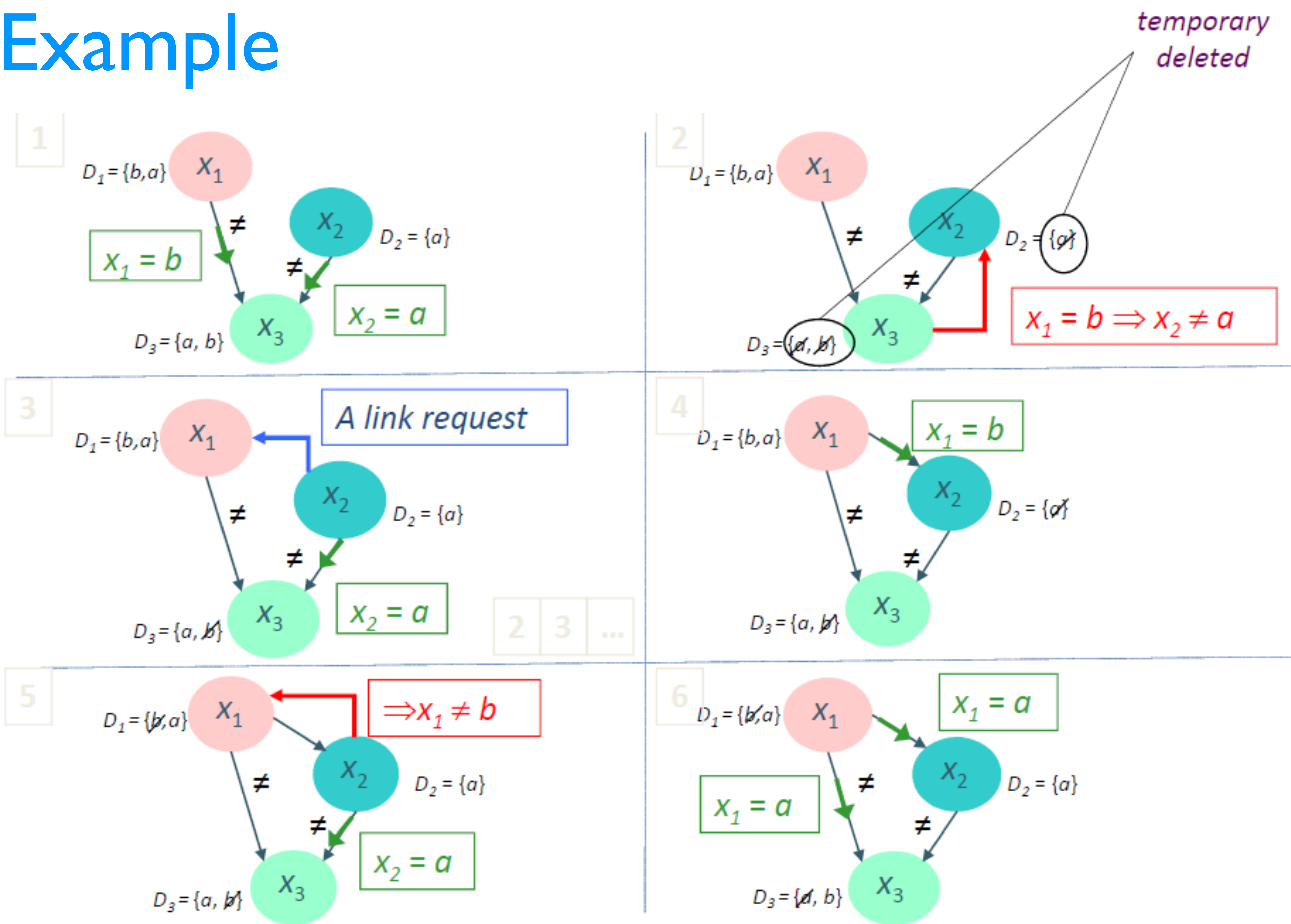
$x_2$
$D_2 = \{a\}$
$x_2 = a$
$x_3$
$D_3 = \{a, b\}$

