



**OPPA European Social Fund
Prague & EU: We invest in your future.**

Expressive Description Logics

Petr Křemen
petr.kremen@fel.cvut.cz

FEL ČVUT

From \mathcal{ALC} to OWL(2)-DL

Final Remarks

From \mathcal{ALC} to OWL(2)-DL

- We have introduced *ALC*, together with a decision procedure. Its expressiveness is higher than propositional calculus, still it is insufficient for many practical applications.
- Let's take a look, how to extend *ALC* while preserving decidability.

- We have introduced ALC , together with a decision procedure. Its expressiveness is higher than propositional calculus, still it is insufficient for many practical applications.
- Let's take a look, how to extend ALC while preserving decidability.

Extending ... \mathcal{ALC} ... (2)

\mathcal{N} (Number restrictions) are used for restricting the number of successors in the given role for the given concept.

syntax (concept)	semantics
$(\geq n R)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^{\mathcal{I}} \} \right \geq n \right\}$
$(\leq n R)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^{\mathcal{I}} \} \right \leq n \right\}$
$(= n R)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^{\mathcal{I}} \} \right = n \right\}$

Example

- Concept $Woman \sqcap (\leq 3 \text{ hasChild})$ denotes women who have at most 3 children.
- What denotes the axiom $Car \sqsubseteq (\geq 4 \text{ hasWheel})$?
- ... and $Bicycle \equiv (= 2 \text{ hasWheel})$?

Extending ... \mathcal{ALC} ... (2)

\mathcal{N} (Number restrictions) are used for restricting the number of successors in the given role for the given concept.

syntax (concept)	semantics
$(\geq n R)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^{\mathcal{I}} \} \right \geq n \right\}$
$(\leq n R)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^{\mathcal{I}} \} \right \leq n \right\}$
$(= n R)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^{\mathcal{I}} \} \right = n \right\}$

Example

- Concept $Woman \sqcap (\leq 3 \text{ hasChild})$ denotes women who have at most 3 children.
- What denotes the axiom $Car \sqsubseteq (\geq 4 \text{ hasWheel})$?
- ... and $Bicycle \equiv (= 2 \text{ hasWheel})$?

Extending ... \mathcal{ALC} ... (2)

\mathcal{N} (Number restrictions) are used for restricting the number of successors in the given role for the given concept.

syntax (concept)	semantics
$(\geq n R)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^{\mathcal{I}} \} \right \geq n \right\}$
$(\leq n R)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^{\mathcal{I}} \} \right \leq n \right\}$
$(= n R)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^{\mathcal{I}} \} \right = n \right\}$

Example

- Concept $Woman \sqcap (\leq 3 \text{ hasChild})$ denotes women who have at most 3 children.
- What denotes the axiom $Car \sqsubseteq (\geq 4 \text{ hasWheel})$?
- ... and $Bicycle \equiv (= 2 \text{ hasWheel})$?

Extending ... *ALC* ... (3)

\mathcal{Q} (Qualified number restrictions) are used for restricting the number of successors *of the given type* in the given role for the given concept.

syntax (concept)	semantics
$(\geq n R C)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^I \wedge b^I \in C^I \} \right \geq n \right\}$
$(\leq n R C)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^I \wedge b^I \in C^I \} \right \leq n \right\}$
$(= n R C)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^I \wedge b^I \in C^I \} \right = n \right\}$

Example

- Concept $Woman \sqcap (\geq 3 \text{ hasChild } Man)$ denotes women who have at least 3 sons.
- What denotes the axiom $Car \sqsubseteq (\geq 4 \text{ hasPart } Wheel)$?
- Which qualified number restrictions can be expressed in *ALC* ?

Extending ... *ALC* ... (3)

Q (Qualified number restrictions) are used for restricting the number of successors *of the given type* in the given role for the given concept.

syntax (concept)	semantics
$(\geq n R C)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^I \wedge b^I \in C^I \} \right \geq n \right\}$
$(\leq n R C)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^I \wedge b^I \in C^I \} \right \leq n \right\}$
$(= n R C)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^I \wedge b^I \in C^I \} \right = n \right\}$

Example

- Concept $Woman \sqcap (\geq 3 \text{ hasChild } Man)$ denotes women who have at least 3 sons.
- What denotes the axiom $Car \sqsubseteq (\geq 4 \text{ hasPart } Wheel)$?
- Which qualified number restrictions can be expressed in *ALC* ?

