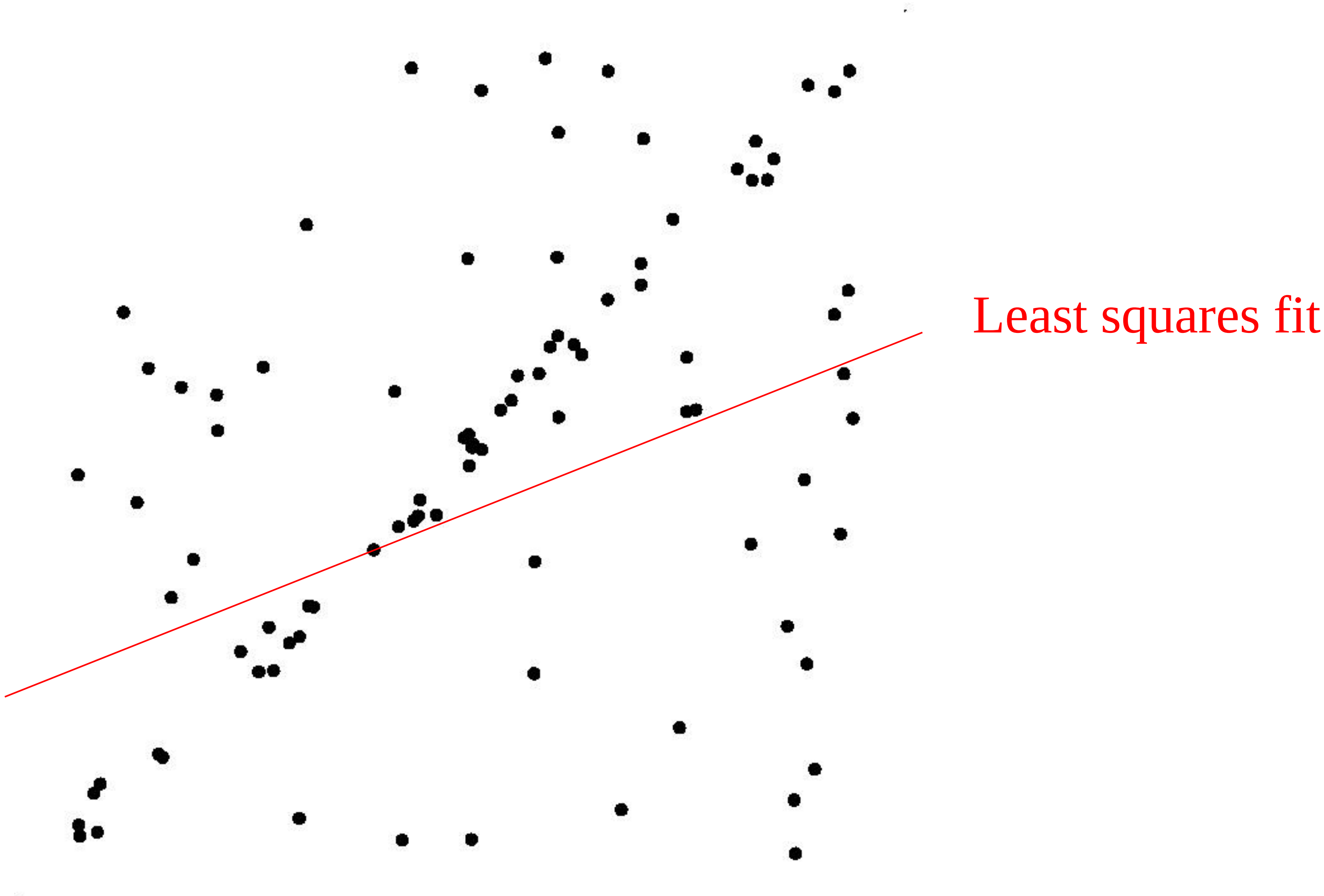
RANSAC

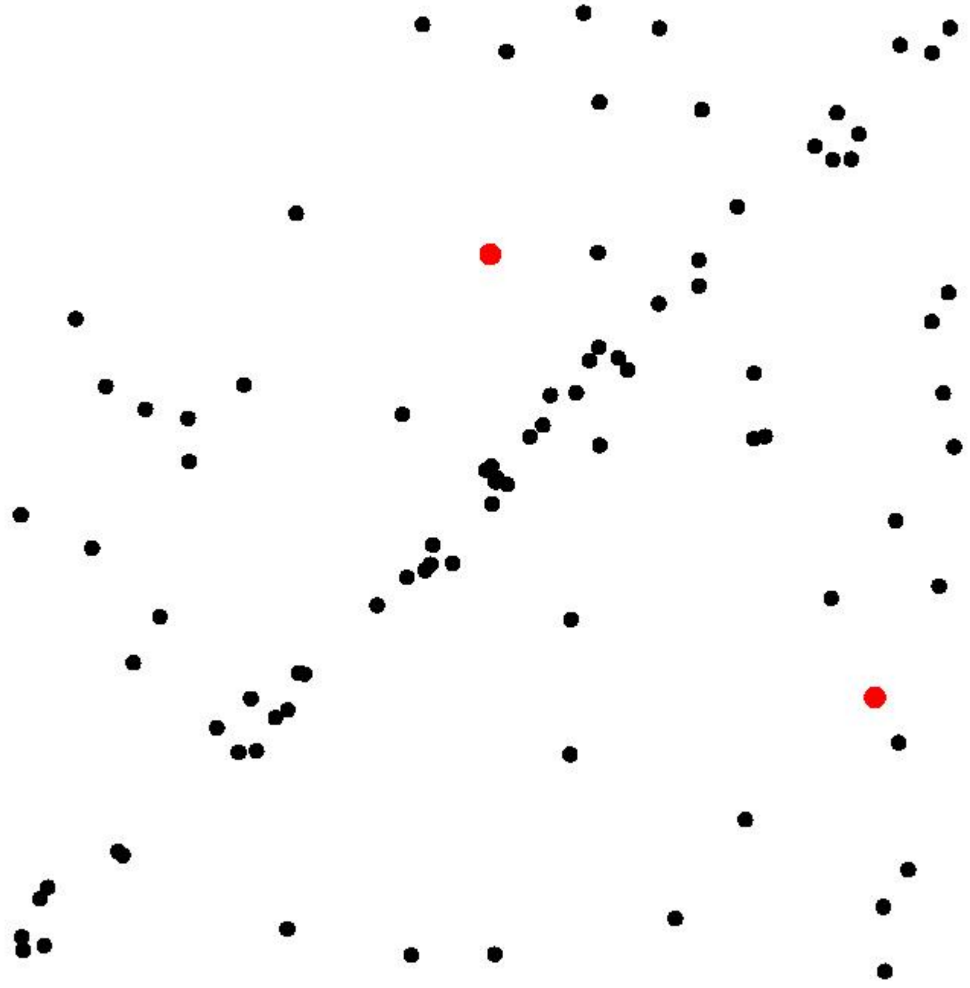
Robust model estimation from data contaminated by outliers