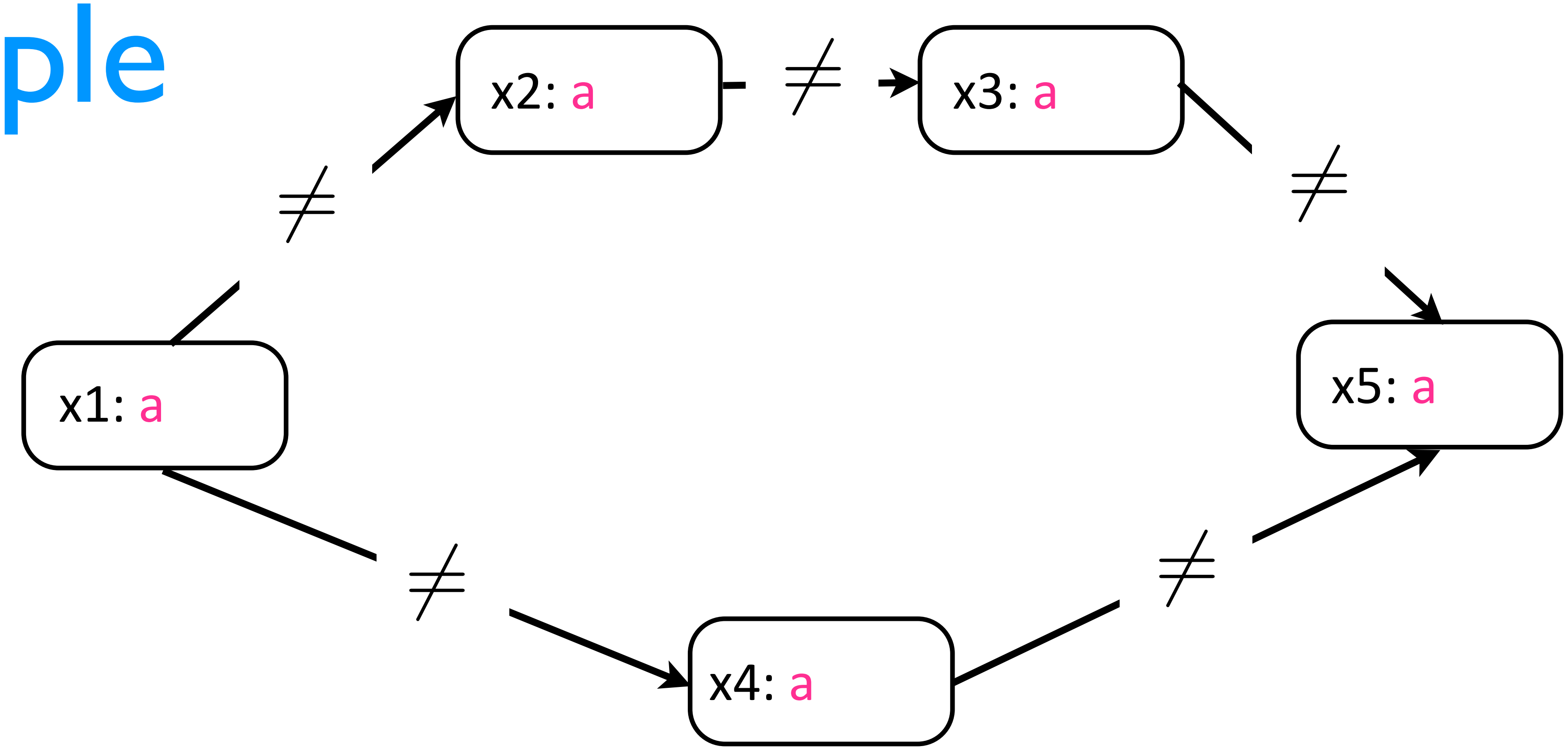
# Example



temporary deleted
1
$D_1 = \{b,a\}$
$x_1 = b$
$D_2 = \{a\}$
$x_2 = a$
$D_3 = \{a, b\}$
2
$D_1 = \{b,a\}$
$D_2 = \{a\}$
$x_1 = b \Rightarrow x_2 \neq a$
$D_3 = \{a, b\}$
3
A link request
$D_1 = \{b,a\}$
$D_2 = \{a\}$
$x_2 = a$
$D_3 = \{a, b\}$
2 3 ...
4
$D_1 = \{b,a\}$
$x_1 = b$
$D_2 = \{a\}$
$D_3 = \{a, b\}$
5
$\Rightarrow x_1 \neq b$
$D_1 = \{b,a\}$
$D_2 = \{a\}$
$x_2 = a$
$D_3 = \{a, b\}$
6
$D_1 = \{b,a\}$
$x_1 = a$
$D_2 = \{a\}$
$x_1 = a$
$D_3 = \{a, b\}$

# Example

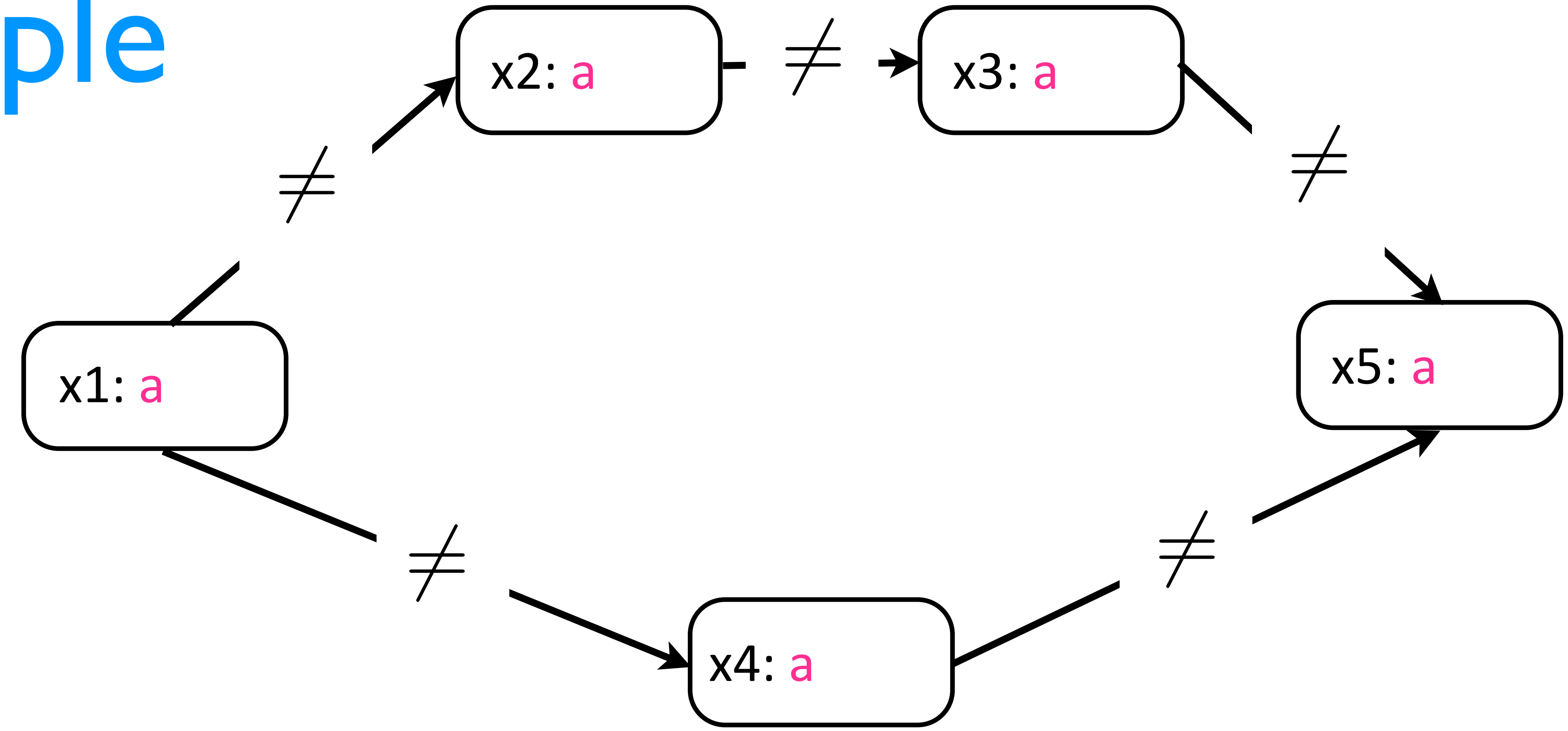





| | message(s) | action |
|---|---|---|
| $a_2$ | OK($x_1$=a) | |
| $a_3$ | OK($x_2$=a) | |
| $a_4$ | OK($x_1$=a) | |
| $a_5$ | OK($x_3$=a) | |
| | OK($x_4$=a) | |

