GRAPHICAL MARKOV MODELS (WS2016)
2. SEMINAR

We consider the following Markov chain model assignments 1-3. The p.d. for sequences $s = (s_1, \ldots, s_n)$ of length n with states $s_i \in K$ is given by:

$$p(s) = p(s_1) \prod_{i=2}^{n} p(s_i \mid s_{i-1}).$$

The transition probabilities $p(s_i \mid s_{i-1})$ and the p.d. $p(s_1)$ for the state of the first element are assumed to be known.

Assignment 1.
a) Suppose that the marginal p.d.s $p(s_i)$ for the states of the i-th element of the sequence are known for all $i = 2, \ldots, n$. Then it is easy to calculate all "inverse" transition probabilities $p(s_{i-1} \mid s_i)$. How?
b) Describe an efficient algorithm for calculating $p(s_i)$ for all $i = 2, \ldots, n$.

Assignment 2. Suppose that there is a special state $k^* \in K$. We want to know how often this state appears on average in a sequence generated by the model. Describe an efficient method for the calculation of this average.
Hint: You may use the fact that the expected value of a sum of random variables is equal to the sum of their expected values. The number of occurrences of the state k^* in a sequence s can be obviously written in the form

$$\delta_{s_1 k^*} + \delta_{s_2 k^*} + \ldots + \delta_{s_n k^*}$$

where δ_{ij} denotes the Kronecker delta.

Assignment 3. Let $A \subset K$ be a subset of states and let $\mathcal{A} = A^n$ denote the set of all sequences s with $s_i \in A$ for all $i = 1, \ldots, n$. Find an efficient algorithm for calculating the probability $p(\mathcal{A})$ of the event $\mathcal{A}$.

Assignment 4. According to Definition 1b of Sec. 1 of the lecture, any Markov chain model can be specified in the form

$$p(s) = \frac{1}{Z} \prod_{i=2}^{n} g_i(s_{i-1}, s_i)$$

with arbitrary functions $g_i \colon K^2 \to \mathbb{R}_+$ and the normalisation constant Z. Find an algorithm for computing the pairwise marginal probabilities $p(s_{i-1} = k, s_i = k')$ for all $k, k' \in K$ and all $i = 2, \ldots, n$ from the given functions g_i, $i = 2, \ldots, n$.