Ondřej Chum

Fitting a Line

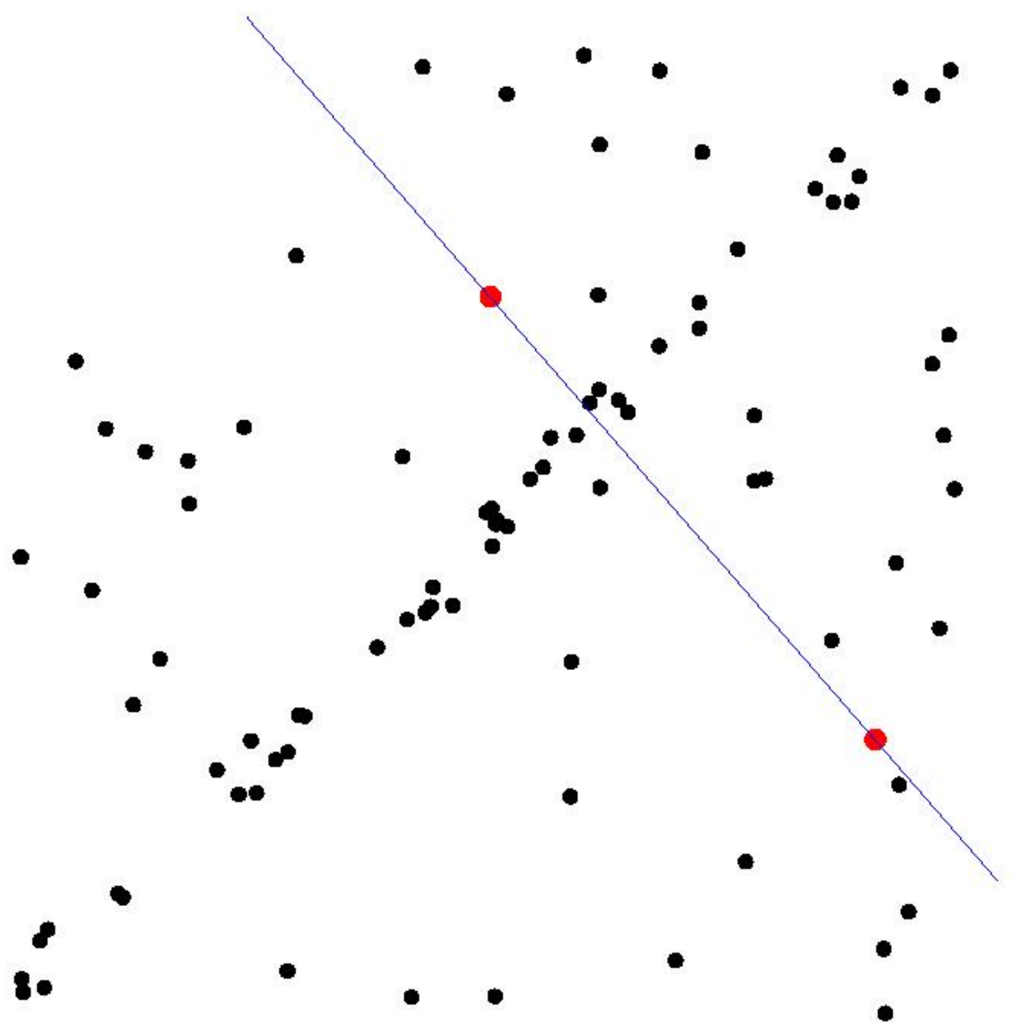


RANSAC



- **Select sample of m points at random**

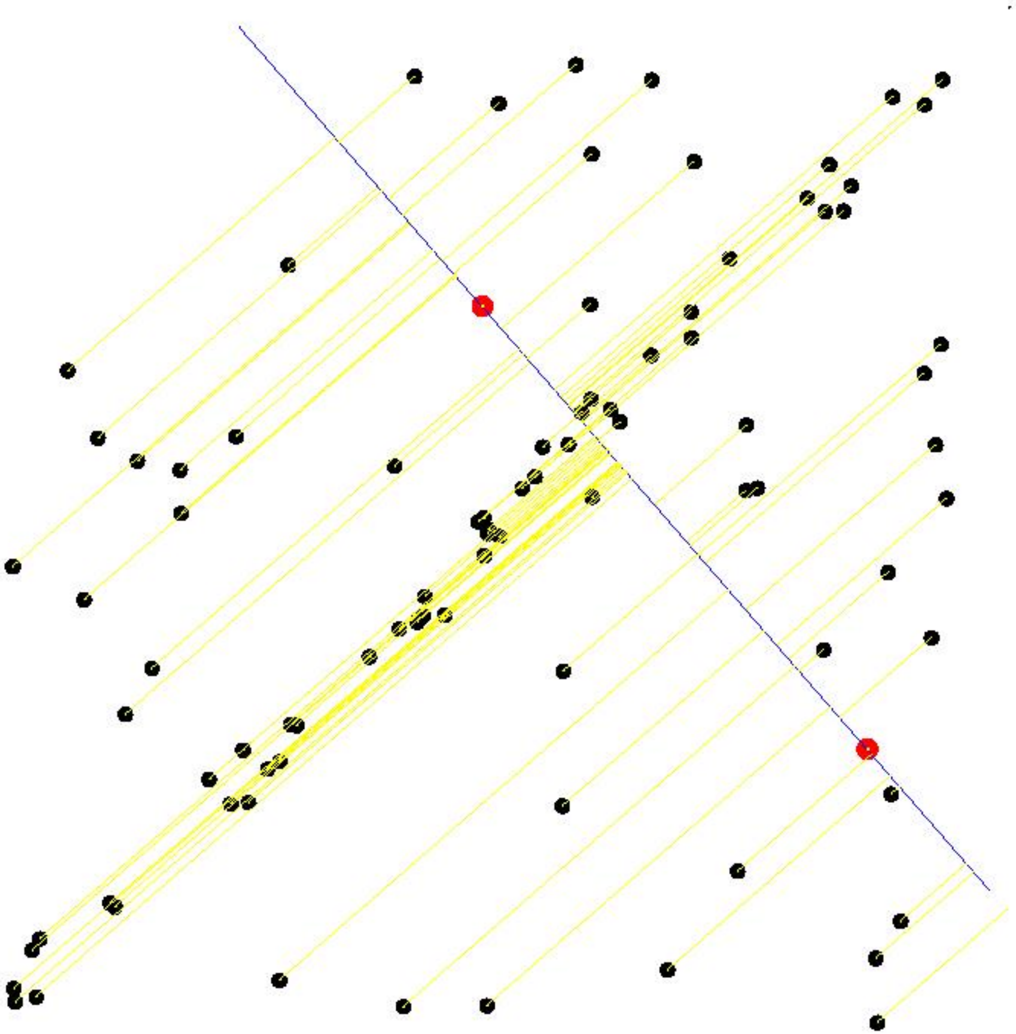
RANSAC



- Select sample of m points at random

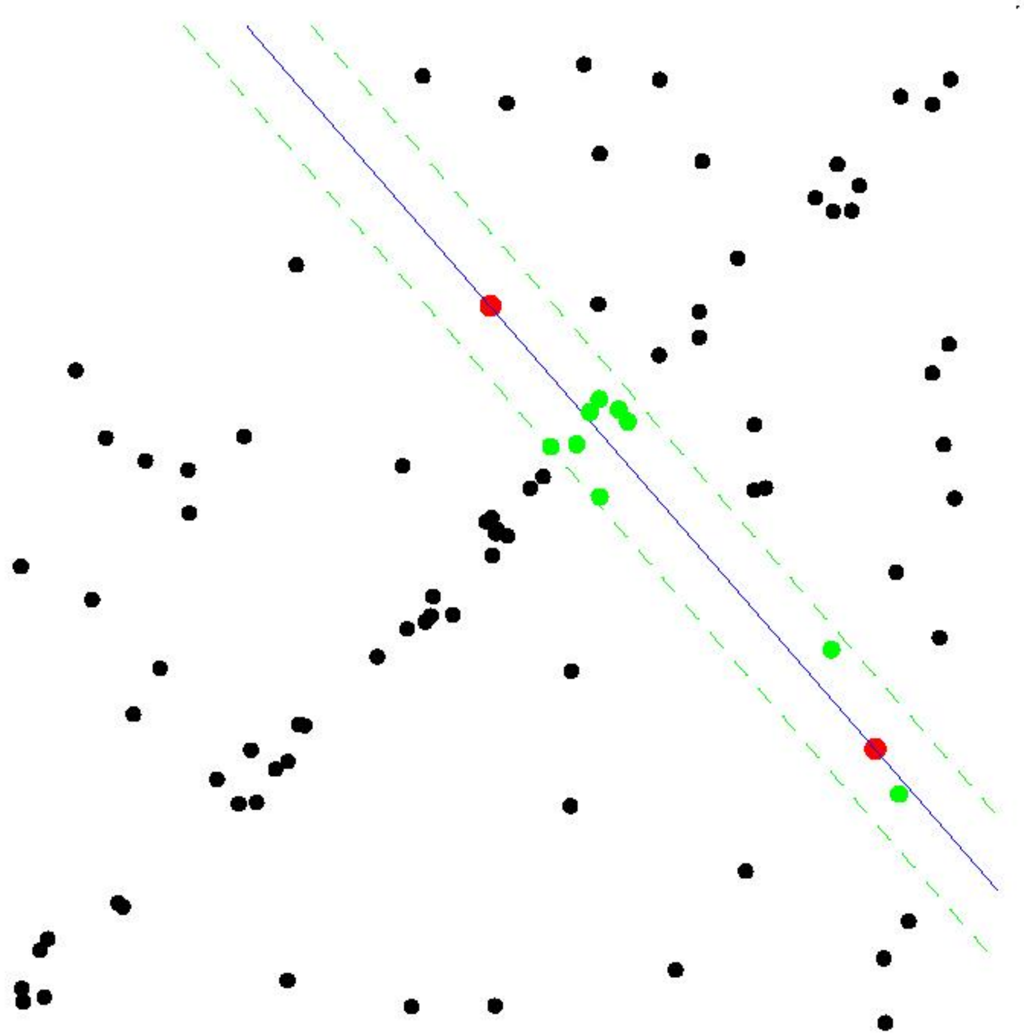
- **Calculate model parameters that fit the data in the sample**

RANSAC



- Select sample of m points at random
- Calculate model parameters that fit the data in the sample
- **Calculate error function for each data point**

