

# Data Collection Planning with Curvature-Constrained Vehicles

## Dubins Traveling Salesman Problem with Neighborhoods (DTSPN) and Dubins Orienteering Problem with Neighborhoods (DOPN)

Jan Faigl

Department of Computer Science  
Faculty of Electrical Engineering  
Czech Technical University in Prague

Lecture 09

### B4M36UIR – Artificial Intelligence in Robotics

# Overview of the Lecture

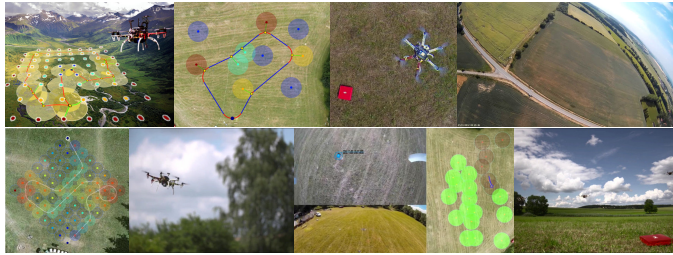
- Part 1 – Data Collection Planning – Aerial Surveillance Missions
  - Dubins Vehicle and Dubins Planning
  - Dubins Touring Problem (DTP)
  - Dubins Traveling Salesman Problem
  - Dubins Traveling Salesman Problem with Neighborhoods
  - Dubins Orienteering Problem
  - Dubins Orienteering Problem with Neighborhoods
  - Planning in 3D – Examples and Motivations
- Part 2 – Bonus HW03b – Data Collection Planning for Surveillance Missions
  - Motivation and Assignment

# Part I

## Part 1 – Data Collection Planning – Aerial Surveillance Missions

# Motivation – Surveillance Missions with Aerial Vehicles

- Provide **curvature-constrained** path to collect the **most valuable measurements** with **shortest possible path/time** or under **limited travel budget**



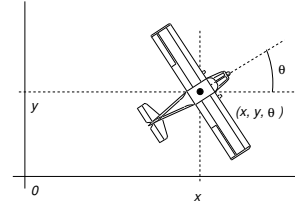
- Formulated as routing problems with Dubins vehicle
  - Dubins Traveling Salesman Problem with Neighborhoods
  - Dubins Orienteering Problem with Neighborhoods

# Dubins Vehicle

- Non-holonomic vehicle such as car-like or aircraft can be modeled as the Dubins vehicle
  - Constant forward velocity
  - Limited minimal turning radius  $\rho$
  - Vehicle state is represented by a triplet  $q = (x, y, \theta)$ , where  $(x, y) \in \mathbb{R}^2$ ,  $\theta \in \mathbb{S}^2$  and thus,  $q \in SE(2)$

The vehicle motion can be described by the equation:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos \theta \\ \sin \theta \\ \frac{u}{\rho} \end{bmatrix}, \quad |u| \leq 1,$$



where  $u$  is the control input.

# Optimal Maneuvers for Dubins Vehicle

- For two states  $q_1 \in SE(2)$  and  $q_2 \in SE(2)$  in the environment **without obstacles**  $\mathcal{W} = \mathbb{R}^2$  the optimal paths can be characterized as one of two main types
  - CCC type: LRL, RLR;
  - CSC type: LSL, LSR, RSL, RSR;
 where S – straight line arc, C – circular arc oriented to left (L) or right (R)
 

*L. E. Dubins (1957) – American Journal of Mathematics*
- The optimal paths are called **Dubins maneuvers**:
  - Constant velocity:  $v(t) = v$  and turning radius  $\rho$
  - 6 types of trajectories connecting any configuration in  $\mathbb{R}^2 \times \mathbb{S}^1$ , i.e.,  $SE(2)$  *without obstacles*
  - The control  $u$  is according to C and S type one of the three possible values  $u \in \{-1, 0, 1\}$

