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Residual Modes for Structure Reduction and Efficient Coupling of Substructures

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***Abstract*—The paper deals with the systematical analysis of the reduction of the large structures using the singular perturbation approximation for the interconnected subsystems. The proportionally damped system is considered in the special modal form identical to the almost balanced form of the model. The residual mode is evaluated for the different types of outputs, namely position, velocity and acceleration form. Two main targets are the efficiency of the algorithms and the accuracy of the coupled reduced model with respect to the original large model. The efficient algorithm and the significant improvement of the accuracy with respect to the interconnection of the truncated subsystems are shown.**

***Index Terms*—Dynamical model reduction, State-space, Residual modes, Singular perturbation approximation, Coupling of substructures.**

INTRODUCTION

The design of the control law of the flexible machines vibration suppression and the flexible system motion control (e.g. [1], [2], [3]) needs an accurate dynamical models of the structure with the reasonably moderate size (order). The need of the efficient control law synthesis accordingly leads to the well known contradictory requests. On the one hand the model should be as accurate as possible, on the other hand it should be as small as possible. The original form of the model is very often a detailed FEM model by far larger than is acceptable in the control design context.

Such structural model is typically reduced either before or after transferring it into the modal space. Therefore we distinguish between a model order reduction based on the chosen physical coordinates (i.e. nodal DOF) e.g. [4], [5] or based on the modal coordinates e.g. [2], [3]. The assembling of the total reduced order model from the substructures can advantageously combine both types of coordinates [6]. The classical reduction methods used within the mechanical engineering comunity takes the first modes selected based on the given frequency range of interest. The more advanced methods take systematically into account the relevancy of particular modes with respect to the inputs and outputs. Such methods are based on the so called balanced model reduction [7], [8] and its frequency weighted variants [9]. In the case of the assembling of reduced substructures the inputs are connecting forces and the outputs are the kinematical quantities of the connecting points [2], [3], [10]. The structural systems with the relatively low proportional damping (which is the case of the majority of the structures in mechanical engineering) can be transformed into the so-called almost balanced form [2], [3]. Such form is obtained by the computationally very cheap way in comparison to the fully balanced reduction obtained by the solution of the Lyapunov equations. The states of the almost balanced model are identical to the special form of the modal states whereas the input-output properties of the fully balanced reduced model are saved. With respect to this fact the modal coordinates/states are considered within the presented paper.

The second important reason of the necessity of efficient and accurate coupling of reduced substructures for dynamical modelling is the changing of kinematical configuration of mechanisms during the motion and consequently the changing of their eigenfrequencies and eigenmodes [4].

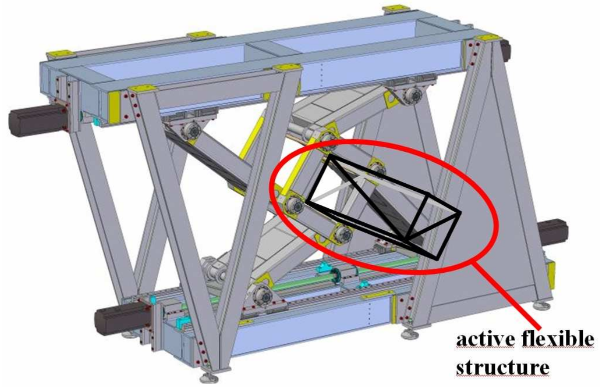

Fig. 1: Parallel redundant machine Sliding Star

The example of such situation is the parallel redundantly actuated machine Sliding Star [11], in the Fig.1 with the scheme of considered active appendage on the end-effector. The positioning within the workspace results in the necessary dynamical model parameterization [4].

The presented paper is devoted to the systematical analysis of the reduction of large structural systems using singular perturbation approximation for interconnected subsystems.

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The two main targets are the efficiency of the algorithms and the accuracy of the coupled reduced model with respect to the original large model.

The paper is organized as follows. In the first paragraph the equations of motion of proportionally damped system are transformed into the special modal form suitable for the set up of almost balanced form of the model. The second paragraph explains the difference between the singular perturbation approximation reduction and truncation reduction. The general formulas for the singular perturbation approximation are applied to the modal form specified within the first paragraph. The computational complexity is significantly reduced. Within the third paragraph the residual mode is evaluated for the different types of outputs, namely position, velocity and acceleration form. The fourth paragraph describes an efficient way of applying of singular perturbation for connected substructures and demonstrates the significant improvement of the accuracy with respect to the interconnection of truncated subsystems.

