DISTRIBUTED CONSTRAINTS SATISFACTION

BE4M36MAS - Multiagent systems

CENTRALIZED CASE

Constraint satisfaction problem

Find an assignment for variables that satisfy given constraints.

- $\mathcal{X} = \{x_1, \ldots, x_n\}$ — set of *variables* to assign
- $\mathcal{D} = \{D_1, \ldots, D_n\}$ — set of *domains* ($x_i \in D_i$)
- $\mathcal{C} = \{C_1, \ldots, C_m\}$ — set of *constraints*

$C_i \subseteq D_{i_1} \times \cdots \times D_{i_r}$ denotes a r-ary constraint over variables $x_{i_1}, \ldots, x_{i_r}$

Constraint satisfaction problem

Solution: n-tuple $(d_1, \cdots, d_n)$, such that:

- $d_i \in D_i$, for $1 \leq i \leq n$
- $(d_{i_1}, \ldots, d_{i_r}) \in C_k$ for every constraint $C_k \subseteq D_{i_1} \times \cdots \times D_{i_r}$

Centralized algorithm

Synchronized backtracking

$v_i \leftarrow$ value from D_i consistent with $(v_1, \ldots, v_{i-1})$;
if *No such* v_i *exists* **then**
    backtrack ;
else if $i = n$ **then**
    stop ;
else
    `ChooseValue`$(x_{i+1}, (v_1, \ldots, v_i))$;
end

Algorithm 1: `ChooseValue`$(x_i, (v_1, \ldots, v_{i-1}))$

Centralized algorithm

Enhancements?

- AC-3 algorithm? (arc consistency - e.g. combinatorial optimization)

DISTRIBUTED CASE

Distributed case

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- $\mathcal{A} = \{A_1, \ldots, A_k\}$ — set of *agents*

Every variable **must be** assigned to one of the agents.

$\rightarrow$ otherwise the DCSP problem is **not fully defined**

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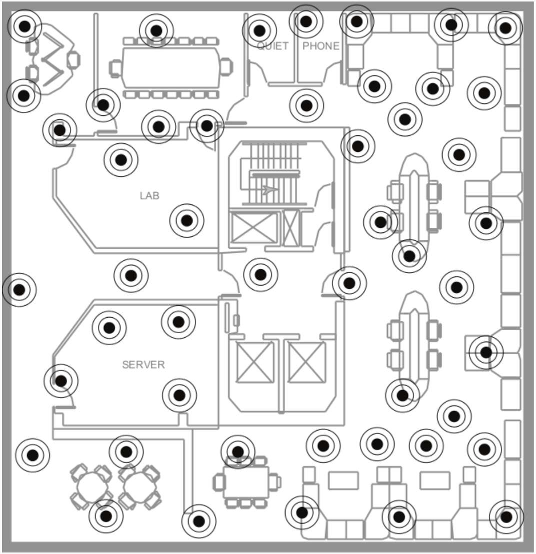
Example



ASYNCHRONOUS BACKTRACKING

What do we assume?

- Every agent controls a single variable
- Constraints are binary

- Messages are delivered in a finite time (but this time may vary randomly)
- Messages from a single agent are delivered in the order they were sent

  $\rightarrow$ imagine an unreliable TCP/IP network

What does an agent initially knows?

- Total ordering of agents (priorities)
- Constraints he is involved in
- Domain of a variable controlled by himself

Data structures

- Current assignment
- Set of outgoing links ($\sim$ who needs to know my assignment)
- Set of incoming links ($\sim$ who will notify me about his assignment)
- **Agent view** — agent's idea about current assignment of other agents

  $\rightarrow$ May be out of sync!
- **Nogood store** — justification of forbidden values in the domain

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Minimalistic example — meeting scheduling

- John needs to arrange a meeting with Bob and Alice
- As all agents, he is a busy guy — both meetings must happen in a single day
- Bob doesn't know about Alice's meeting and vice versa

Minimalistic example — meeting scheduling

$\mathcal{A} = \{\text{Alice}, \text{Bob}, \text{John}\}$

$\mathcal{X} = \{x_{\text{Alice}}, x_{\text{Bob}}, x_{\text{John}}\}$
Agent i controls variable x_i.

$\mathcal{D} = \{D_{\text{Alice}}, D_{\text{Bob}}, D_{\text{John}}\}$
$D_{\text{Alice}} = \{\text{Mon}, \text{Thu}\}$
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$\mathcal{C} = \{x_{\text{Bob}} = x_{\text{John}}, x_{\text{Alice}} = x_{\text{John}}\}$

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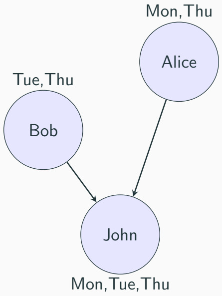


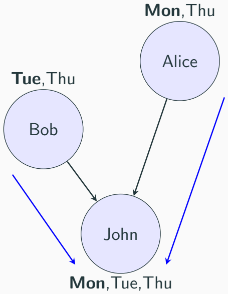
Minimalistic example — meeting scheduling

Alice: $\varnothing$ Bob: $\varnothing$ John: $\varnothing$

Let's all propose a date and see what happens!

