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Mean shift ¹

Tomáš Svoboda, svoboda@cmp.felk.cvut.cz

Czech Technical University in Prague, Center for Machine Perception

<http://cmp.felk.cvut.cz>

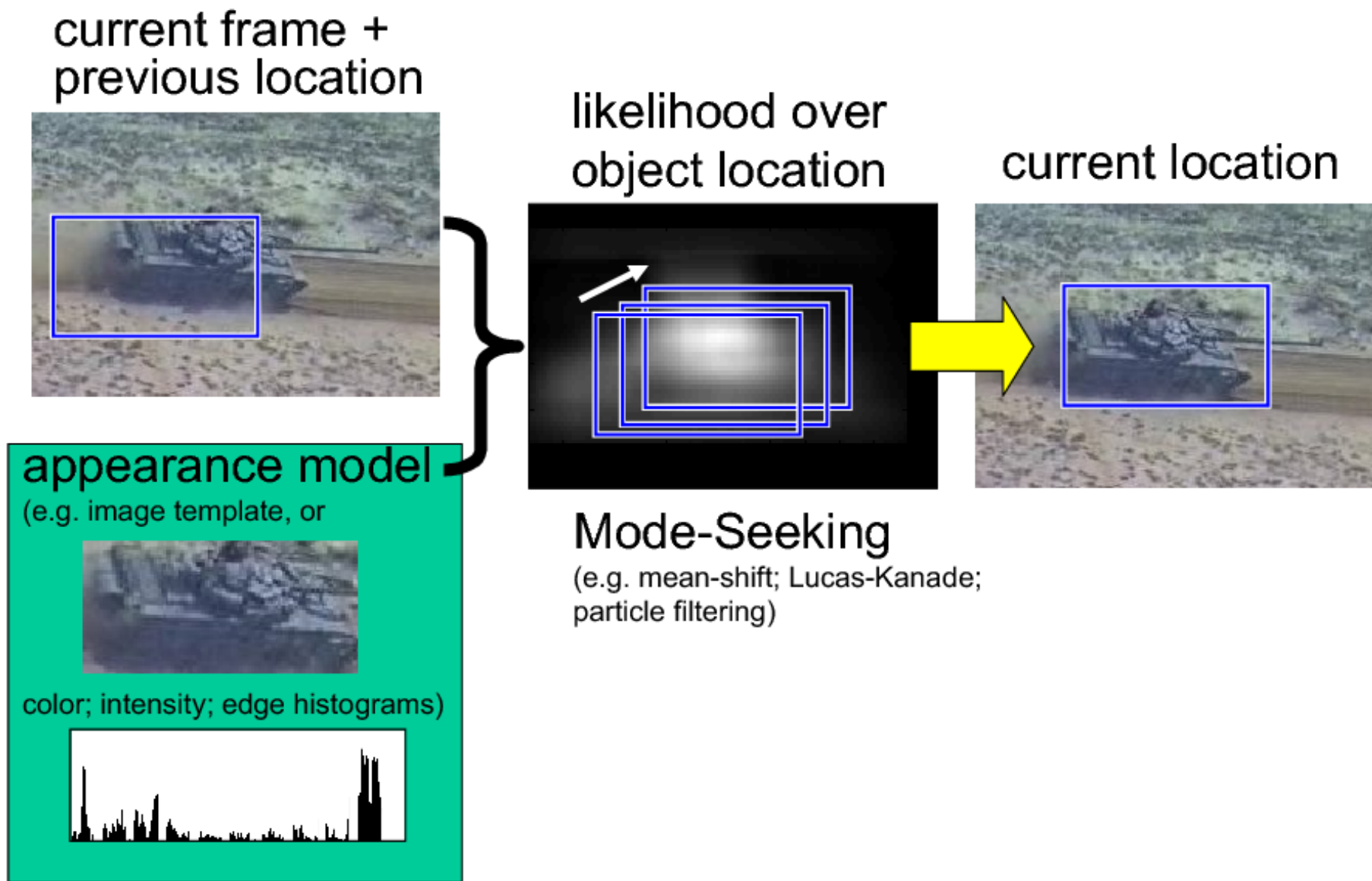
Last update: April 8, 2013

Talk Outline

- ◆ appearance based tracking
- ◆ patch similarity using histogram
- ◆ tracking by mean shift
- ◆ experiments, discussion

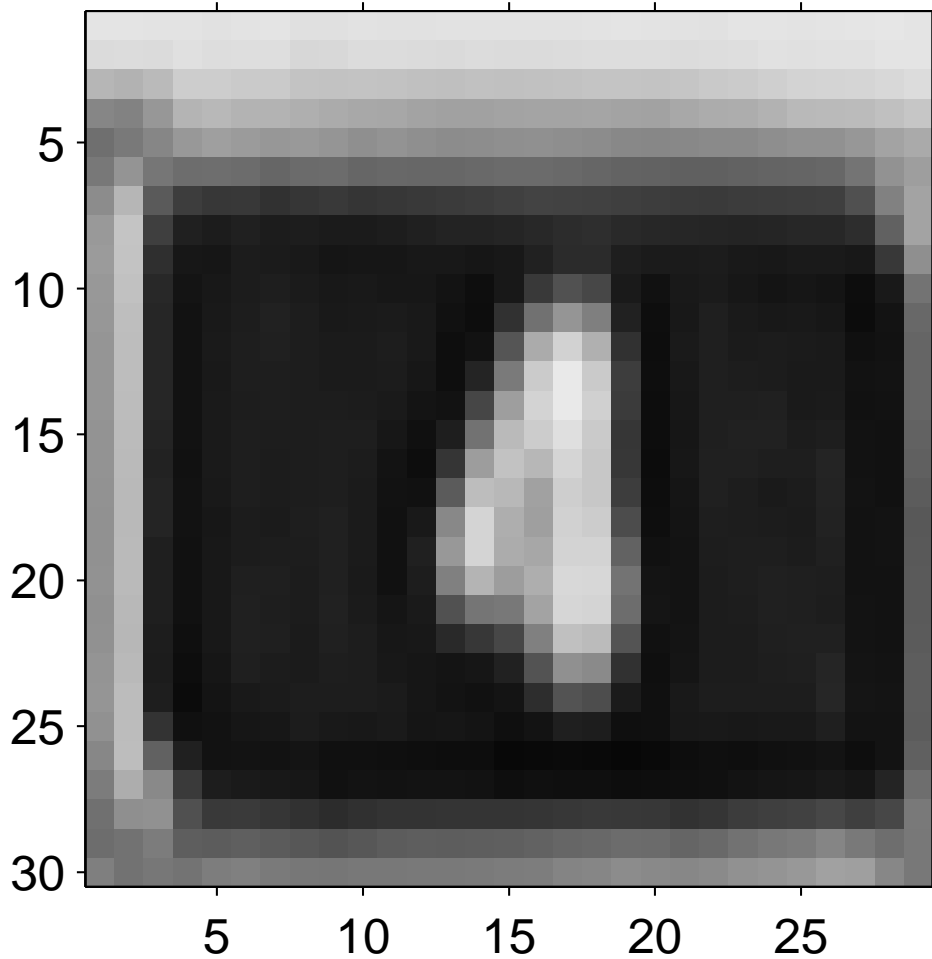
¹Please note that the lecture will be accompanied by several sketches and derivations on the blackboard and few live-interactive demos in Matlab

Appearance based tracking

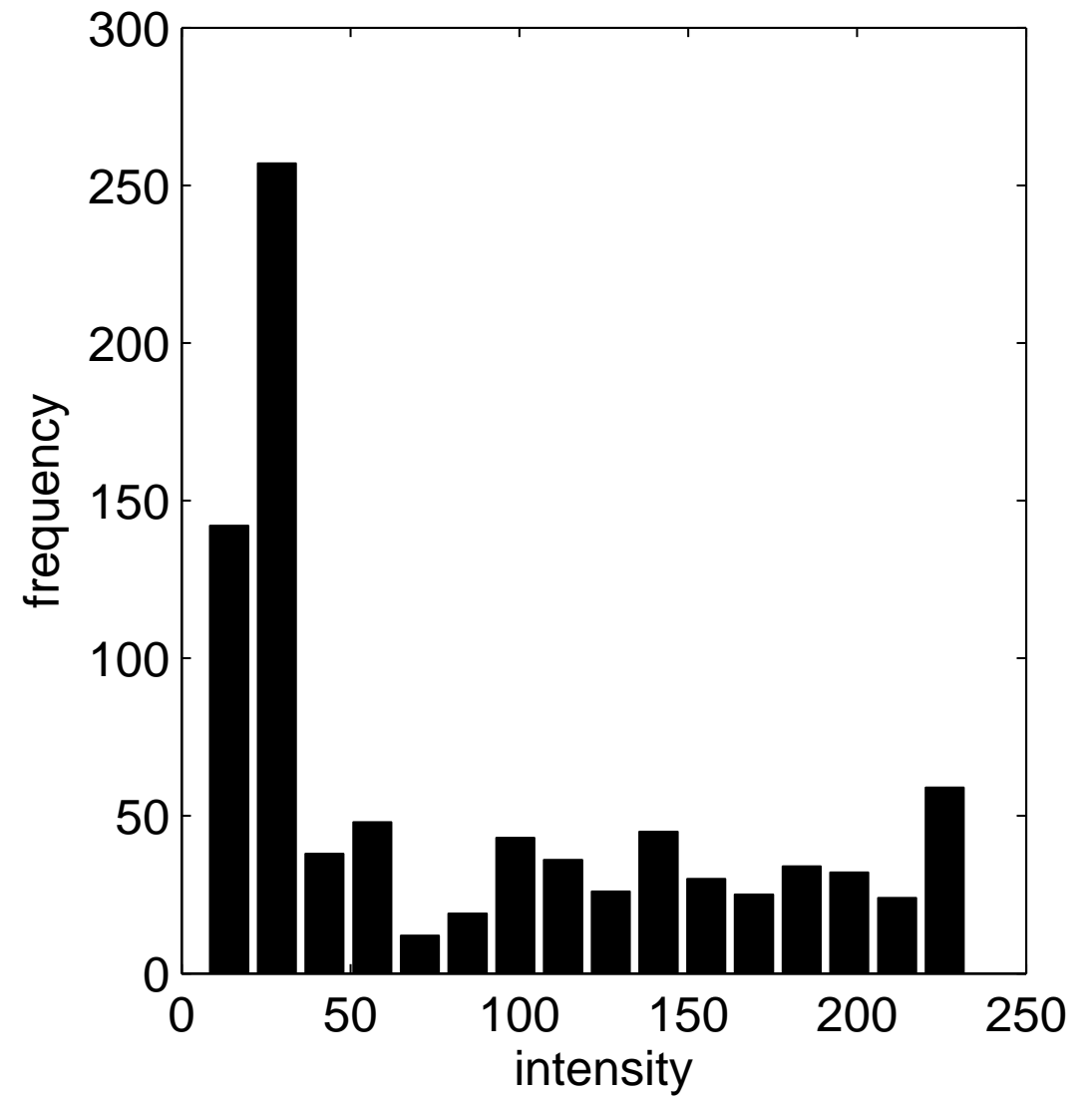


Histogram based representation

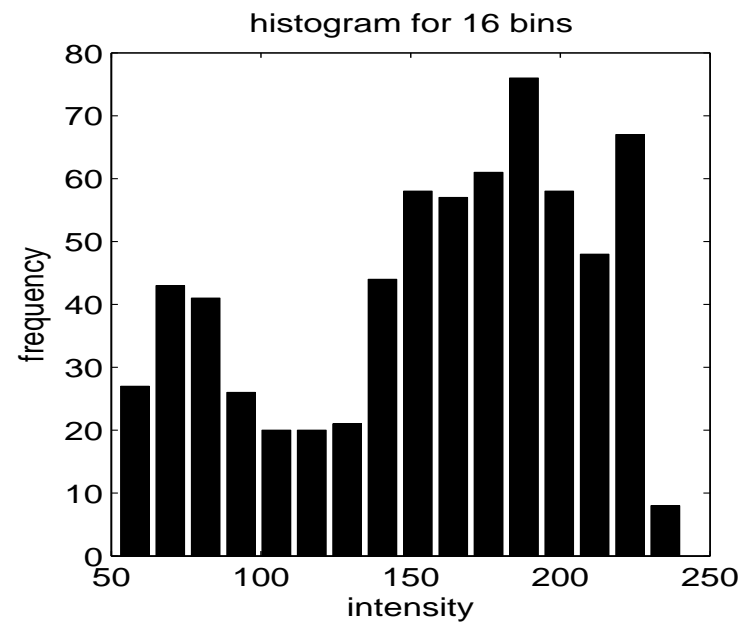
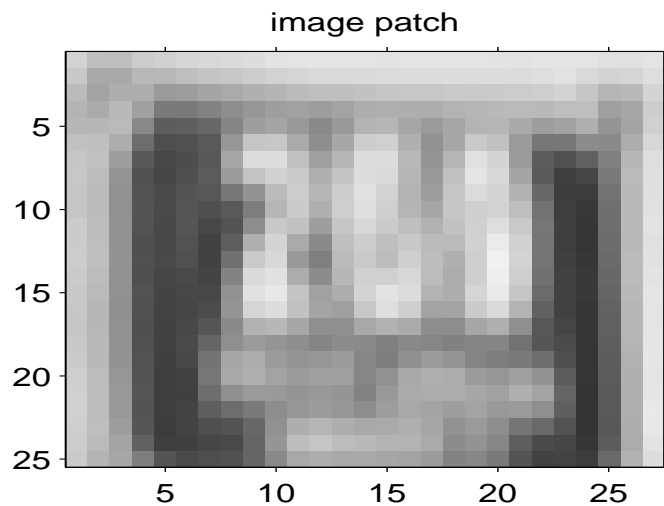
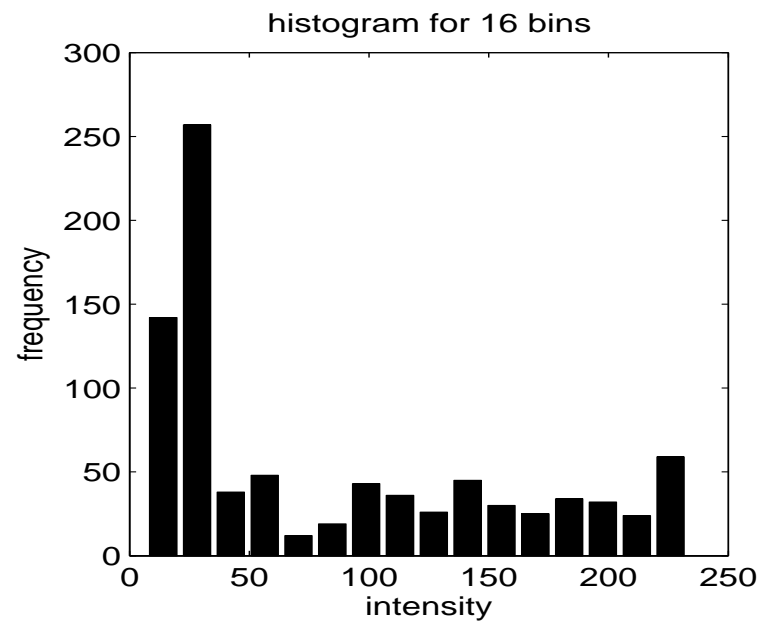
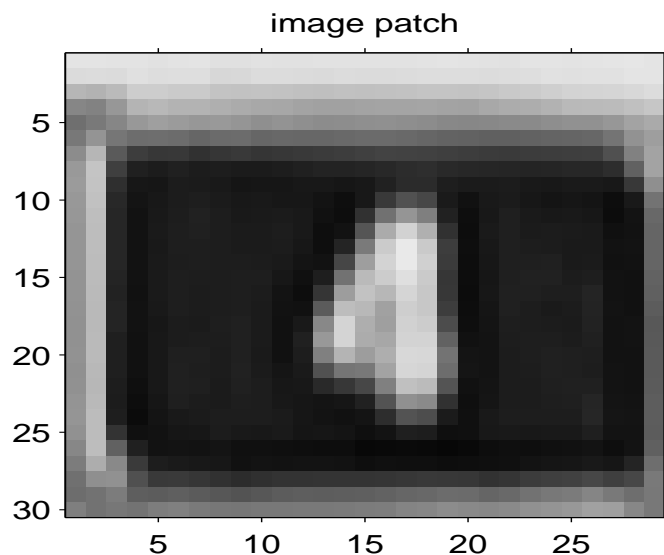
image patch



histogram for 16 bins



Patch comparison



histogram difference

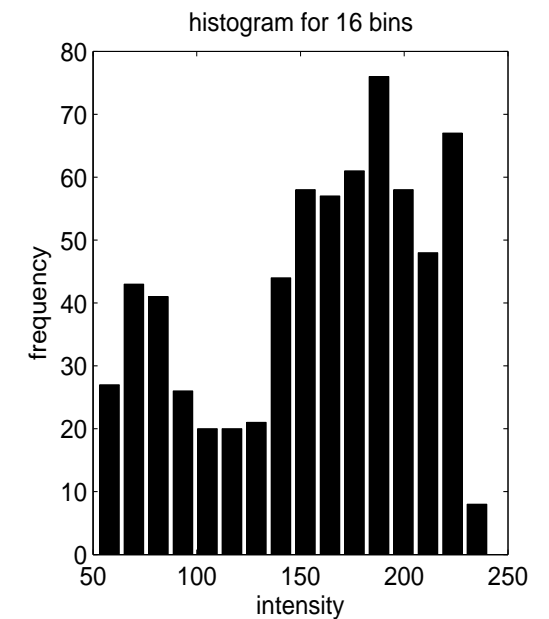
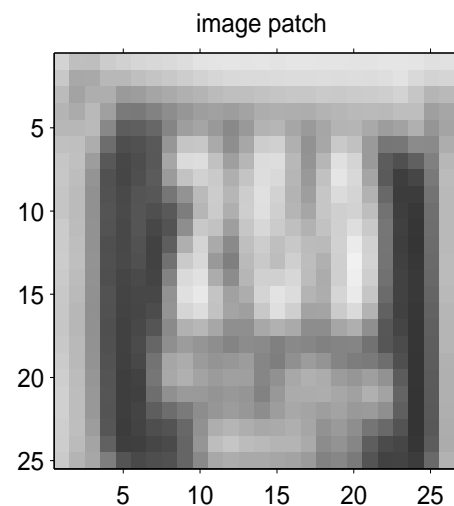
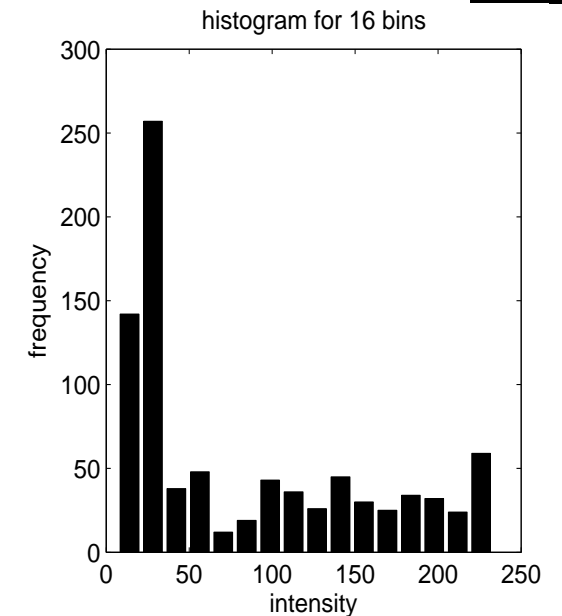
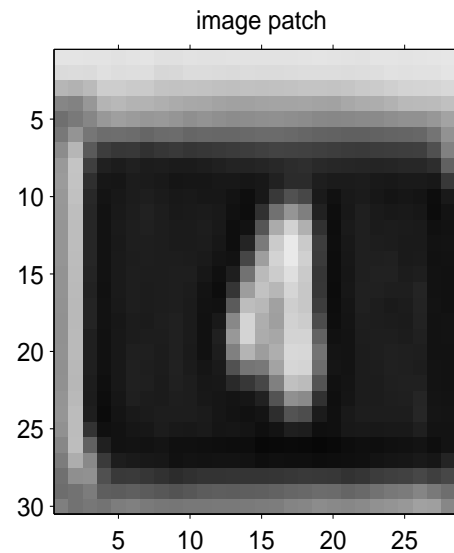
assume **normalized** histograms, i.e

$$\sum_{u=1}^m p_u = 1$$

$$d = \sqrt{1 - \rho[p, q]}$$

where $\rho[p, q]$ is the **Bhattacharyya coefficient**

$$\rho[p, q] = \sum_{u=1}^m \sqrt{p_u q_u}$$

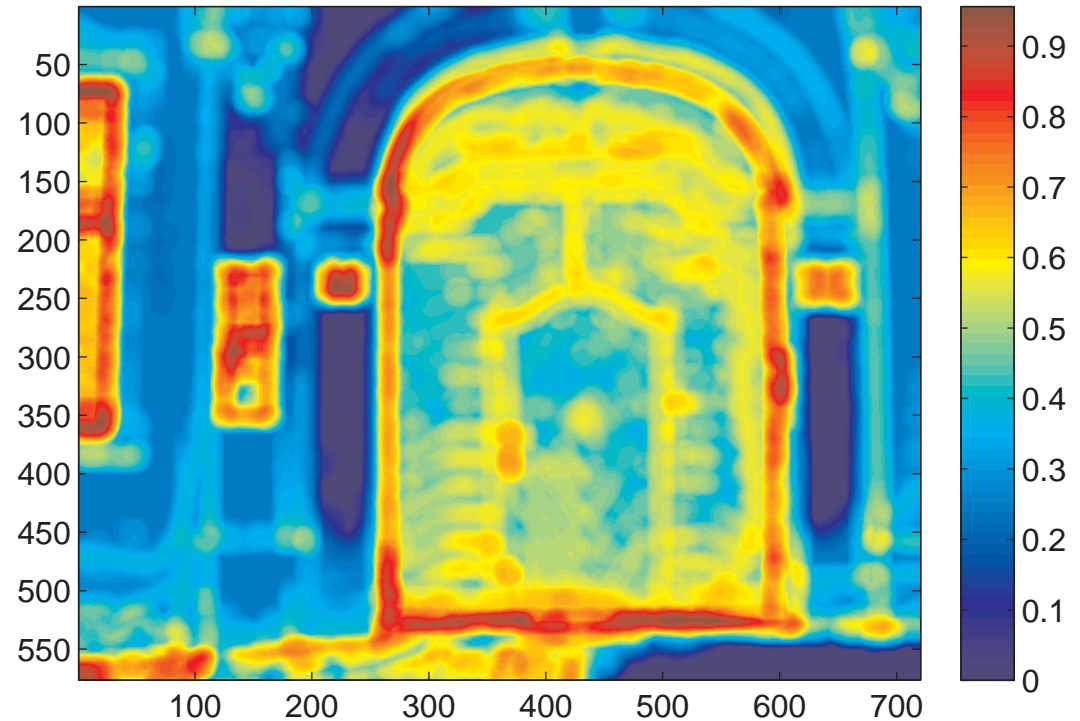


Similarity measured by the Bhattacharyya coefficient

Example of an input image



Similarity surface of the frame 206



The object is the “4” plate and the model is histogram of image intensities.