I. MODAL STATE SPACE FORM OF STRUCTURAL SYSTEM

Let starts with the well known equation of the linear mechanical system.

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f} \qquad (1.1)$$

Concerning modal transformation $\mathbf{x} = \mathbf{V}\mathbf{q}$ and left multiplication by modal matrix $\mathbf{V}^T$ equation (1.1) leads to

$$\mathbf{V}^T\mathbf{M}\mathbf{V}\ddot{\mathbf{q}} + \mathbf{V}^T\mathbf{B}\mathbf{V}\dot{\mathbf{q}} + \mathbf{V}^T\mathbf{K}\mathbf{V}\mathbf{q} = \mathbf{V}^T\mathbf{f}\,. \qquad (1.2)$$

The proportional damping and distinct eigenfrequencies are considered. The elastic part (which belongs to nonzero eigenfrequencies) of the modal matrix $\mathbf{V}_{elas}$ has been evaluated concerning the well known mass matrix normalization

$$\mathbf{V}_{elas}^T\mathbf{M}\mathbf{V}_{elas} = \mathbf{I}\,. \qquad (1.3)$$

The diagonal matrix of the structures eigenfrequencies $\mathbf{\Omega}$ [rad/s] is then computed as

$$\mathbf{V}_{elas}^T\mathbf{K}\mathbf{V}_{elas} = \mathbf{\Omega}^2\,. \qquad (1.4)$$

Concerning proportional damping, also the third part of the equation system is diagonalized

$$\mathbf{V}_{elas}^T\mathbf{B}\mathbf{V}_{elas} = 2\mathbf{b}_d\mathbf{\Omega}\,, \qquad (1.5)$$

where $\mathbf{b}_d$ is the diagonal matrix of the modal damping ratios of separate eigenmodes. The modal form of the state space model can be set up in different variants e.g. [1], [3] more appropriate for transformation to state-space form and corresponding also to the so called almost-balanced form. The state vector of considered modal description has special form $\mathbf{z} = [\ldots \quad \mathbf{z}_i \quad \ldots]^T$, where $\mathbf{z}_i = [\Omega_i q_{mi}, \dot{q}_{mi}]^T$, q_{mi} and $\dot{q}_{mi}$ are modal elastic coordinates and modal elastic velocities. The state space system matrix for such coordinates has the block diagonal form

$$\mathbf{A} = \begin{bmatrix} 0 & \Omega_1 & 0 & 0 & \ldots & 0 \\ -\Omega_1 & -2b_{d1}\Omega_1 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \Omega_2 & \ldots & 0 \\ 0 & 0 & -\Omega_2 & -2b_{d2}\Omega_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots & -2b_{dN}\Omega_N \end{bmatrix}. \qquad (1.6)$$

N is the total number of degrees of freedom. The input matrix $\mathbf{B}$ for considered force inputs ($\mathbf{u} = \mathbf{f}$) is

$$\mathbf{B} = \begin{bmatrix} \mathbf{0}^T \\ \mathbf{V}_1^T \\ \mathbf{0}^T \\ \mathbf{V}_2^T \\ \ldots \\ \mathbf{V}_N^T \end{bmatrix} \qquad (1.7)$$

Together the dynamic equation in the state space form is

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}\mathbf{u} \qquad (1.8)$$

II. SINGULAR PERTURBATION APPROXIMATION AND RESIDUAL MODE OF STRUCTURE

The so called residual mode known from the modal system description can be derived as a special case of the singular perturbation approximation. Let the original general state space model is described as follows

$$SS_{orig} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{B}_1 \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{B}_2 \\ \mathbf{C}_1 & \mathbf{C}_2 & \mathbf{D} \end{bmatrix}. \qquad (2.1)$$

Then the reduced order state space model obtained by **state truncation** of any type is

$$SS_{trun} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{B}_1 \\ \mathbf{C}_1 & \mathbf{D} \end{bmatrix}. \qquad (2.2)$$