Extending ... *ALC* ... (3)

Q (Qualified number restrictions) are used for restricting the number of successors of *the given type* in the given role for the given concept.

syntax (concept)	semantics
$(\geq n R C)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^I \wedge b^I \in C^I \} \right \geq n \right\}$
$(\leq n R C)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^I \wedge b^I \in C^I \} \right \leq n \right\}$
$(= n R C)$	$\left\{ a \mid \left \{ b \mid (a, b) \in R^I \wedge b^I \in C^I \} \right = n \right\}$

Example

- Concept $Woman \sqcap (\geq 3 \text{ hasChild } Man)$ denotes women who have at least 3 sons.
- What denotes the axiom $Car \sqsubseteq (\geq 4 \text{ hasPart } Wheel)$?
- Which qualified number restrictions can be expressed in *ALC* ?

Extending ... *ALC* ... (4)

- (Nominals) can be used for naming a concept elements explicitly.

syntax (concept)	semantics
$\{a_1, \dots, a_n\}$	$\{a_1^I, \dots, a_n^I\}$

Example

- Concept $\{MALE, FEMALE\}$ denotes a gender concept that must be interpreted with at most two elements. Why at most ?
- *Continent* \equiv
 $\{EUROPE, ASIA, AMERICA, AUSTRALIA, AFRICA, ANTARCTICA\}$
?

Extending ... *ALC* ... (4)

- (Nominals) can be used for naming a concept elements explicitly.

syntax (concept)	semantics
$\{a_1, \dots, a_n\}$	$\{a_1^I, \dots, a_n^I\}$

Example

- Concept $\{MALE, FEMALE\}$ denotes a gender concept that must be interpreted with at most two elements. Why at most ?
- *Continent* \equiv
 $\{EUROPE, ASIA, AMERICA, AUSTRALIA, AFRICA, ANTARCTICA\}$
?

\mathcal{I} (Inverse roles) are used for defining role inversion.

$$\frac{\text{syntax (role)}}{R^-} \quad \frac{\text{semantics}}{(R^{\mathcal{I}})^{-1}}$$

Example

- Role $maDite^-$ denotes the relationship $maRodice$.
- What denotes axiom $Person \sqsubseteq (= 2 \text{ hasChild}^-)$?
- What denotes axiom $Person \sqsubseteq \exists \text{ hasChild}^- \cdot \exists \text{ hasChild} \cdot \top$?

\mathcal{I} (Inverse roles) are used for defining role inversion.

$$\frac{\text{syntax (role)}}{R^-} \quad \frac{\text{semantics}}{(R^{\mathcal{I}})^{-1}}$$

Example

- Role $maDite^-$ denotes the relationship $maRodice$.
- What denotes axiom $Person \sqsubseteq (= 2 \text{ hasChild}^-)$?
- What denotes axiom $Person \sqsubseteq \exists \text{ hasChild}^- \cdot \exists \text{ hasChild} \cdot \top$?

\mathcal{I} (Inverse roles) are used for defining role inversion.

syntax (role)	semantics
R^-	$(R^{\mathcal{I}})^{-1}$

Example

- Role $maDite^-$ denotes the relationship $maRodice$.
- What denotes axiom $Person \sqsubseteq (= 2 \text{ hasChild}^-)$?
- What denotes axiom $Person \sqsubseteq \exists \text{ hasChild}^- \cdot \exists \text{ hasChild} \cdot \top$?

Extending ... \mathcal{ALC} ... (6)

\cdot^{trans} (Role transitivity axiom) denotes that a role is transitive.
Attention – it is not a transitive closure operator.

syntax (axiom)	semantics
$trans(R)$	$R^{\mathcal{I}}$ is transitive

Example

- Role *isPartOf* can be defined as transitive, while role *hasParent* is not. What about roles *hasPart*, *hasPart*⁻, *hasGrandFather*⁻ ?
- What is a transitive closure of a relationship ? What is the difference between a transitive closure of *hasDirectBoss* ^{\mathcal{I}} and *hasBoss* ^{\mathcal{I}} .