$$s(\mathbf{y}) = \sum_{u=1}^m \sqrt{p_u(\mathbf{y})q_u}$$

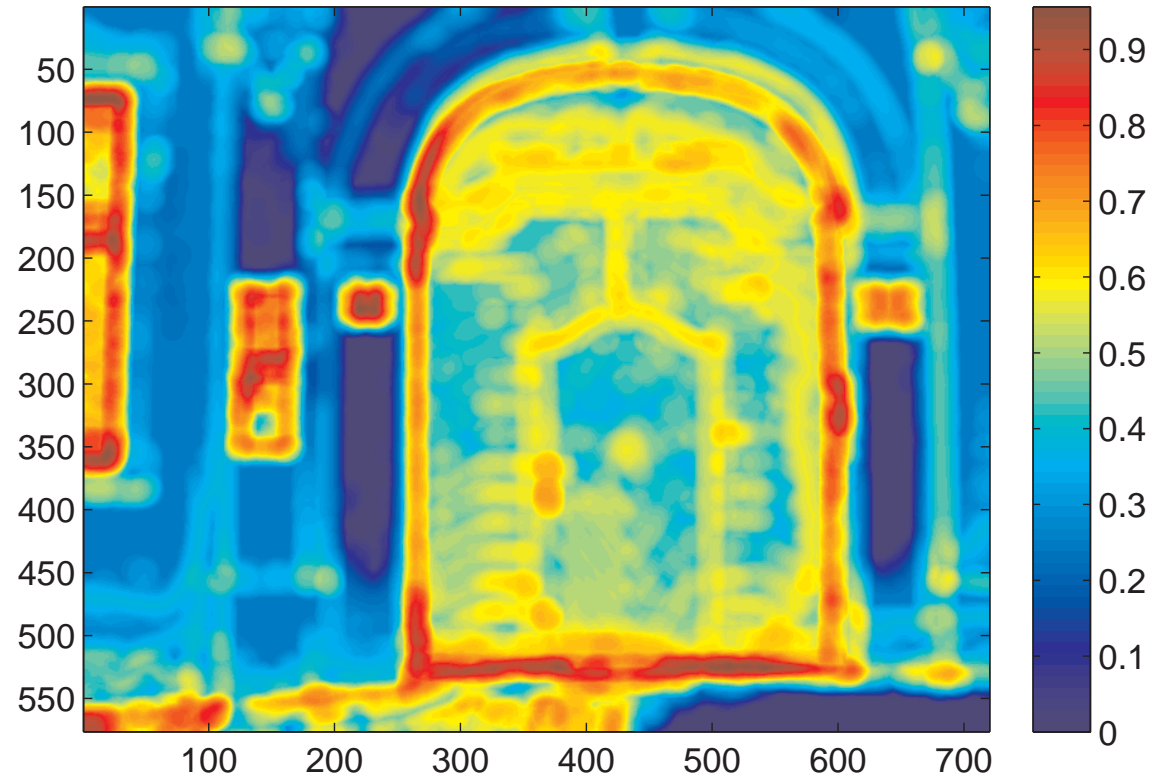
where $p(\mathbf{y})$ is the histogram of image patch at position \mathbf{y} and q is the histogram of the template.

Problem: finding modes in probability density

Example of an input image



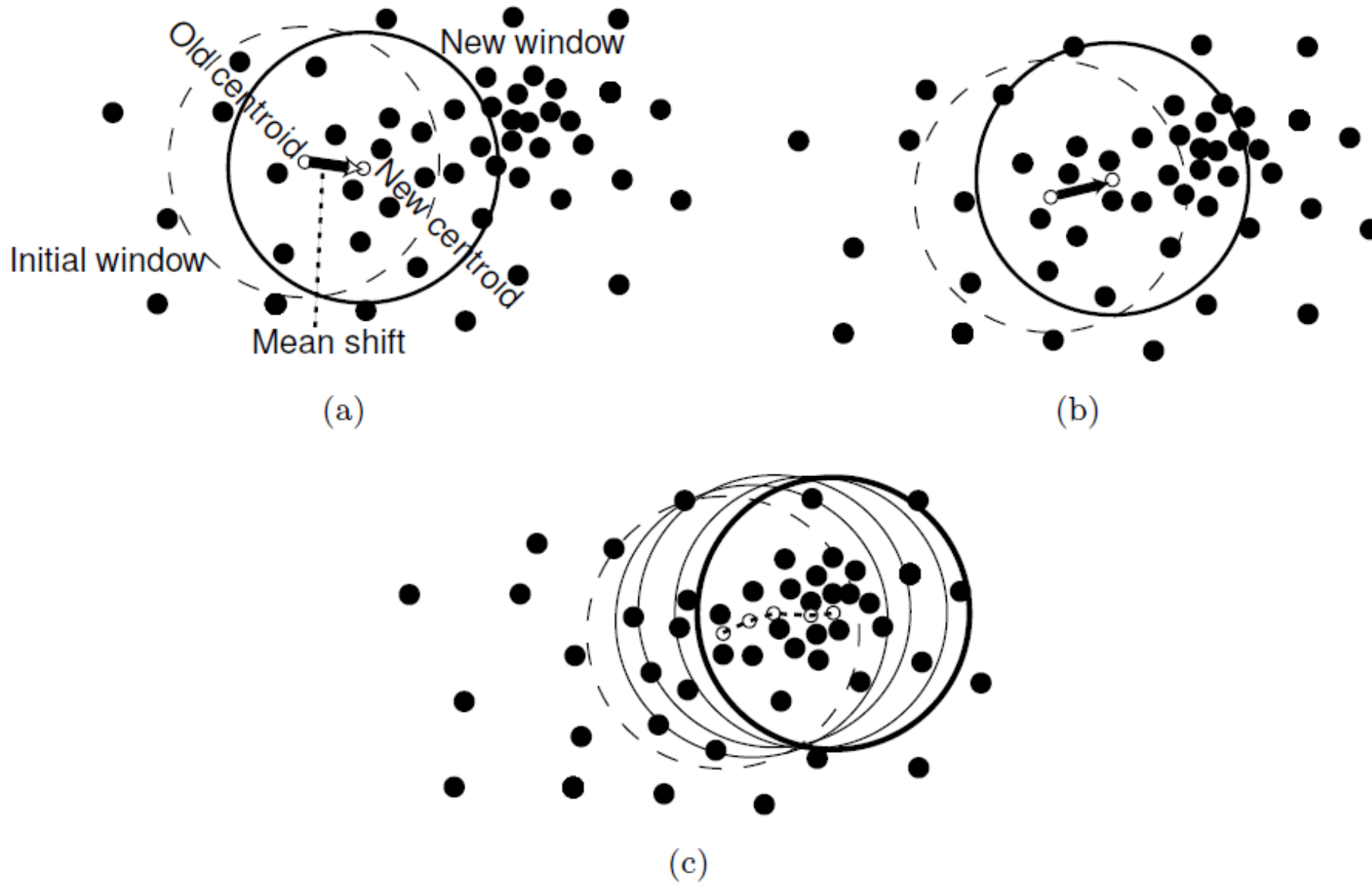
Similarity surface of the frame 206



- ◆ the complete enumeration of similarity surface can be costly,
- ◆ can we do it faster and more elegantly?

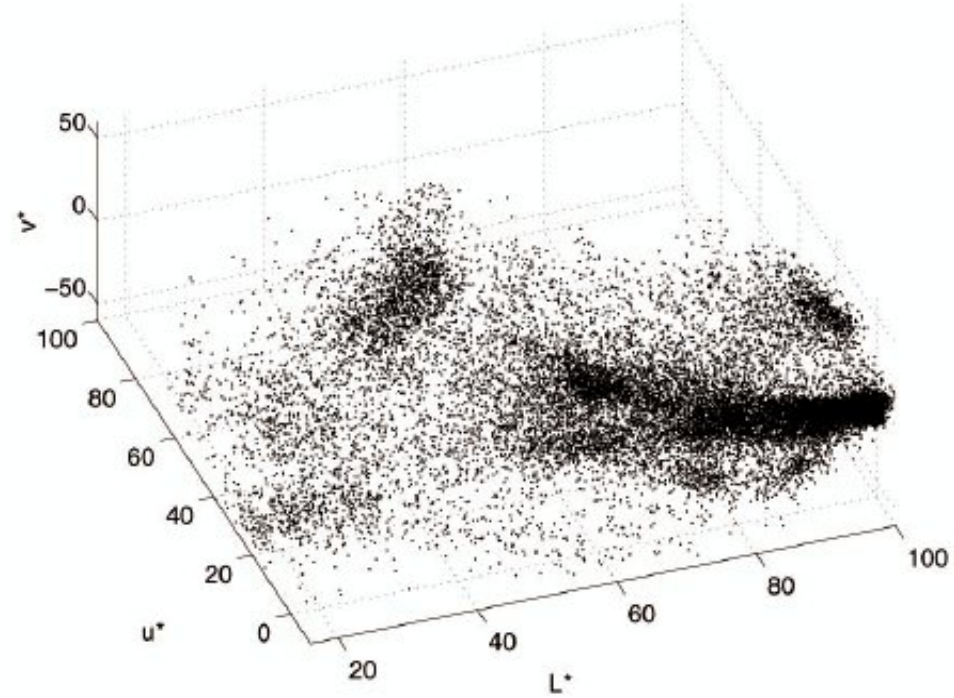
Density Gradient Estimation

Mean shift procedure



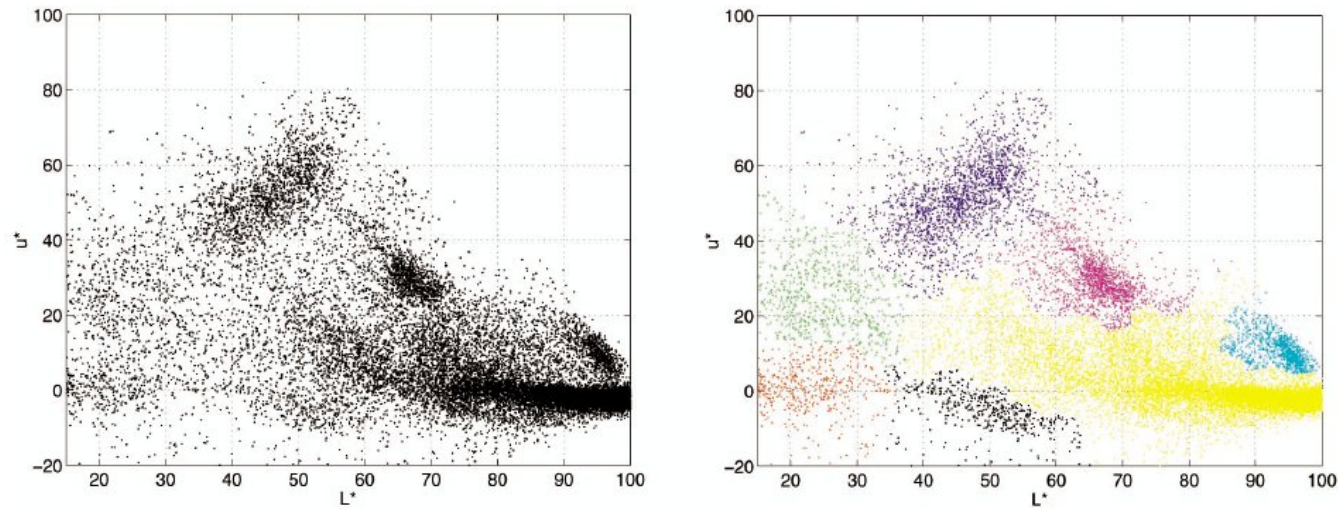
³Figure borrowed from [4]

Meanshift segmentation of colours - color distribution



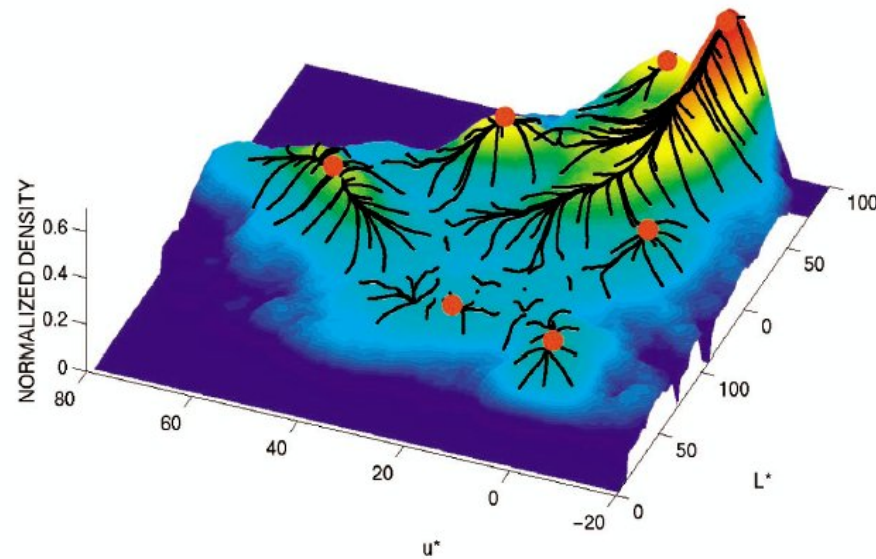
4

Meanshift segmentation of colours - color modes seeking



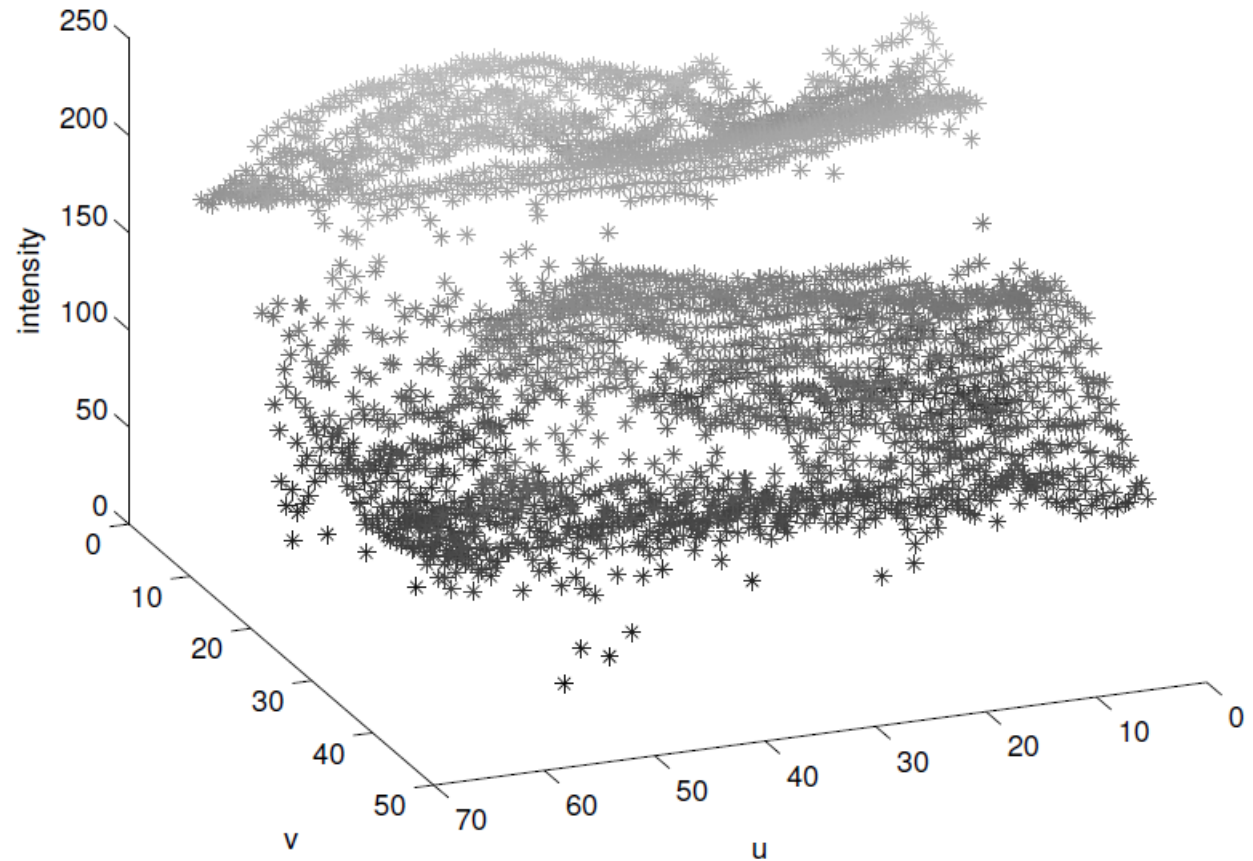
(a)

(b)



(c)

Mean shift segmentation - intensity and space

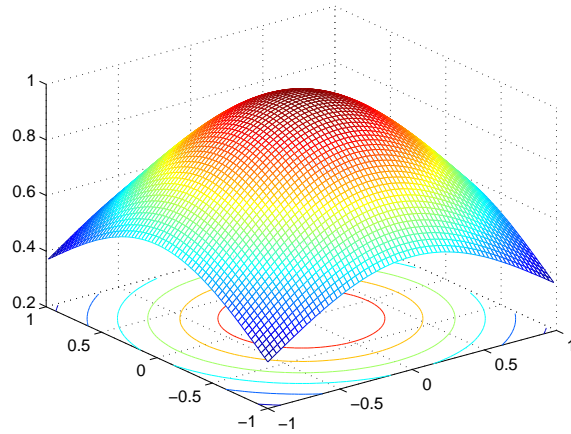


u, v are here spatial pixel coordinates

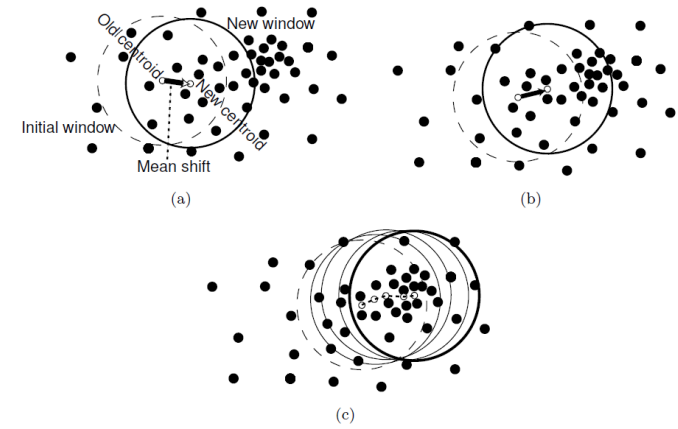
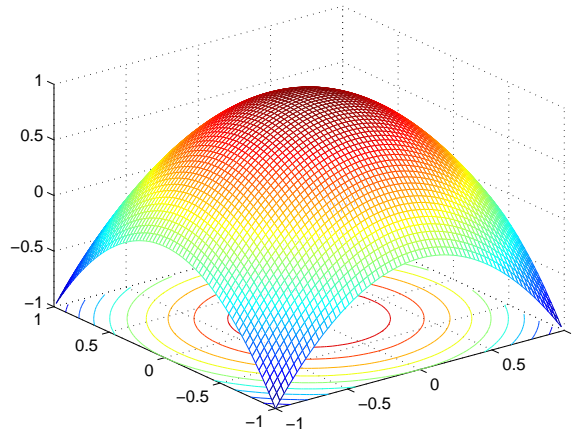
different normalization for intensity and spatial coordinates

Multivariate kernel density estimator

Normal kernel, $K_N(\mathbf{x}) = \exp(-\frac{1}{2}\|\mathbf{x}\|^2)$



Epanechnikov kernel, $K_E(\mathbf{x}) = 1 - \|\mathbf{x}\|^2$

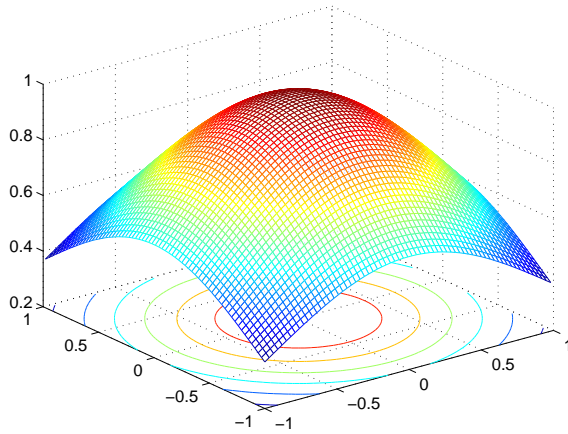


Given n data points \mathbf{x}_i in d -dimensional space R^d .