The **singular perturbation approximation** (SPA) variant [8] of reduced model preserves the DC-gains of an original system and generally can be written as

$$SS_{spa} = \begin{bmatrix} \mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{A}_{21} & \mathbf{B}_1 - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{B}_2 \\ \mathbf{C}_1 - \mathbf{C}_2\mathbf{A}_{22}^{-1}\mathbf{A}_{21} & \mathbf{D} - \mathbf{C}_2\mathbf{A}_{22}^{-1}\mathbf{B}_2 \end{bmatrix} \qquad (2.3)$$

The preserving of transfer functions for DC-gains and for low frequencies is important in context of the low order model approximation as well as for the preserving of model behaviour for the motion control (preserving rigid body motion components [1]). The flexible mechanical system with proportional damping enables simplification of the general operations from (2.3) using modal form (1.6)-(1.8).

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} = diag(\mathbf{A}_{mi}), \mathbf{A}_{mi} = \begin{bmatrix} 0 & \Omega_i \\ -\Omega_i & -2b_{di}\Omega_i \end{bmatrix}. \qquad (2.4)$$

The special form of the system matrix in given modal description (1.6)-(1.8) simplifies the reduced state space model (2.3) coming from SPA. Firstly the matrices $\mathbf{A}_{12} = \mathbf{0}, \mathbf{A}_{21} = \mathbf{0}$, so that

$$SS_{\mathrm{mod}\,spa} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{B}_1 \\ \mathbf{C}_1 & \mathbf{D} - \mathbf{C}_2\mathbf{A}_{22}^{-1}\mathbf{B}_2 \end{bmatrix}. \qquad (2.5)$$

Secondly the simple block diagonal structure leads to a very simple form of the inversed submatrix $\mathbf{A}_{22}^{-1}$. Let n is the number of DOF/eigenmodes in ROM and N the number of DOF/eigenmodes in original model before reduction. So that the original submatrix $\mathbf{A}_{22}$ is

$$\mathbf{A}_{22} = \begin{bmatrix} 0 & \Omega_{n+1} & 0 & 0 & \dots & 0 & 0 \\ -\Omega_{n+1} & -2b_{n+1}\Omega_{n+1} & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \Omega_{n+2} & \dots & 0 & 0 \\ 0 & 0 & -\Omega_{n+2} & -2b_{n+2}\Omega_{n+2} & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 0 & \Omega_N \\ 0 & 0 & 0 & 0 & \dots & -\Omega_N & -2b_N\Omega_N \end{bmatrix}, \qquad (2.6)$$

and the inversed submatrix $\mathbf{A}_{22}^{-1}$ has again block diagonal structure

$$\mathbf{A}_{22}^{-1} = \begin{bmatrix} -2b_{n+1}/\Omega_{n+1} & -1/\Omega_{n+1} & 0 & 0 & \dots & 0 & 0 \\ 1/\Omega_{n+1} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & -2b_{n+2}/\Omega_{n+2} & -1/\Omega_{n+2} & \dots & 0 & 0 \\ 0 & 0 & 1/\Omega_{n+2} & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & -2b_N/\Omega_N & -1/\Omega_N \\ 0 & 0 & 0 & 0 & \dots & 1/\Omega_N & 0 \end{bmatrix}. \qquad (2.7)$$

This presents very advantageous simplification, seeing that N is typically very high number of DOF of the original model coming from FEM modeling. All around the additional part of the feedthrough matrix $\mathbf{D}$ for structural systems with proportional damping owing to singular perturbation is $\Delta\mathbf{D} = -\mathbf{C}_2\mathbf{A}_{22}^{-1}\mathbf{B}_2$ including simple block diagonal form of $\mathbf{A}_{22}^{-1}$ obtainable without numerical effort of the general matrix inversion.

III. RESIDUAL MODE FOR STRUCTURAL SYSTEM WITH DIFFERENT OUTPUTS

The particular form of $\mathbf{D}$ and $\Delta\mathbf{D}$ will be further studied for different variants of the mechanical outputs namely positions, velocities and accelerations. The general form of the output equation is

$$\mathbf{y} = \mathbf{C}\mathbf{z} + \mathbf{D}\mathbf{u}\,. \qquad (3.1)$$

In the case of position output of the whole system $\mathbf{y} = \mathbf{x} = \mathbf{V}\mathbf{q}$ or using state vector $\mathbf{z}$

$$\mathbf{y} = \lfloor \mathbf{V}_1 / \Omega_1 \mathbf{0} \mathbf{V}_2 / \Omega_2 \mathbf{0} \dots \mathbf{V}_N / \Omega_N \mathbf{0} \rfloor \mathbf{z} \qquad (3.2)$$