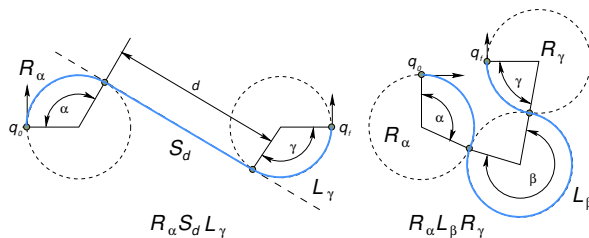
# Parametrization of Dubins Maneuvers

- Parametrization of each trajectory phase:

$$\{L_\alpha R_\beta L_\gamma, R_\alpha L_\beta R_\gamma, L_\alpha S_d L_\gamma, L_\alpha S_d R_\gamma, R_\alpha S_d L_\gamma, R_\alpha S_d R_\gamma\}$$

for  $\alpha \in [0, 2\pi)$ ,  $\beta \in (\pi, 2\pi)$ ,  $d \geq 0$

Notice the prescribed orientation at  $q_0$  and  $q_1$ .



# Planning with Dubins vehicle

- The optimal path connecting two configurations can be found analytically
 

*E.g., for UAVs that usually operates in environment without obstacles*
- The Dubins maneuvers can also be used in randomized-sampling based motion planners, such as RRT, in the control based sampling
- Dubins vehicle model can be considered in the multi-goal path planning
  - Surveillance, inspection or monitoring missions to periodically visits given target locations (areas)
- Dubins Touring Problem DTP**

Given a sequence of locations, what is the shortest path visiting the locations, i.e., what are headings of the vehicle at the locations.
- Dubins Traveling Salesman Problem DTSP**

Given a set of locations, what is the shortest Dubins path that visits each location exactly once and returns to the origin location.
- Dubins Orienteering Problem**

Given a set of locations, each with associated reward, what is the Dubins path visiting the most rewarding locations and not exceeding given travel budget.

# Dubins Planning

- Minimal turning radius  $\rho$
  - Constant forward velocity  $v$
  - State of the Dubins vehicle is  $q = (x, y, \theta)$ ,  $q \in SE(2)$ ,  $(x, y) \in \mathbb{R}^2$  and  $\theta \in \mathbb{S}^1$
- $$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos \theta \\ \sin \theta \\ \frac{u}{\rho} \end{bmatrix}$$
- 
- 
- Optimal path connecting  $q_1 \in SE(2)$  and  $q_2 \in SE(2)$  consists only of straight line arcs and arcs with the maximal curvature, i.e., two types of maneuvers CCC and CSC
  - The solution can be found analytically (Dubins, 1957)
  - The main difficulty is to determine the vehicle headings for the given sequence of waypoints**

## Dubins Touring Problem – DTP

- For a sequence of the  $n$  waypoint locations  $P = (p_1, \dots, p_n)$ ,  $p_i \in \mathbb{R}^2$ , the **Dubins Touring Problem (DTP)** stands to determine the **optimal headings**  $T = \{\theta_1, \dots, \theta_n\}$  at the waypoints  $q_i$  such that

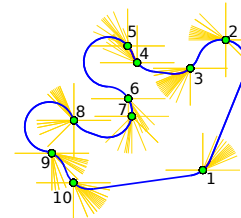
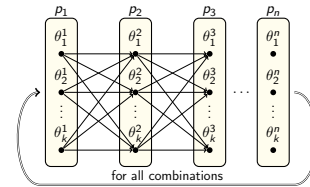
$$\begin{aligned} \text{minimize } T \quad & \mathcal{L}(T, P) = \sum_{i=1}^{n-1} \mathcal{L}(q_i, q_{i+1}) + \mathcal{L}(q_n, q_1) \\ \text{subject to} \quad & q_i = (p_i, \theta_i), \theta_i \in [0, 2\pi), p_i \in P, \end{aligned}$$

where  $\mathcal{L}(q_i, q_j)$  is the length of the optimal Dubins maneuver connecting  $q_i$  with  $q_j$

