GRAPHICAL MARKOV MODELS (WS2016)
5. SEMINAR

Assignment 1. Consider the task of finding the most probable sequence of (hidden) states for a (Hidden) Markov model on a chain.
a) Show that the Dynamic Programming approach applied for this task can be interpreted as an equivalent transformation (re-parametrisation) of the model.
b) Show that the transformed functions (potentials) encode an explicit description of *all* optimisers of the problem

Assignment 2. Consider a GRF for binary valued labellings $x\colon V \to \{0,1\}$ of a graph (V,E) given by

$$p(x) = \frac{1}{Z}\exp\Big[\sum_{ij\in E} u_{ij}(x_i,x_j) + \sum_{i\in V} u_i(x_i)\Big].$$

Show that is is always possible to find an equivalent transformation (re-parametrisation)

$$u_{ij} \to \tilde{u}_{ij}, \quad u_i \to \tilde{u}_i$$

such that the new pairwise functions $\tilde{u}_{ij}$ have the form

$$\tilde{u}_{ij}(x_i,x_j) = \alpha_{ij}|x_i - x_j|$$

with some real numbers $\alpha_{ij} \in \mathbb{R}$.

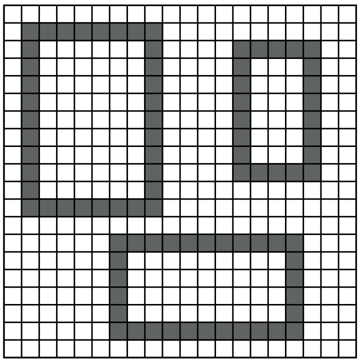
Assignment 3. Transform the *Travelling Salesman Problem* into a $(\min,+)$-problem.
Hints:
a) represent a route by a binary valued $n \times n$ matrix, where rows encode cities and columns encode the position of the city in the route,
b) consider matrix elements as graph vertices and introduce edges and predicates on them in order to forbid labellings which do not represent a valid tour,
c) finally, introduce edges and edge functions to represent the length of the route.

Assignment 4. Consider the language L of all b/w images $x\colon D \to \{b,w\}$ containing an arbitrary number of non-overlapping and non-touching one pixel wide rectangular frames (see figure).
a) Prove that L is not expressible by a locally conjunctive predicate

$$x \in L \quad \text{if and only if} \quad f(x) = \bigwedge_{c\in\mathcal{C}} f_c(x_c) = 1$$

with predicates f_c, defined on image fragments x_c, where $c \subset D$ have bounded size $|c| < |D|$.
b) Show that L can be expressed by introducing a field $s\colon D \to K$ of non-terminal symbols, a locally conjunctive predicate on them and pixel-wise predicates g relating the non-terminal and terminal symbol in each pixel

$$x \in L \quad \text{if and only if} \quad \bigvee_{s\in K^D}\Big[\bigwedge_{c\in\mathcal{C}} f_c(s_c) \wedge \bigwedge_{i\in D} g(x_i,s_i)\Big] = 1$$



Find a suitable structure $\mathcal{C}$, an alphabet of non-terminal symbols K and predicates f_c, g.