Logical reasoning and programming, task III

(December 11, 2017)

Problem

Your task is to modify the following `leantap` Prolog program in such a way that it is able to produce a tableau proof tree (at the last additional 6th argument of original predicate `prove/5`) that exactly correspond to the original `leantap` computation.

```
% prove(+Fml,+UnExp,+Lits,+FreeV,+VarLim)

prove((A,B),UnExp,Lits,FreeV,VarLim) :- !,
   prove(A,[B|UnExp],Lits,FreeV,VarLim).

prove((A;B),UnExp,Lits,FreeV,VarLim) :- !,
    prove(A,UnExp,Lits,FreeV,VarLim),
    prove(B,UnExp,Lits,FreeV,VarLim).

prove(all(X,Fml),UnExp,Lits,FreeV,VarLim) :- !,
    \+ length(FreeV,VarLim),
    copy_term((X,Fml,FreeV),(X1,Fml1,FreeV)),
    append(UnExp,[all(X,Fml)],UnExp1),
    prove(Fml1,UnExp1,Lits,[X1|FreeV],VarLim).

prove(Lit,_,[L|Lits],_,_) :-
    ( Lit = -Neg; -Lit = Neg ) -> ( unify_with_occurs_check(Neg,L)
                                  ; prove(Lit,[],Lits,_,_) ).

prove(Lit,[Next|UnExp],Lits,FreeV,VarLim) :-
    prove(Next,UnExp,[Lit|Lits],FreeV,VarLim).
```

Tableau Proof Format

The tableau proof tree format is a Prolog repesentation of Semantic Tableau tree. Every node in the tree has its unique *identifier* that can be Prolog integer or Prolog atom. Every tableau rule can add one (in case of `add_rule`, `all_rule`, and `closed_by`) or two (in case of `or_rule`) leaves to the current working node. Every node has the following format:

`node`(*identifier*,*corresponding NNF formula*,*used rule*,*list of its sub-nodes/sons*)

where *used rule* can be a term `add_rule`, `or_rule`, `all_rule`, or `closed_by` with arguments that contain identifiers as references to previous nodes that are needed by this rule to infer the current node. The root of the Semantic Tablueau tree has a node with `top` identifier. Our tableau proof tree does not have this node but you can refer `top` as identifier. The *corresponding NNF formula*, that is inferred by `closed_by` rule, is `false`.

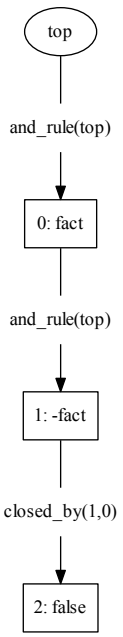
The tableau proof tree starts with a list of sub-nodes of `top`. All substitutions, that are needed, are already applied on the resulting tableau proof tree.

Program

You are supposed to upload a program `leantap2.pl`, in an archive, containing a predicate `prove/6`, where the last argument is the output tableau proof and the other arguments correspond one to one to the original `leantap` implementation. Of course, you can use additional predicates in your solution.

Example 1

```
?- prove((fact,-fact), [], [], [], 0, Proof).
Proof = [node(1, fact, and_rule(top),
              [node(2, -fact, and_rule(top),
                    [node(3, false, closed_by(2, 1), [])])
              ])
        ]
```



Example 2

```
?- prove((all(X,p(X)) , (-p(c);-p(d))), [], [], [], 3, Proof).
Proof = [node(0, all(B, p(B)), and_rule(top), [node(2, p(c), all_rule(0), [
         node(3, (-p(c);-p(d)), and_rule(top), [node(4, -p(c), or_rule(3), [
         node(5, false, closed_by(4, 2), [])]), node(6, -p(d), or_rule(3), [
         node(7, p(d), all_rule(0), [node(8, false, closed_by(7, 6), [])
         ])])])])])]
```