Bob $\rightarrow$ John:
Ok?(Bob $\rightarrow$ Tue)

Alice $\rightarrow$ John:
Ok?(Alice $\rightarrow$ Mon)



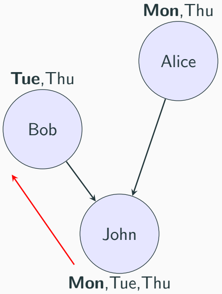
Alice: $\varnothing$ Bob: $\varnothing$ John: $\{\text{Alice} \rightarrow \text{Mon}, \text{Bob} \rightarrow \text{Tue}\}$

Minimalistic example — meeting scheduling

Alice: $\varnothing$ Bob: $\varnothing$ John: $\{\text{Alice} \to \text{Mon}, \text{Bob} \to \text{Tue}\}$

John: Argh, I wanted to have both meetings in one day :-(Let's make them change their minds...

John $\to$ Bob:
$\text{Nogood}(\{\text{Bob} \to \text{Tue}, \text{Alice} \to \text{Mon}\})$



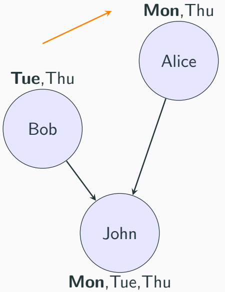
Alice: $\varnothing$ Bob: $\{\text{Alice} \to \text{Mon}\}$ John: $\{\text{Alice} \to \text{Mon}\}$

Minimalistic example — meeting scheduling

Alice: $\varnothing$ Bob: $\{\text{Alice} \to \text{Mon}\}$ John: $\{\text{Alice} \to \text{Mon}\}$

Bob: Who is that Alice? I've never heard of her.

Bob $\to$ Alice:
AddLink(Alice $\to$ Bob)



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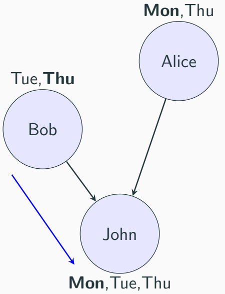
Bob: John told me that the meeting cannot happen on Tuesday if Alice opts for Monday. Let's try Thursday then...

Bob $\to$ John:
Ok?(Bob $\to$ Thu)



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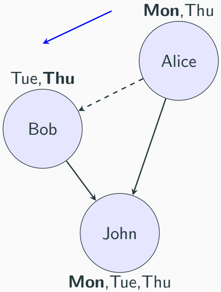
Minimalistic example — meeting scheduling

Alice: $\varnothing$ Bob: $\{\text{Alice} \to \text{Mon}\}$ John: $\{\text{Alice} \to \text{Mon}, \text{Bob} \to \text{Thu}\}$

Alice: Bob, why are you so curious?

Alice $\to$ Bob:

Ok?(Alice $\to$ Mon)



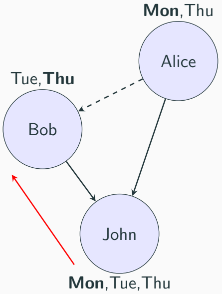
Alice: $\varnothing$ Bob: $\{\text{Alice} \to \text{Mon}\}$ John: $\{\text{Alice} \to \text{Mon}, \text{Bob} \to \text{Thu}\}$

Minimalistic example — meeting scheduling

Alice: $\varnothing$ Bob: $\{\text{Alice} \to \text{Mon}\}$ John: $\{\text{Alice} \to \text{Mon}, \text{Bob} \to \text{Thu}\}$

John: They tried it again. Alright, one more try...

John $\to$ Bob:
Nogood($\{\text{Bob} \to \text{Thu}, \text{Alice} \to \text{Mon}\}$)



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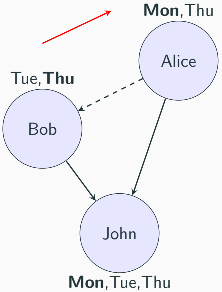
Bob: I have run out of options. It's up to Alice now...

Bob $\to$ Alice:
Nogood($\{\text{Alice} \to \text{Mon}\}$)



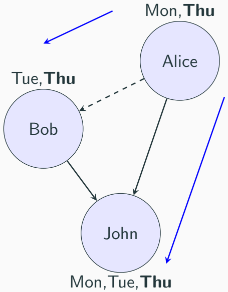
Alice: $\varnothing$ Bob: $\varnothing$ John: $\{\text{Alice} \to \text{Mon}\}$

Minimalistic example — meeting scheduling

Alice: $\varnothing$ Bob: $\varnothing$ John: $\{\text{Alice} \to \text{Mon}\}$

Alice: I have one more option, let's try Thursday.

Alice $\to$ Bob, John:
$\color{blue}{\text{Ok?}(\{\text{Alice} \to \text{Thu}\})}$

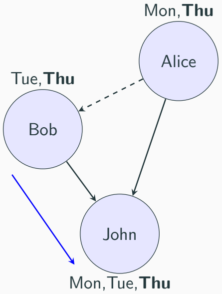


Alice: $\varnothing$ Bob: $\{\text{Alice} \to \text{Thu}\}$ John: $\{\text{Alice} \to \text{Thu}\}$

Minimalistic example — meeting scheduling

Alice: $\varnothing$ Bob: $\{\text{Alice} \rightarrow \text{Thu}\}$ John: $\{\text{Alice} \rightarrow \text{Thu}\}$

John: Finally. Thursday seems like a viable option.



