GRAPHICAL MARKOV MODELS (WS2016)
3. SEMINAR

Assignment 1.* Suppose that a regular language $\mathcal{L}$ of strings over the finite alphabet Σ is described by a non-deterministic finite-state machine. Given a string $y \notin \mathcal{L}$, the task is to find the string $x \in \mathcal{L}$ with smallest Hamming distance to y, i.e.

$$x^* = \arg\min_{x \in \mathcal{L}} d_h(x, y),$$

where d_h denotes the Hamming distance. Construct an efficient algorithm for solving this task.

Assignment 2. Let $\boldsymbol{x}$ be a grey value image of size $n \times m$, where x_{ij} denotes the grey value of the pixel with coordinates (i, j). The task is to segment such images into an upper and lower part by a boundary represented as a sequence of height values $s_j \in \{1, 2, \ldots, n\}$ for all $j = 1, 2, \ldots, m$.

The prior probability for boundaries is assumed to be a homogeneous Markov chain such that $p(s_j \mid s_{j-1}) = 0$ if $|s_j - s_{j-1}| > 1$. The appearance model for columns $\boldsymbol{x}_j$ given the boundary value s_j, is assumed to be conditional independent

$$p(\boldsymbol{x}_j \mid s_j) = \prod_{i \leqslant s_j} p_1(x_{ij}) \cdot \prod_{i > s_j} p_2(x_{ij}),$$

where $p_1()$ and $p_2()$ are two distributions for grey values.
a) Deduce an efficient algorithm for determining the most probable boundary.
b*) Suppose that the loss function $\ell(s, s')$ for incorrectly recognised boundaries is defined by

$$\ell(s, s') = \sum_{j=1}^{m} (s_j - s'_j)^2.$$

Formulate the segmentation task for this case. Deduce an efficient inference algorithm.

Assignment 3. Consider an HMM for pairs of sequences $(\boldsymbol{x}, \boldsymbol{s})$, $\boldsymbol{x} \in F^n$, $\boldsymbol{s} \in K^n$ with unknown transition probabilities and emission probabilities. It is however known that the model is homogeneous. An i.i.d. sample of training data $\mathcal{T} = \{(\boldsymbol{x}^j, \boldsymbol{s}^j) \mid j = 1, \ldots, m\}$ is given for learning. Modify the formulae for the maximum likelihood estimate (see Sec. 7 in lecture notes) for this situation. Prove correctness.

Assignment 4. Consider the following variation of the previous assignment. The observable features are no longer discrete – instead, they are real numbers, i.e., $F = \mathbb{R}$. It is known, that the emission probabilities are Gaussian distributions. For each hidden state $k \in K$ there is a Gaussian p.d.

$$p(x \mid k) = \frac{1}{\sqrt{2\pi}\sigma_k} e^{-\frac{(x-\mu_k)^2}{2\sigma_k^2}}$$

with unknown mean and variance. Derive formulae for learning these model parameters.