Extending ... \mathcal{ALC} ... (6)

\cdot ^{trans} (Role transitivity axiom) denotes that a role is transitive.
Attention – it is not a transitive closure operator.

syntax (axiom)	semantics
$trans(R)$	$R^{\mathcal{I}}$ is transitive

Example

- Role *isPartOf* can be defined as transitive, while role *hasParent* is not. What about roles *hasPart*, *hasPart*⁻, *hasGrandFather*⁻ ?
- What is a transitive closure of a relationship ? What is the difference between a transitive closure of *hasDirectBoss* ^{\mathcal{I}} and *hasBoss* ^{\mathcal{I}} .

\mathcal{H} (Role hierarchy) serves for expressing role hierarchies (taxonomies) – similarly to concept hierarchies.

syntax (axiom)	semantics
$R \sqsubseteq S$	$R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$

Example

- Role *hasMother* can be defined as a special case of the role *hasParent*.
- What is the difference between a concept hierarchy $Mother \sqsubseteq Parent$ and role hierarchy $hasMother \sqsubseteq hasParent$.

\mathcal{H} (Role hierarchy) serves for expressing role hierarchies (taxonomies) – similarly to concept hierarchies.

syntax (axiom)	semantics
$R \sqsubseteq S$	$R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$

Example

- Role *hasMother* can be defined as a special case of the role *hasParent*.
- What is the difference between a concept hierarchy $Mother \sqsubseteq Parent$ and role hierarchy $hasMother \sqsubseteq hasParent$.

Extending ... \mathcal{ALC} ... (8)

\mathcal{R} (role extensions) serve for defining expressive role constructs, like role chains, role disjunctions, etc.

syntax	semantics
$R \circ S \sqsubseteq P$	$R^{\mathcal{I}} \circ S^{\mathcal{I}} \sqsubseteq P^{\mathcal{I}}$
$Dis(R, R)$	$R^{\mathcal{I}} \cap S^{\mathcal{I}} = \emptyset$
$\exists R \cdot Self$	$\{a \mid (a, a) \in R^{\mathcal{I}}\}$

Example

- How would you define the role *hasUncle* by means of *hasSibling* and *hasParent* ?
- how to express that *R* is transitive, using a role chain ?
- Whom does the following concept denote
 $Person \sqcap \exists likes \cdot Self$?

Extending ... *ALC* ... (8)

\mathcal{R} (role extensions) serve for defining expressive role constructs, like role chains, role disjunctions, etc.

syntax	semantics
$R \circ S \sqsubseteq P$	$R^{\mathcal{I}} \circ S^{\mathcal{I}} \sqsubseteq P^{\mathcal{I}}$
$Dis(R, R)$	$R^{\mathcal{I}} \cap S^{\mathcal{I}} = \emptyset$
$\exists R \cdot Self$	$\{a \mid (a, a) \in R^{\mathcal{I}}\}$

Example

- How would you define the role *hasUncle* by means of *hasSibling* and *hasParent* ?
- how to express that *R* is transitive, using a role chain ?
- Whom does the following concept denote
Person \sqcap $\exists likes \cdot Self$?

Extending ... *ALC* ... (8)

\mathcal{R} (role extensions) serve for defining expressive role constructs, like role chains, role disjunctions, etc.

syntax	semantics
$R \circ S \sqsubseteq P$	$R^{\mathcal{I}} \circ S^{\mathcal{I}} \sqsubseteq P^{\mathcal{I}}$
$Dis(R, R)$	$R^{\mathcal{I}} \cap S^{\mathcal{I}} = \emptyset$
$\exists R \cdot Self$	$\{a \mid (a, a) \in R^{\mathcal{I}}\}$

Example

- How would you define the role *hasUncle* by means of *hasSibling* and *hasParent* ?
- how to express that *R* is transitive, using a role chain ?
- Whom does the following concept denote
 $Person \sqcap \exists likes \cdot Self$?