# Example

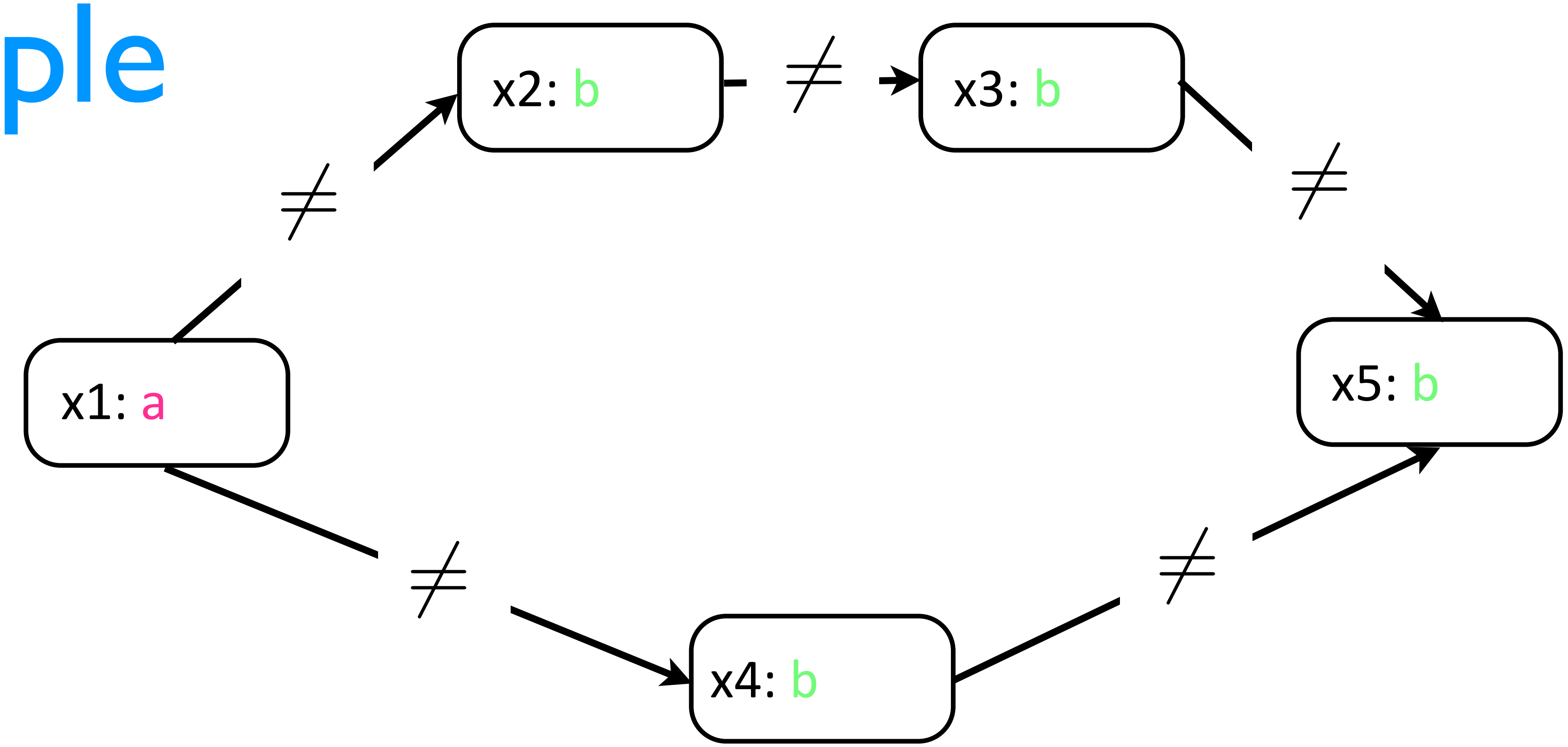





| | message(s) | action |
|---|---|---|
| $a_2$ | OK($x_1$=a) | $x_2 \leftarrow$ b |
| $a_3$ | OK($x_2$=a) | $x_3 \leftarrow$ b |
| $a_4$ | OK($x_1$=a) | $x_4 \leftarrow$ b |
| $a_5$ | OK($x_3$=a) | $x_5 \leftarrow$ b |
| | OK($x_4$=a) | |

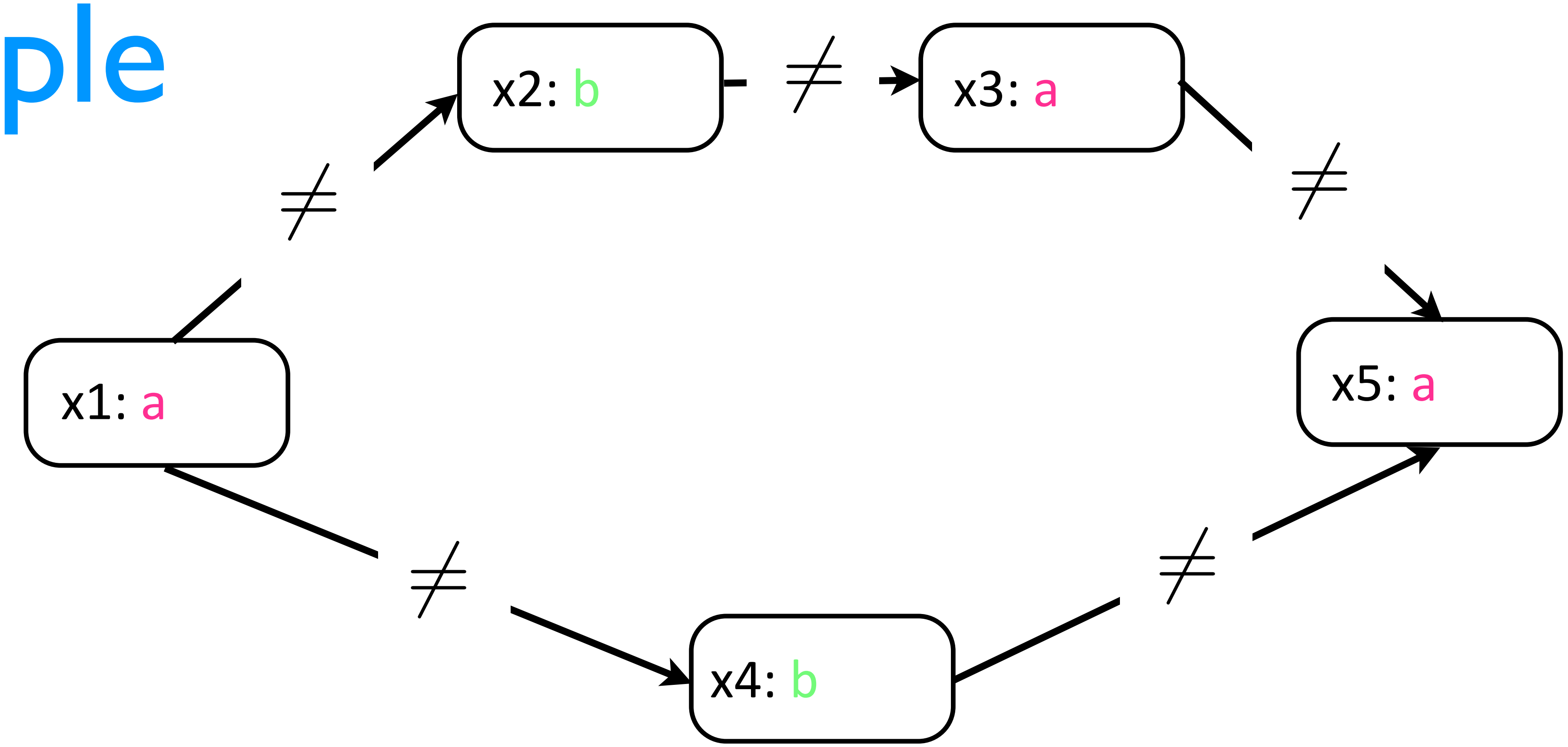
# Example






| | message(s) | action |
|---|---|---|
| $a_3$ | $\mathtt{OK}(x_2{=}\mathtt{b})$ | $x_3 \leftarrow \mathtt{a}$ |
| $a_5$ | $\mathtt{OK}(x_3{=}\mathtt{b})$ | $x_5 \leftarrow \mathtt{a}$ |
| | $\mathtt{OK}(x_4{=}\mathtt{b})$ | |

