GRAPHICAL MARKOV MODELS (WS2014)
5. SEMINAR

Assignment 1. Consider the task of finding the most probable sequence of (hidden) states for a (Hidden) Markov model on a chain.
a) Show that the Dynamic Programming approach applied for this task can be interpreted as an equivalent transformation (re-parametrisation) of the model.
b) Show that the transformed functions (potentials) encode an explicit description of *all* optimisers of the problem

Assignment 2. Let (V, E) be an undirected tree and let s_i, $i \in V$ be K-valued random variables associated with the vertices of that tree. Their joint distribution is a Markov model given by

$$p(s) = \frac{1}{Z} \prod_{ij \in E} g_{ij}(s_i, s_j),$$

where g_{ij} are non-negative factors.
a) Find an algorithm for computing all marginal probabilities $p(s_i)$, $s_i \in K$, $i \in V$ of the model. *Hint:* See sec. 11 of the lecture, fill in details.
b) Is it possible to parallelise the algorithm?
c) Modify the algorithm such that it computes "max-marginals" of the distribution. The latter are defined by

$$m(s_i = k) = \max_{s\colon s_i = k} p(s).$$

Assignment 3. Consider a GRF for binary valued labellings $x\colon V \to \{0, 1\}$ of a graph (V, E) given by

$$p(x) = \frac{1}{Z} \exp\Big[\sum_{ij \in E} u_{ij}(x_i, x_j) + \sum_{i \in V} u_i(x_i)\Big].$$

Show that is is always possible to find an equivalent transformation (re-parametrisation)

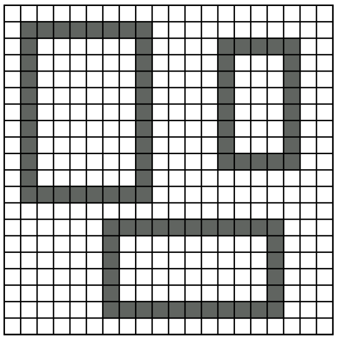
$$u_{ij} \to \tilde{u}_{ij}, \quad u_i \to \tilde{u}_i$$

such that the new pairwise functions $\tilde{u}_{ij}$ have the form

$$\tilde{u}_{ij}(x_i, x_j) = \alpha_{ij} |x_i - x_j|$$

with some reals $\alpha_{ij} \in \mathbb{R}$.

Assignment 4. Consider the language L of all b/w images $x\colon D \to \{b, w\}$ containing an arbitrary number of non-overlapping and non-touching one pixel wide rectangular frames (see figure).



a) Prove that L is not expressible by a locally conjunctive predicate

$$x \in L \quad \text{if and only if} \quad f(x) = \bigwedge_{c \in \mathcal{C}} f_c(x_c) = 1$$

with predicates f_c, defined on image fragments x_c, where $c \subset D$ have bounded size $|c| \leqslant m$.

b) Show that L can be expressed by introducing a field $s \colon D \to K$ of non-terminal symbols, a locally conjunctive predicate for them and pixel-wise predicates g "connecting" the non-terminal and terminal symbol in each pixel

$$x \in L \quad \text{if and only if} \quad \bigvee_{s \in K^D} \Big[\bigwedge_{c \in \mathcal{C}} f_c(s_c) \wedge \bigwedge_{i \in D} g(x_i, s_i) \Big] = 1$$

Find a suitable structure $\mathcal{C}$, alphabet of non-terminal symbols K and predicates f_c, g.