- From the previously introduced extensions, two prominent decidable supersets of \mathcal{ALC} can be constructed:
 - \mathcal{SHOIN} is a description logics that backs OWL-DL.
 - \mathcal{SROIQ} is a description logics that backs OWL2-DL.
 - Both OWL-DL and OWL2-DL are semantic web languages – they extend the corresponding description logics by:
 - \mathcal{R} (role chains)
 - \mathcal{D} (disjoint domain)
 - \mathcal{F} (functional roles)
 - \mathcal{I} (inverse roles)
 - \mathcal{N} (nominals)
 - \mathcal{Q} (qualified disjunction)
 - \mathcal{S} (self-roles)
 - \mathcal{O} (overlapping domains)

- From the previously introduced extensions, two prominent decidable supersets of \mathcal{ALC} can be constructed:
 - \mathcal{SHOIN} is a description logics that backs OWL-DL.
 - \mathcal{SROIQ} is a description logics that backs OWL2-DL.
 - Both OWL-DL and OWL2-DL are semantic web languages – they extend the corresponding description logics by:

- From the previously introduced extensions, two prominent decidable supersets of \mathcal{ALC} can be constructed:
 - \mathcal{SHOIN} is a description logics that backs OWL-DL.
 - \mathcal{SROIQ} is a description logics that backs OWL2-DL.
 - Both OWL-DL and OWL2-DL are semantic web languages – they extend the corresponding description logics by:
 - syntactic sugar – axioms NegativeObjectPropertyAssertion, PropertyChain, AllDisjoint, etc.
 - *existential restrictions* – *transitive* *symmetric* *reflexive*
 - *transitive* *symmetric* *reflexive* *irreflexive* *antisymmetric*
 - *transitive* *symmetric* *reflexive* *irreflexive* *antisymmetric* *inverse*

- From the previously introduced extensions, two prominent decidable supersets of \mathcal{ALC} can be constructed:
 - \mathcal{SHOIN} is a description logics that backs OWL-DL.
 - \mathcal{SROIQ} is a description logics that backs OWL2-DL.
 - Both OWL-DL and OWL2-DL are semantic web languages – they extend the corresponding description logics by:
 - syntactic sugar – axioms NegativeObjectPropertyAssertion, AllDisjoint, etc.
 - extralogical constructs – imports, annotations
 - data types – XSD datatypes are used

- From the previously introduced extensions, two prominent decidable supersets of \mathcal{ALC} can be constructed:
 - \mathcal{SHOIN} is a description logics that backs OWL-DL.
 - \mathcal{SROIQ} is a description logics that backs OWL2-DL.
 - Both OWL-DL and OWL2-DL are semantic web languages – they extend the corresponding description logics by:
 - syntactic sugar – axioms NegativeObjectPropertyAssertion, AllDisjoint, etc.
 - extralogical constructs – imports, annotations
 - data types – XSD datatypes are used

- From the previously introduced extensions, two prominent decidable supersets of \mathcal{ALC} can be constructed:
 - \mathcal{SHOIN} is a description logics that backs OWL-DL.
 - \mathcal{SROIQ} is a description logics that backs OWL2-DL.
 - Both OWL-DL and OWL2-DL are semantic web languages – they extend the corresponding description logics by:
 - syntactic sugar – axioms NegativeObjectPropertyAssertion, AllDisjoint, etc.
 - extralogical constructs – imports, annotations
 - data types – XSD datatypes are used

- From the previously introduced extensions, two prominent decidable supersets of \mathcal{ALC} can be constructed:
 - \mathcal{SHOIN} is a description logics that backs OWL-DL.
 - \mathcal{SROIQ} is a description logics that backs OWL2-DL.
 - Both OWL-DL and OWL2-DL are semantic web languages – they extend the corresponding description logics by:
 - syntactic sugar – axioms NegativeObjectPropertyAssertion, AllDisjoint, etc.
 - extralogical constructs – imports, annotations
 - data types – XSD datatypes are used

- What is the impact of the extensions to the automated reasoning procedure ? The introduced tableau algorithm for \mathcal{ALC} has to be adjusted as follows:
 - additional inference rules reflecting the semantics of newly added constructs ($\mathcal{O}, \mathcal{N}, \mathcal{Q}$)
 - definition of R -neighbourhood of a node in a completion graph. R -neighbourhood notion generalizes simple tests of two nodes being connected with an edge, e.g. in \exists -rule. ($\mathcal{H}, \mathcal{R}, \mathcal{I}$)
 - new conditions for direct clash detection
 - more strict blocking conditions (blocking over graph structures).
- This results in significant computation blowup – from EXPTIME (\mathcal{ALC}) to