# Example



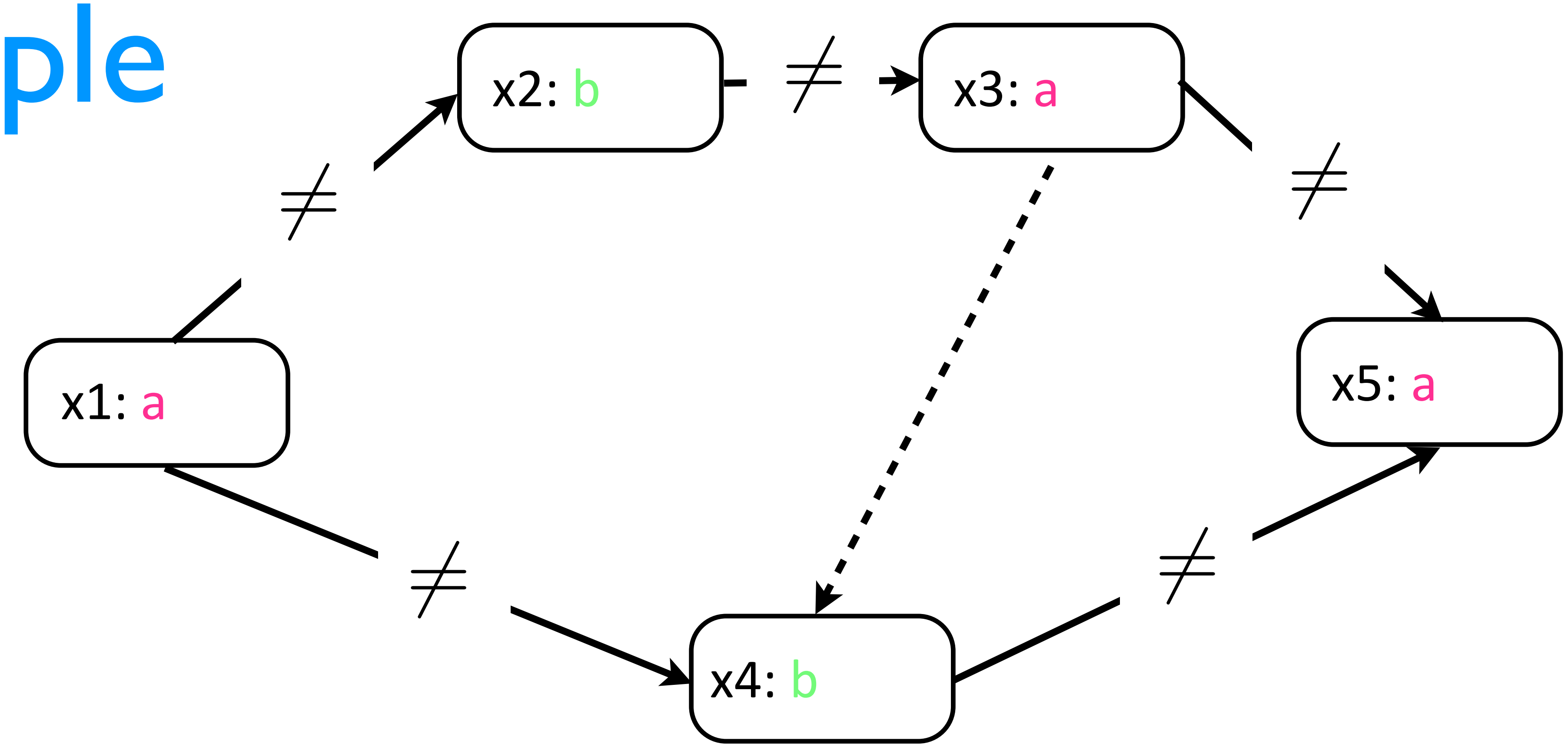

| | message(s) | action |
|---|---|---|
| $a_5$ | OK($x_3$=a) | inconsistent! |

$$x_3 = \text{a} \Rightarrow x_5 \neq \text{a}$$
$$x_4 = \text{b} \Rightarrow x_5 \neq \text{b}$$

$a_5$ sends a nogood to $a_4$:

v = b, cond $= (x_3 = \text{a})$, tag $= x_5$ cost = 1

# Example




| | message(s) | action |
|---|---|---|
| $a_5$ | OK($x_3$=a) | inconsistent! |

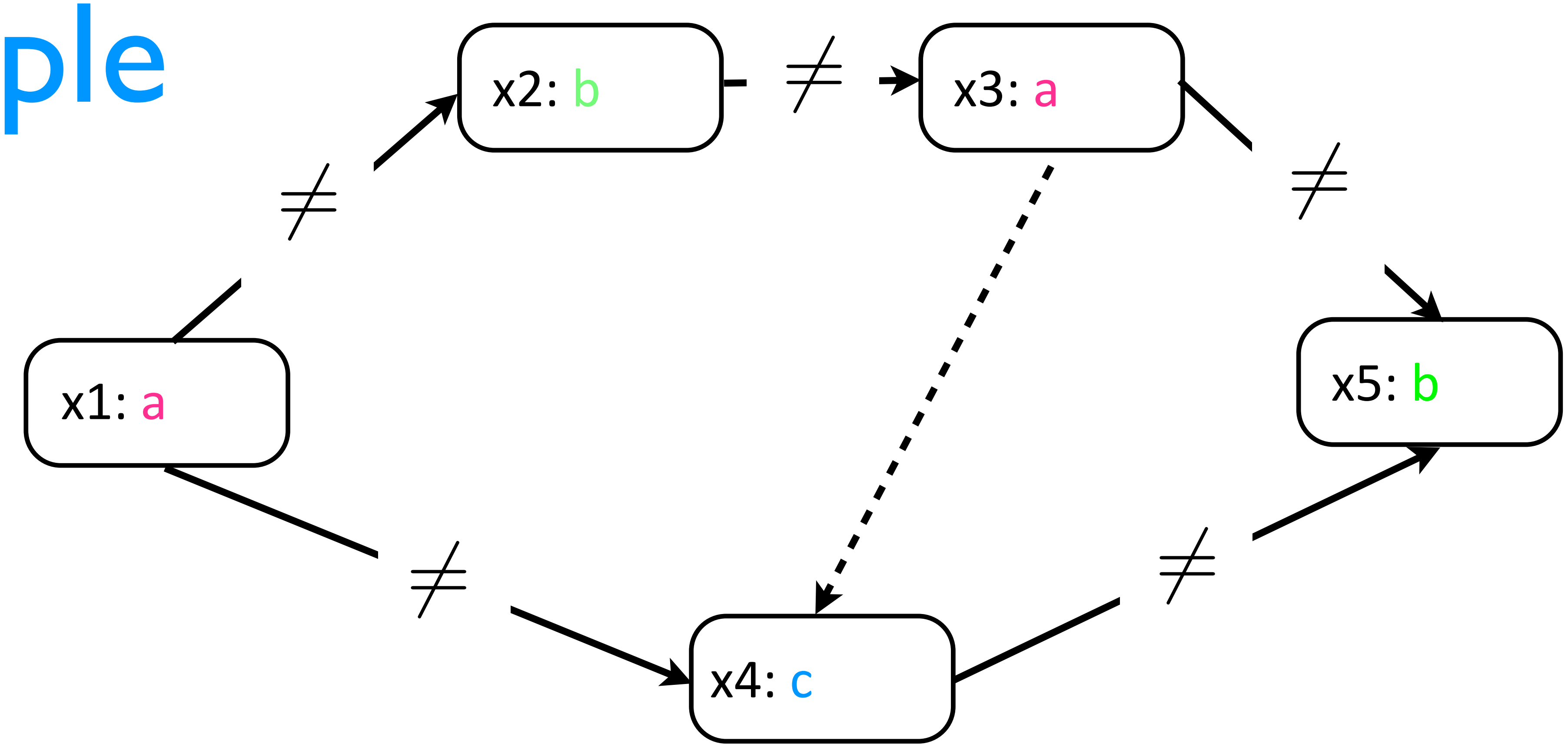
$$x_3 = a \Rightarrow x_5 \neq a$$
$$x_4 = b \Rightarrow x_5 \neq b$$

$a_5$ sends a nogood to $a_4$:

v = b, cond $= (x_3 = \text{a})$, tag $= x_5$ cost = 1

# Example






| | message(s) | action |
|---|---|---|
| $a_5$ | OK($x_3$=a) | inconsistent! |

$$x_3 = \mathrm{a} \Rightarrow x_5 \neq \mathrm{a}$$
$$x_4 = \mathrm{b} \Rightarrow x_5 \neq \mathrm{b}$$