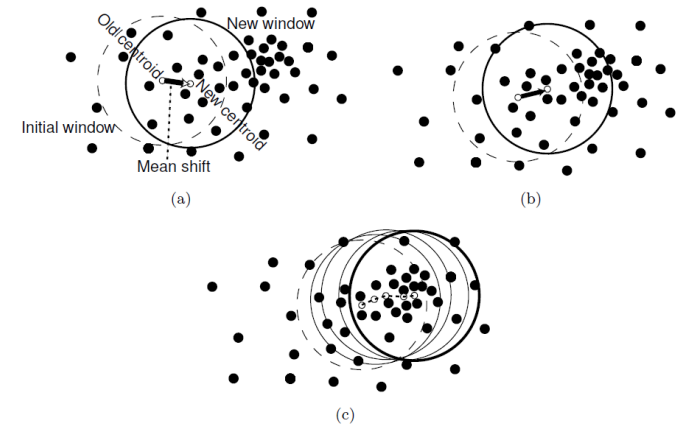
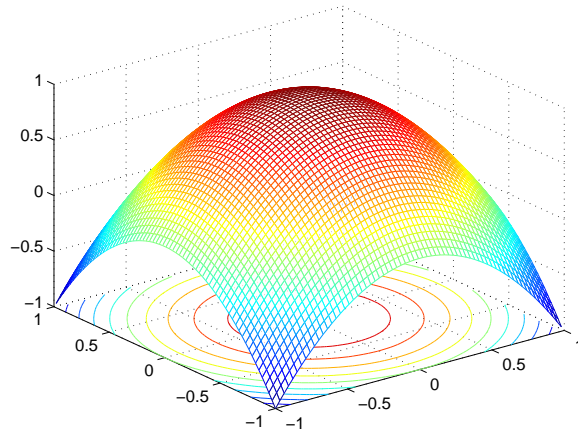
$$\tilde{f}_{h,K}(\mathbf{x}) = \frac{1}{nh^d} \sum_{i=1}^n K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right)$$

Multivariate kernel density estimator

Normal kernel, $K_N(\mathbf{x}) = \exp(-\frac{1}{2}\|\mathbf{x}\|^2)$



Epanechnikov kernel, $K_E(\mathbf{x}) = 1 - \|\mathbf{x}\|^2$



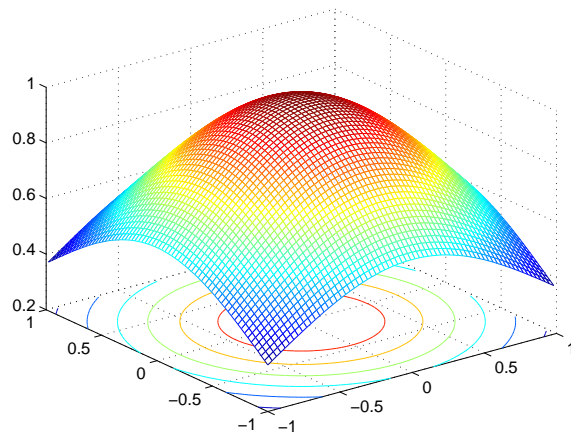
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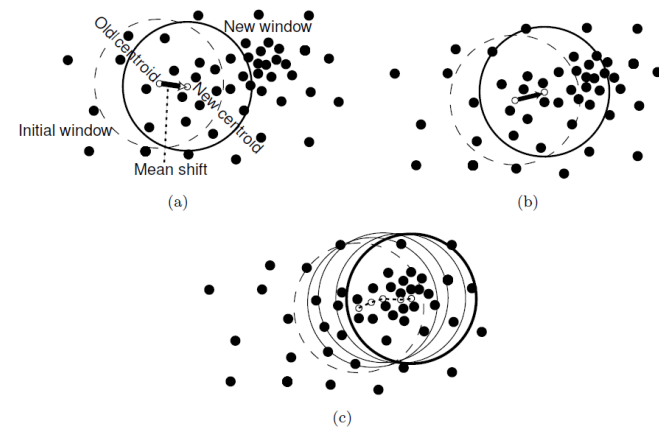
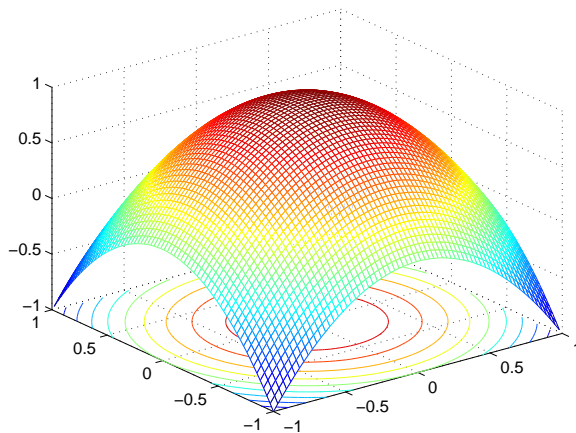
- ◆ looking for extremum of $f_{h,K}(\mathbf{x})$

Multivariate kernel density estimator

Normal kernel, $K_N(\mathbf{x}) = \exp(-\frac{1}{2}\|\mathbf{x}\|^2)$



Epanechnikov kernel, $K_E(\mathbf{x}) = 1 - \|\mathbf{x}\|^2$



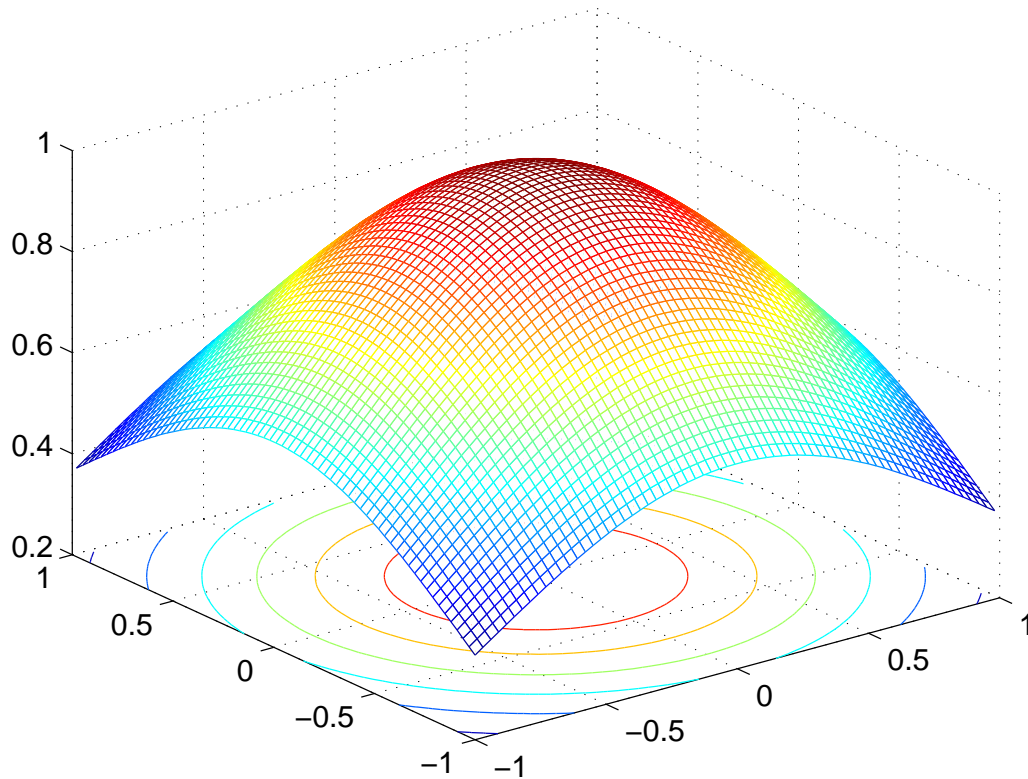
Given n data points \mathbf{x}_i in d -dimensional space R^d .

$$\tilde{f}_{h,K}(\mathbf{x}) = \frac{1}{nh^d} \sum_{i=1}^n K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right)$$

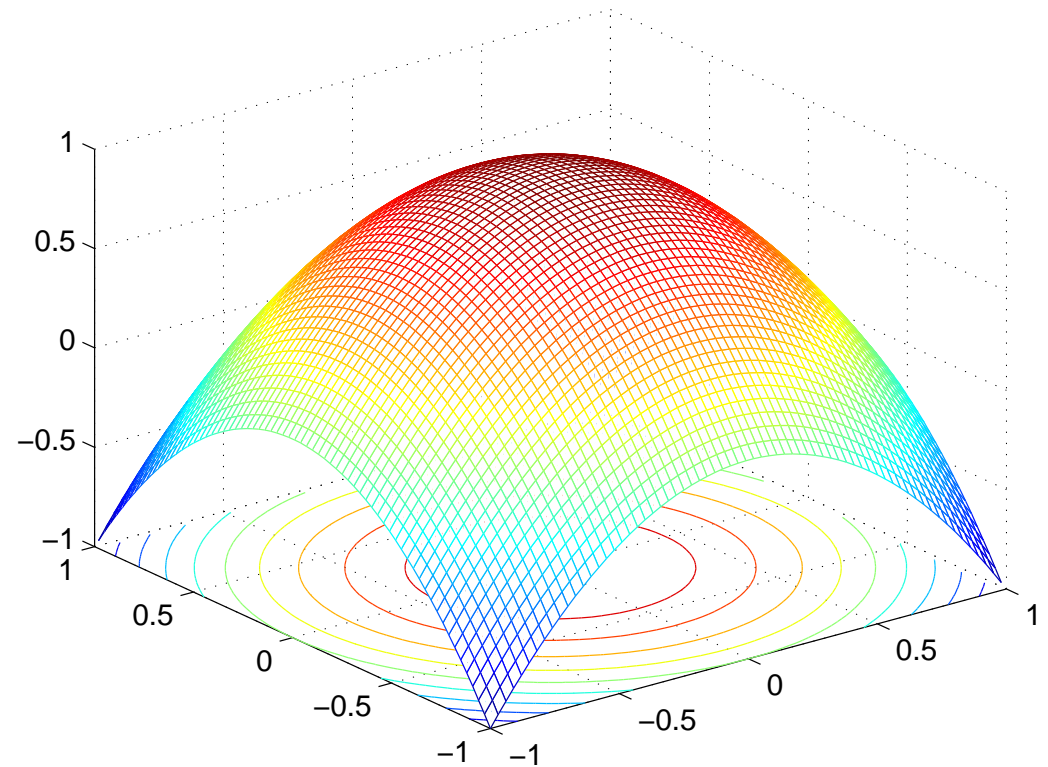
- ◆ looking for extremum of $f_{h,K}(\mathbf{x})$
- ◆ gradient $\nabla f_{h,K}(\mathbf{x}) = \mathbf{0}$

Kernels

Normal kernel, $K_N(\mathbf{x}) = \exp(-\frac{1}{2}\|\mathbf{x}\|^2)$



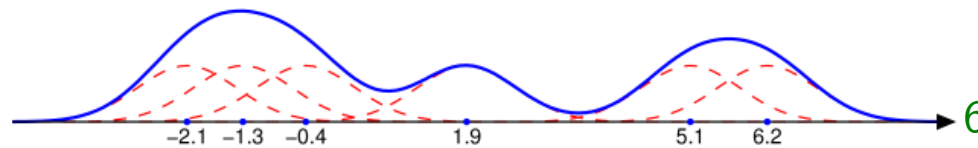
Epanechnikov kernel, $K_E(\mathbf{x}) = 1 - \|\mathbf{x}\|^2$



Can be seen as membership function.

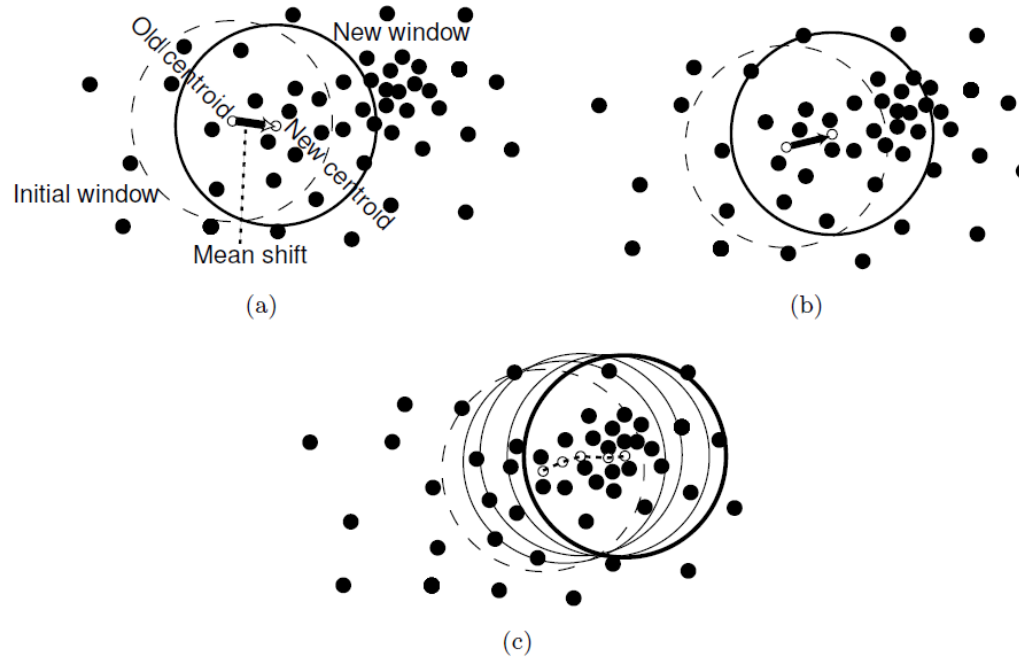
Remind **Kernel density estimation** (Parzen method).

Remember convolution?



⁶Taken from http://en.wikipedia.org/wiki/Kernel_density_estimation

Mean-shift iterations

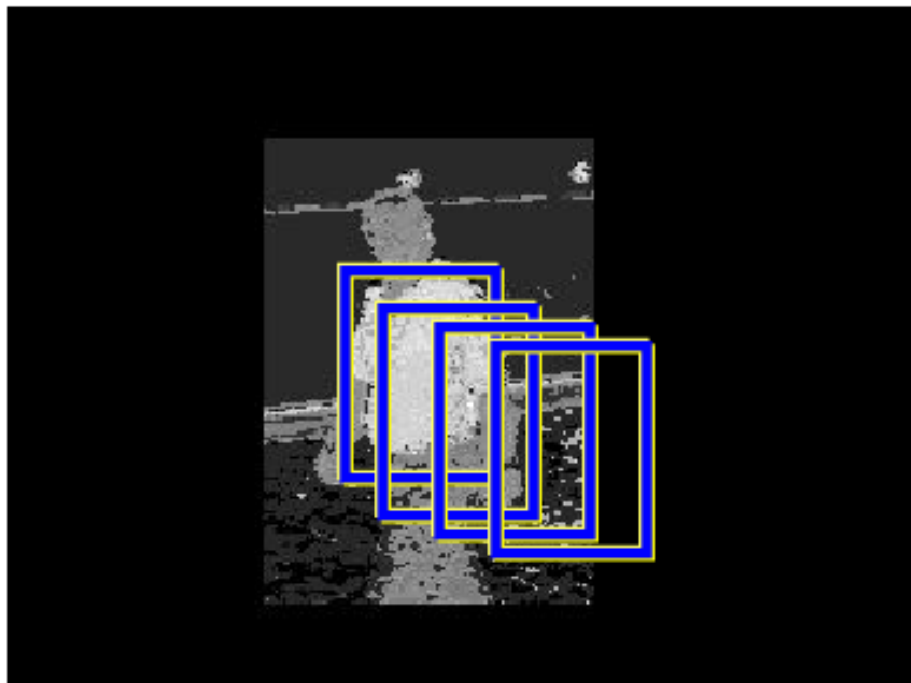


Assuming a reasonable differentiable kernel K , iterate till convergence:

$$\mathbf{y}_{k+1} = \frac{\sum_{i=1}^n \mathbf{x}_i g(\|\mathbf{y}_k - \mathbf{x}_i\|^2)}{\sum_{i=1}^n g(\|\mathbf{y}_k - \mathbf{x}_i\|^2)}$$

g is the derivative of kernel profile.

Mean-shift tracking - ratio histogram



Ratio histogram:

$$r_u = \min \left(\frac{q_u}{p_u}, 1 \right)$$

where q is the histogram of the target and p is the histogram of the **current** frame. $w_i = r_{b(\mathbf{x}_i)}$ (just binning)

Image intensities (or colors) are transformed into **weights**, w_i , by back projection of the ratio histogram. Mean-shift iterations:

$$\mathbf{y}_{k+1} = \frac{\sum_{i=1}^n w_i \mathbf{x}_i g(\|\mathbf{y}_k - \mathbf{x}_i\|^2)}{\sum_{i=1}^n w_i g(\|\mathbf{y}_k - \mathbf{x}_i\|^2)}$$

Mean-shift tracking - Bhattacharya coefficient

model, coordinates \mathbf{x}_i^* centered at $\mathbf{0}$:

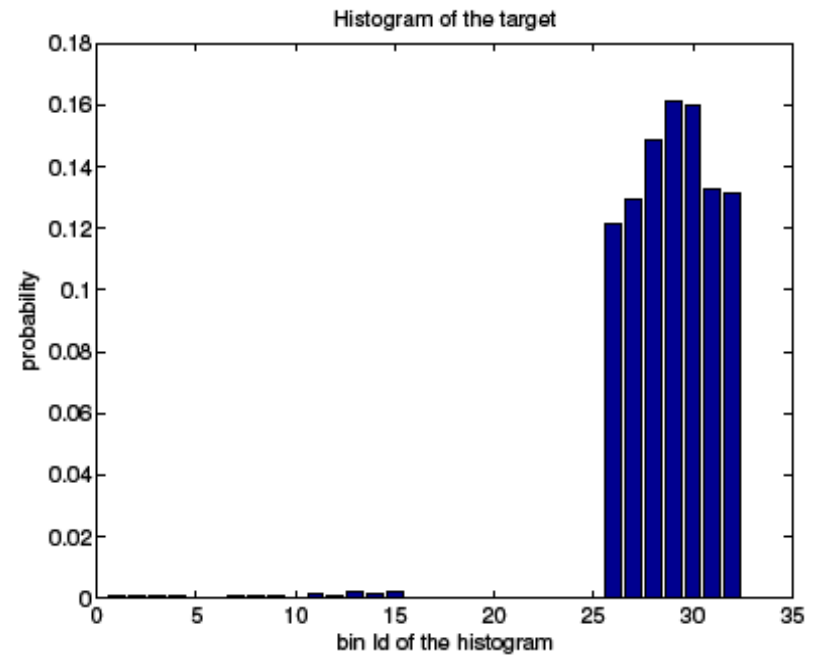
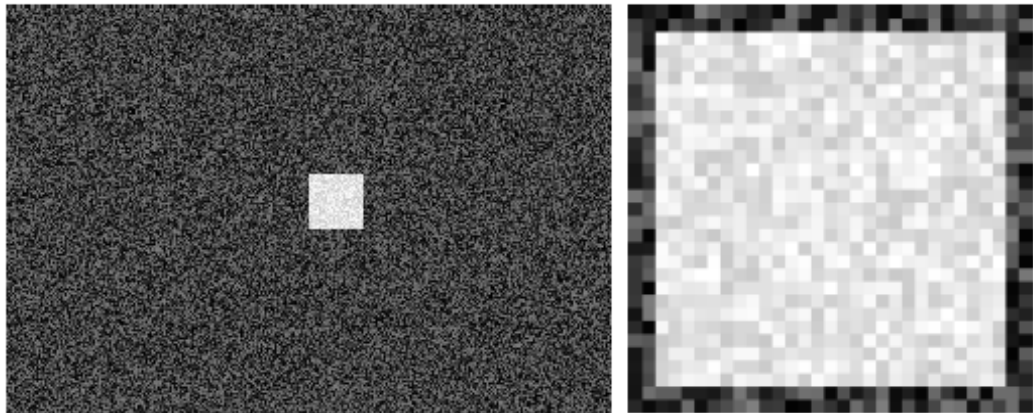
$$q_u = C \sum_{i=1}^n k(\|\mathbf{x}_i^*\|^2) \delta(b(\mathbf{x}_i^*) - u)$$

target candidate centered at \mathbf{y} :

$$p_u(\mathbf{y}) = C_h \sum_{i=1}^{n_h} k\left(\left\|\frac{\mathbf{y} - \mathbf{x}_i}{h}\right\|^2\right) \delta(b(\mathbf{x}_i) - u)$$

ms tracking - object and its model

Example of an input image

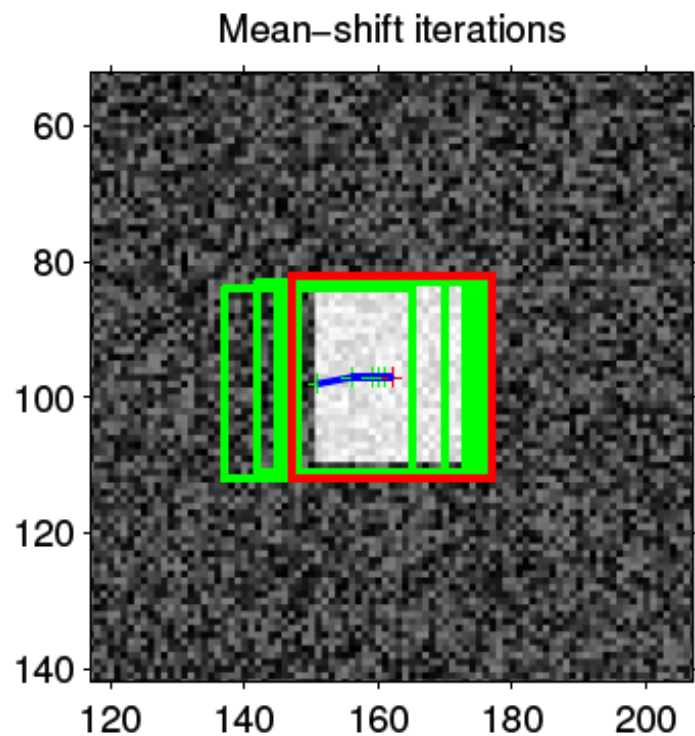


(c)