- The DTP is a **continuous optimization problem**
- The term  $\mathcal{L}(q_n, q_1)$  is for the possibly closed tour that can be for example requested in the TSP with Dubins vehicle, a.k.a. DTSP
- On the other, when the DTP can also be utilized for open paths such as solutions of the OP with Dubins vehicle*
- In some cases, it may be suitable to relaxed the heading at the first/last locations in finding closed tours (i.e., solving DTSP)

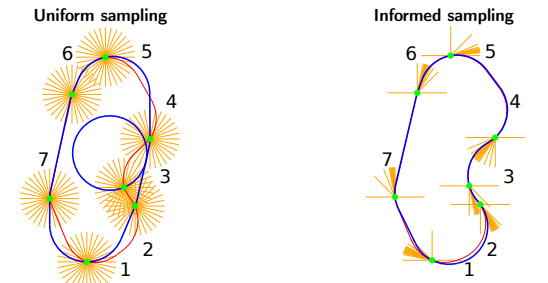
## Sampling-based Solution of the DTP

- For a closed sequence of the waypoint locations  $P = (p_1, \dots, p_n)$
- We can sample possible heading values at each location  $i$  into a discrete set of  $k$  headings, i.e.,  $\Theta^i = \{\theta_1^i, \dots, \theta_k^i\}$  and create a graph of all possible Dubins maneuvers



- For a set of heading samples, the optimal solution can be found by a forward search of the graph in  $O(nk^3)$
- For open sequence we do not need to evaluate all possible initial headings, and the complexity is  $O(nk^2)$*
- The key is to **determined the most suitable heading samples per each waypoint**

## Example of Heading Sampling – Uniform vs. Informed



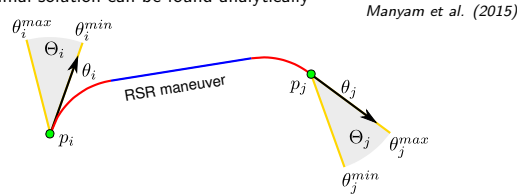
Uniform sampling:  $N = 224, T_{cpu} = 128$  ms,  $\mathcal{L} = 19.8, \mathcal{L}_U = 13.8$

Informed sampling:  $N = 128, T_{cpu} = 76$  ms,  $\mathcal{L} = 14.4, \mathcal{L}_U = 14.2$

- $N$  is the total number of samples, i.e., 32 samples per waypoint for uniform sampling
- $\mathcal{L}$  is the length of the tour (blue) and  $\mathcal{L}_U$  is the lower bound (red) determined as a solution of the **Dubins Interval Problem (DIP)**

## Dubins Interval Problem (DIP)

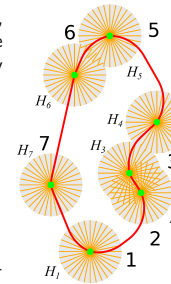
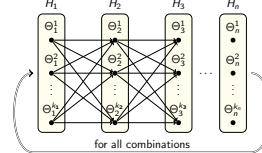
- Dubins Interval Problem (DIP)** is a generalization of Dubins maneuvers to the shortest path connecting two points  $p_i$  and  $p_j$
- In the DIP, an leaving interval  $\Theta_i$  at  $p_i$  and arrival interval  $\Theta_j$  at  $p_j$  are allowed
- The optimal solution can be found analytically



- Solution of the DIP is a tight lower bound for the DTP
- Solution of the DIP is not a feasible solution of the DTP
- Notice, for  $\Theta_i = \Theta_j = (0, 2\pi)$  the optimal maneuver for DIP is straight line segment*

## Lower Bound of the DTP

- For a discrete set of heading intervals  $\mathcal{H} = \{H_1, \dots, H_n\}$ , where  $H_i = \{\Theta_i^1, \Theta_i^2, \dots, \Theta_i^{k_i}\}$ , a similar graph as for the DTP can be constructed with edge cost determined by the solution of the associated DIP