$a_5$ sends a nogood to $a_4$:

v = b, cond $= (x_3 = \mathrm{a})$, tag $= x_5$ cost = 1

# Example



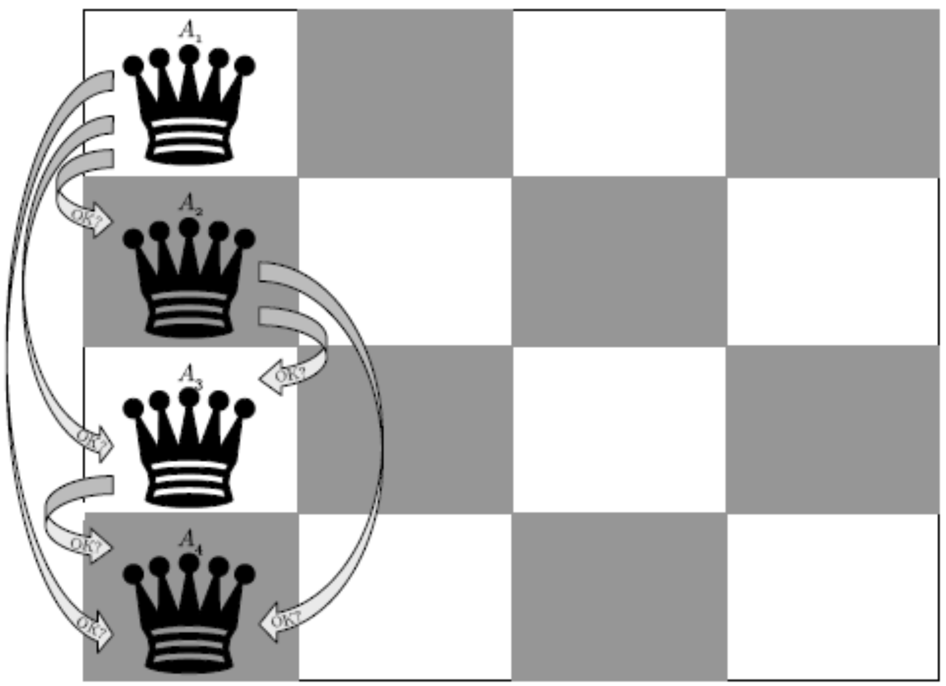



Figure 1.6: Cycle 1 of ABT for four queens. All agents are active.

# Example

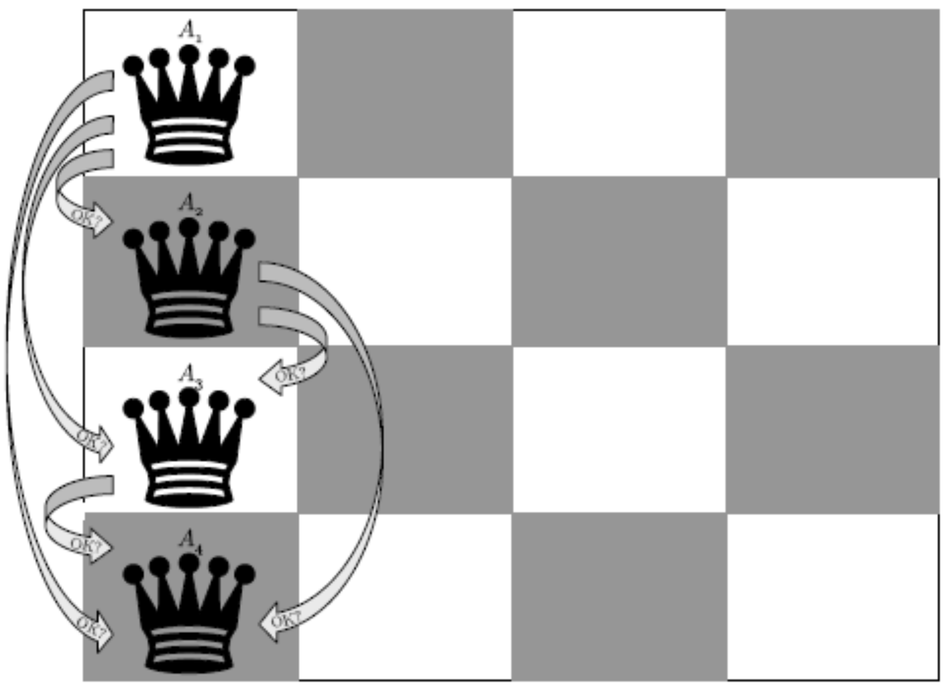



Figure 1.6: Cycle 1 of ABT for four queens. All agents are active.

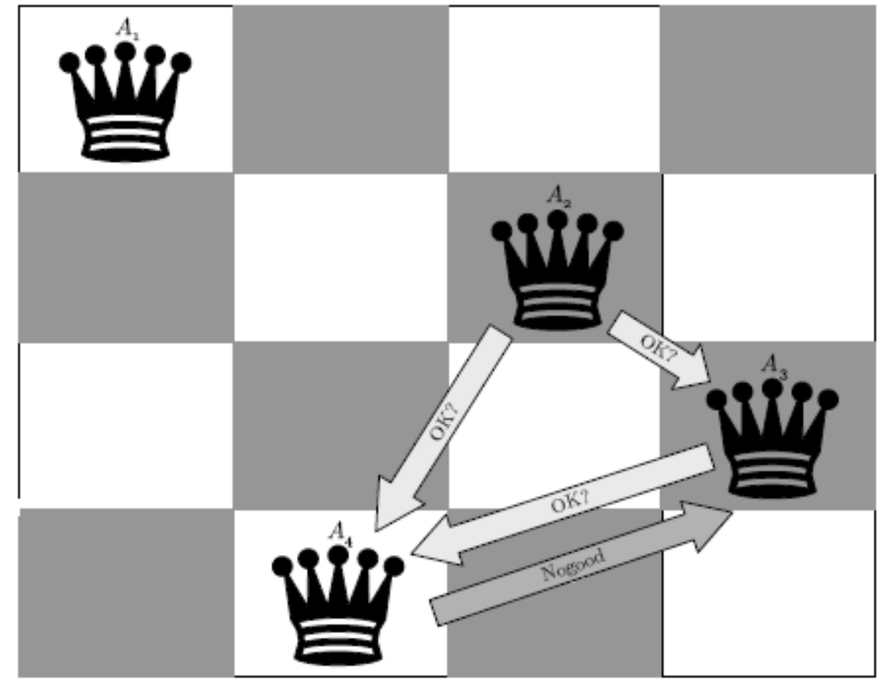



Figure 1.7: Cycle 2 of ABT for four queens. $A_2$, $A_3$ and $A_4$ are active. The Nogood message is $A_1 = 1 \wedge A_2 = 1 \rightarrow A_3 \neq 1$.

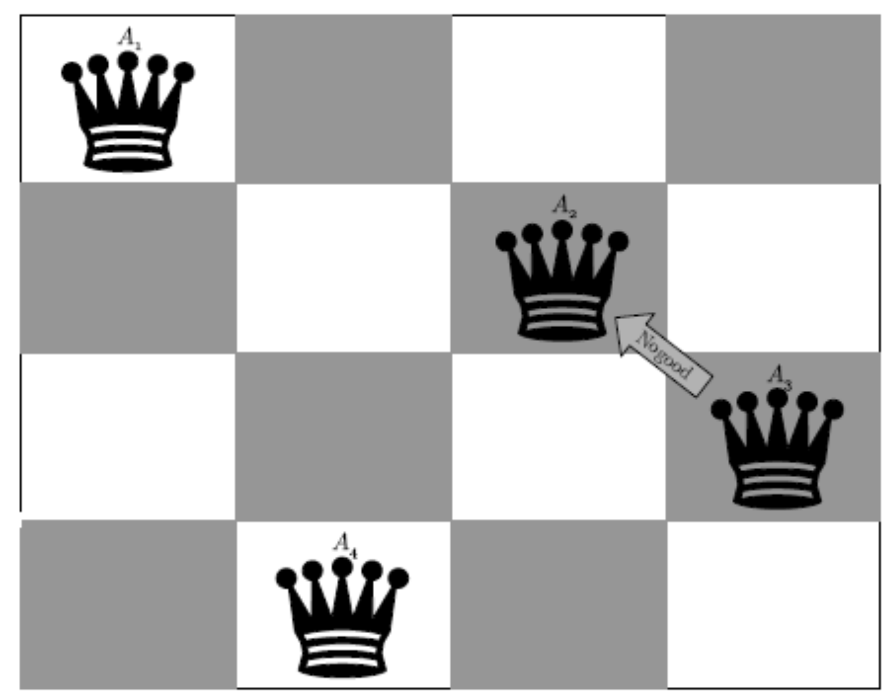



Figure 1.8: Cycle 3. Only $A_3$ is active. The Nogood message is $A_1 = 1 \rightarrow A_2 \neq 3$.