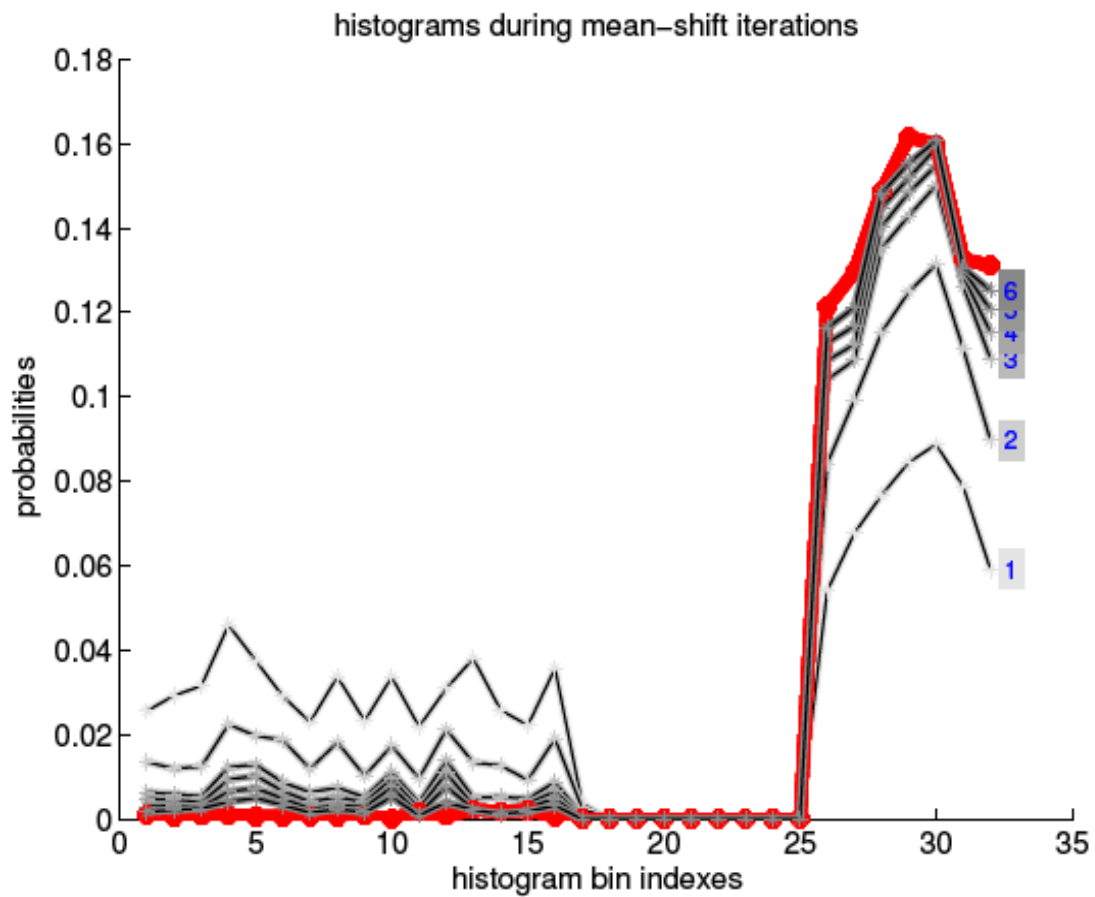
7

⁷Figure from [5]

ms tracking - iterations



(a)



(b)

References

Mean-shift originally from [3].

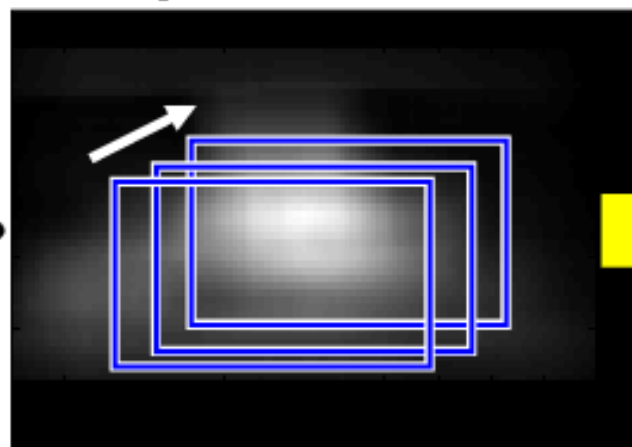
- [1] Robert Collins. CSE/EE486 Computer Vision I. slides, web page.
<http://www.cse.psu.edu/~rcollins/CSE486/>. Robert kindly gave general permission to reuse the material.
- [2] Dorin Comaniciu and Peter Meer. Mean shift: A robust approach toward feature space analysis. *IEEE Transactions on Pattern Analysis and Machine Analysis*, 24(5):603–619, May 2002.
- [3] Keinosuke Fukunaga and Larry D. Hostetler. The estimation of the gradient of a density function, with applications in pattern recognition. *IEEE Transactions on Information Theory*, 21(1):32–40, January 1975.
- [4] Milan Šonka, Václav Hlaváč, and Roger Boyle. *Image Processing, Analysis and Machine Vision*. Thomson, 3rd edition, 2007.
- [5] Tomáš Svoboda, Jan Kybic, and Václav Hlaváč. *Image Processing, Analysis and Machine Vision. A MATLAB Companion*. Thomson, 2007. Accompanying www site <http://visionbook.felk.cvut.cz>.

End

current frame +
previous location



likelihood over
object location



current location



appearance model

(e.g. image template, or



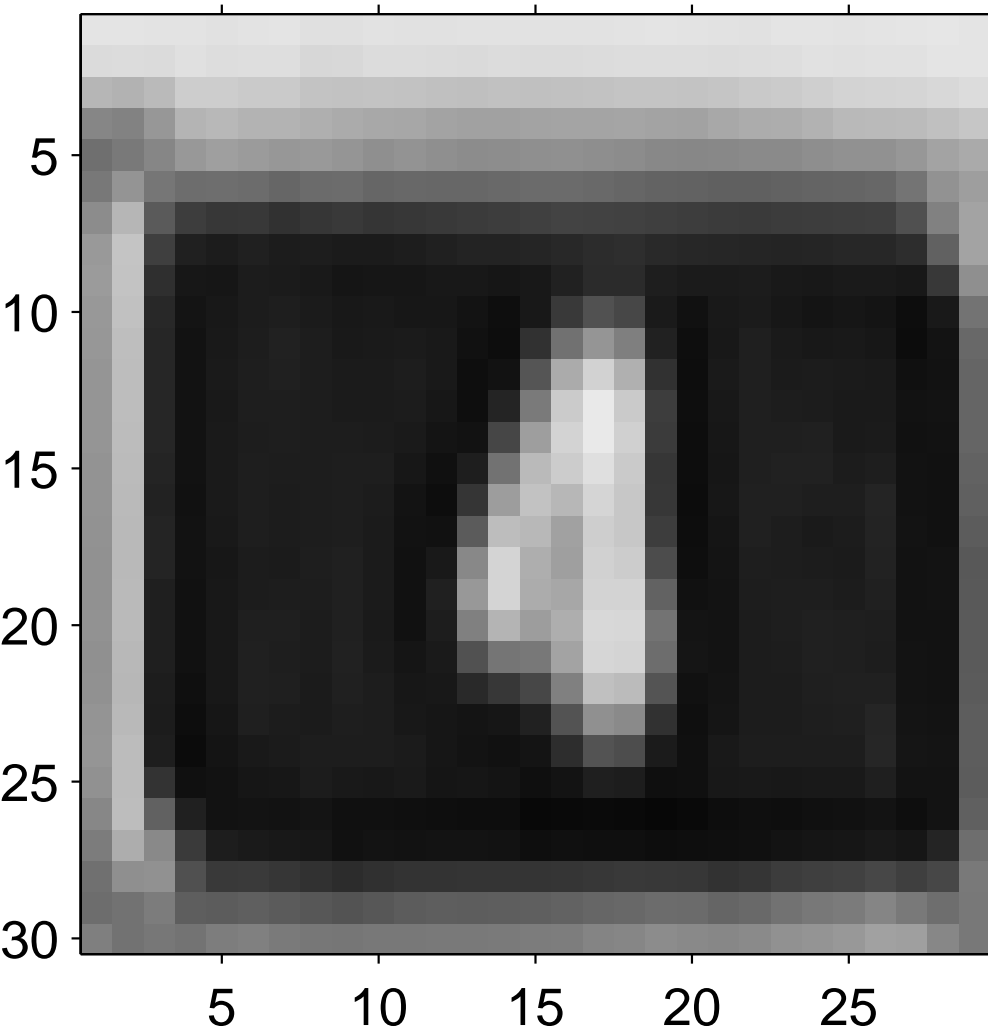
color; intensity; edge histograms)



Mode-Seeking

(e.g. mean-shift; Lucas-Kanade;
particle filtering)

image patch



histogram for 16 bins

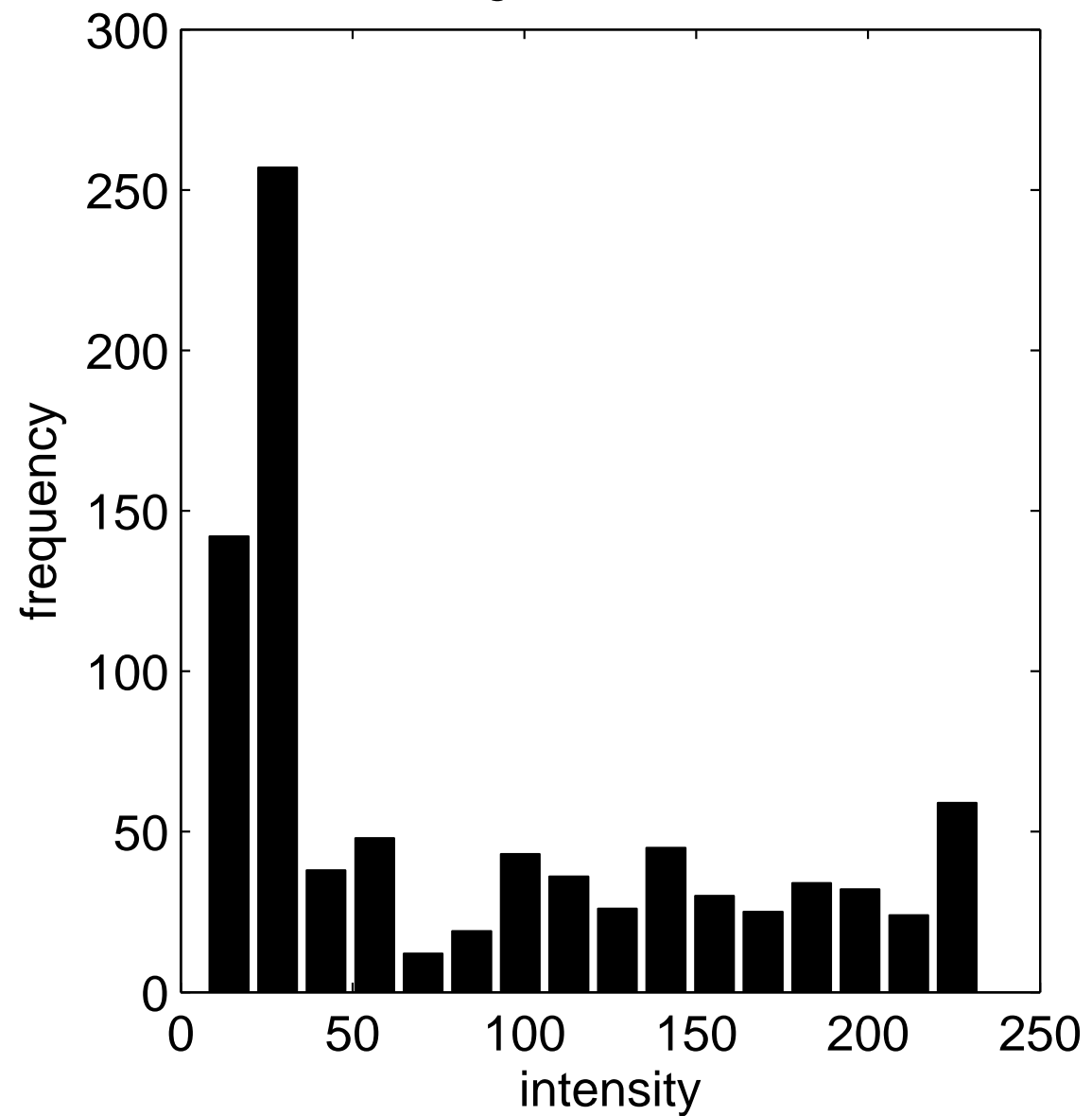
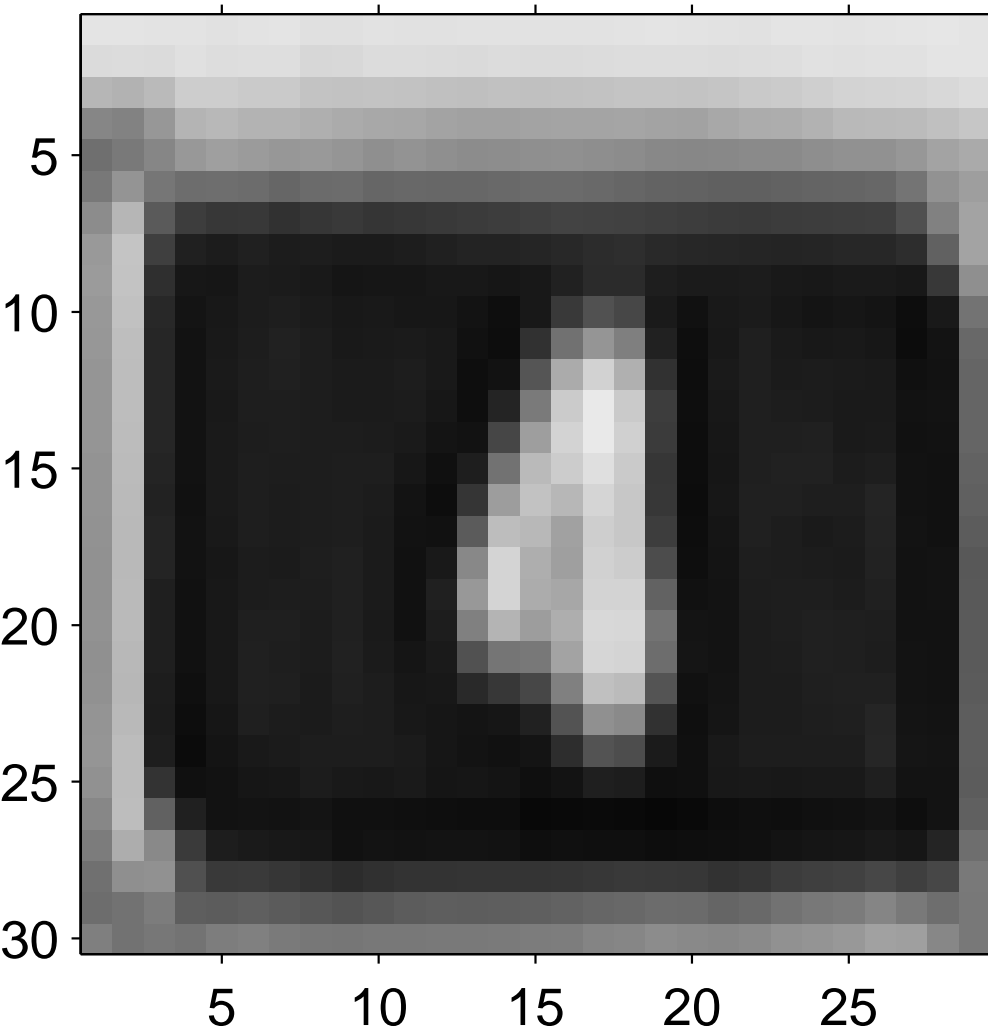


image patch



histogram for 16 bins

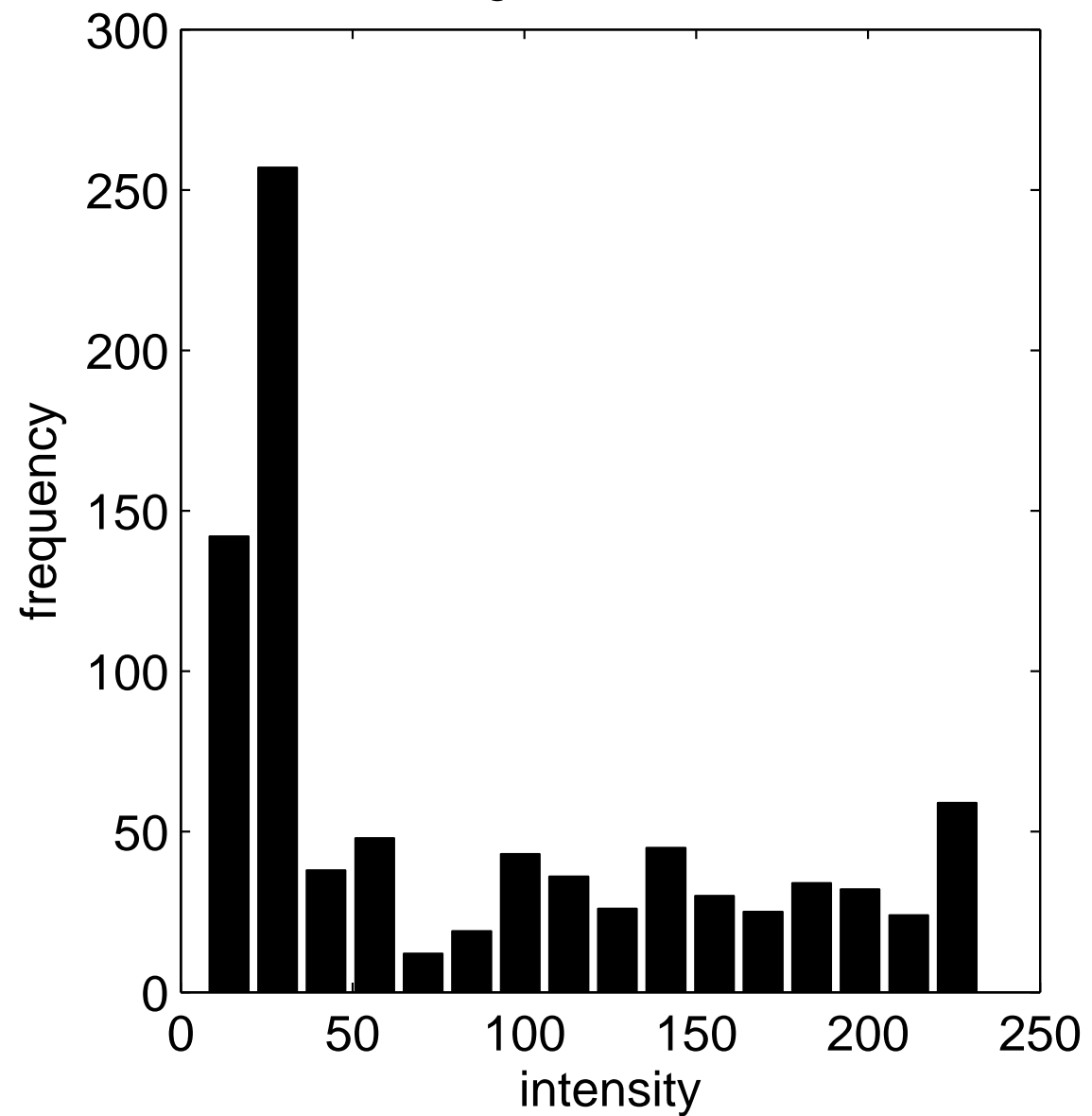
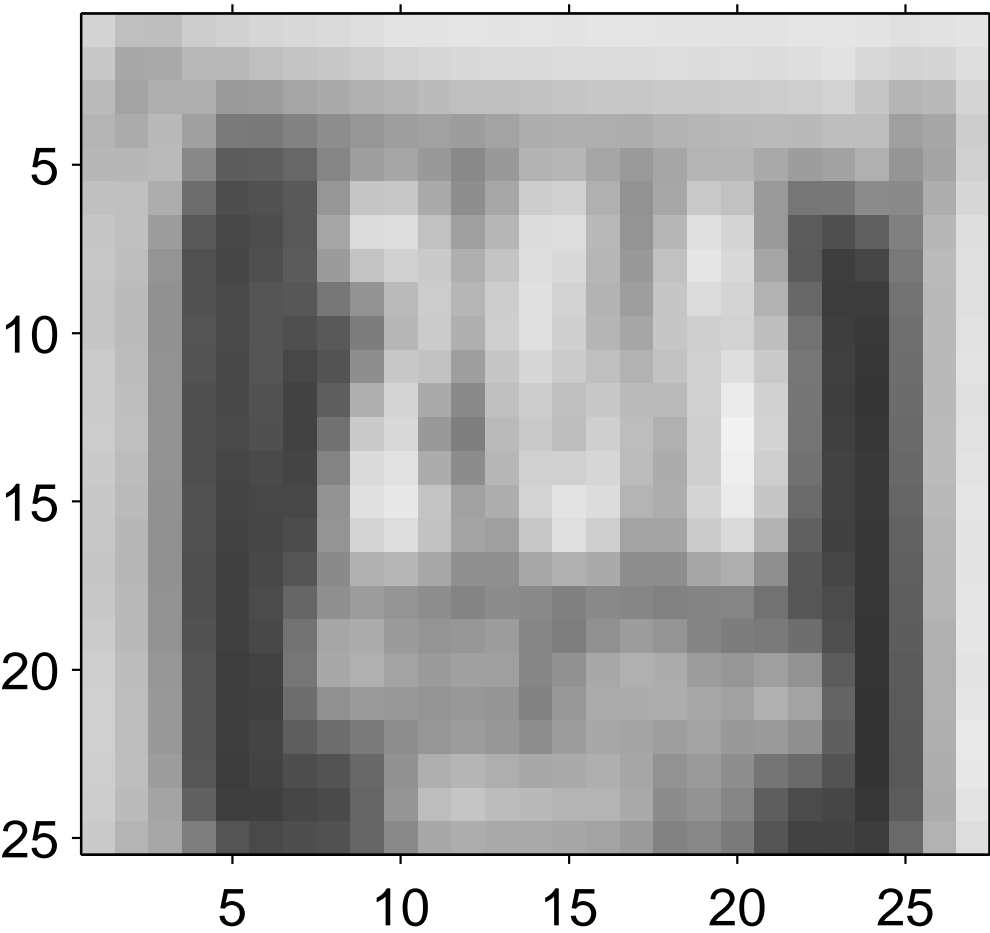