Alice: $\varnothing$ Bob: $\{\text{Alice} \rightarrow \text{Thu}\}$ John: $\{\text{Alice} \rightarrow \text{Thu}, \text{Bob} \rightarrow \text{Thu}\}$

Asynchronous backtracking

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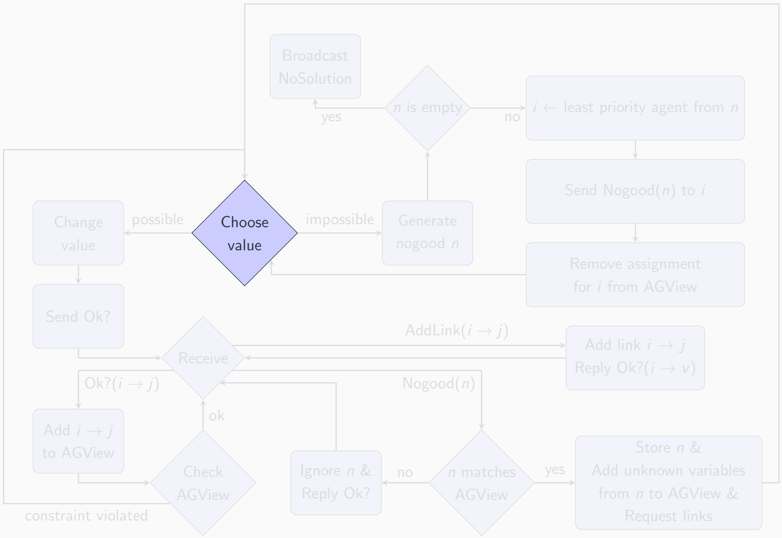

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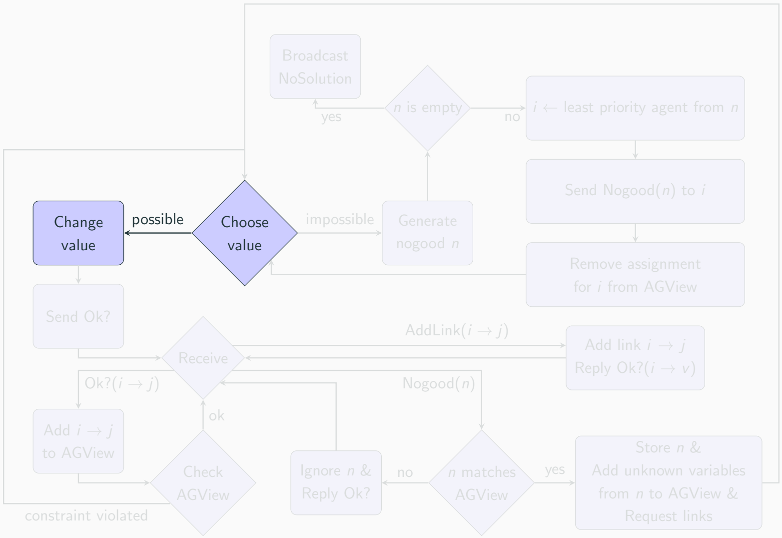

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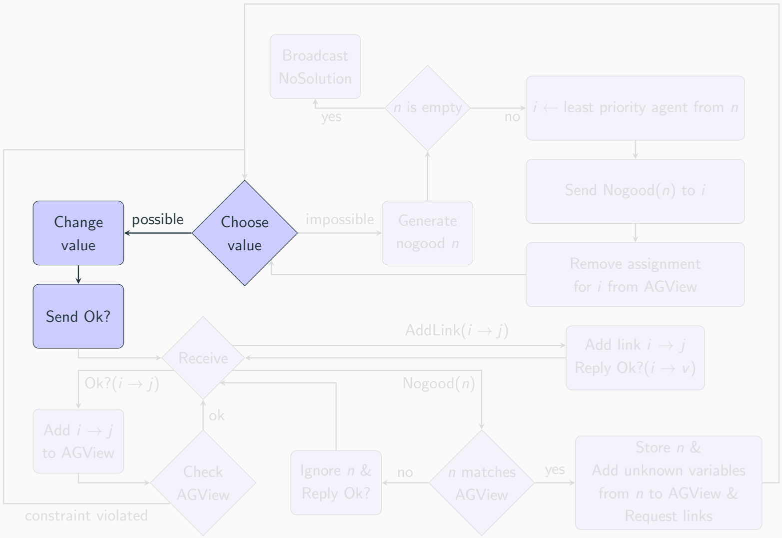

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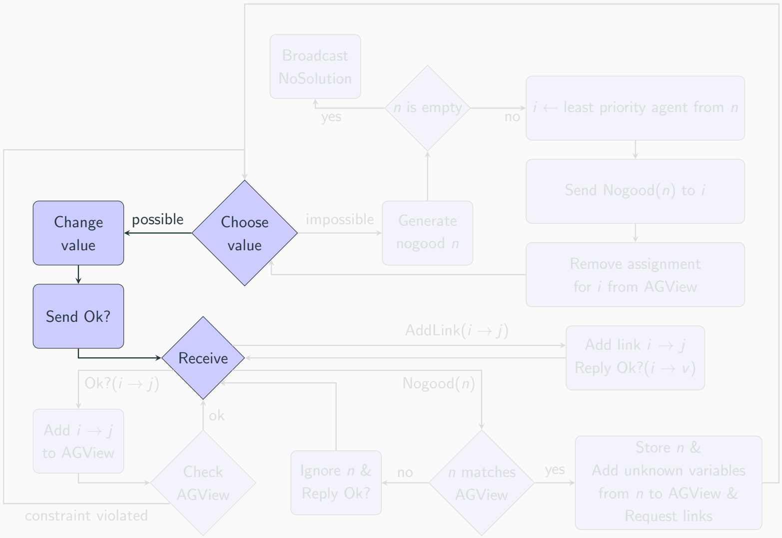