# Example

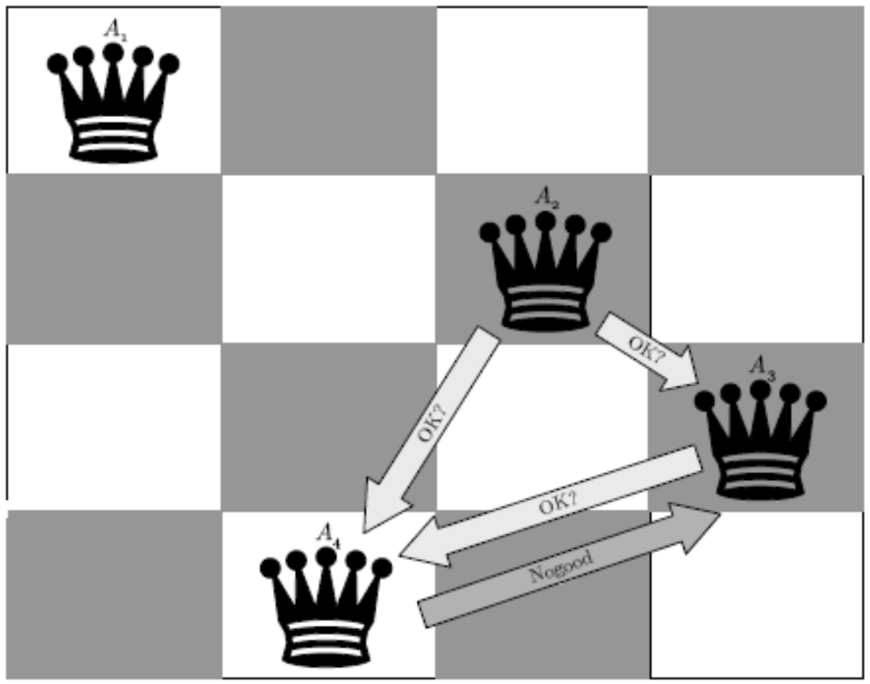



Figure 1.7: Cycle 2 of ABT for four queens. $A_2$, $A_3$ and $A_4$ are active. The Nogood message is $A_1 = 1 \wedge A_2 = 1 \rightarrow A_3 \neq 1$.

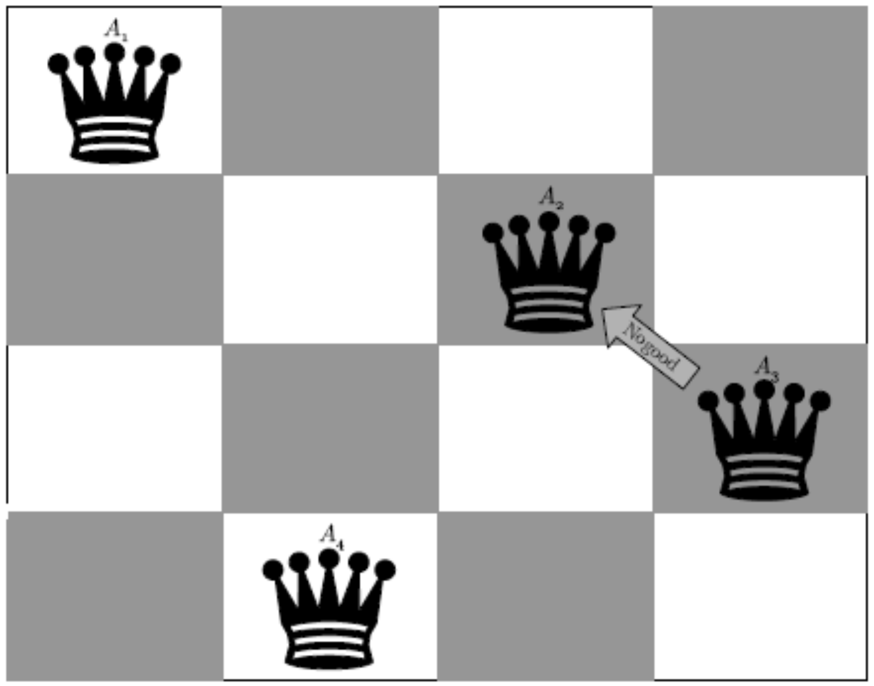



Figure 1.8: Cycle 3. Only $A_3$ is active. The Nogood message is $A_1 = 1 \rightarrow A_2 \neq 3$.

# Example



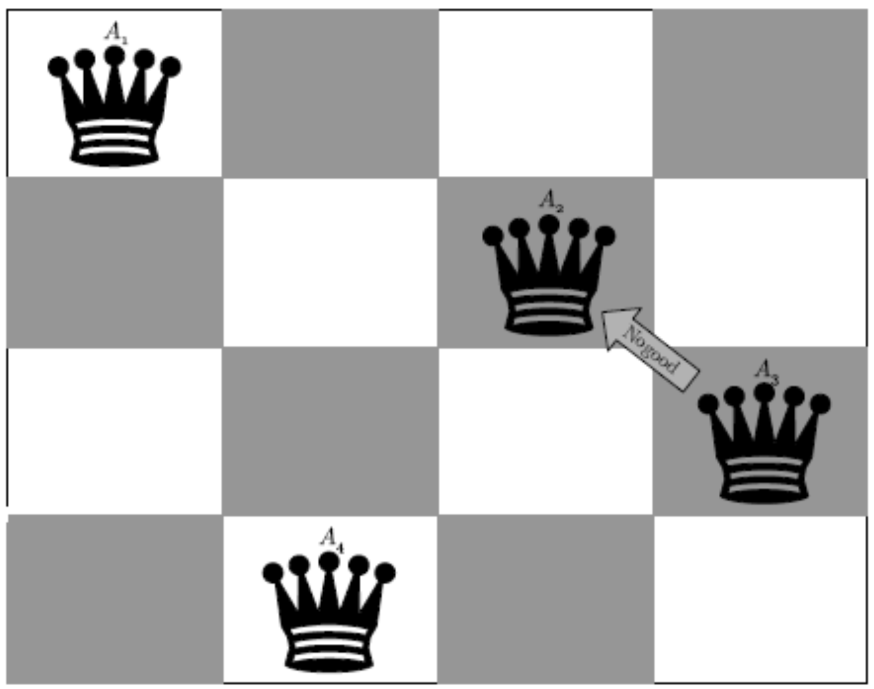



Figure 1.8: Cycle 3. Only $A_3$ is active. The Nogood message is $A_1 = 1 \rightarrow A_2 \neq 3$.