image patch



histogram for 16 bins

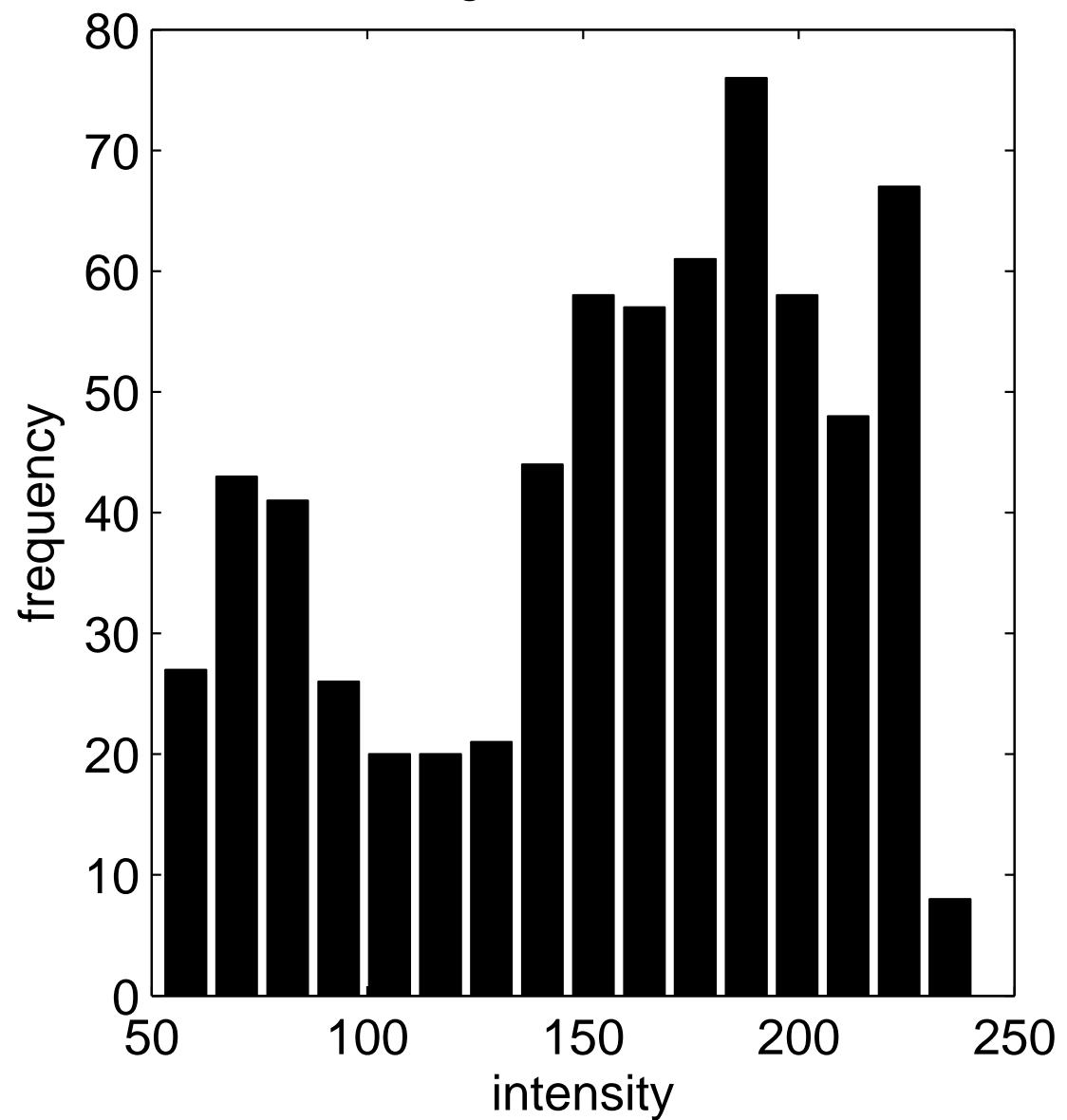
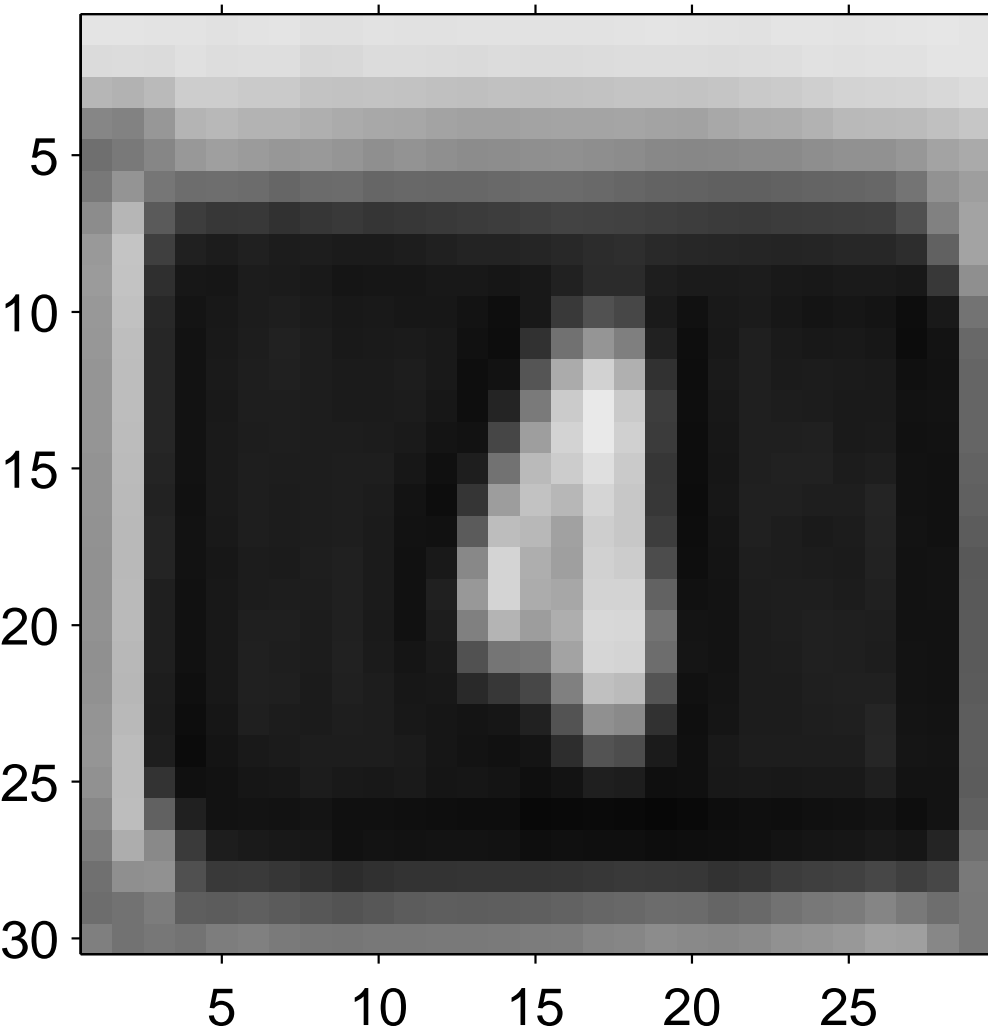


image patch



histogram for 16 bins

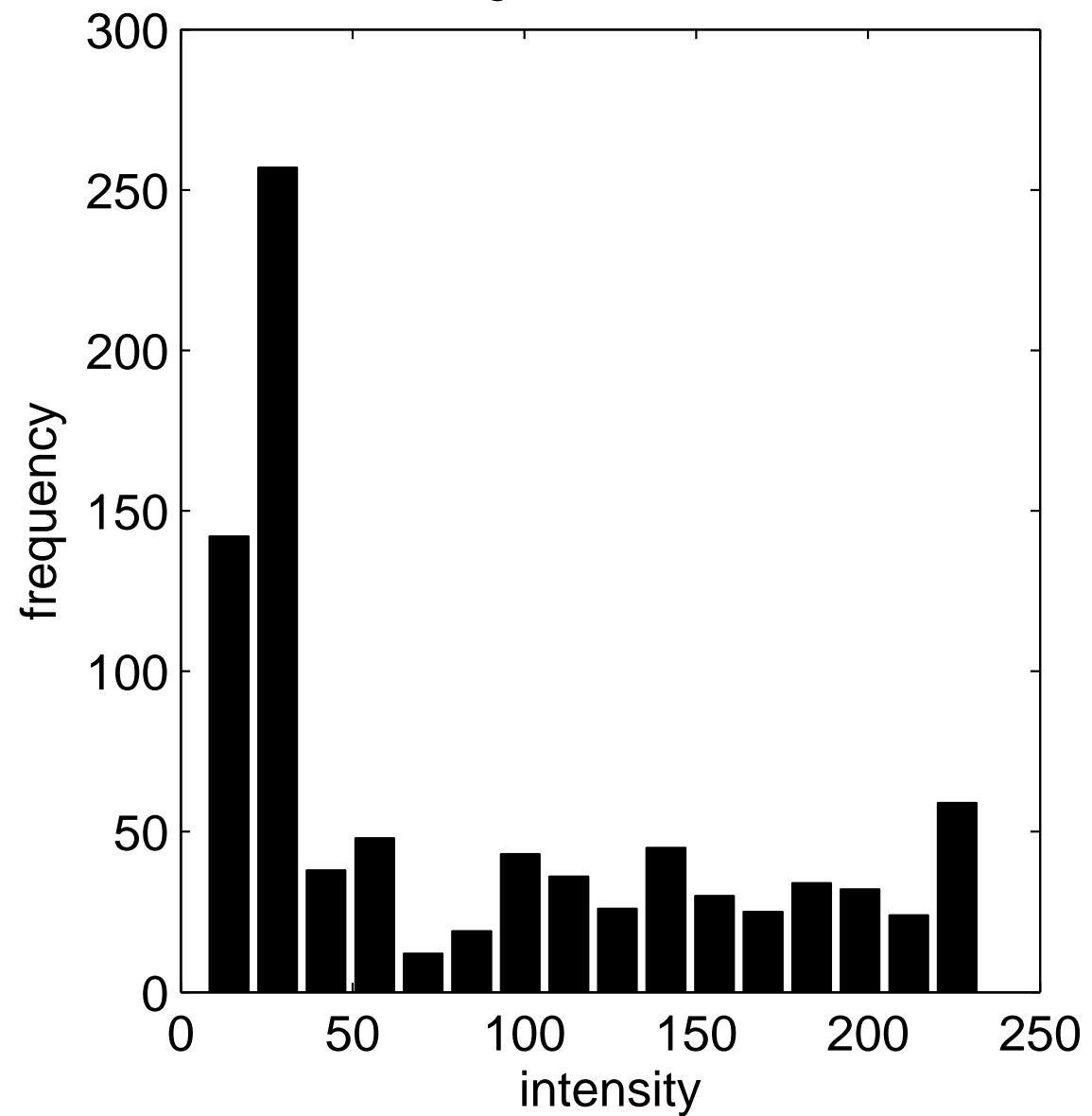
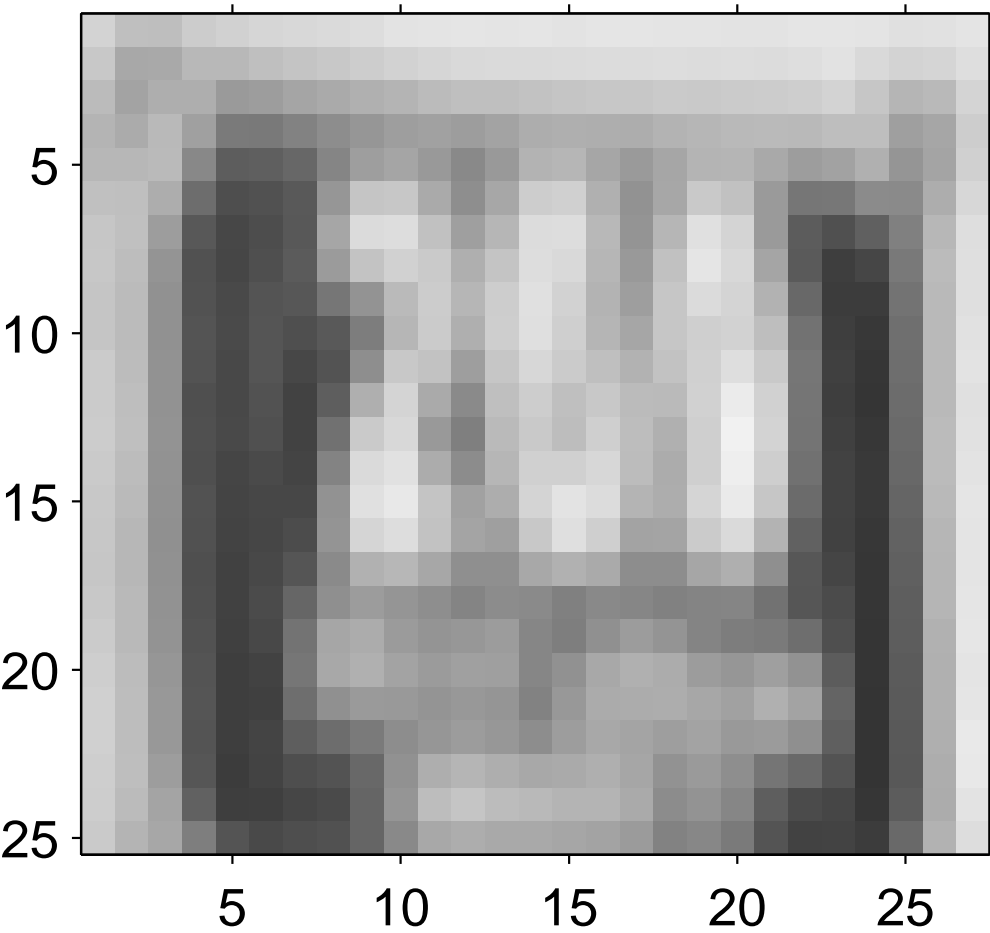
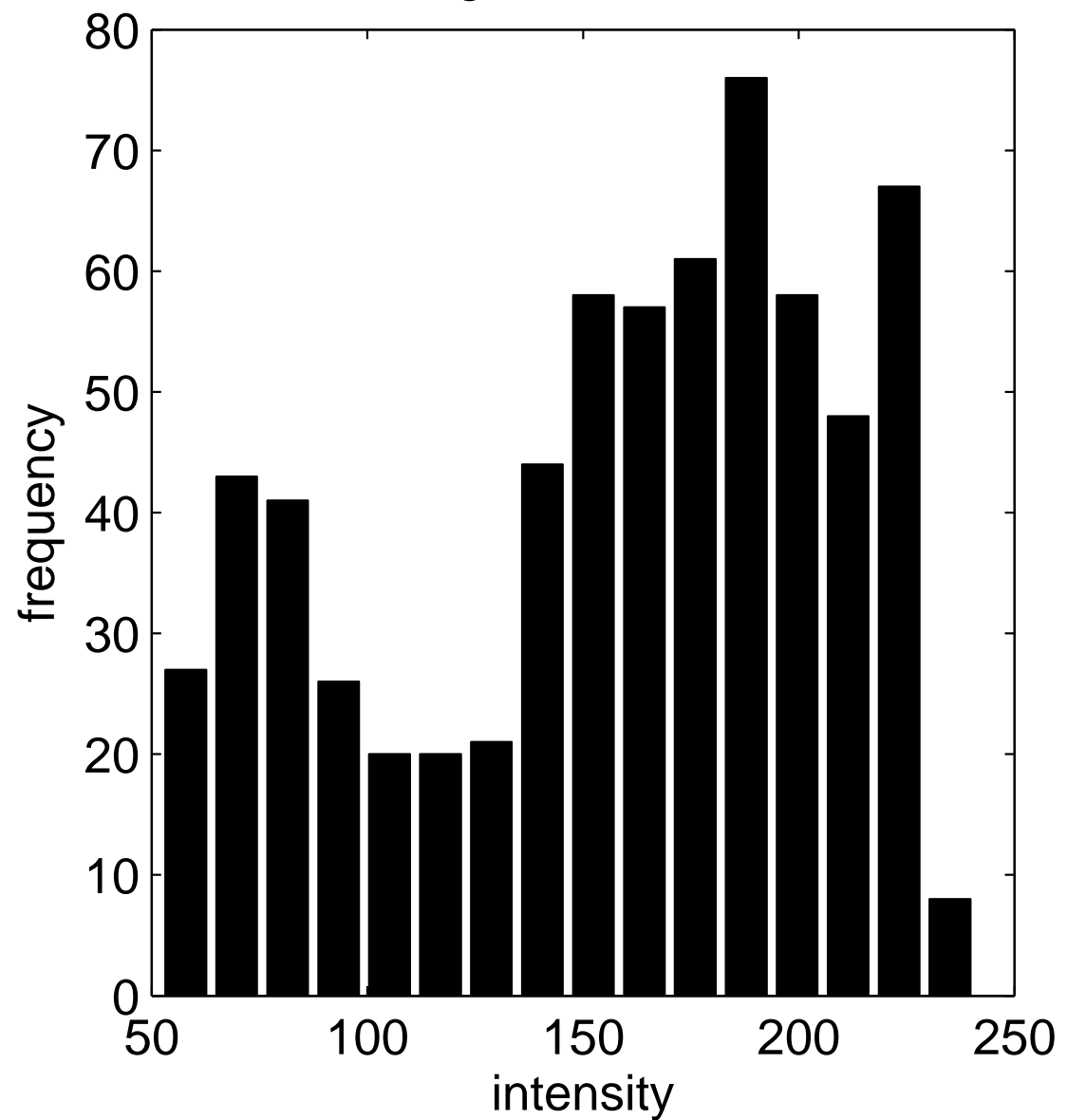