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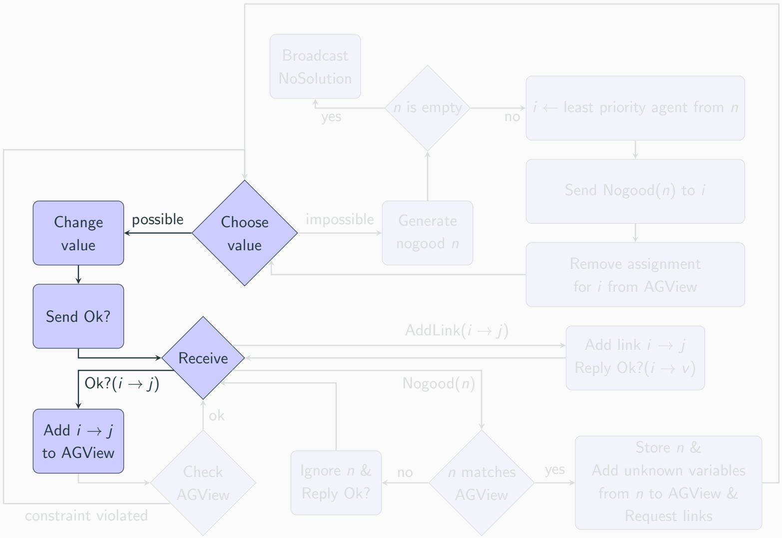

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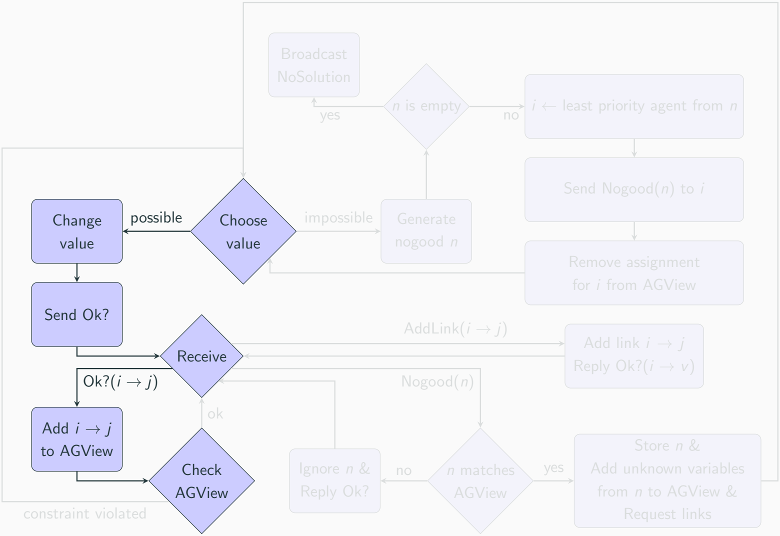

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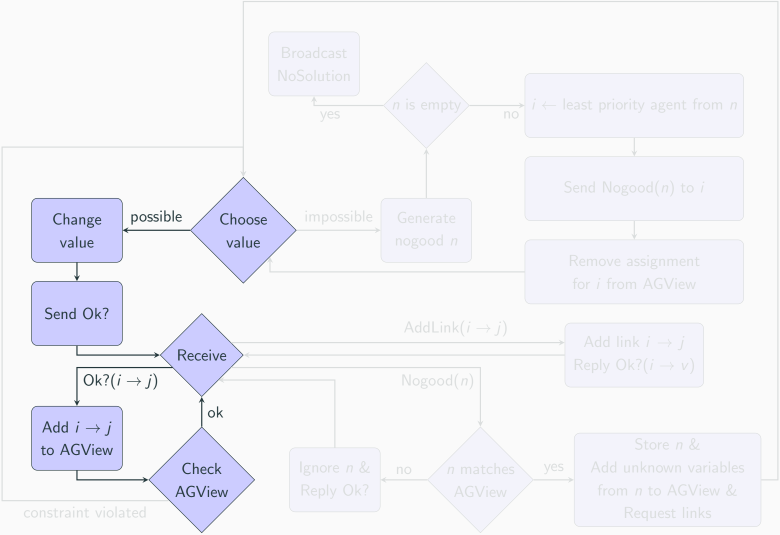

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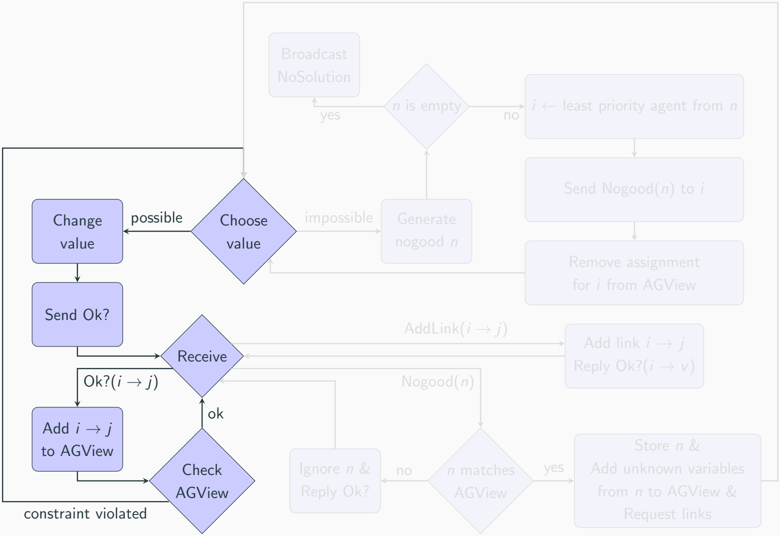