RANSAC



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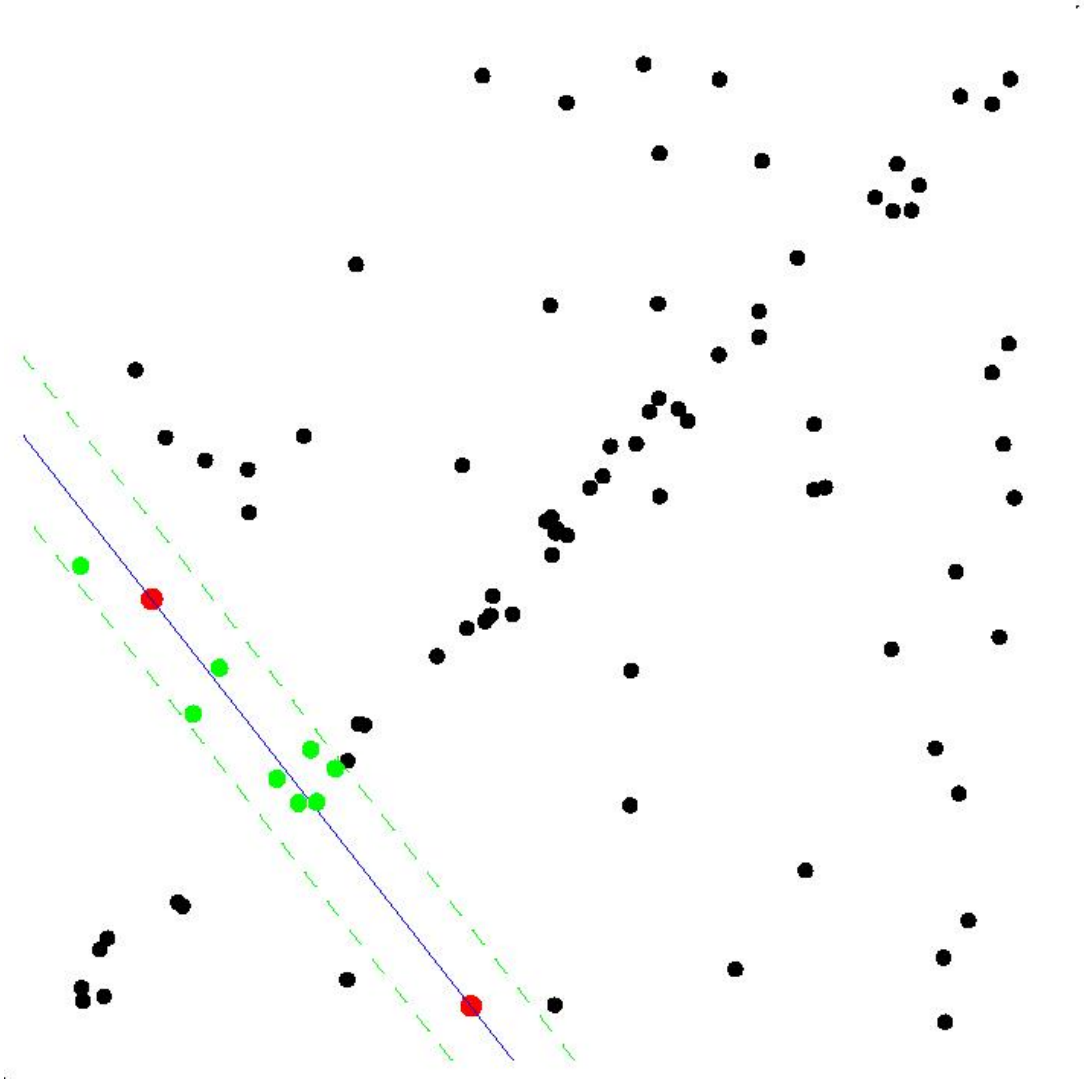
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- **Select data that support current hypothesis**

RANSAC



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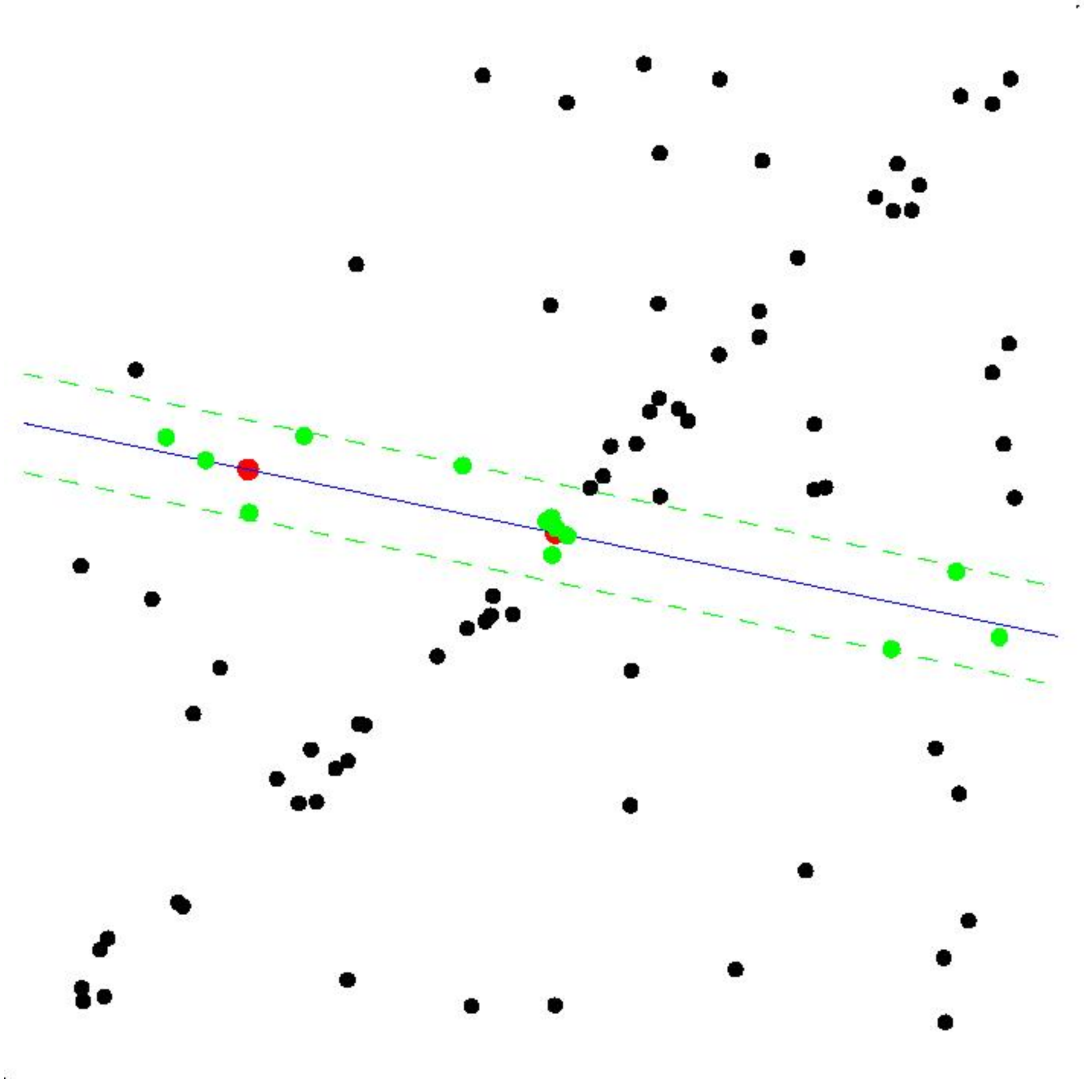
RANSAC