Therefore the corresponding **position output matrices** of the whole system are

$$C_{POS} = \begin{bmatrix} V_1 / \Omega_1 & 0 & V_2 / \Omega_2 & 0 & \dots & V_N / \Omega_N & 0 \end{bmatrix}. \qquad (3.3)$$
$$D_{POS} = [0]$$

In the case of velocity output of the whole system $\mathbf{y} = \dot{\mathbf{x}} = \mathbf{V}\dot{\mathbf{q}}$ or using state vector $\mathbf{z}$

$$\mathbf{y} = \lfloor \mathbf{0} \quad \mathbf{V}_1 \quad \mathbf{0} \quad \mathbf{V}_2 \quad \dots \quad \mathbf{0} \quad \mathbf{V}_N \rfloor \mathbf{z}\,. \qquad (3.4)$$

Therefore the corresponding **velocity output matrices** of the whole system are

$$\mathbf{C}_{VEL} = \begin{bmatrix} \mathbf{0} & \mathbf{V}_1 & \mathbf{0} & \mathbf{V}_2 & \dots & \mathbf{0} & \mathbf{V}_N \end{bmatrix}. \qquad (3.5)$$
$$\mathbf{D}_{VEL} = [\mathbf{0}]$$

The acceleration output $\mathbf{y} = \ddot{\mathbf{x}} = \mathbf{V}\ddot{\mathbf{q}}$ cannot be evaluated only from states, the state derivatives must be substituted from the matrix dynamic equation. Based on equations (1.2)-(1.5)

$$\mathbf{y} = \ddot{\mathbf{x}} = \mathbf{V}\ddot{\mathbf{q}} = \mathbf{V}\left(-2\mathbf{b}_d\boldsymbol{\Omega}\dot{\mathbf{q}} - \boldsymbol{\Omega}^2\mathbf{q} + \mathbf{V}^T\mathbf{f}\right) = \qquad (3.6)$$
$$= -2\mathbf{V}\mathbf{b}_d\boldsymbol{\Omega}\dot{\mathbf{q}} - \mathbf{V}\boldsymbol{\Omega}^2\mathbf{q} + \mathbf{V}\mathbf{V}^T\mathbf{f}$$

or using state vector $\mathbf{z}$

$$\mathbf{y} = \lfloor -\mathbf{V}_1\Omega_1 - 2\mathbf{V}_1 b_{d1}\Omega_1 - \mathbf{V}_2\Omega_2 - 2\mathbf{V}_2 b_{d2}\Omega_2 \dots - \mathbf{V}_N\Omega_N - 2\mathbf{V}_N b_{dN}\Omega_N \rfloor \mathbf{z} + \qquad (3.7)$$
$$+ \left(\sum_{i=1}^{N} \mathbf{V}_i \mathbf{V}_i^T \right) \mathbf{u}$$

Therefore the corresponding **acceleration output matrices** of the whole system are

$$\mathbf{C}_{ACC} = \left[-\mathbf{V}_1\Omega_1 - 2\mathbf{V}_1 b_{d1}\Omega_1 - \mathbf{V}_2\Omega_2 - 2\mathbf{V}_2 b_{d2}\Omega_2 \ldots - \mathbf{V}_N\Omega_N - 2\mathbf{V}_N b_{dN}\Omega_N\right]$$
$$\mathbf{D}_{ACC} = \left[\sum_{i=1}^{N}\mathbf{V}_i\,\mathbf{V}_i^T\right] \tag{3.8}$$

The reduction of the model based on truncation (1.1) does not require any further operations, computation of reduction based on singular perturbation approximation requires evaluation of $\Delta\mathbf{D} = -\mathbf{C}_2\mathbf{A}_{22}^{-1}\mathbf{B}_2$ for matrices (1.7), (1.2), (3.3), (3.5) and (3.8). The evaluation of the matrix $\mathbf{D}_{\mathrm{SPA}}$ for different types of the outputs can be simply developed by the symbolic operations (e.g. in Matlab). Based on this the following results summarized in table has been obtained. The most important result of this analysis is the simple and clear form of the residualization (singular perturbation approximation) also for the acceleration outputs.