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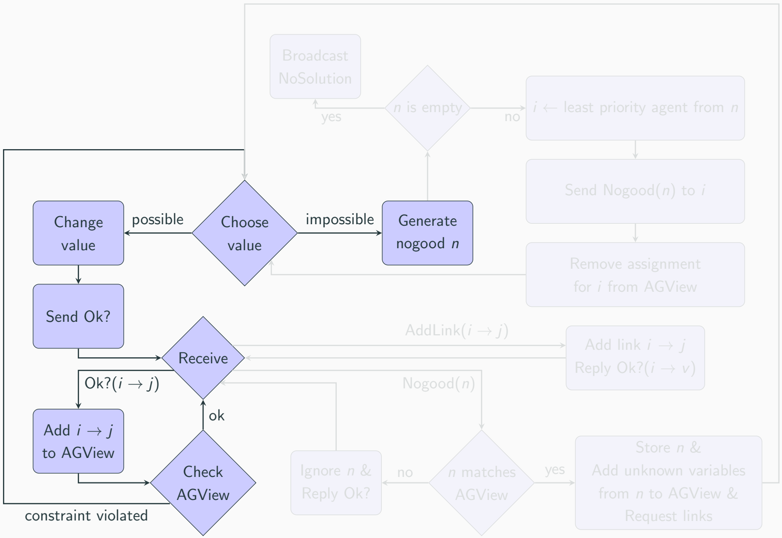

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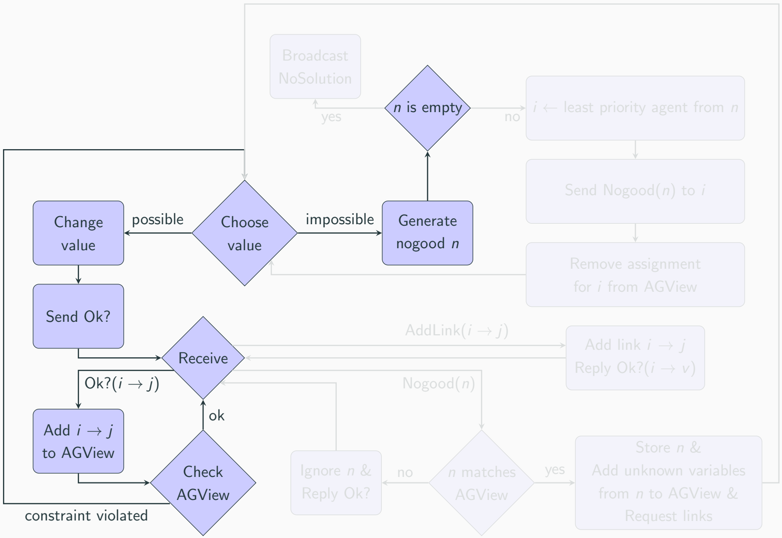

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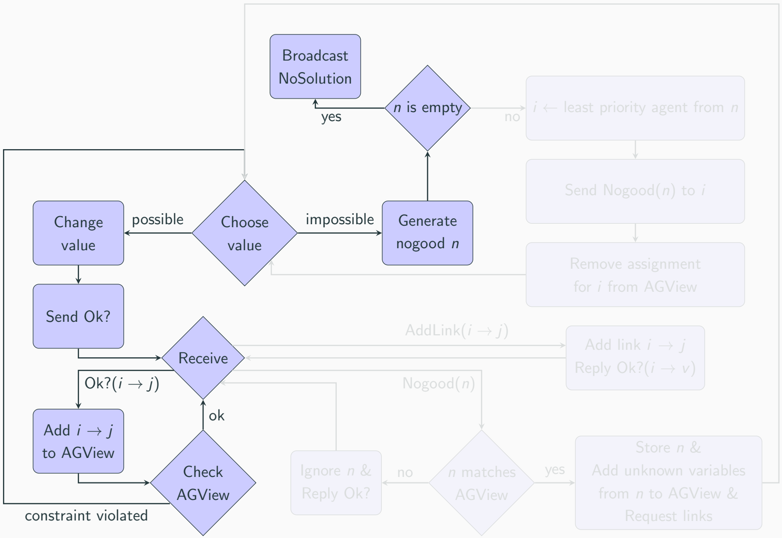

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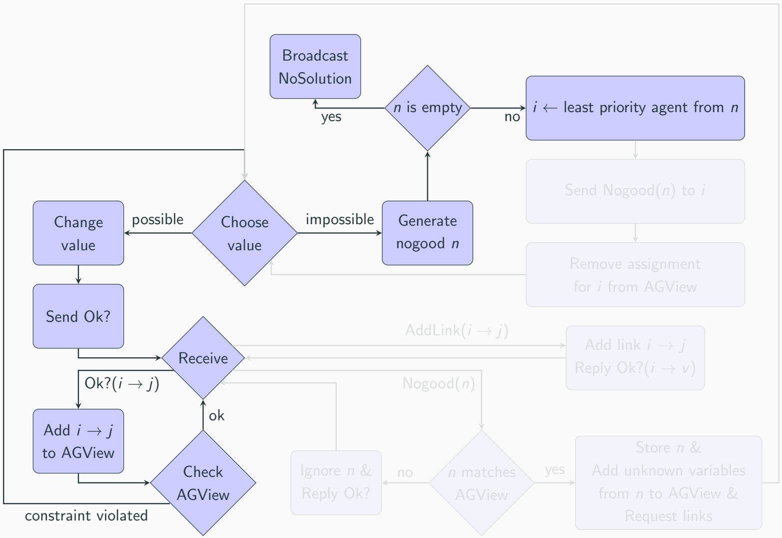