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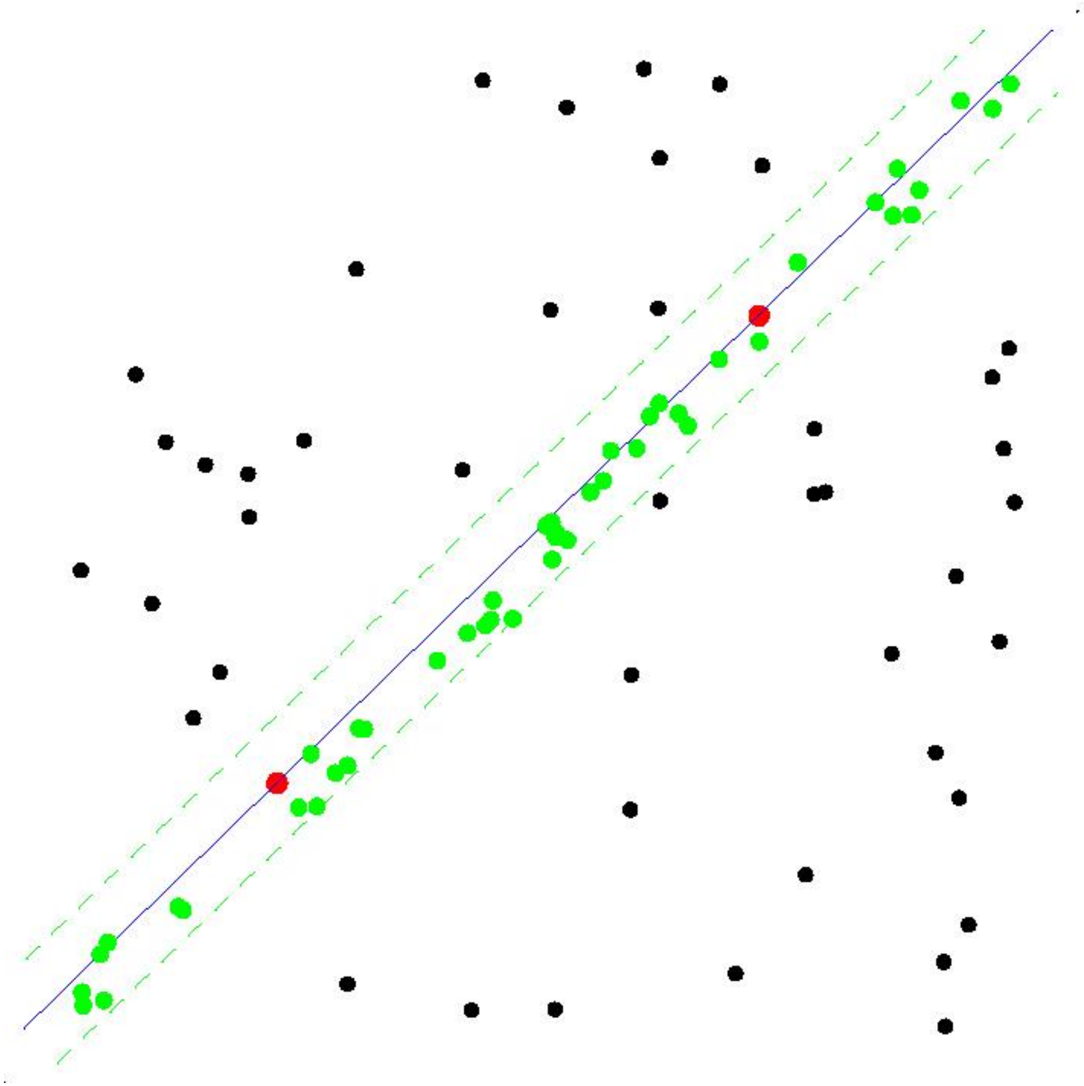
RANSAC



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- **Repeat sampling**

How Many Samples?

On average

N	… number of points
I	… number of inliers
m	… size of the sample

$$\text{P(good)} = \frac{\binom{I}{m}}{\binom{N}{m}} = \prod_{j=0}^{m-1} \frac{I-j}{N-j}$$

mean time before the success

$E(k) = 1 / \text{P(good)}$

How Many Samples?

With confidence p

How large k?

. . . to hit at least one pair of points on the line l with probability larger than p (0.95)

Equivalently

. . . the probability of not hitting any pair of points on l is $\leq 1 - p$

How Many Samples?