	truncation (TRU)	residualization (SPA)
$\mathbf{D}_{POS}$ ($\mathbf{y}=\mathbf{x}$)	$\mathbf{D}_{POS,TRU} = \mathbf{0}$	$\mathbf{D}_{POS,SPA} = \sum_{i=n+1}^{N}\frac{\mathbf{V}_i\mathbf{V}_i^T}{\Omega_i^2}$
$\mathbf{D}_{VEL}$ ($\mathbf{y}=\dot{\mathbf{x}}$)	$\mathbf{D}_{VEL,TRU} = \mathbf{0}$	$\mathbf{D}_{VEL,SPA} = \mathbf{0}$
$\mathbf{D}_{ACC}$ ($\mathbf{y}=\ddot{\mathbf{x}}$)	$\mathbf{D}_{ACC,TRU} = \sum_{i=1}^{N}\mathbf{V}_i\,\mathbf{V}_i^T$	$\mathbf{D}_{ACC,SPA} = \sum_{i=1}^{n}\mathbf{V}_i\,\mathbf{V}_i^T$

Feedthrough matrices of structure for different reduction method and different outputs

Matrix $\mathbf{D}_{POS,SPA}$ can be further simplified thanks to the property of the static flexibility matrix $\mathbf{G} = \mathbf{K}^{-1}$. This matrix can be composed from the eigenmodes matrix $\mathbf{V}$ and diagonal eigenfrequencies matrix $\mathbf{\Omega}$. Firstly for the matrix inversion can be generally written, that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ and consequently also $(\mathbf{ABC})^{-1} = \mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1}$. Concerning equation (1.4) for flexible modes it is valid simultaneously, that $\mathbf{\Omega}^2 = \mathbf{V}^T\mathbf{K}\mathbf{V}$. Further starting from the identity can be stepwise concluded

$$\begin{aligned}
\mathbf{K}^{-1} &= \mathbf{K}^{-1}\\
\mathbf{K}^{-1} &= \mathbf{V}\mathbf{V}^{-1}\mathbf{K}^{-1}\left(\mathbf{V}^T\right)^{-1}\mathbf{V}^T\\
\mathbf{K}^{-1} &= \mathbf{V}\left(\mathbf{V}^{-1}\mathbf{K}^{-1}\left(\mathbf{V}^T\right)^{-1}\right)\mathbf{V}^T\\
\mathbf{K}^{-1} &= \mathbf{V}\left(\mathbf{V}^T\mathbf{K}\mathbf{V}\right)^{-1}\mathbf{V}^T\\
\mathbf{G} = \mathbf{K}^{-1} &= \mathbf{V}\left(\mathbf{\Omega}^2\right)^{-1}\mathbf{V}^T = \mathbf{V}\mathbf{\Omega}^{-2}\mathbf{V}^T = \sum_{i=1}^{N}\frac{\mathbf{V}_i\mathbf{V}_i^T}{\Omega_i^2}
\end{aligned} \tag{3.9}$$

Therefore finally we get

$$\mathbf{D}_{POS,SPA} = \sum_{i=n+1}^{N}\frac{\mathbf{V}_i\mathbf{V}_i^T}{\Omega_i^2} = \sum_{k=1}^{N}\frac{\mathbf{V}_k\mathbf{V}_k^T}{\Omega_k^2} - \sum_{j=1}^{n}\frac{\mathbf{V}_j\mathbf{V}_j^T}{\Omega_j^2} = \mathbf{G} - \sum_{j=1}^{n}\frac{\mathbf{V}_j\mathbf{V}_j^T}{\Omega_j^2} \tag{3.10}$$

It means, that for the evaluation of SPA feedthrough matrices for the acceleration as well as for the position outputs we need only the reduced system eigenmodes of number n instead of all N eigenmodes of the original large system.

