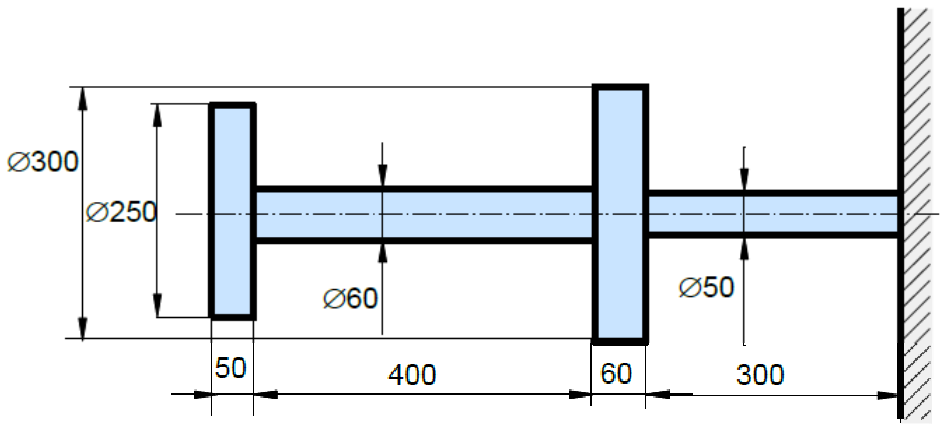
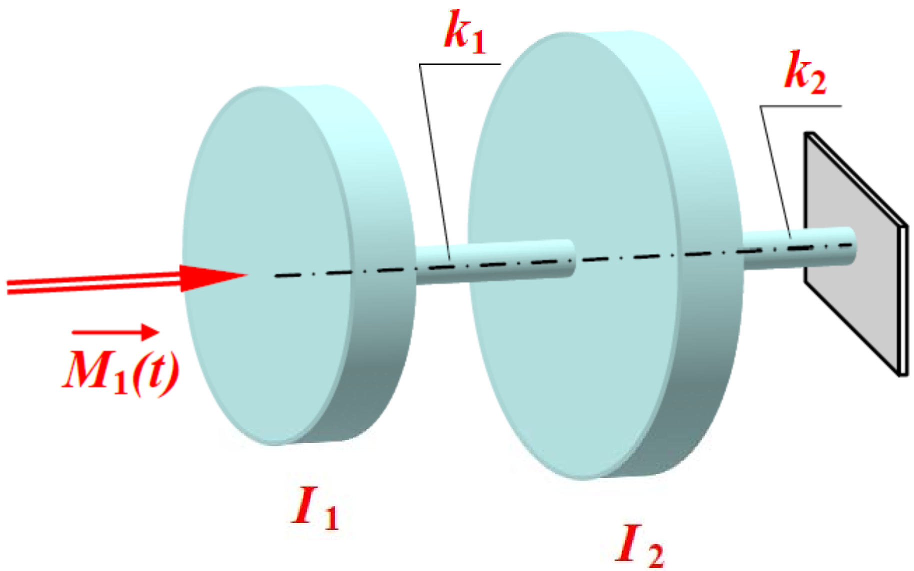
Example 3:

For the system in the picture find out its eigenfrequencies, shapes of eigenmodes and free and forced vibration.





Known parameters:

$M_1(t)=M_0.\sin(\omega t)$,

$M_0= 5$ Nm, $\omega= 1300\ s^{-1}$,

t=0: $\varphi_1(0)= \varphi_{10}=0{,}001$ rad,

$\varphi_2(0)= \varphi_{20}=0{,}003$ rad,

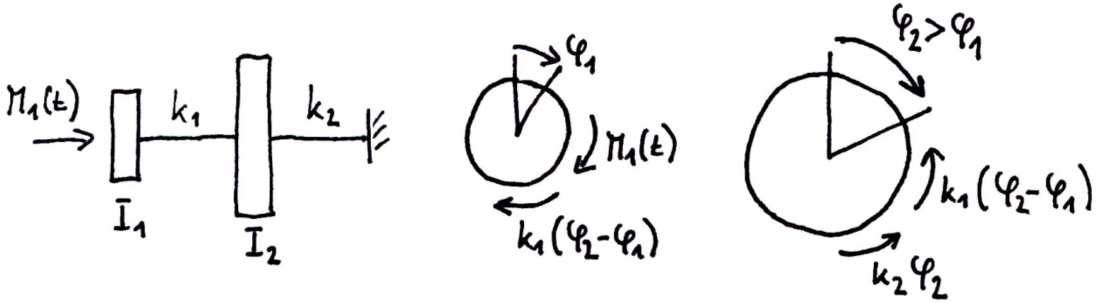
$\omega_1(0)= \omega_{10}=0\ s^{-1}$,

$\omega_2(0)= \omega_{20}=0\ s^{-1}$,

$I_1= 0{,}17$ kgm^2, $I_2= 0{,}38$ kgm^2,

$k_1= 2{,}61e^5$ Nm/rad,

$k_1= 1{,}68e^5$ Nm/rad,



$$I_1\ddot{\varphi}_1 = M_1(t) + k_1(\varphi_2-\varphi_1)$$
$$I_2\ddot{\varphi}_2 = -k_1(\varphi_2-\varphi_1) - k_2\varphi_2$$

$$\underbrace{\begin{bmatrix} I_1 & \emptyset \\ \emptyset & I_2 \end{bmatrix}}_{\underline{M}} \underbrace{\begin{bmatrix} \ddot{\varphi}_1 \\ \ddot{\varphi}_2 \end{bmatrix}}_{\ddot{\underline{x}}} + \underbrace{\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1+k_2 \end{bmatrix}}_{\underline{K}} \underbrace{\begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix}}_{\underline{x}} = \begin{bmatrix} M_0 \cdot \sin(\omega t) \\ \emptyset \end{bmatrix} = \underbrace{\begin{bmatrix} M_0 \\ \emptyset \end{bmatrix}}_{\underline{f_0}} \sin(\omega t) \rightarrow \underline{M}\,\ddot{\underline{x}} + \underline{K}\,\underline{x} = \underline{f_0}\cdot\sin(\omega t)$$

Řešení ve tvaru:

$$\underline{x} = \underline{x_h} + \underline{x_p}$$

Volné kmitání $\underline{M}\,\ddot{\underline{x}} + \underline{K}\,\underline{x} = \underline{\emptyset}$, $\underline{x_h} = \underline{u}\cdot\sin\Omega t \rightarrow (\underline{K} - \lambda\underline{M})\,\underline{u} = \underline{\emptyset}$

$\det(\underline{K} - \lambda\underline{M}) \overset{!}{=} \emptyset \rightarrow \lambda_{1,2} \Rightarrow \begin{matrix}\Omega_1 = \sqrt{\lambda_1} \\ \Omega_2 = \sqrt{\lambda_2}\end{matrix}$ Ω ... vlastní úhlová frekvence

$(\underline{K} - \lambda_i\underline{M})\,\underline{u}_i = \underline{\emptyset} \Rightarrow \underline{u}$ vlastní tvary

Vynucené kmitání $\underline{M}\,\ddot{\underline{x}} + \underline{K}\,\underline{x} = \underline{f_0}\cdot\sin\omega t$, $\underline{x_p} = \underline{v}\cdot\sin\omega t \rightarrow \underline{v} = (\underline{K} - \omega^2\underline{M})^{-1}\cdot\underline{f_0}$

Celkové řešení

$$\varphi_1 = a_1 u_{11}\cos\Omega_1 t + b_1 u_{11}\sin\Omega_1 t + a_2 u_{12}\cos\Omega_2 t + b_2 u_{12}\sin\Omega_2 t + v_1\sin\omega t$$
$$\varphi_2 = a_1 u_{21}\cos\Omega_1 t + b_1 u_{21}\sin\Omega_1 t + a_2 u_{22}\cos\Omega_2 t + b_2 u_{22}\sin\Omega_2 t + v_2\sin\omega t$$

konstanty a_1, a_2, b_1, b_2 z počátečních podmínek