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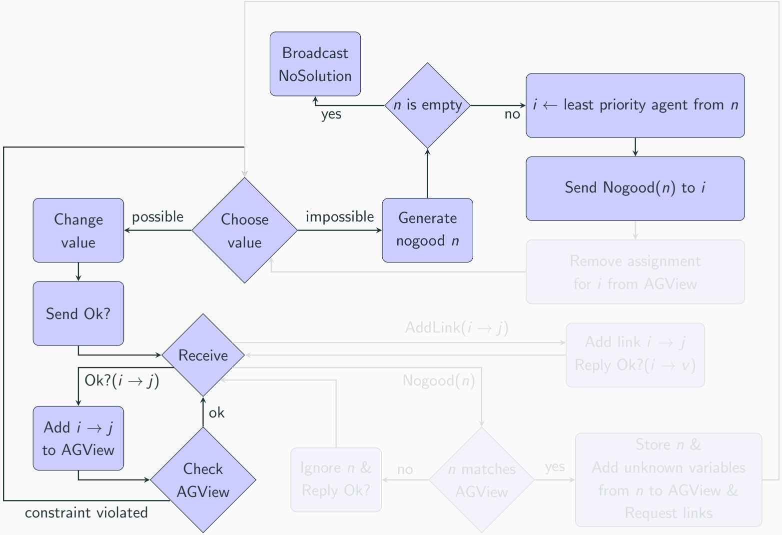

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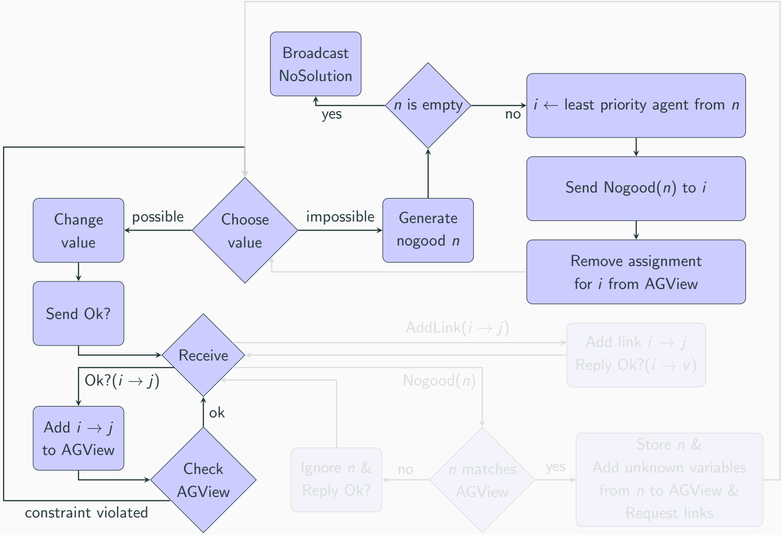

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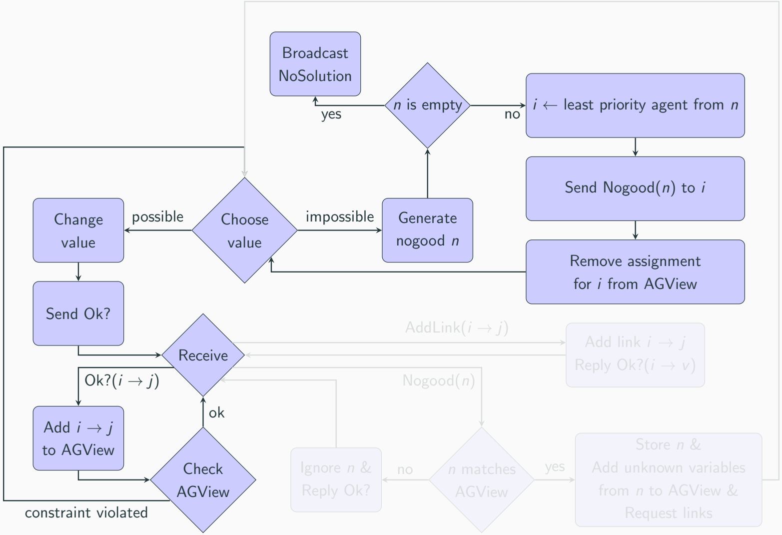

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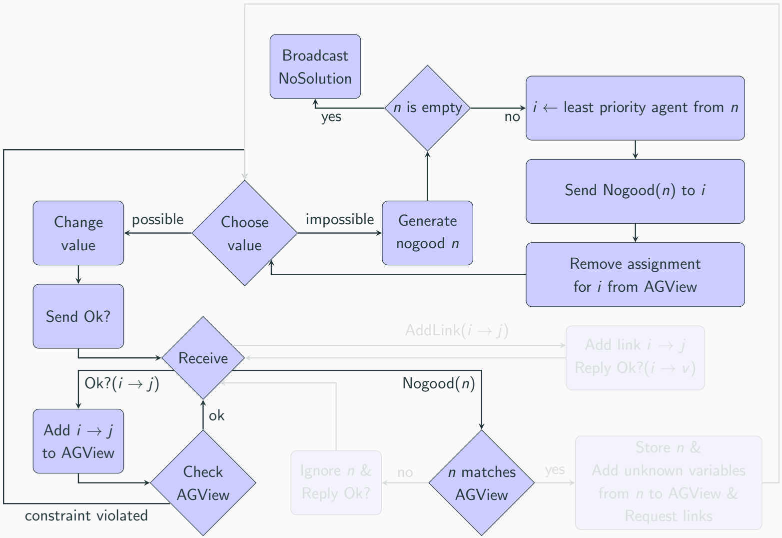