IV. RESIDUAL MODE FOR COUPLING OF STRUCTURAL SUBSYSTEMS

The computer aided design of machines often make use of analysis of dynamical properties of composed system compact of simplified models of particular subsystems. The example of such task can be the optimization of mechanical machine properties in many positions across the workspace [4]. The interconnection of models of two mechanical parts (subsystems) is an analogy of the interconnection of the model of mechanical system and the controller in the feedback loop. It is well known in the structural control context, that the closeness of the resulting behaviour of the model and the original composed mechatronic system is significantly influenced by the feedthrough components. The unrealistic values of the “system zeros” caused by neglecting of the feedthrough components are discussed e.g. in [1]. The wrong positions of the model zeros consequently induced unrealistic dynamic properties of the composed mechatronical system with the feedbacks. The lesson learned is, that also in the context of subsystem modelling of structures the feedthrough components shouldn’t be neglected. The interconnection of the substructures into one entity can be seen as the special version of the feedback loop closure.

The optimization of the system dynamical properties using computations with interconnected simplified substructures takes into account two main goals – closeness of the simplified and original system together with computational efficiency.

The idea of interconnection will be further demonstrated using simple scheme (Fig. 2). Let the subsystems I and II are connected in points A and B by two springs k_x, k_y. From the point of view of equations (1.8) and (3.1) the input vector u of each subsystem is composed from interconnection forces in springs, whereas the output vector y is composed from displacements of point A or B ($[x_A\; y_A]^T$, $[x_B\; y_B]^T$).

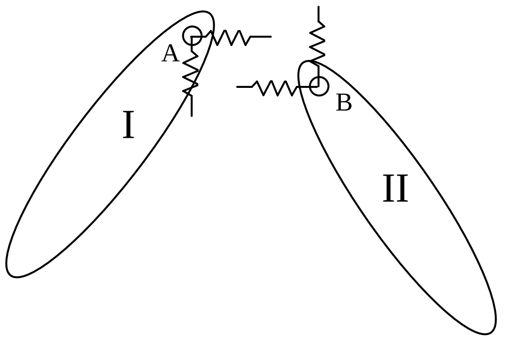

Fig. 2: A simple scheme of two connected bodies.

From displacement of connected points can be derived forces in springs like

$$F_x = k_x({}^{II}x_B - {}^{I}x_A)$$
$$F_y = k_y({}^{II}y_B - {}^{I}y_A) \tag{4.1}$$

Each body (system) is described by the state-space in modal coordinates. The body I will be described as follows

$$\begin{bmatrix} {}^{I}\Omega_1 {}^{I}\dot{q}_1 \\ {}^{I}\ddot{q}_1 \\ \vdots \\ {}^{I}\Omega_1 {}^{I}\dot{q}_j \\ {}^{I}\ddot{q}_j \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 & {}^{I}\Omega_1 & 0 & 0 & 0 & 0 \\ -{}^{I}\Omega_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & {}^{I}\Omega_j & 0 \\ 0 & 0 & \dots & -{}^{I}\Omega_j & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & \ddots \end{bmatrix} \begin{bmatrix} {}^{I}\Omega_1 {}^{I}\dot{q}_1 \\ {}^{I}\ddot{q}_1 \\ \vdots \\ {}^{I}\Omega_1 {}^{I}\dot{q}_j \\ {}^{I}\ddot{q}_j \\ \vdots \end{bmatrix} + \begin{bmatrix} 0 \\ {}^{I}B_1 \\ \vdots \\ 0 \\ {}^{I}B_j \\ \vdots \end{bmatrix} \begin{bmatrix} {}^{I}u \end{bmatrix} \tag{4.2}$$

Outputs of the state-space system are coordinates of the connecting point of the body "A"

$$\begin{bmatrix} {}^{I}x_A \\ {}^{I}y_A \end{bmatrix} = \begin{bmatrix} \frac{{}^{I}V_{xA1}}{{}^{I}\Omega_1} & 0 & \cdots & \frac{{}^{I}V_{xAj}}{{}^{I}\Omega_2} & 0 & \cdots \\ \frac{{}^{I}V_{yA1}}{{}^{I}\Omega_1} & 0 & \cdots & \frac{{}^{I}V_{yAj}}{{}^{I}\Omega_j} & 0 & \cdots \end{bmatrix} \begin{bmatrix} {}^{I}\Omega_1 {}^{I}q_1 \\ {}^{I}\dot{q}_1 \\ \vdots \\ {}^{I}\Omega_j {}^{I}q_j \\ {}^{I}\dot{q}_j \\ \vdots \end{bmatrix} + \begin{bmatrix} {}^{I}D_{SUB,SPA} \end{bmatrix} [u] \tag{4.3}$$

Where **u** contains interconnection forces

$${}^{I}u = \begin{bmatrix} F_x \\ F_y \end{bmatrix} \tag{4.4}$$

and

$${}^{II}u = \begin{bmatrix} -F_x \\ -F_y \end{bmatrix} \tag{4.5}$$

There is the second body (II) described in the analogy with the previous descriptions of the body (I).

