STRUCTURED MODEL LEARNING (SS2015)
4. SEMINAR

Assignment 1. Consider a K-valued Gibbs random field $S = \{S_i \mid i \in V\}$ on a complete bipartite graph $(V = V_1 \cup V_2, E = E_{12})$. Its joint probability distribution is given by

$$p(\boldsymbol{s}) = \frac{1}{Z(\boldsymbol{u})} \exp\Big[\sum_{i \in V_1} \sum_{j \in V_2} u_{ij}(s_i, s_j)\Big].$$

Suppose we want to learn its parameters $\boldsymbol{u}$ from given training data $\mathcal{T}_\ell = \{\boldsymbol{s}^j \mid j = 1, \ldots, \ell\}$ by using pseudo-likelihood maximisation. Derive the corresponding formula for the gradient of the pseudo-likelihood.

Assignment 2. Consider the task of unsupervised learning of a Hidden Markov Model, i.e. a Gibbs random field on a comb-like graph given by

$$p(\boldsymbol{x}, \boldsymbol{s}) = \frac{1}{Z(\boldsymbol{u}, \boldsymbol{v})} \exp\Big[\sum_{i=2}^{n} u_i(s_{i-1}, s_i) + \sum_{i=1}^{n} v_i(x_i, s_i)\Big]$$

where the random variables S_i and X_i, $i = 1, \ldots, n$ are both finite valued. The task consists in estimating the parameters $\boldsymbol{u}$ and $\boldsymbol{v}$ given training data $\mathcal{T}_\ell = \{\boldsymbol{x}^j \mid j = 1, \ldots, \ell\}$, which have been i.i.d. generated from the (unknown) model. Show that both sub-steps of the EM-algorithm (which is a particular case of a DC-algorithm) are tractable for this model class. Find algorithms for solving them.

Assignment 3. Consider the binary valued Gibbs random field on a ring as defined in Assignment 4 of the previous seminar. Use the Gibbs sampler you have implemented there and generate a training sample. Try to estimate the model parameters by implementing pseudo-likelihood maximisation for this model.