GRAPHICAL MARKOV MODELS (WS2013)
4. SEMINAR

Assignment 1. Consider a GRF for binary valued labellings $x\colon V \to \{0,1\}$ of a graph (V,E) given by

$$p(x) = \frac{1}{Z} \exp\Bigl[\sum_{ij \in E} u_{ij}(x_i, x_j) + \sum_{i \in V} u_i(x_i)\Bigr].$$

Show that is is always possible to find an equivalent transformation (reparametrisation)

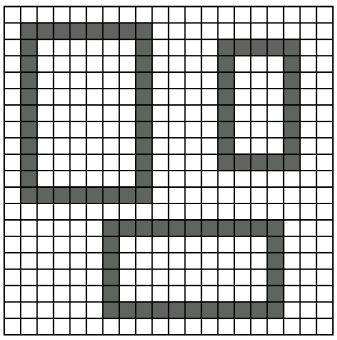
$$u_{ij} \to \tilde{u}_{ij}, \quad u_i \to \tilde{u}_i$$

such that the new pairwise functions $\tilde{u}_{ij}$ have the form

$$\tilde{u}_{ij}(x_i, x_j) = \alpha_{ij} |x_i - x_j|$$

with some reals $\alpha_{ij} \in \mathbb{R}$.

Assignment 2. Consider the language L of all b/w images $x\colon D \to \{b, w\}$ containing an arbitrary number of non-overlapping and non-touching one pixel wide rectangular frames (see figure).



a) Prove that L is not expressible by a locally conjunctive predicate

$$x \in L \quad \text{if and only if} \quad f(x) = \bigwedge_{c \in \mathcal{C}} f_c(x_c) = 1$$

with predicates f_c, defined on image fragments x_c, where $c \subset D$ have bounded size $|c| \leqslant m$.

b) Show that L can be expressed by introducing a field $s\colon D \to K$ of non-terminal symbols, a locally conjunctive predicate for these and pixel-wise predicates g "connecting" the non-terminal and terminal symbol in each pixel

$$x \in L \quad \text{if and only if} \quad \bigvee_{s \in K^D} \Bigl[\bigwedge_{c \in \mathcal{C}} f_c(s_c) \wedge \bigwedge_{i \in D} g(x_i, s_i)\Bigr] = 1$$

Find a suitable structure $\mathcal{C}$, alphabet of non-terminal symbols K and predicates f_c, g.

Assignment 3. Consider the class of (Min,+)-problems for K-valued labellings $x\colon V \to K$ of an undirected graph (V, E):

$$x^* = \operatorname*{arg\,min}_{x \in K^V} \Big[\sum_{ij \in E} u_{ij}(x_i, x_j) + \sum_{i \in V} u_i(x_i) \Big]$$

where u_{ij} and u_i are real valued functions. Show that this class is NP-hard by proving that every instance of a max-clique problem can be reduced to some instance of an (Min,+)-problem.

Assignment 4. Transform the *Travelling Salesman Problem* into a (Min,+)-problem.
Hints:
a) represent a tour by a binary valued $n \times n$ matrix, where rows code the cities and columns the position of the city in the tour,
b) consider matrix elements as graph vertices and introduce edges and predicates on them in order to penalise labellings which do not represent a valid tour,
c) finally, introduce edges and edge functions to represent the tour length.