image patch



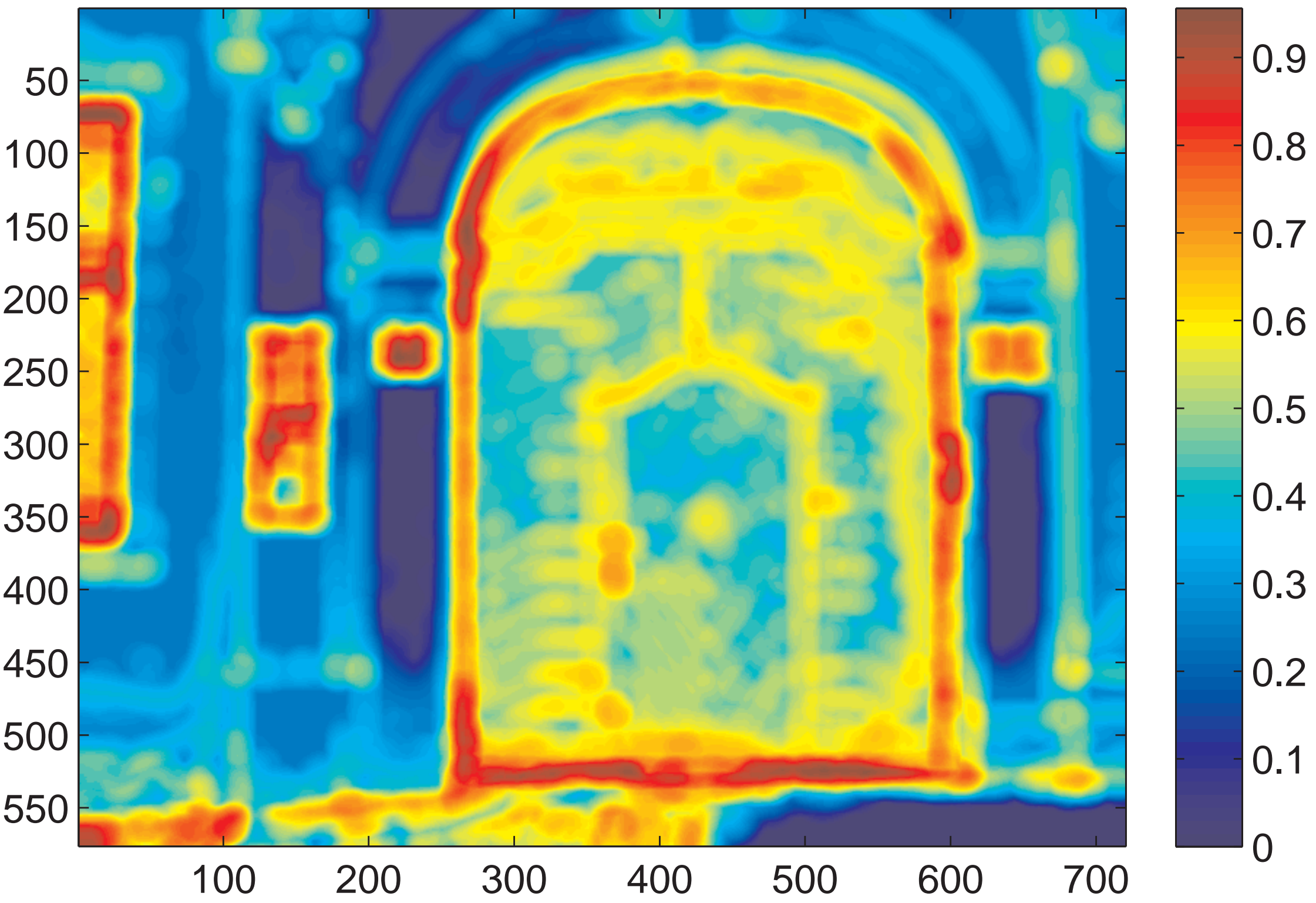
histogram for 16 bins



Example of an input image



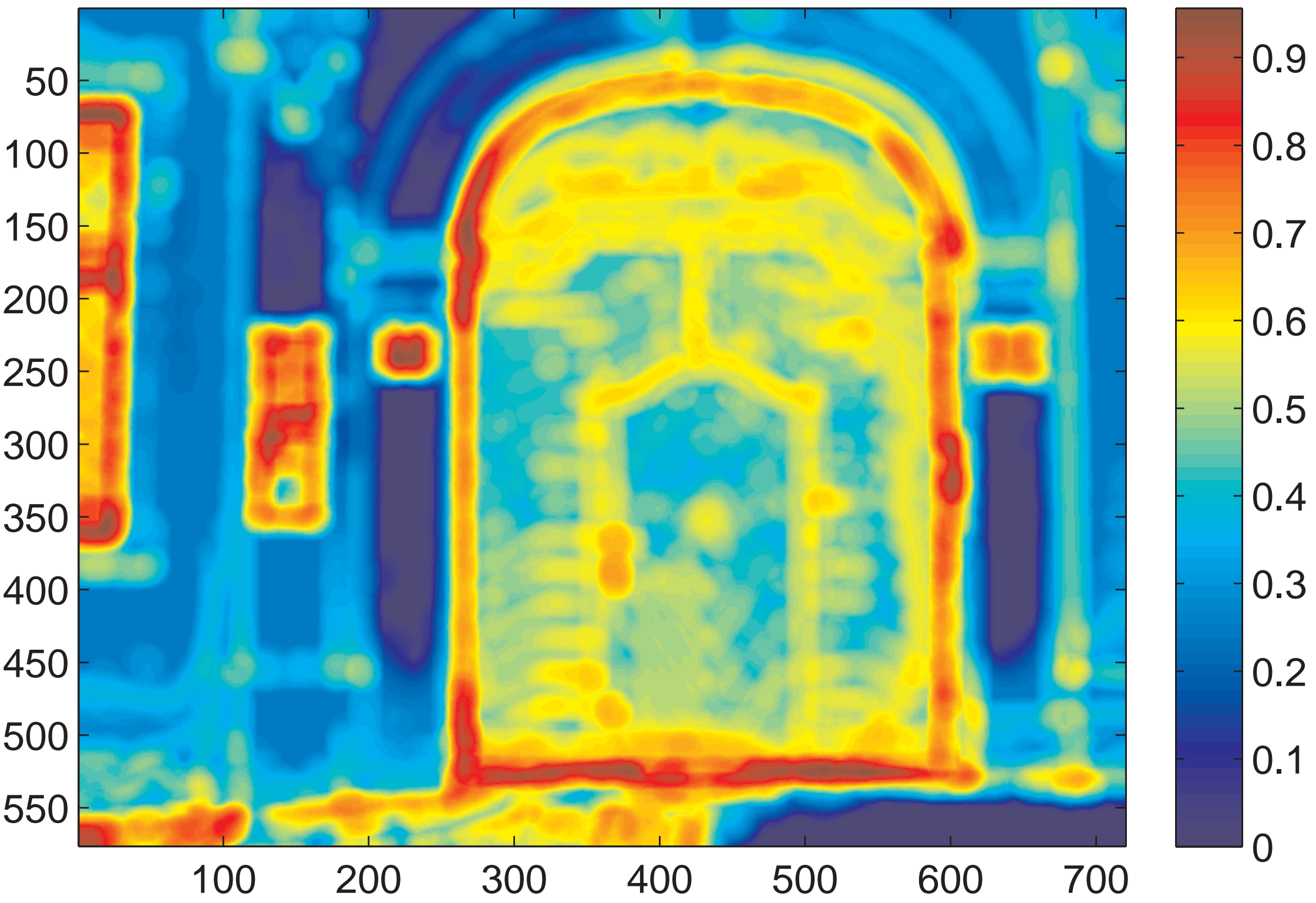
Similarity surface of the frame 206

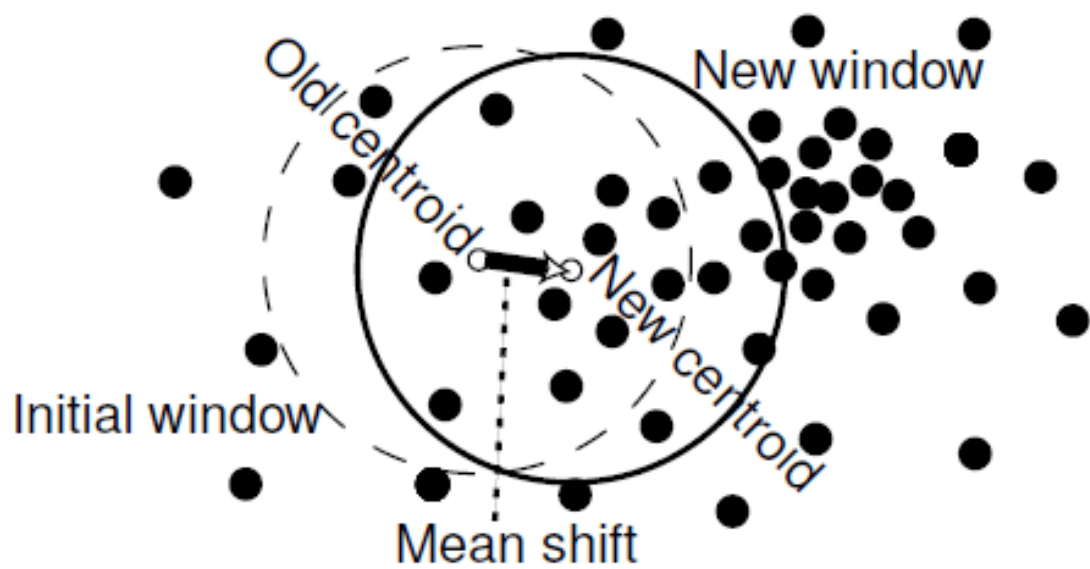


Example of an input image

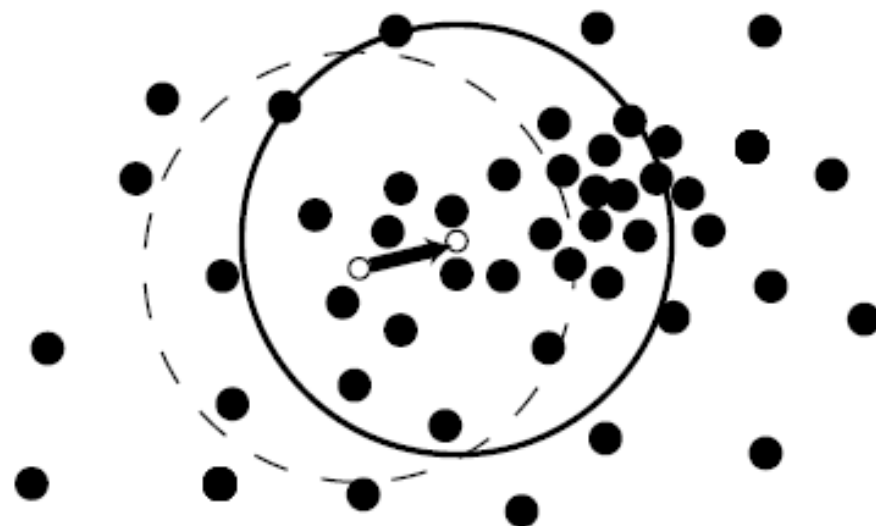


Similarity surface of the frame 206

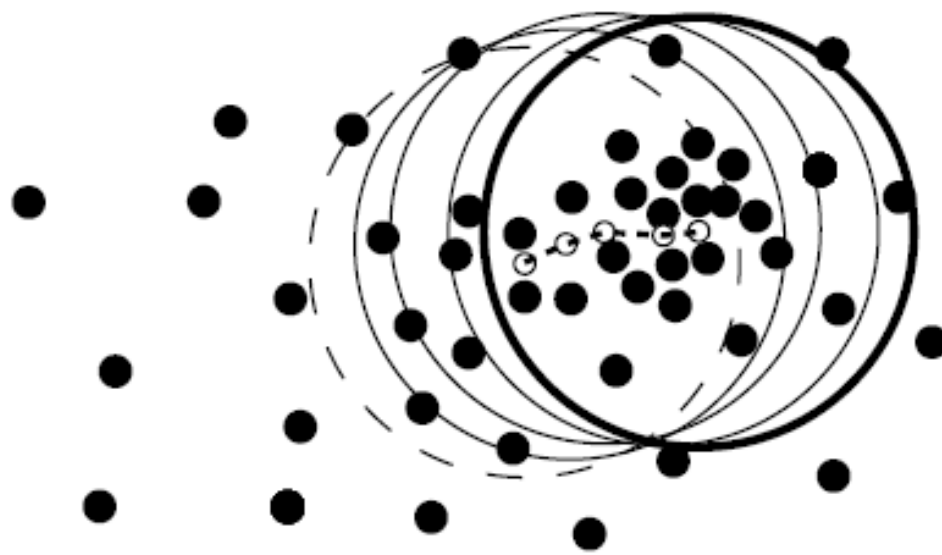




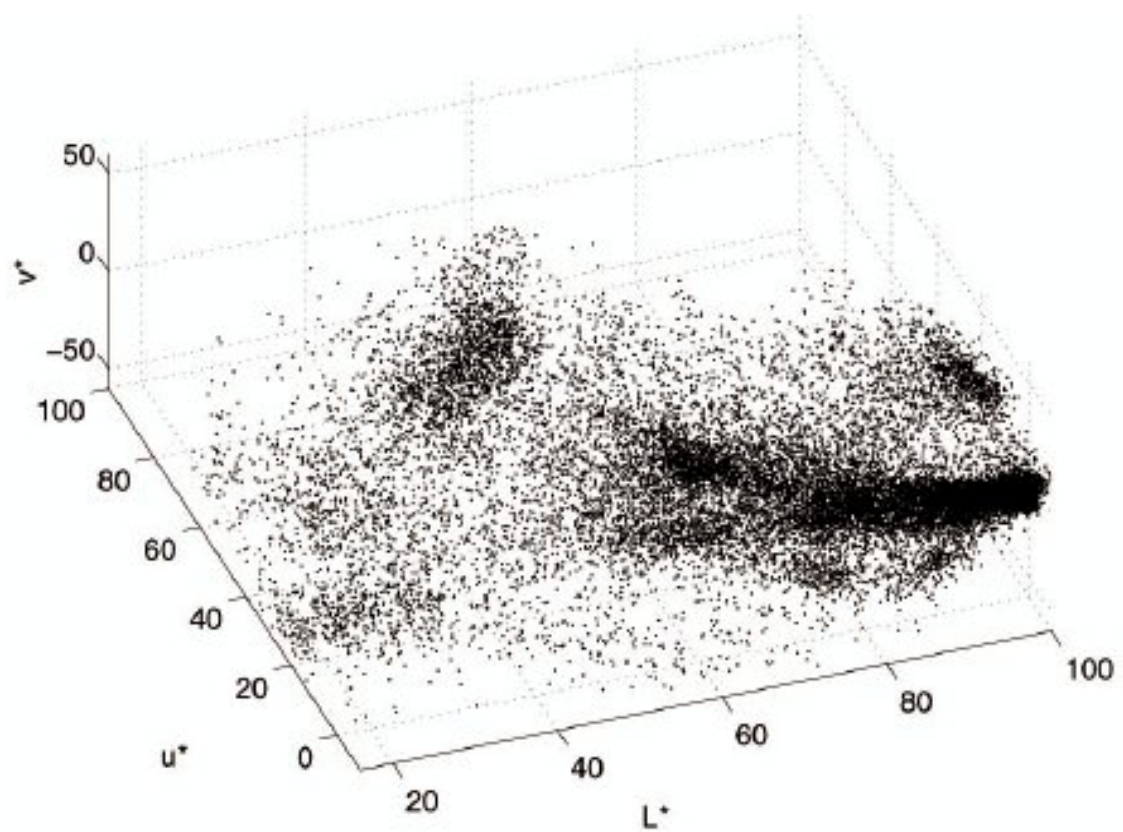
(a)

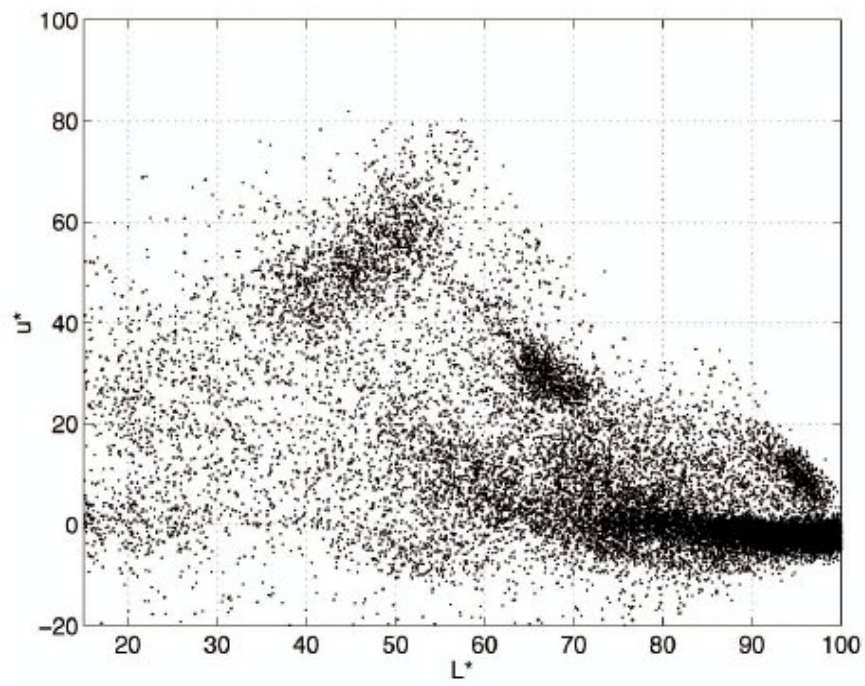


(b)

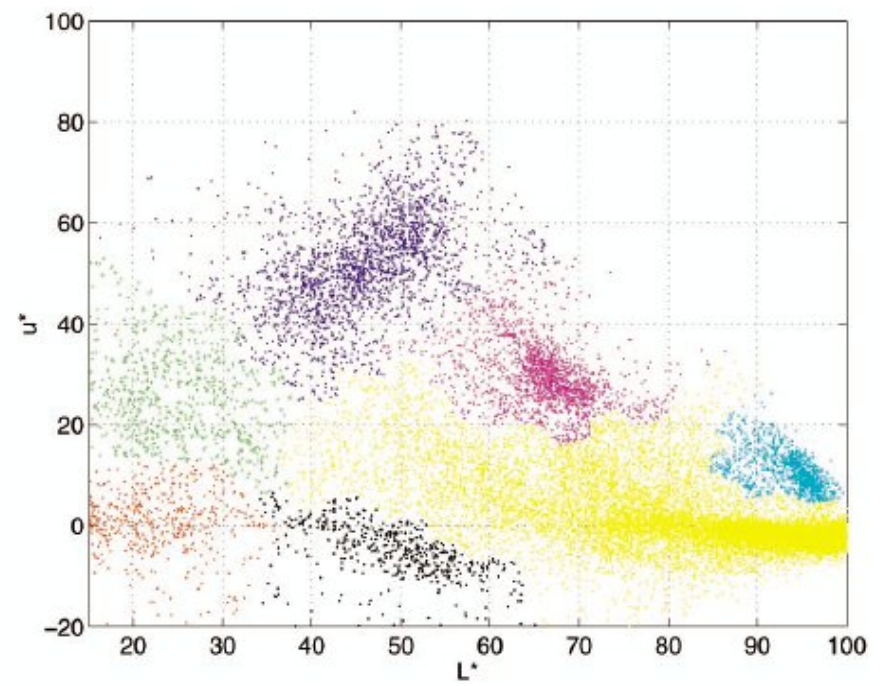


(c)

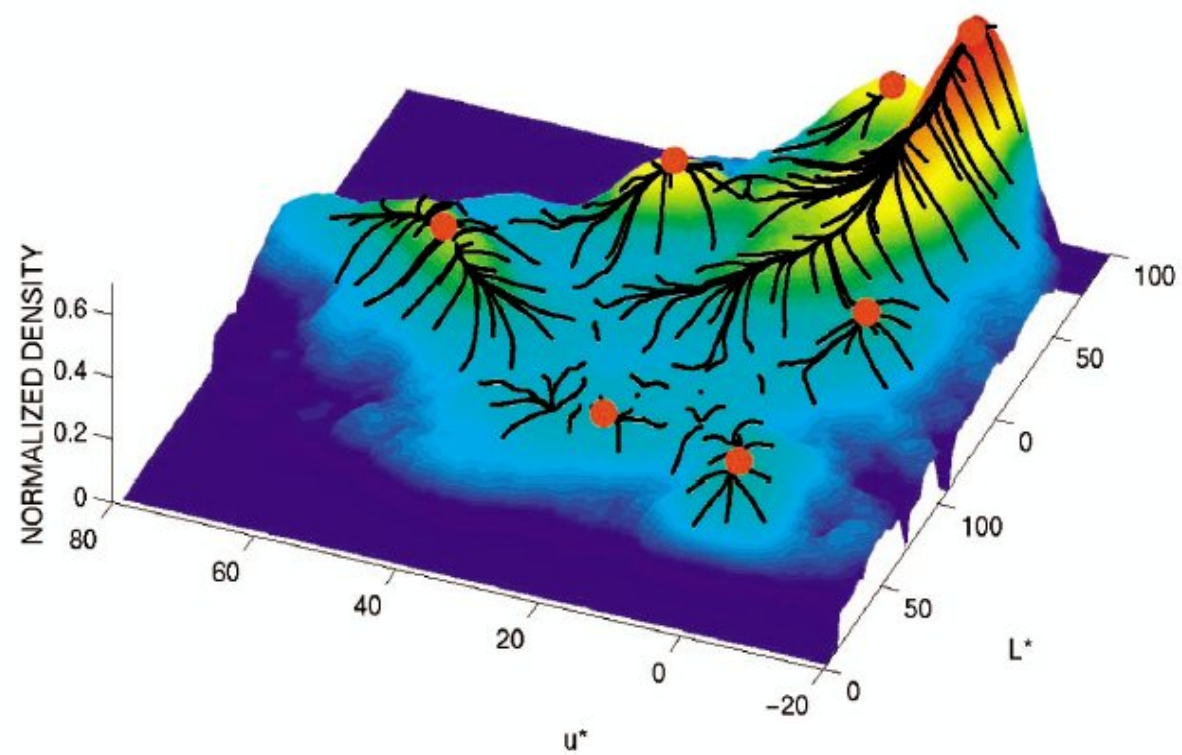




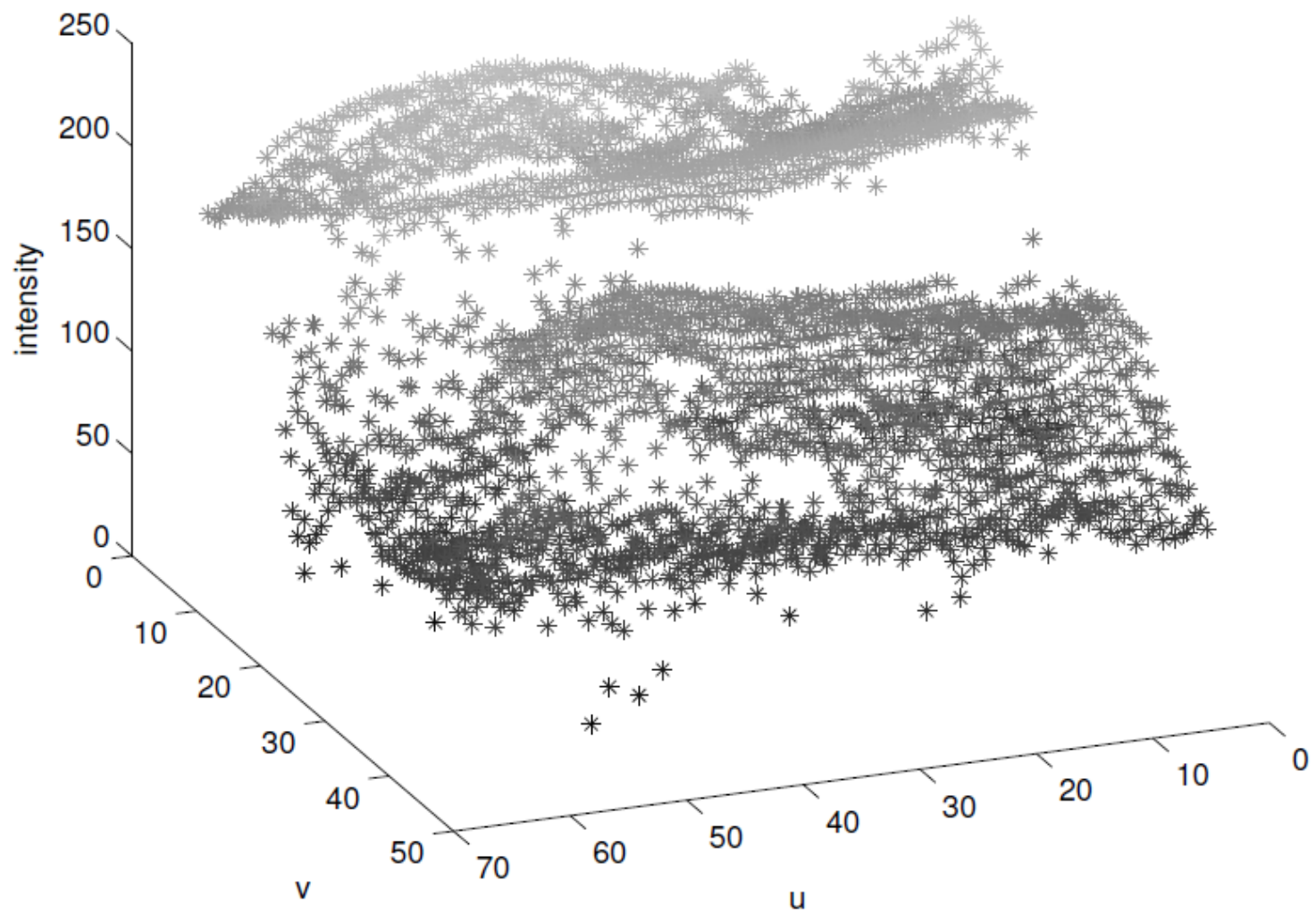
(a)