- What is the impact of the extensions to the automated reasoning procedure ? The introduced tableau algorithm for \mathcal{ALC} has to be adjusted as follows:
 - additional inference rules reflecting the semantics of newly added constructs ($\mathcal{O}, \mathcal{N}, \mathcal{Q}$)
 - definition of *R-neighbourhood* of a node in a completion graph. R-neighbourhood notion generalizes simple tests of two nodes being connected with an edge, e.g. in \exists -rule. ($\mathcal{H}, \mathcal{R}, \mathcal{I}$)
 - new conditions for direct clash detection
 - more strict blocking conditions (blocking over graph structures).
- This results in significant computation blowup – from EXPTIME (\mathcal{ALC}) to

- What is the impact of the extensions to the automated reasoning procedure ? The introduced tableau algorithm for \mathcal{ALC} has to be adjusted as follows:
 - additional inference rules reflecting the semantics of newly added constructs ($\mathcal{O}, \mathcal{N}, \mathcal{Q}$)
 - definition of *R-neighbourhood* of a node in a completion graph. R-neighbourhood notion generalizes simple tests of two nodes being connected with an edge, e.g. in \exists -rule. ($\mathcal{H}, \mathcal{R}, \mathcal{I}$)
 - new conditions for direct clash detection
 - more strict blocking conditions (blocking over graph structures).
- This results in significant computation blowup – from EXPTIME (\mathcal{ALC}) to

Extending \mathcal{ALC} – Reasoning

- What is the impact of the extensions to the automated reasoning procedure ? The introduced tableau algorithm for \mathcal{ALC} has to be adjusted as follows:
 - additional inference rules reflecting the semantics of newly added constructs ($\mathcal{O}, \mathcal{N}, \mathcal{Q}$)
 - definition of *R-neighbourhood* of a node in a completion graph. R-neighbourhood notion generalizes simple tests of two nodes being connected with an edge, e.g. in \exists -rule. ($\mathcal{H}, \mathcal{R}, \mathcal{I}$)
 - new conditions for direct clash detection
 - more strict blocking conditions (blocking over graph structures).
- This results in significant computation blowup – from EXPTIME (\mathcal{ALC}) to

- What is the impact of the extensions to the automated reasoning procedure ? The introduced tableau algorithm for \mathcal{ALC} has to be adjusted as follows:
 - additional inference rules reflecting the semantics of newly added constructs ($\mathcal{O}, \mathcal{N}, \mathcal{Q}$)
 - definition of *R-neighbourhood* of a node in a completion graph. R-neighbourhood notion generalizes simple tests of two nodes being connected with an edge, e.g. in \exists -rule. ($\mathcal{H}, \mathcal{R}, \mathcal{I}$)
 - new conditions for direct clash detection
 - more strict blocking conditions (blocking over graph structures).
- This results in significant computation blowup – from EXPTIME (\mathcal{ALC}) to
 - NEXPTIME for \mathcal{SHOIN}

- What is the impact of the extensions to the automated reasoning procedure ? The introduced tableau algorithm for \mathcal{ALC} has to be adjusted as follows:
 - additional inference rules reflecting the semantics of newly added constructs ($\mathcal{O}, \mathcal{N}, \mathcal{Q}$)
 - definition of R -neighbourhood of a node in a completion graph. R -neighbourhood notion generalizes simple tests of two nodes being connected with an edge, e.g. in \exists -rule. ($\mathcal{H}, \mathcal{R}, \mathcal{I}$)
 - new conditions for direct clash detection
 - more strict blocking conditions (blocking over graph structures).
- This results in significant computation blowup – from EXPTIME (\mathcal{ALC}) to
 - NEXPTIME for \mathcal{SHOIN}
 - N2EXPTIME for \mathcal{SROIQ}

- What is the impact of the extensions to the automated reasoning procedure ? The introduced tableau algorithm for \mathcal{ALC} has to be adjusted as follows:
 - additional inference rules reflecting the semantics of newly added constructs ($\mathcal{O}, \mathcal{N}, \mathcal{Q}$)
 - definition of R -neighbourhood of a node in a completion graph. R -neighbourhood notion generalizes simple tests of two nodes being connected with an edge, e.g. in \exists -rule. ($\mathcal{H}, \mathcal{R}, \mathcal{I}$)
 - new conditions for direct clash detection
 - more strict blocking conditions (blocking over graph structures).
- This results in significant computation blowup – from EXPTIME (\mathcal{ALC}) to
 - NEXPTIME for \mathcal{SHOIN}
 - N2EXPTIME for \mathcal{SROIQ}

