GRAPHICAL MARKOV MODELS (WS2012)
5. SEMINAR

Assignment 1. Let y^i, $i \in I$ be a finite set of points in $\mathbb{R}^n$. The task is to find the point x closest to all of them, i.e.

$$x^* = \arg\min_{x \in \mathbb{R}^n} \max_{i \in I} \|x - y^i\|^2.$$

Interpret the task and its solution geometrically. Derive and algorithm for solving this task.
Hint: Apply the saddle point theorem discussed in the lecture.

Assignment 2. Derive an EM-algorithm for learning the following conditional independent probability model (sometimes called "naive Bayes probabilistic model")

$$p(x, y, k) = p(x|k)p(y|k)p(k),$$

where $x \in X$, $y \in Y$, $k \in K$ and all three sets are finite. The training data provided for learning are independent realisations of pairs (x, y) with the empirical distribution $p^*(x, y)$ (the corresponding values of k are not observed).

Assignment 3. Consider an HMM $p(x, s) = p(x|s)p(s)$ on an acyclic graph $\mathcal{G} = (V, E)$, where

$$p(s) = \frac{\prod_{ij \in E} p(s_i, s_j)}{\prod_{i \in V} p(s_i)^{n_i - 1}}, \quad p(x|s) = \prod_{i \in V} p(x_i|s_i).$$

Given the observed field x, we want to calculate posterior marginal probabilities $p(s_i|x)$ for *all* vertices $i \in V$ and all states $s_i \in K$. Can you find an efficient algorithm for this task?

Assignment 4. (Discussion) Get familiar with the definition of Bayesian networks. Compare them with Markov Random Fields and Gibbs Random Fields defined on undirected graphs.