With confidence *p*

N	… number of point
I	… number of inliers
m	… size of the sample

$$P(\text{good}) = \frac{\binom{I}{m}}{\binom{N}{m}} = \prod_{j=0}^{m-1} \frac{I-j}{N-j}$$

$$P(\text{bad}) = 1 - P(\text{good})$$

$$P(\text{bad } k \text{ times}) = \left(1 - P(\text{good})\right)^k$$

How Many Samples?

With confidence p

$$\text{P(bad } k \text{ times)} = \big(1 - \text{P(good)}\big)^k \leq 1 - p$$

$$k \log \big(1 - \text{P(good)}\big) \leq \log(1 - p)$$

$$k \geq \log(1 - p) \,/\, \log \big(1 - \text{P(good)}\big)$$

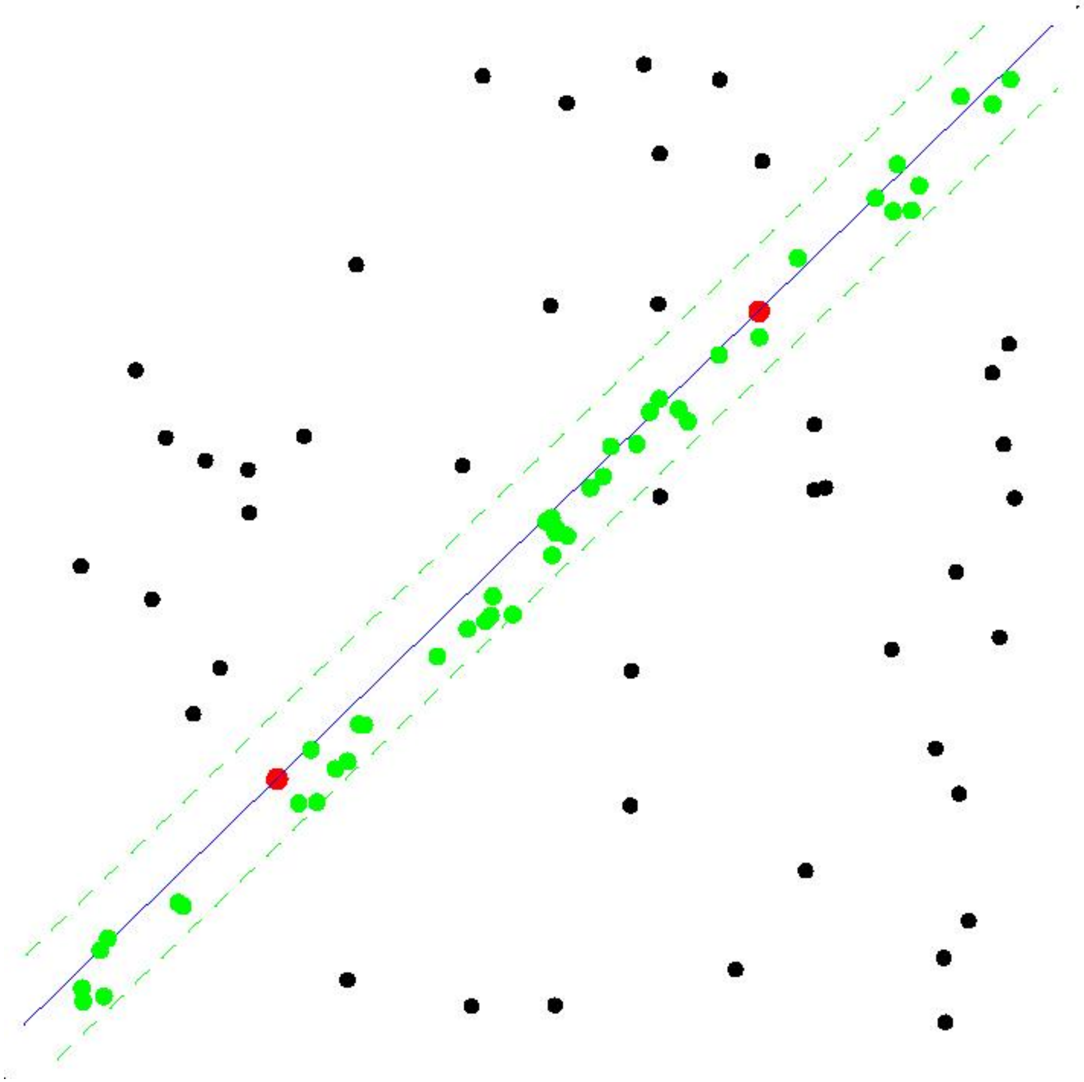
How Many Samples

I / N [%]

Size of the sample m

	15%	20%	30%	40%	50%	70%
2	132	73	32	17	10	4
4	5916	1871	368	116	46	11
7	$1.75 \cdot 10^6$	$2.34 \cdot 10^5$	$1.37 \cdot 10^4$	1827	382	35
8	$1.17 \cdot 10^7$	$1.17 \cdot 10^6$	$4.57 \cdot 10^4$	4570	765	50
12	$2.31 \cdot 10^{10}$	$7.31 \cdot 10^8$	$5.64 \cdot 10^6$	$1.79 \cdot 10^5$	$1.23 \cdot 10^4$	215
18	$2.08 \cdot 10^{15}$	$1.14 \cdot 10^{13}$	$7.73 \cdot 10^9$	$4.36 \cdot 10^7$	$7.85 \cdot 10^5$	1838
30	∞	∞	$1.35 \cdot 10^{16}$	$2.60 \cdot 10^{12}$	$3.22 \cdot 10^9$	$1.33 \cdot 10^5$
40	∞	∞	∞	$2.70 \cdot 10^{16}$	$3.29 \cdot 10^{12}$	$4.71 \cdot 10^6$

RANSAC



$$k = \frac{\log(1-p)}{\log\left(1 - \frac{I}{N}\frac{I-1}{N-1}\right)}$$

k … number of samples drawn

N … number of data points

I … time to compute a single model

p … confidence in the solution (.95)

RANSAC [Fischler, Bolles '81]