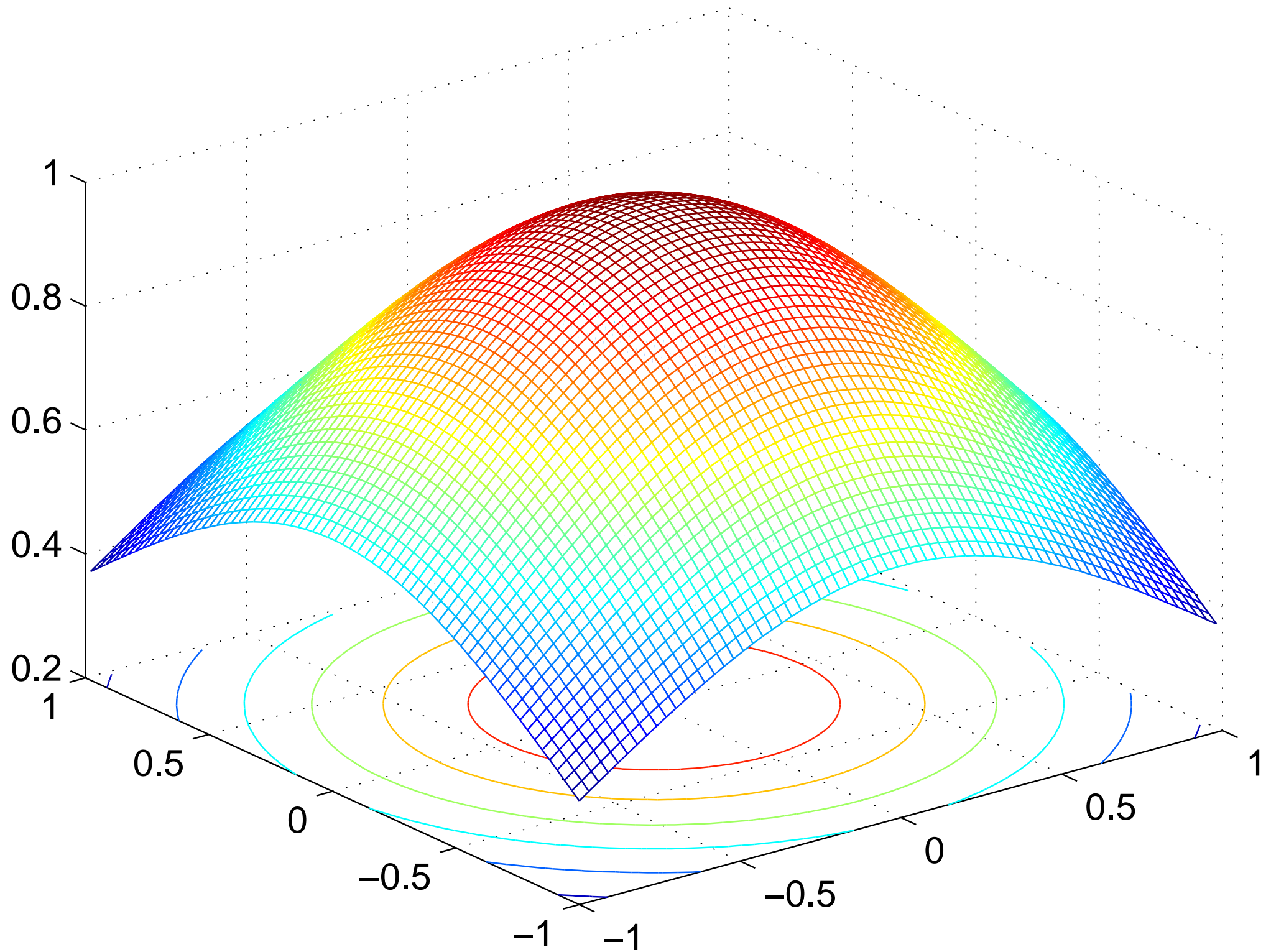
(b)



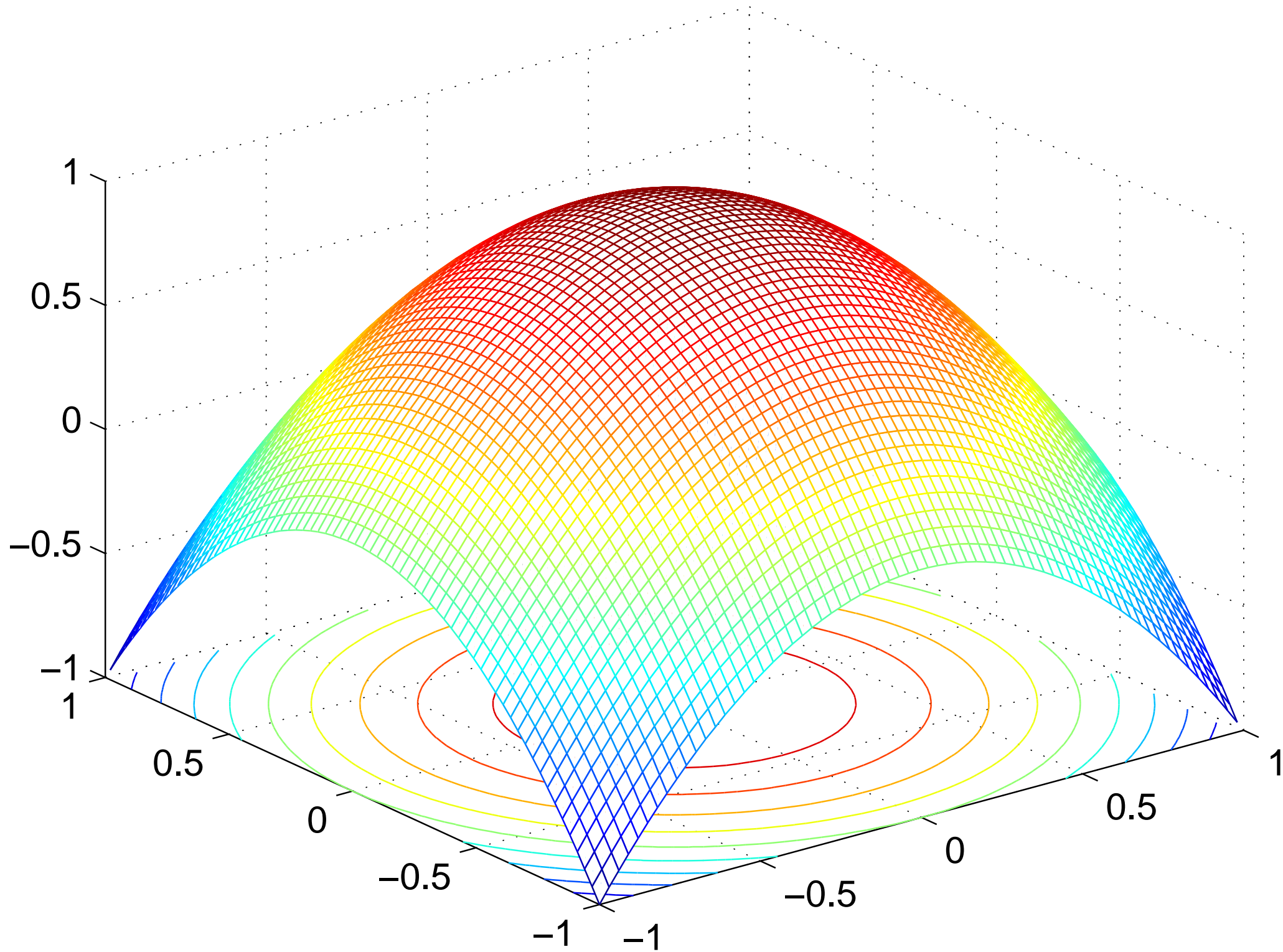
(c)

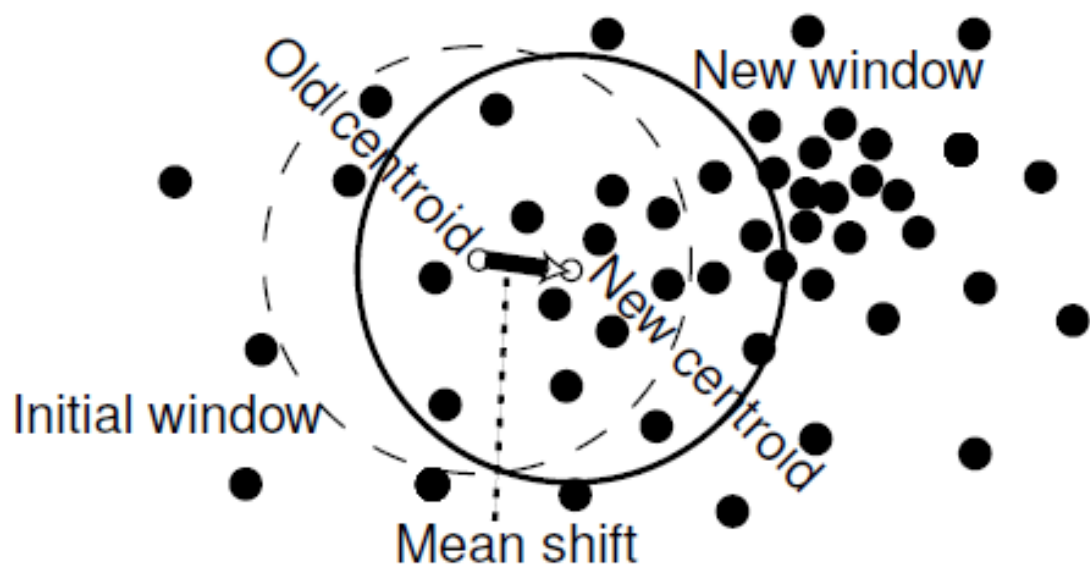


Normal kernel, $K_N(\mathbf{x}) = \exp(-\frac{1}{2}\|\mathbf{x}\|^2)$

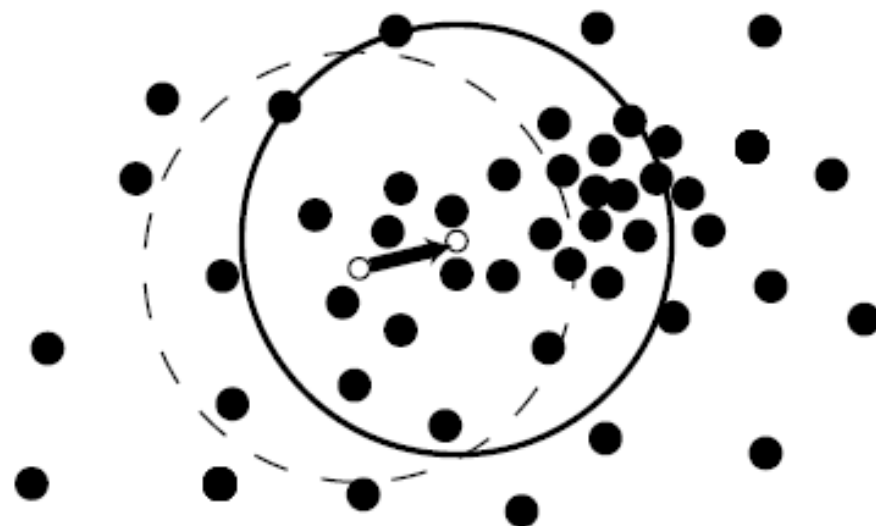


Epanechnikov kernel, $K_E(\mathbf{x}) = 1 - \|\mathbf{x}\|^2$

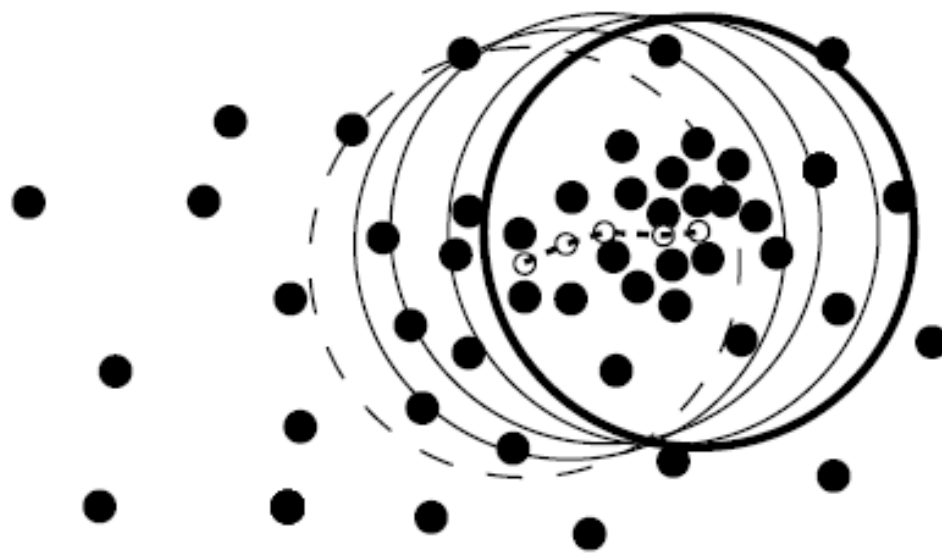




(a)

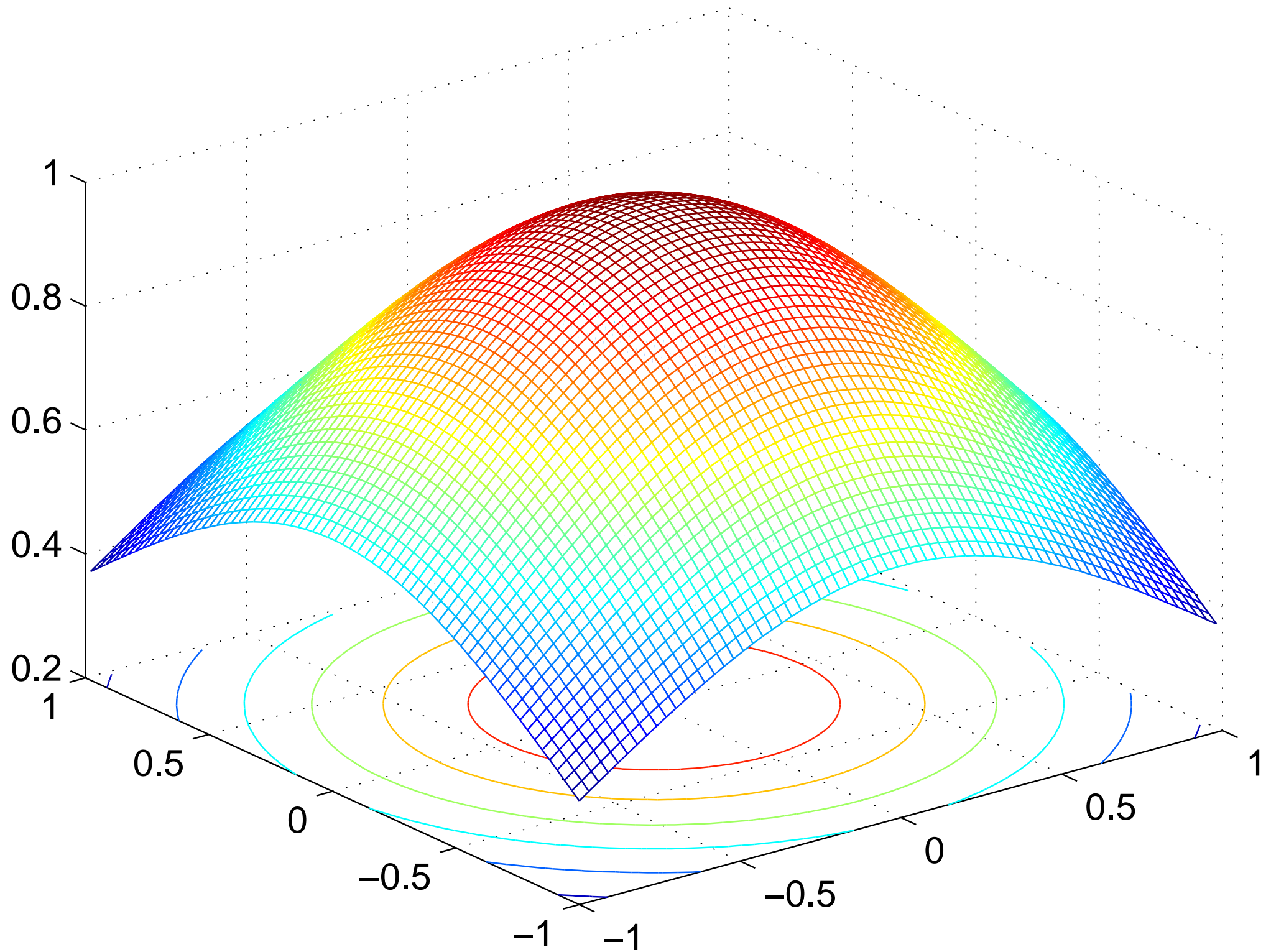


(b)