All deformations in the static analysis on the body can be determined using the stiffness matrix **K**, vector of forces **f** and vector of deformations **x** from the equation

$$Kx = f \tag{4.6}$$

like

$$x = K^{-1} f = Gf \tag{4.7}$$

where **G** is the compliance matrix of the propriate body.

Sparse matrices are generally used for FEM models. Only a few components from matrix **G** are needed and they can be solved from term

$$\mathbf{KG} = \mathbf{I} \tag{4.8}$$

The procedure of computation for only one desired component of the **G** matrix is simple for sparse matrices, because the vector of forces contains zeros and only one nonzero element as follows

$$\mathbf{Kx} = \begin{bmatrix} 0 & \dots & 1 & 0 & \dots \end{bmatrix}^T \tag{4.9}$$

It is a system of linear equations which can be solved fast. Determining the part $D_{SUB,SPA}$ is also easily (3.10), because it compounds of a small part of **G** matrix and a subspace of $\sum_{j=1}^{n} \frac{\mathbf{V}_j \mathbf{V}_j^T}{\Omega_j^2}$ corresponding to coordinates of points A and B and slave eigenmodes.

Coordinates of the body II can be evaluated analogically with term (4.3). We will become four equations for coordinates $[x_A \ y_A]^T$ and $[x_B \ y_B]^T$.

Equation (4.3) providing coordinates of the point "A" will be completed with equations for the point "B". Terms ${}^{I}D_{SUB,SPA}$ and ${}^{II}D_{SUB,SPA}$ will be divided into four elements

$${}^{I}D_{SUB,SPA} = \begin{bmatrix} {}^{I}D_{11} & {}^{I}D_{12} \\ {}^{I}D_{21} & {}^{I}D_{22} \end{bmatrix} \tag{4.10}$$

and

$${}^{II}D_{SUB,SPA} = \begin{bmatrix} {}^{II}D_{11} & {}^{II}D_{12} \\ {}^{II}D_{21} & {}^{II}D_{22} \end{bmatrix} \tag{4.11}$$

These four equations will be rewritten in the form

$$\begin{bmatrix} {}^{I}x_A \\ {}^{I}y_A \\ {}^{II}x_B \\ {}^{II}y_B \end{bmatrix} = \begin{bmatrix} \frac{{}^{I}V_{xA1}}{{}^{I}\Omega_1} & 0 & \dots & \frac{{}^{I}V_{xAm}}{{}^{I}\Omega_m} & 0 & 0 & 0 & \dots & 0 & 0 \\ \frac{{}^{I}V_{yA1}}{{}^{I}\Omega_1} & 0 & \dots & \frac{{}^{I}V_{yAm}}{{}^{I}\Omega_m} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & \frac{{}^{II}V_{xB1}}{{}^{II}\Omega_1} & 0 & \dots & \frac{{}^{II}V_{xBn}}{{}^{II}\Omega_n} & 0 \\ 0 & 0 & \dots & 0 & 0 & \frac{{}^{II}V_{yB1}}{{}^{II}\Omega_1} & 0 & \dots & \frac{{}^{II}V_{yBn}}{{}^{II}\Omega_1} & 0 \end{bmatrix} \begin{bmatrix} {}^{I}\Omega_1 {}^{I}q_1 \\ {}^{I}\dot{q}_1 \\ \vdots \\ {}^{I}\Omega_m {}^{I}q_m \\ {}^{I}\dot{q}_m \\ {}^{II}\Omega_1 {}^{II}q_1 \\ {}^{II}\dot{q}_1 \\ \vdots \\ {}^{II}\Omega_n {}^{II}q_n \\ {}^{II}\dot{q}_n \end{bmatrix} +$$
$$+ \begin{bmatrix} -k_x {}^{I}D_{11} & -k_y {}^{I}D_{12} & k_x {}^{I}D_{11} & k_y {}^{I}D_{12} \\ -k_x {}^{I}D_{21} & -k_y {}^{I}D_{22} & k_x {}^{I}D_{21} & k_y {}^{I}D_{22} \\ k_x {}^{II}D_{11} & k_y {}^{II}D_{12} & -k_x {}^{II}D_{11} & -k_y {}^{II}D_{12} \\ k_x {}^{II}D_{21} & k_y {}^{II}D_{22} & -k_x {}^{II}D_{21} & -k_y {}^{II}D_{22} \end{bmatrix} \begin{bmatrix} {}^{I}x_A \\ {}^{I}y_A \\ {}^{II}x_B \\ {}^{II}y_B \end{bmatrix} \tag{4.12}$$