- What is the impact of the extensions to the automated reasoning procedure ? The introduced tableau algorithm for \mathcal{ALC} has to be adjusted as follows:
 - additional inference rules reflecting the semantics of newly added constructs ($\mathcal{O}, \mathcal{N}, \mathcal{Q}$)
 - definition of R -neighbourhood of a node in a completion graph. R -neighbourhood notion generalizes simple tests of two nodes being connected with an edge, e.g. in \exists -rule. ($\mathcal{H}, \mathcal{R}, \mathcal{I}$)
 - new conditions for direct clash detection
 - more strict blocking conditions (blocking over graph structures).
- This results in significant computation blowup – from EXPTIME (\mathcal{ALC}) to
 - NEXPTIME for \mathcal{SHOIN}
 - N2EXPTIME for \mathcal{SROIQ}

Final Remarks

Other extensions

Modal Logic introduces *modal operators* – possibility/necessity, used in multiagent systems.

Example

- (\Box represents e.g. the “believes” operator of an agent)

$$\Box(\text{Man} \sqsubseteq \text{Person} \sqcap \nabla \text{hasFather} \cdot \text{Man}) \quad (1)$$

- As \mathcal{ALC} is a syntactic variant to a multi-modal propositional logic, where each role represents the accessibility relations between worlds in Kripke structures, the previous example can be transformed to the modal logic as:

$$\Box(\text{Man} \rightarrow \text{Person} \wedge \Box_{\text{hasFather}} \text{Man}) \quad (2)$$

Vague Knowledge - fuzzy, probabilistic and possibilistic extensions (see [HPS05]).

Data Types (\mathcal{D}) allow integrating a data domain (numbers, strings), e.g.

$\text{Person} \sqcap \exists \text{hasAge} \cdot 23$ represents the concept describing “23-years old persons”.

Other extensions

Modal Logic introduces *modal operators* – possibility/necessity, used in multiagent systems.

Example

- (\Box represents e.g. the "believe" operator of an agent)

$$\Box(\text{Man} \sqsubseteq \text{Person} \sqcap \forall \text{hasFather} \cdot \text{Man}) \quad (1)$$

- As \mathcal{ALC} is a syntactic variant to a multi-modal propositional logic, where each role represents the accessibility relations between worlds in Kripke structure, the previous example can be transformed to the modal logic as:

- $$\Box(\text{Man} \Rightarrow \text{Person} \wedge \Box_{\text{hasFather}} \text{Man}) \quad (2)$$

Vague Knowledge - fuzzy, probabilistic and possibilistic extensions (see [HPS05]).

Data Types (\mathcal{D}) allow integrating a data domain (numbers, strings), e.g.

$\text{Person} \sqcap \exists \text{hasAge} \cdot 23$ represents the concept describing "23-years old persons".

Other extensions

Modal Logic introduces *modal operators* – possibility/necessity, used in multiagent systems.

Example

- (\Box represents e.g. the "believe" operator of an agent)

$$\Box(\text{Man} \sqsubseteq \text{Person} \sqcap \forall \text{hasFather} \cdot \text{Man}) \quad (1)$$

- As \mathcal{ALC} is a syntactic variant to a multi-modal propositional logic, where each role represents the accessibility relations between worlds in Kripke structure, the previous example can be transformed to the modal logic as:
-

$$\Box(\text{Man} \Rightarrow \text{Person} \wedge \Box_{\text{hasFather}} \text{Man}) \quad (2)$$

Vague Knowledge - fuzzy, probabilistic and possibilistic extensions (see [HPS05]).

Data Types (\mathcal{D}) allow integrating a data domain (numbers, strings), e.g. $\text{Person} \sqcap \exists \text{hasAge} \cdot 23$ represents the concept describing "23-years old persons".

Other extensions

Modal Logic introduces *modal operators* – possibility/necessity, used in multiagent systems.