In: $U = \{x_i\}$ set of data points, $|U| = N$

$f(S): S \to p$ function f computes model parameters p given a sample S from U

$\rho(p, x)$ the cost function for a single data point x

Out: p^* p^*, parameters of the model maximizing the cost function

k := 0

Repeat until P{better solution exists} < η (a function of C^* and no. of steps k)

k := k + 1

I. Hypothesis

(1) select randomly set $S_k \subset U$, sample size $|S_k| = m$

(2) compute parameters $p_k = f(S_k)$

II. Verification

(3) compute cost $C_k = \sum_{x \in U} \rho(p_k, x)$

(4) if $C^* < C_k$ then $C^* := C_k$, $p^* := p_k$

end

Advanced RANSAC

In: $U = \{x_i\}$ set of data points, $|U| = N$

$f(S) : S \rightarrow p$ function f computes model parameters p given a sample S from U

$\rho(p, x)$ the cost function for a single data point x

Out: p^* p^*, parameters of the model maximizing the cost function

k := 0

Repeat until P{better solution exists} < η (a function of C^* and no. of steps k)

k := k + 1

Non-uniform sampling in PROSAC

I. Hypothesis

(1) select randomly set $S_k \subset U$, sample size $|S_k| = m$

(2) compute parameters $p_k = f(S_k)$

Many models are bad, no need to verify all data points – RANDOMIZED RANSAC

II. Verification

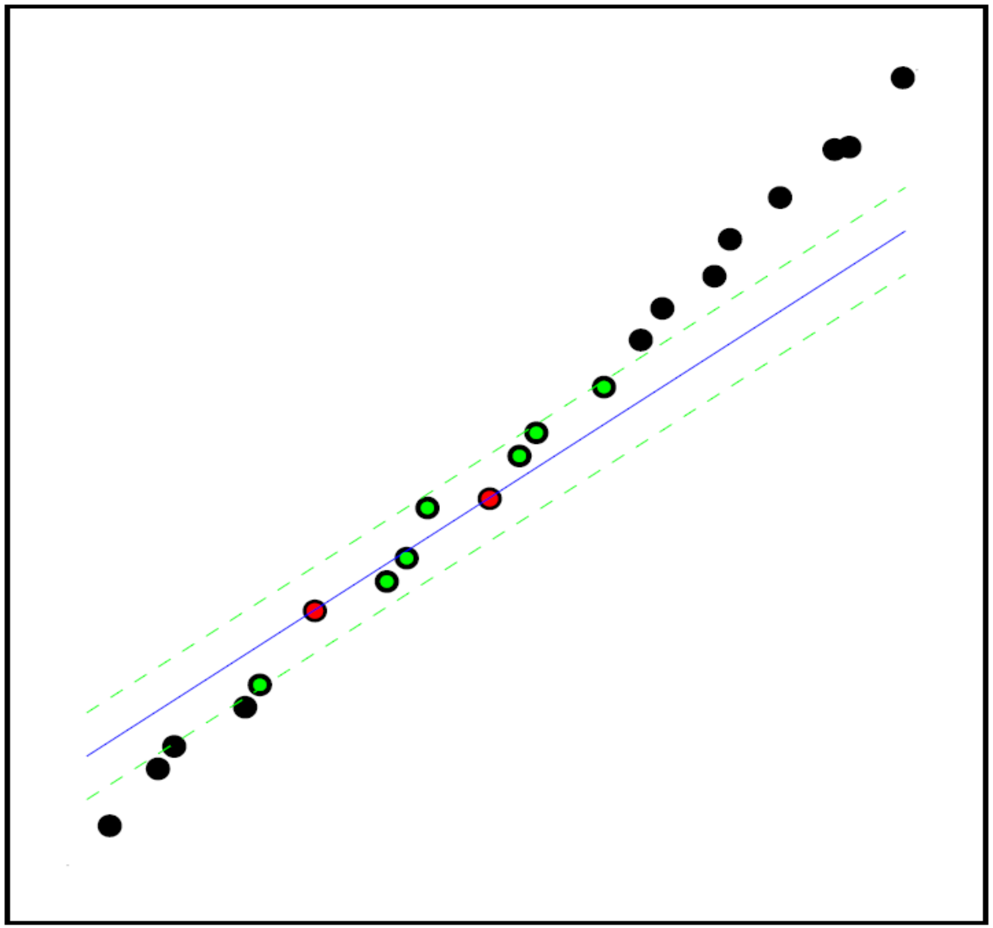
(3) compute cost $C_k = \sum_{x \in U} \rho(p_k, x)$

(4) if $C^* < C_k$ then $C^* := C_k$, $p^* := p_k$

end

Improving precision by Local Optimization

RANSAC Makes an Invalid Assumption



Not every all-inlier sample gives a model consistent with all inliers

↓

Lower number of inliers is detected

↓

RANSAC runs longer

Solution: Local Optimisation Step

Repeat k times
1. Hypothesis generation
2. Model verification
   2b. If model best-so-far Execute (Local) Optimisation

Inner RANSAC + Re-weighted least squares:

- Samples are drawn from the set of data points consistent with the best-so-far hypothesis
- New models are verified on all data points
- Samples can contain more than minimal number of data points since consistent points include almost entirely inliers

