GRAPHICAL MARKOV MODELS (WS2013)
5. SEMINAR

Assignment 1. Let K be a completely ordered finite set (you may assume K to be an interval of integers, $K \subset \mathbb{Z}$ where appropriate). Consider real valued functions on K^n.
a) Prove that a conical linear combination

$$f(\kappa) = \sum_{i=1}^{m} \alpha_i g_i(\kappa), \quad \alpha_i \geqslant 0 \; \forall i$$

of submodular functions $g_i \colon K^n \to \mathbb{R}$ is a submodular function.
b) Let f, g be submodular functions on K^n. Is their point-wise maximum a submodular function? And, their minimum?
c) Consider the function $g \colon K^2 \to \mathbb{R}$ defined by

$$g(k, k') = \alpha(k - k')^2.$$

For which α is it submodular?
d) Prove that truncated norm

$$g(k, k') = \min(M, |k - k'|)$$

is not submodular.

Assignment 2. Consider a general (Min,+)-problem for K-valued labellings s of a graph (V, E)

$$\sum_{i \in V} u_i(s_i) + \sum_{ij \in E} u_{ij}(s_i, s_j) \to \min_{s \in K^{|V|}} .$$

Maximising a lower bound w.r.t. reparametrisations can be equivalently written as an LP task

$$\begin{aligned} & \sum_{i \in V} c_i + \sum_{ij \in E} c_{ij} \to \max_{c, \psi} \\ \text{s.t.} \quad & c_i + \sum_{j \in \mathcal{N}_i} \psi_{ij}(k) \leqslant u_i(k) && \forall i \in V, \; \forall k \in K \\ & c_{ij} - \psi_{ij}(k) - \psi_{ji}(k') \leqslant u_{ij}(k, k') && \forall ij \in E, \; \forall k, k' \in K \end{aligned}$$

a) Find its dual task and interpret it.
b**) Try to prove that there is no gap between the optimal value of the dual LP task and the optimal value of the discrete (Min,+)-task if the latter is submodular. *Hint:* Try to prove that the optimal set of the dual LP task contains at least one integer valued minimiser.

Assignment 3. Consider a (Min,+)-problem as in the previous assignment, but now for an acyclic graph (you may consider a chain). You know of course, that such a task can be solved by Dynamic Programming. Show that this corresponds to an equivalent transformation (reparametrisation). This reparametrisation maximises the lower bound and there is no gap to the optimal value of the (Min,+)-problem.