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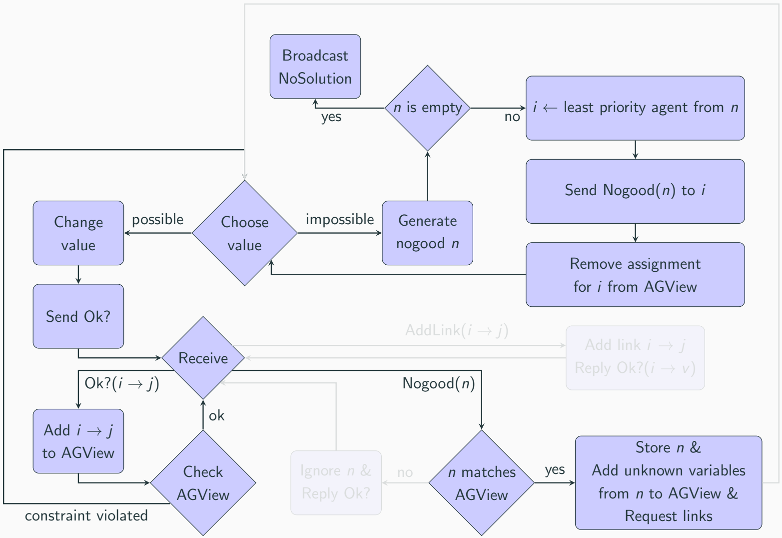
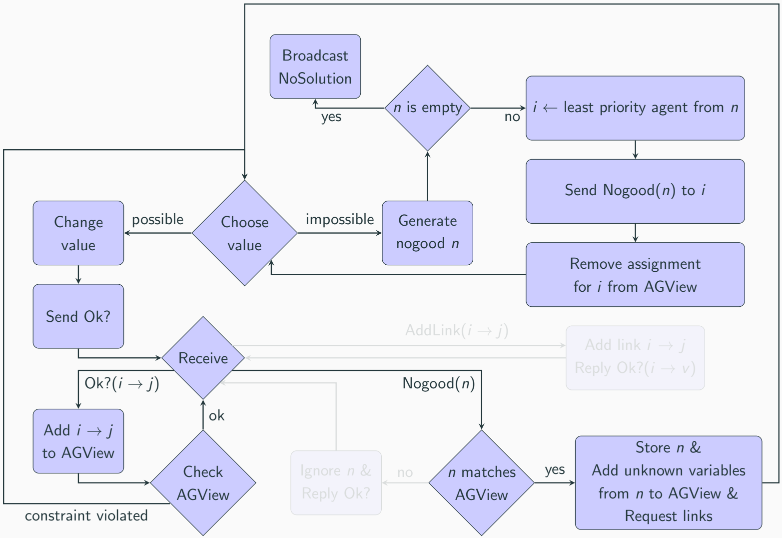
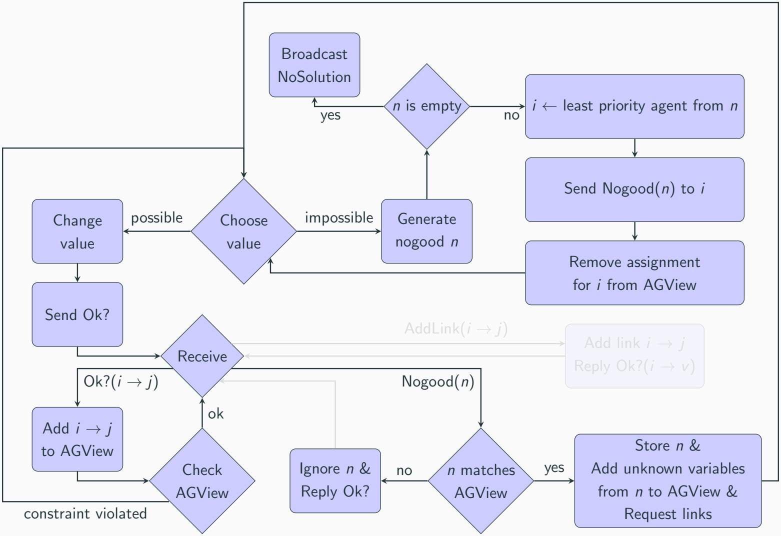



Broadcast NoSolution
yes
n is empty
no
i ← least priority agent from n
Send Nogood(n) to i
Remove assignment for i from AGView
Change value
possible
Choose value
impossible
Generate nogood n
Send Ok?
Receive
AddLink(i → j)
Add link i → j
Reply Ok?(i → v)
Ok?(i → j)
Nogood(n)
ok
Add i → j to AGView
Check AGView
Ignore n &
Reply Ok?
no
n matches AGView
yes
Store n &
Add unknown variables from n to AGView &
Request links
constraint violated