Example

- (\Box represents e.g. the "believe" operator of an agent)

$$\Box(\text{Man} \sqsubseteq \text{Person} \sqcap \forall \text{hasFather} \cdot \text{Man}) \quad (1)$$

- As \mathcal{ALC} is a syntactic variant to a multi-modal propositional logic, where each role represents the accessibility relations between worlds in Kripke structure, the previous example can be transformed to the modal logic as:

$$\Box(\text{Man} \Rightarrow \text{Person} \wedge \Box_{\text{hasFather}} \text{Man}) \quad (2)$$

Vague Knowledge - fuzzy, probabilistic and possibilistic extensions (see [HPS05]).

Data Types (\mathcal{D}) allow integrating a data domain (numbers, strings), e.g. $\text{Person} \sqcap \exists \text{hasAge} \cdot 23$ represents the concept describing "23-years old persons".

Other extensions

Modal Logic introduces *modal operators* – possibility/necessity, used in multiagent systems.

Example

- (\Box represents e.g. the "believe" operator of an agent)

$$\Box(Man \sqsubseteq Person \sqcap \forall hasFather \cdot Man) \quad (1)$$

- As \mathcal{ALC} is a syntactic variant to a multi-modal propositional logic, where each role represents the accessibility relations between worlds in Kripke structure, the previous example can be transformed to the modal logic as:
-

$$\Box(Man \Rightarrow Person \wedge \Box_{hasFather} Man) \quad (2)$$

Vague Knowledge - fuzzy, probabilistic and possibilistic extensions (see [HPS05]).

Data Types (\mathcal{D}) allow integrating a data domain (numbers, strings), e.g.

$Person \sqcap \exists hasAge \cdot 23$ represents the concept describing "23-years old persons".

RacerPro (<http://www.racer-systems.com>) is a commercial LISP-based system for OWL-DL and SWRL (also available in client/server version).

Pellet (<http://www.mindswap.org>) is an open-source Java OWL2-DL engine.

Jena <http://jena.sourceforge.net/> is an open-source Java framework and API for OWL and RDF(S).

FaCT++ <http://owl.man.ac.uk/factplusplus/> is a DL reasoner for *SHOIQ* written in C++.

and other ... KAON2, FOWL, Kris

RacerPro (<http://www.racer-systems.com>) is a commercial LISP-based system for OWL-DL and SWRL (also available in client/server version).

Pellet (<http://www.mindswap.org>) is an open-source Java OWL2-DL engine.

Jena <http://jena.sourceforge.net/> is an open-source Java framework and API for OWL and RDF(S).

FaCT++ <http://owl.man.ac.uk/factplusplus/> is a DL reasoner for *SHOIQ* written in C++.

and other ... KAON2, FOWL, Kris

RacerPro (<http://www.racer-systems.com>) is a commercial LISP-based system for OWL-DL and SWRL (also available in client/server version).

Pellet (<http://www.mindswap.org>) is an open-source Java OWL2-DL engine.

Jena <http://jena.sourceforge.net/> is an open-source Java framework and API for OWL and RDF(S).

FaCT++ <http://owl.man.ac.uk/factplusplus/> is a DL reasoner for *SHOIQ* written in C++.

and other ... KAON2, FOWL, Kris

RacerPro (<http://www.racer-systems.com>) is a commercial LISP-based system for OWL-DL and SWRL (also available in client/server version).

Pellet (<http://www.mindswap.org>) is an open-source Java OWL2-DL engine.

Jena <http://jena.sourceforge.net/> is an open-source Java framework and API for OWL and RDF(S).

FaCT++ <http://owl.man.ac.uk/factplusplus/> is a DL reasoner for *SHOIQ* written in C++.

and other ... KAON2, FOWL, Kris

RacerPro (<http://www.racer-systems.com>) is a commercial LISP-based system for OWL-DL and SWRL (also available in client/server version).

Pellet (<http://www.mindswap.org>) is an open-source Java OWL2-DL engine.

Jena <http://jena.sourceforge.net/> is an open-source Java framework and API for OWL and RDF(S).

FaCT++ <http://owl.man.ac.uk/factplusplus/> is a DL reasoner for *SHOIQ* written in C++.

and other ... KAON2, FOWL, Kris



**OPPA European Social Fund
Prague & EU: We invest in your future.**