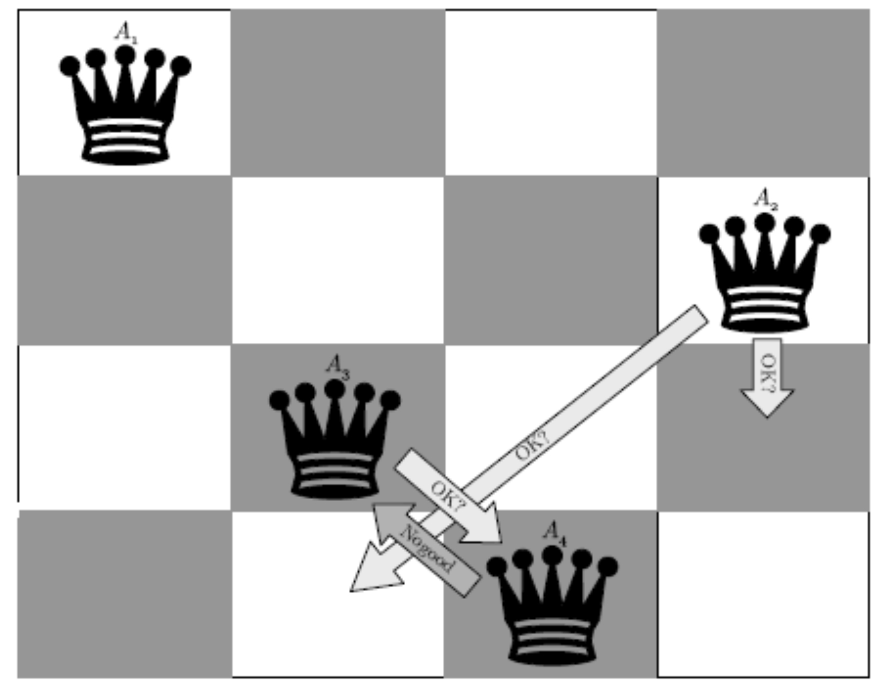



Figure 1.9: Cycles 4 and 5. $A_2$, $A_3$ and $A_4$ are active. The Nogood message is $A_1 = 1 \wedge A_2 = 4 \rightarrow A_3 \neq 4$.

# Example

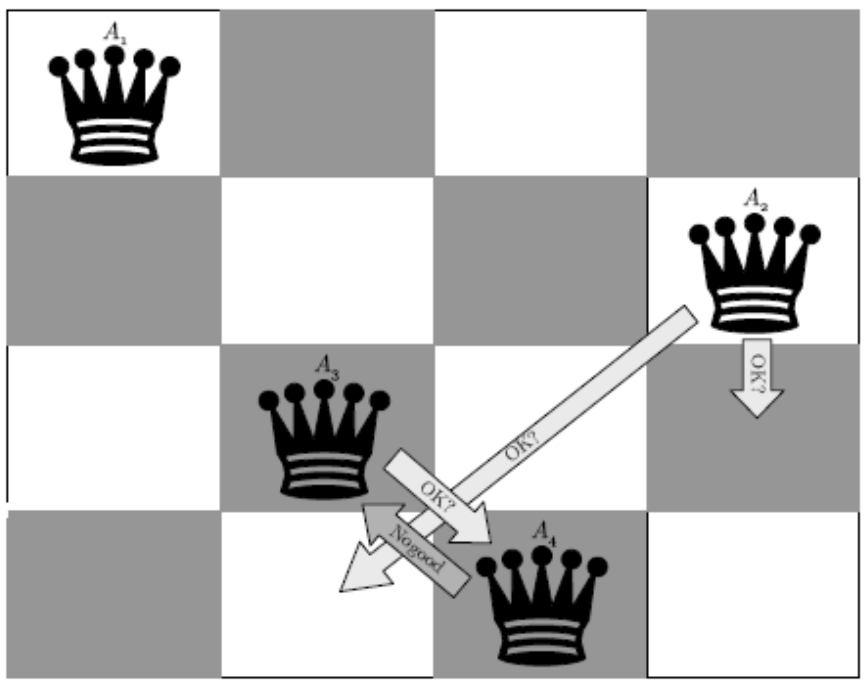



Figure 1.9: Cycles 4 and 5. $A_2$, $A_3$ and $A_4$ are active. The Nogood message is $A_1 = 1 \wedge A_2 = 4 \rightarrow A_3 \neq 4$.

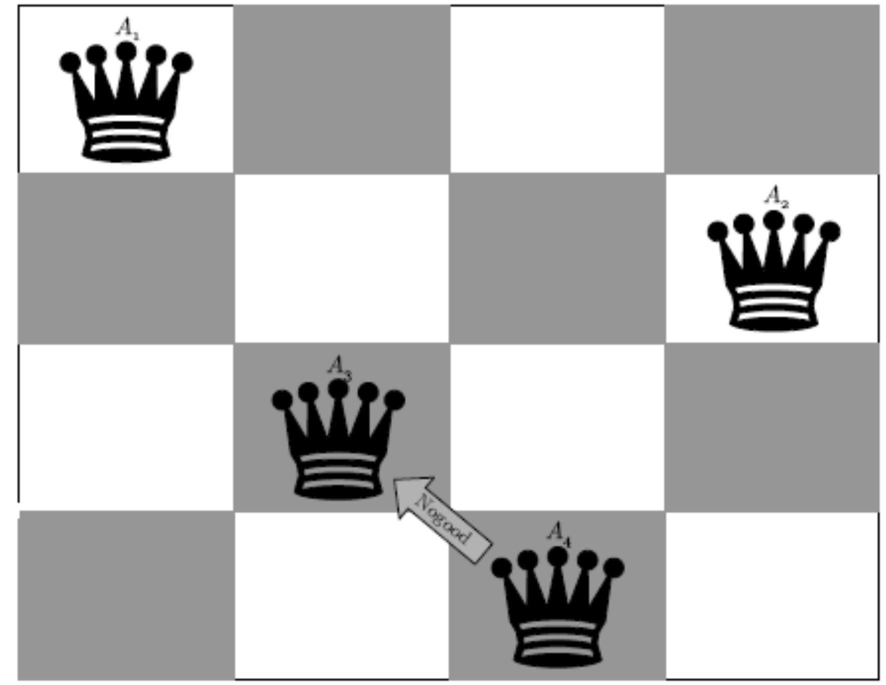



Figure 1.10: Cycle 6. Only $A_4$ is active. The Nogood message is $A_1 = 1 \wedge A_2 = 4 \rightarrow A_3 \neq 2$.

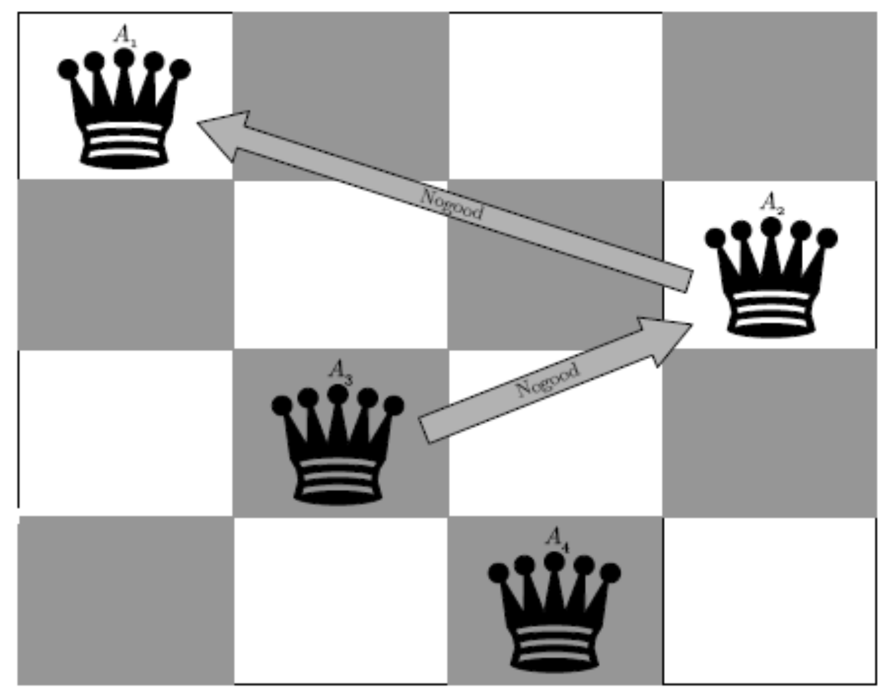



Figure 1.11: Cycles 7 and 8. $A_3$ is active in the first cycle and $A_2$ is active in the second. The Nogood messages are $A_1 = 1 \rightarrow A_2 \neq 4$ and $A_1 \neq 1$.

