Logical reasoning and programming, lab session IV

(October 23, 2017)

IV.1 Formalize the following scenario in Prolog:

> Basil owns Fawlty Towers. Basil and Sybil are married. Polly and Manuel are employees at Fawlty Towers. Smith and Jones are guests at Fawlty Towers. All hotel-owners and their spouses serve all guests at the hotel. All employees at a hotel serve all guests at the hotel. All employees dislike the owner of the workplace. Basil dislikes Manuel.

Then ask the queries “Who serves who?” and “Who dislikes who?”.

IV.2 We can represent natural numbers as numerals using 0 and the successor function s, where s(0) is 1, s(s(0)) is 2 etc.

Assume the predicate `plus/3` from the lecture

```
plus(0,X,X).
plus(s(X),Y,s(Z)):-plus(X,Y,Z).
```

and a variant `plus2/3` (using an accumulator)

```
plus2(0,X,X).
plus2(s(X),Y,Z):-plus2(X,s(Y),Z).
```

Test their behavior on some inputs. Do they produce always the same results? How would you define predicate `minus/3` from them? How many variants are there?

(Hint: try `?-plus(X,Y,s(s(0))).` and `?-plus2(X,Y,s(s(0))).`)

IV.3 Give the Herbrand universe and Herbrand base of the following program:

```
p(f(X)):-q(X, g(X)).
q(a, g(b)).
q(b, g(b)).
```

Give the least Herbrand model (minimal model) of the program.

IV.4 Consider the Herbrand universe consisting of constants a, b, c and d. Let $\mathcal{H}$ be the Herbrand interpretation

$$\{\, p(a), p(b), q(a), q(b), q(c), q(d) \,\}.$$

Which of the following formulae are true in $\mathcal{H}$?

(a) $\forall X p(X)$,

(b) $\forall X q(X)$,

(c) $\exists X (q(X) \wedge p(X))$,

(d) $\forall X (q(X) \rightarrow p(X))$,

(e) $\forall X(p(X) \rightarrow q(X))$.

IV.5 Recall that clauses with at most one positive literal, say $A \leftarrow B_1, \ldots, B_n$, are called definite. A program is definite if it contains only definite clauses. Prove the model intersection property: Let M be a non-empty family of Herbrand models of a definite program P. Then $\bigcap M$ is also a Herbrand model of P.

(Hint: Take two Herbrand models of P, say H_1 and H_2, and assume that $H_1 \cap H_2$ is not a Herbrand model of P. Hence it fails for a definite clause in P. Is that possible? Use the fact that H_1 and H_2 are both models of this clause!)

IV.6 The model intersection property from the previous exercise does not hold for indefinite clauses. Why?