Asynchronous backtracking



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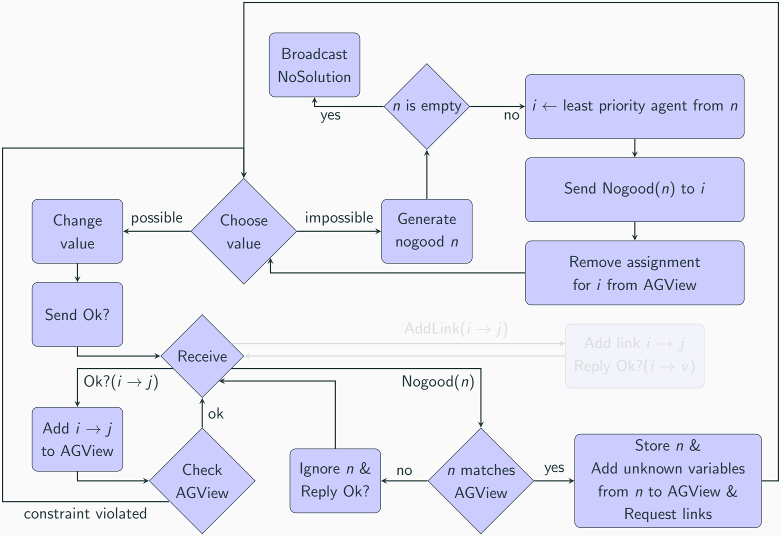

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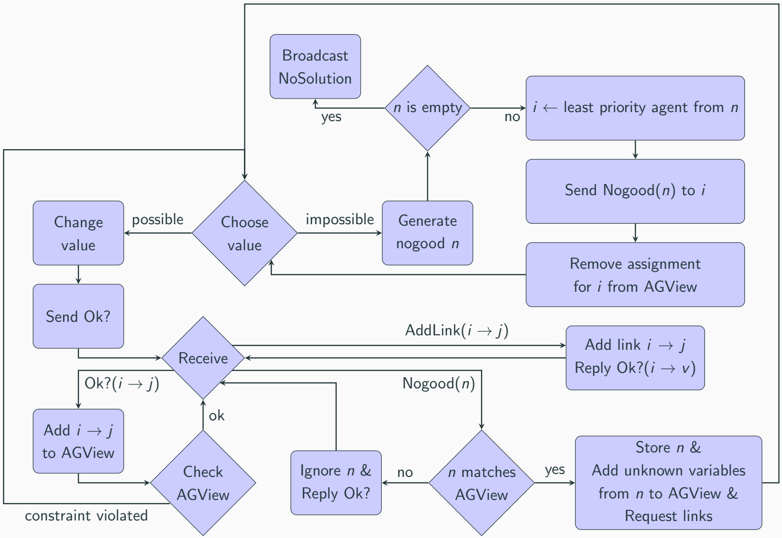


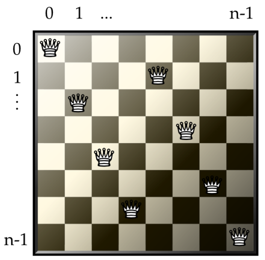
ASSIGNMENT

n-Queens problem in a distributed way

n queens from a $n \times n$ world had a serious dispute:

- They don't want to know of each other (i.e. no queen wants to have any other in her line of sight)
- They don't talk to each other except for few formal messages Ok? Nogood AddLink

Help them to find their place in the world!



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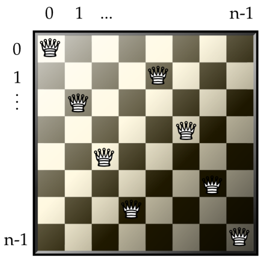

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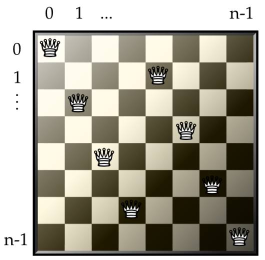
- They don't want to know of each other (i.e. no queen wants to have any other in her line of sight)
- They don't talk to each other except for few formal messages Ok? Nogood AddLink

Help them to find their place in the world!

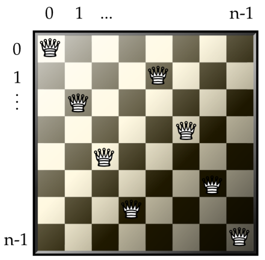


n-Queens problem in a distributed way

n queens from a $n \times n$ world had a serious dispute:

- They don't want to know of each other (i.e. no queen wants to have any other in her line of sight)
- They don't talk to each other except for few formal messages
  Ok? Nogood AddLink

Help them to find their place in the world!



n-Queens problem in a distributed way

Every agent controls **one queen** and decides about her position within its row.

In the end, one of the following has to happen:

- One of the agents reports that no solutions exists
- Each queen reports her position in her row (i.e. a column in which it is located)

  $\uparrow$ of course correctly ;-)

n-Queens problem in a distributed way

Any **asynchronous** and **distributed** solution is acceptable (e.g. ABT).

$\rightarrow$ No centralized knowledge allowed!

$\rightarrow$ No synchonization!

$\rightarrow$ No hardcoded solutions!

n-Queens problem in a distributed way

Total: **12 points**

- Solve 3×3 chessboard problem with 3 queens (3 points)
- Solve 4×4 chessboard problem with 4 queens (2 points)
- Solve 8×8 chessboard problem with 8 queens (2 points)
- Solve 12×12 chessboard problem with 12 queens (3 points)

n-Queens problem in a distributed way

Guaranteed termination detection (**1 point**)

- How to detect *quiescence* in an algorithmic way?
- You may want to get inspired by other DCSP/DCOP algorithms.

Quiescence should be discovered using **local knowledge** only.

$\rightarrow$ Sending whole solution to a single agent for verification is not an option!

n-Queens problem in a distributed way

Report (**1 point**)

- How is the n-queens problem modeled as a DCSP? (variables, domains, constraints, agents)
- How is the ABT algorithm customized for the n-queens problem?
- How do you determine priorities between agents?
- How do you detect that the search has terminated?

ABT EXAMPLE