# Example

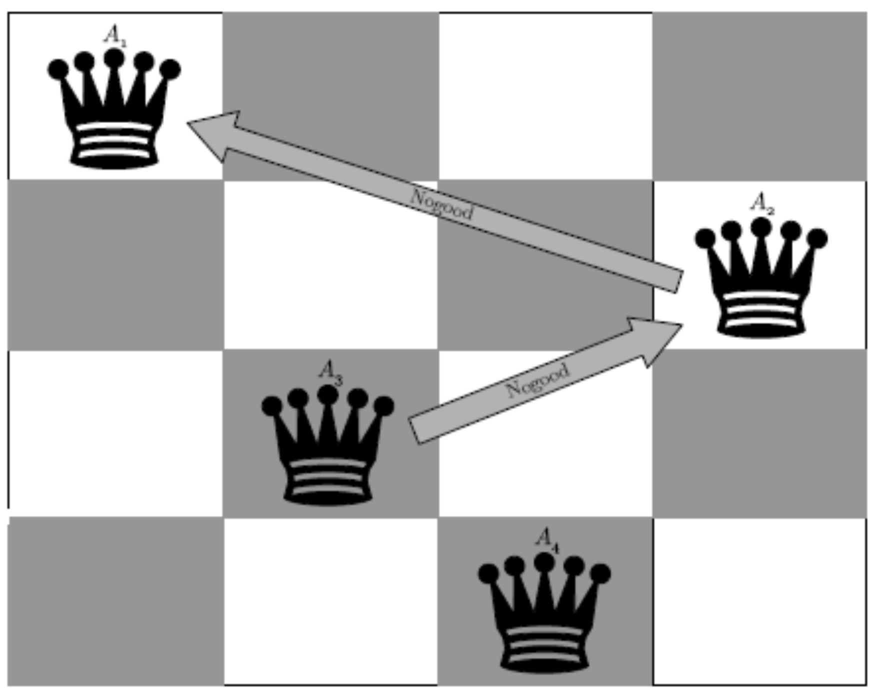



Figure 1.11: Cycles 7 and 8. $A_3$ is active in the first cycle and $A_2$ is active in the second. The Nogood messages are $A_1 = 1 \rightarrow A_2 \neq 4$ and $A_1 \neq 1$.

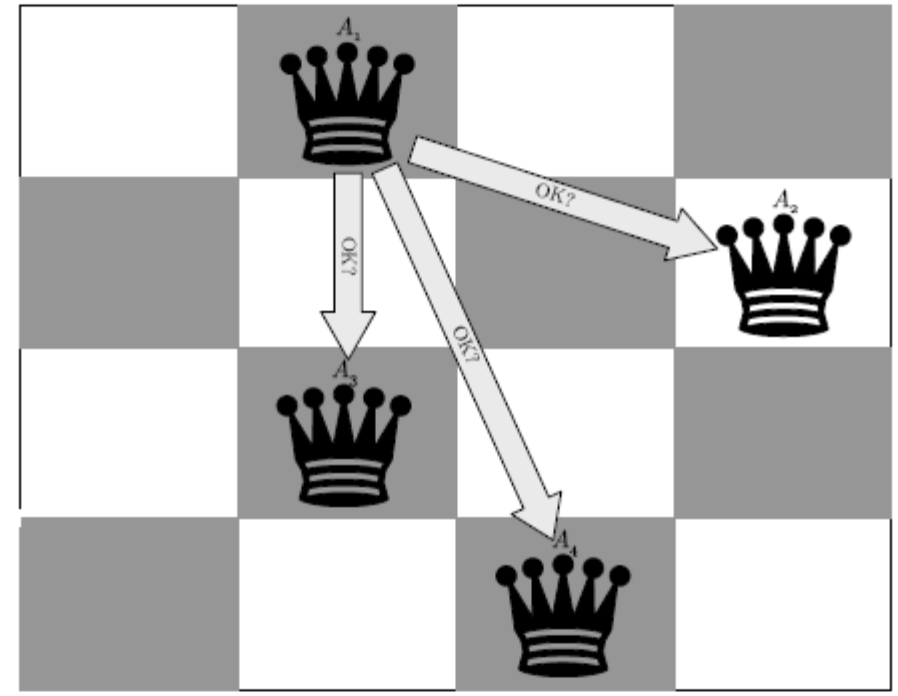



Figure 1.12: Cycle 9. Only $A_1$ is active.

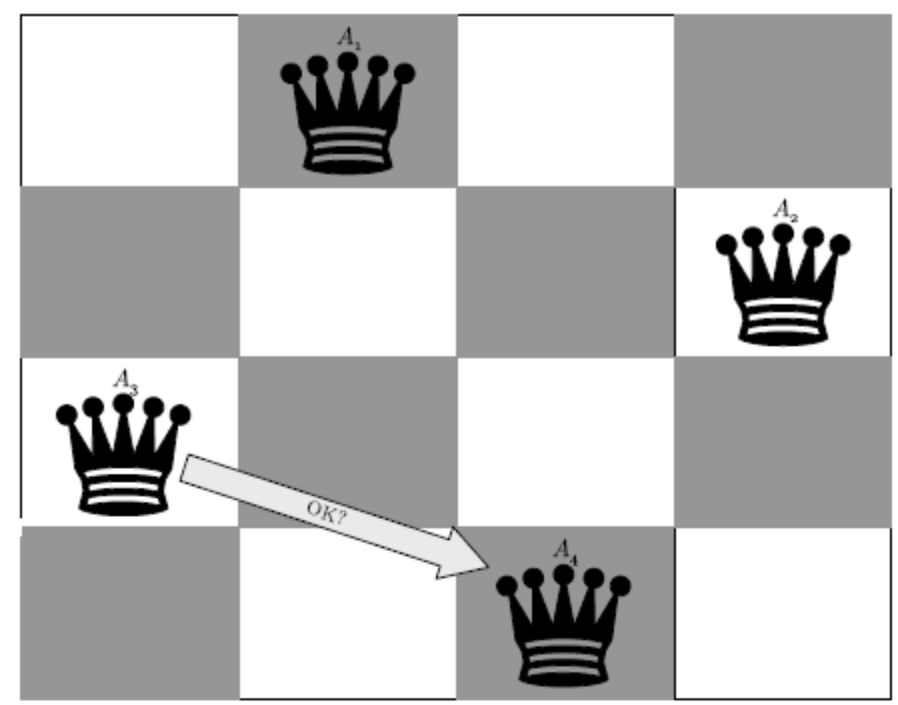



Figure 1.13: Cycle 10. Only $A_3$ is active.

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