symbolically

$$x_{AB} = V_{AB} q_{AB} + D_{AB} x_{AB} \tag{4.13}$$

where "m" is the number of eigenmodes of the system I and "n" of the system II.

This term is resolved by the rearranging of the equation

$$x_{AB} = (I - D_{AB})^{-1} V_{AB} q_{AB} \quad (4.14)$$

These coordinates are after resolving substituted into the interconnection forces (4.1) to the original coupled systems of bodies in inputs ${}^{I}u$ and ${}^{II}u$ (4.2).

Two bodies were generated (Fig. 3) by FEM and after connection are used for the simple practical demonstration of the properties.

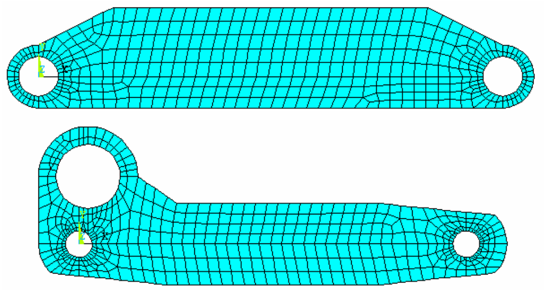

Fig. 3: Connected bodies

Both bodies are described in modal coordinates by spectral and eigenmodes matrices. These matrices were used for two types of reduction. The first reduction is reduction by truncation (TRU) of slave modes and the second by reduction including the residual term (SPA). Interconnecting forces are also generated by these methods. There are observed high differences if the TRU or SPA systems are used (Fig. 4).

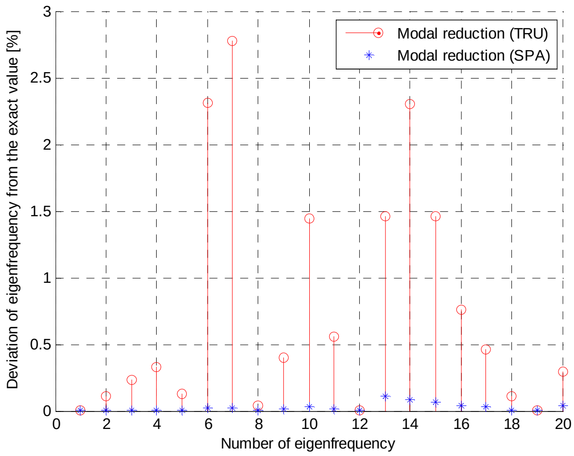

Fig. 4: Erors of first 20 eigenfrequencies for connected bodies modeled by two types of reductions SPA/TRU

It is clearly shown (Fig. 4) that the system composed from the SPA reduced parts reaches by far better results than with the TRU reduced parts. The number of modes of the reduced parts is the same (30) for both versions (SPA/TRU).

V.CONCLUSION

The systematical analysis of the reduction of the large structures using the singular perturbation approximation (SPA) for the interconnected subsystems has been presented. The proportionally damped systems have been considered in the special modal form identical to the almost balanced form of the model. The residual modes have been efficiently evaluated for the different types of outputs, namely position, velocity and acceleration form. The evaluation of SPA feedthrough matrices for the acceleration as well as for the position outputs we need only the reduced system eigenmodes instead of all eigenmodes of the original large system. Two main targets of the paper are the efficiency of the algorithms and the accuracy of the coupled reduced model with respect to the original large model. The efficient algorithm of the connection of subsystems is presented and the significant improvement of the accuracy with respect to the interconnection of the truncated subsystems have been demonstrated.

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