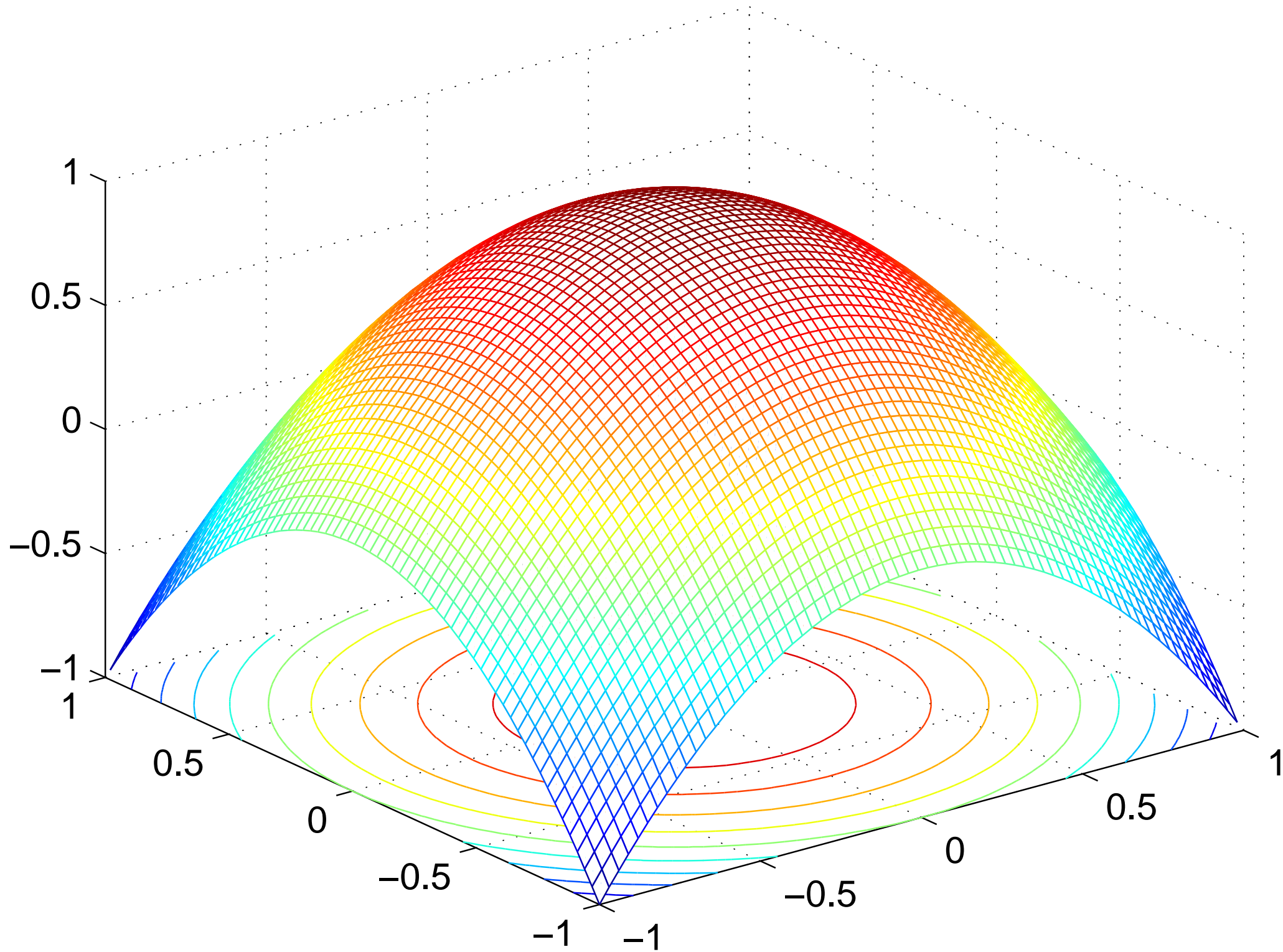


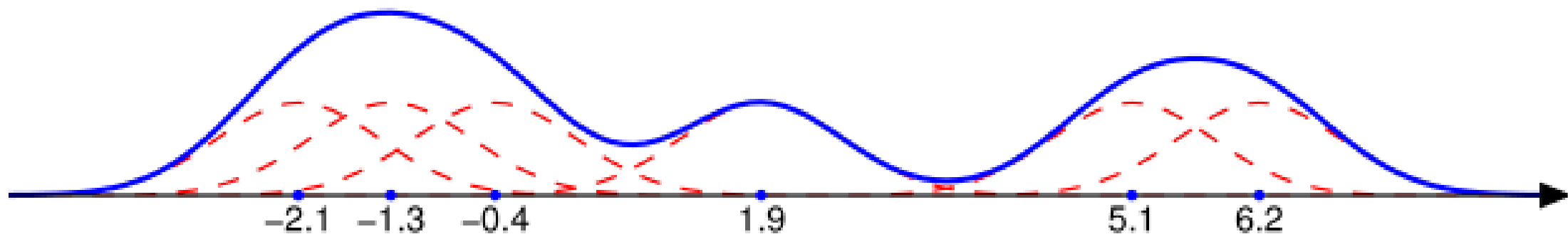
(c)

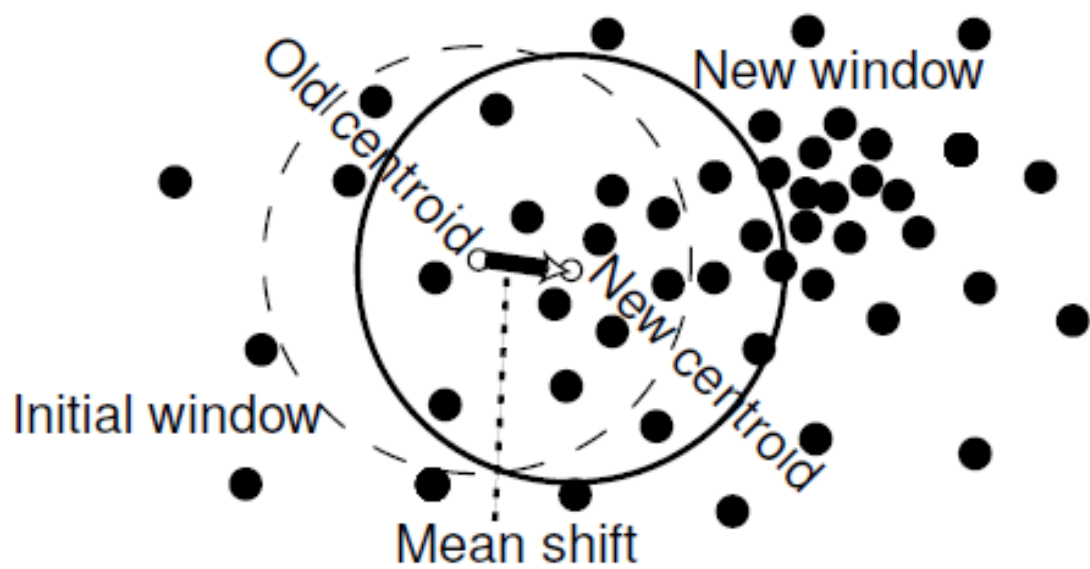
Normal kernel, $K_N(\mathbf{x}) = \exp(-\frac{1}{2}\|\mathbf{x}\|^2)$



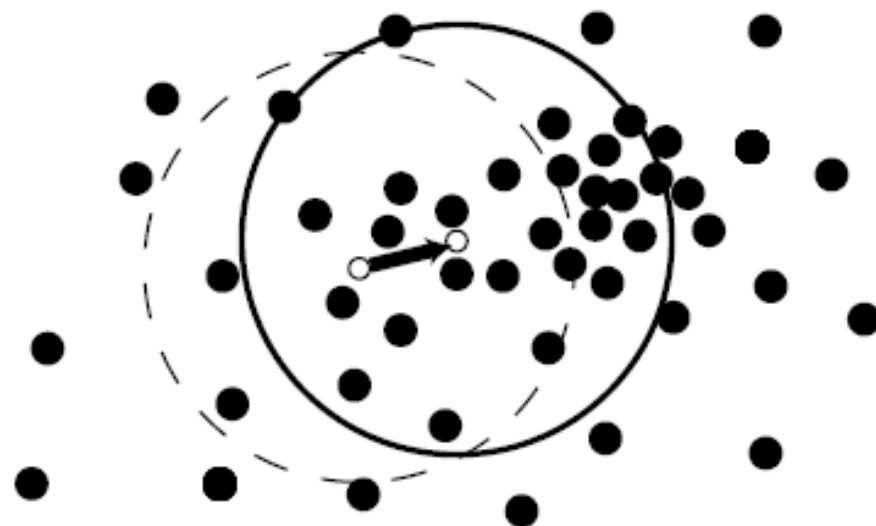
Epanechnikov kernel, $K_E(\mathbf{x}) = 1 - \|\mathbf{x}\|^2$



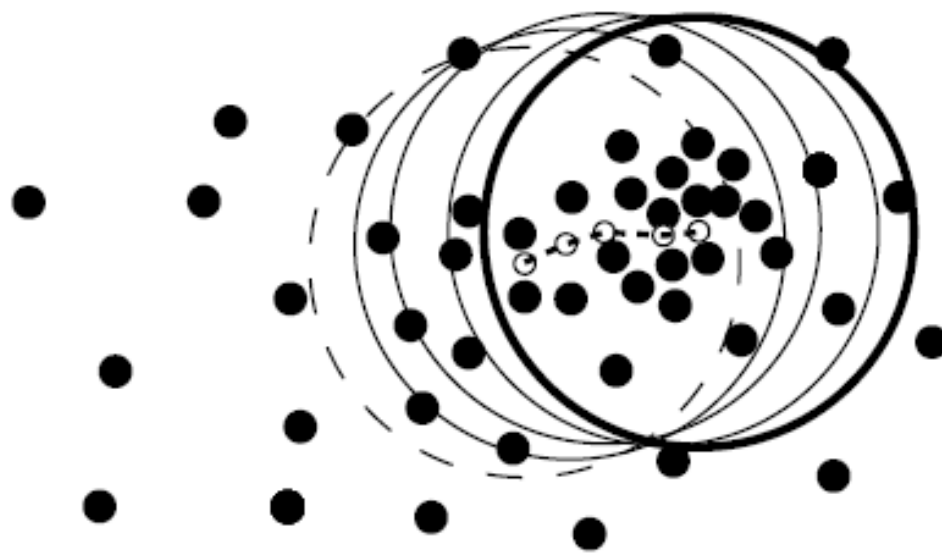




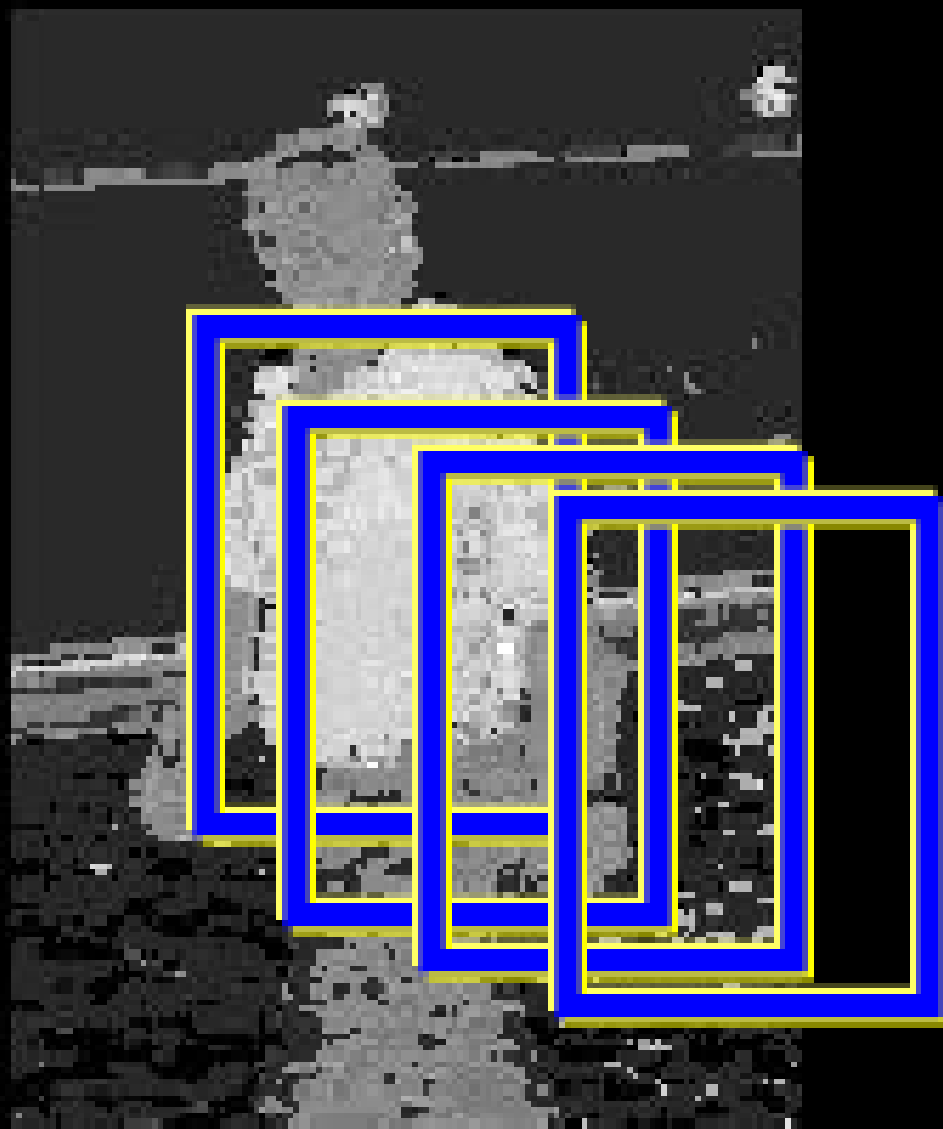
(a)



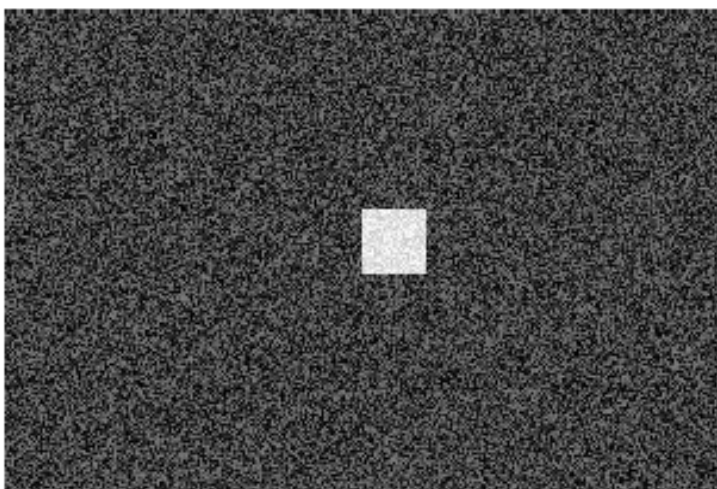
(b)



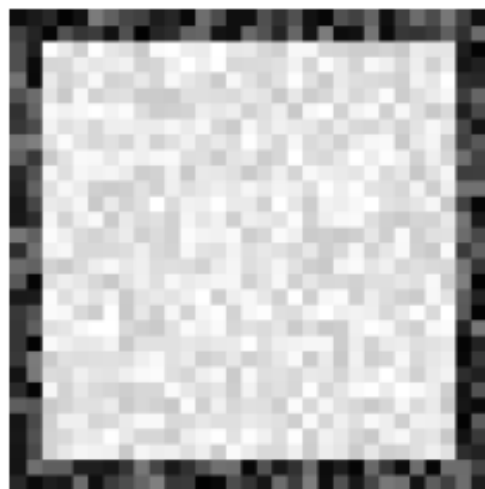
(c)



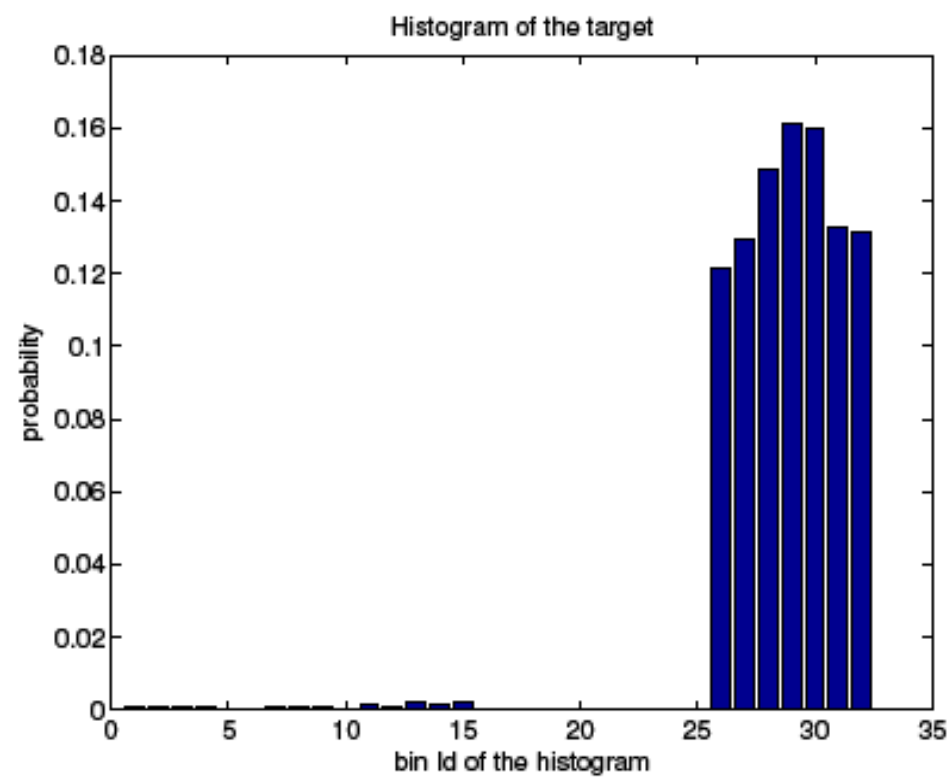
Example of an input image



(a)

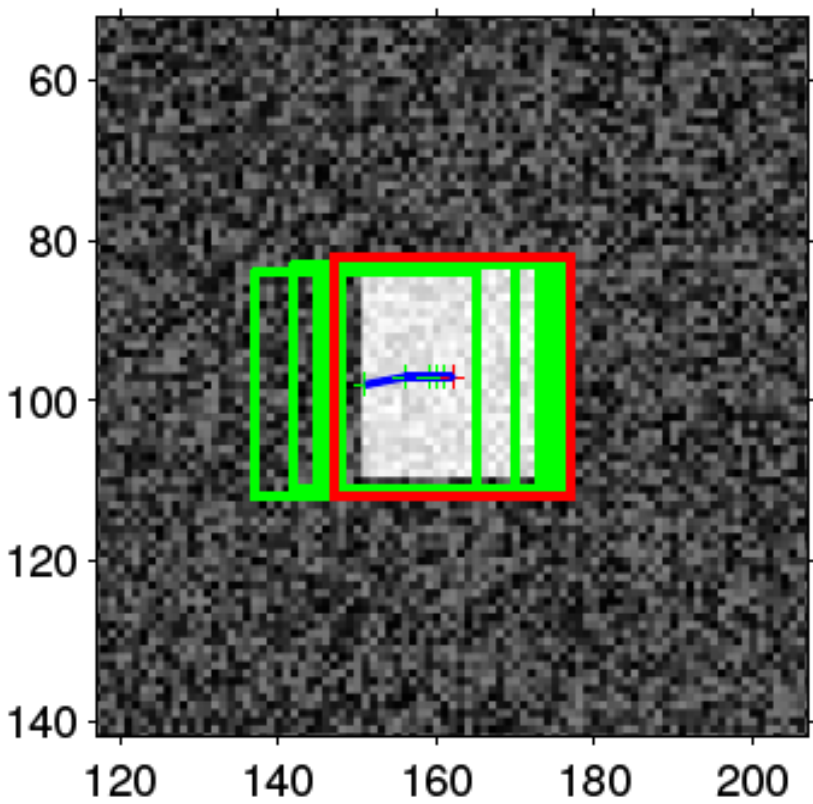


(b)



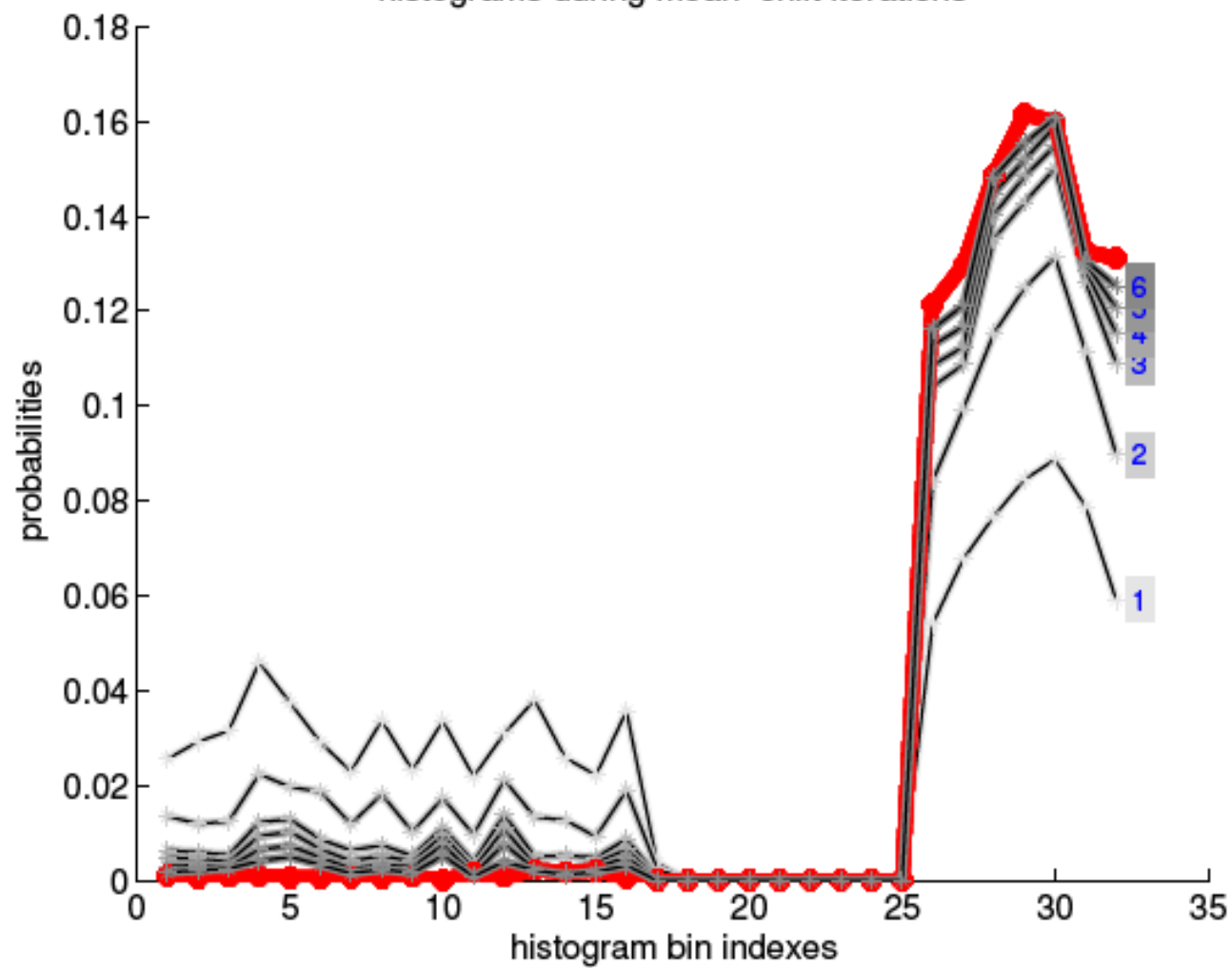
(c)

Mean-shift iterations



(a)

histograms during mean-shift iterations



(b)



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