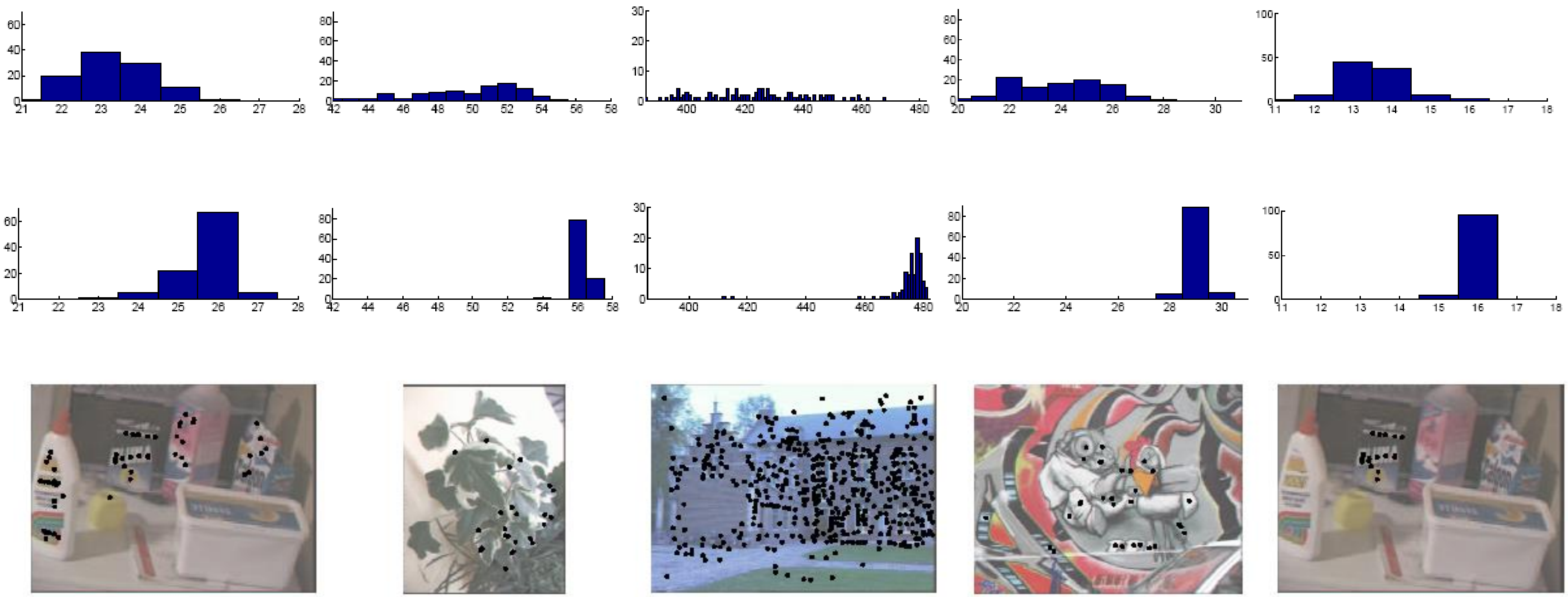
How often?

$$\sum_{l=1}^{k} P_l = \sum_{l=1}^{k} \frac{1}{l} \leq \int_{1}^{k} \frac{1}{x}\, dx + 1 = \log k + 1$$

Conclusion: the LO step 2b is executed rarely, does not influence running time significantly

Validation: Two-view Geometry Estimation

Histograms of the number of inliers returned over 100 executions of RANSAC (top) and LO-RANSAC (bottom)



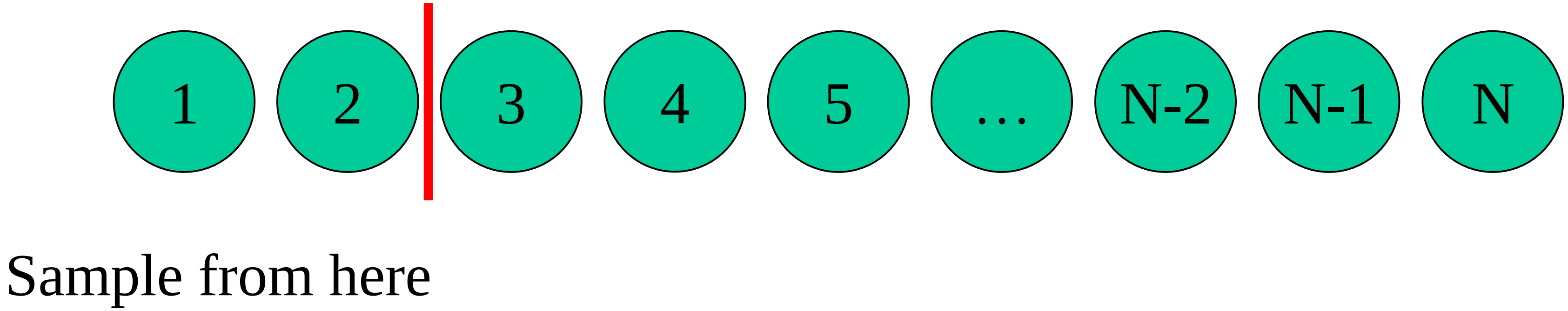
Result:

(i) variation of the number of inliers significantly reduced

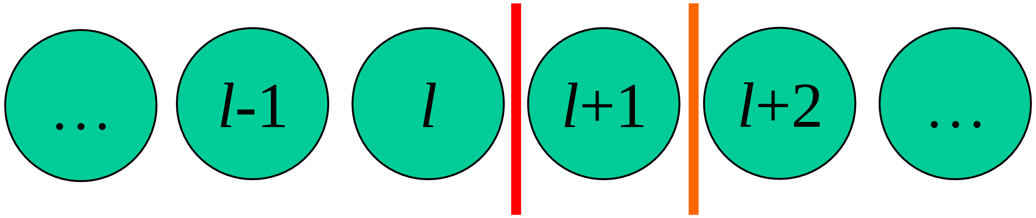
(ii) speed-up up to 3 times (for 7pt EG and 4pt homography est.)

PROSAC – PROgressive SAmple Consensus

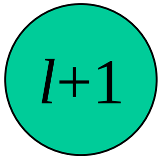
- Not all correspondences are created equally
- Some are better than others
- Sample from the best candidates first

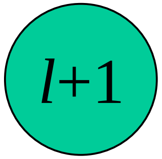


PROSAC Samples



Draw T_l samples from (1 … l)
Draw T_{l+1} samples from (1 … l+1)

Samples from (1 … l) that are not from (1 … l+1) contain 

Draw T_{l+1} - T_l samples of size m-1 and add 

Conclusions

- RANSAC is a standard tool in computer vision
- it is a simple procedure
  - hypothesize and verify loop
- handles large number of outliers
- a number of advanced strategies to
  - increase the stability
  - speed up
- Vanilla RANSAC never used in practice