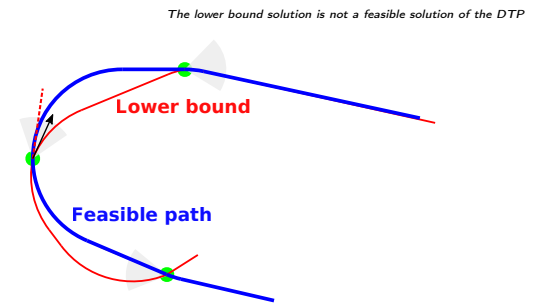


- The forward search of the graph with dense samples provides a **tight lower bound of the DTP**

*Manyam and Rathinam, 2015*

## Lower Bound and Feasible Solution of the DTP

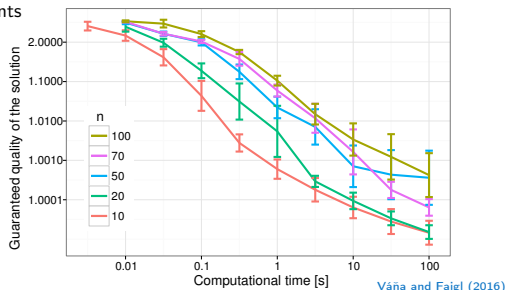
- The arrival and departure angles may not be the same



- DTP solution** – use any particular heading of each interval in the lower bound solution

## The DIP-based Sampling of Headings in the DTP

- A similar forward search graph as for the DTP can be used for heading intervals instead of particular headings
- Using DIP for a sequence of waypoints is a **lower bound** of the DTP
- It can be used to inform how to splitting heading intervals
- The ratio between the lower bound and feasible solution of the DTP provides estimation of the solution quality, e.g. for problems with  $n$  waypoints



## Informed Sampling of Headings in Solution of the DTP

- Iterative refinement of the heading intervals  $\mathcal{H}$  up to the angular resolution  $\epsilon_{req}$ . The angular resolution is gradually decreased for promising intervals

```

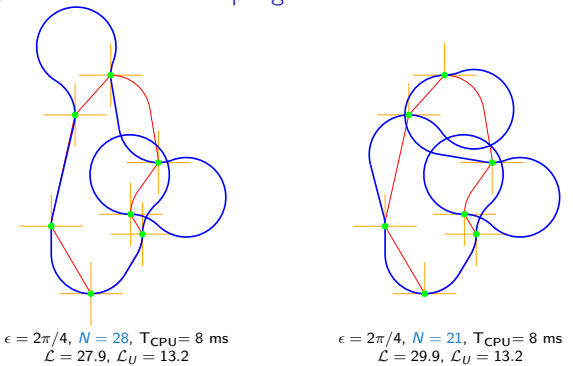
Algorithm 1: Iterative Informed Sampling-based DTP Algorithm
Vinput: P – Target locations to be visited
Vinput:  $\epsilon_{req}$  – Requested angular resolution
Vinput:  $\alpha_{req}$  – Requested quality of the solution
Vinput: T – A tour visiting the targets
 $\epsilon = 2\pi$  // initial angular resolution
 $\mathcal{H} = \text{createIntervals}(P, \epsilon)$  // initial intervals;
 $\mathcal{L}_L = 0$  // init lower bound;
 $\mathcal{L}_U = \infty$  // init upper bound;
while  $\epsilon > \epsilon_{req}$  and  $\mathcal{L}_U / \mathcal{L}_L > \alpha_{req}$  do
     $\epsilon = \epsilon / 2$ 
     $(N, \mathcal{L}_L) = \text{refineDTP}(P, \epsilon, N)$ ;
     $(T, \mathcal{L}_U) = \text{solveDTP}(P, N)$ ;
end
return T;
    
```

*Faigl, Váňa et al. (2017)*

- refineDTP** – divide the intervals of the lower bound solution
- solveDTP** – solve DTP using the heading from the refined intervals
- It simultaneously provides **feasible and lower bound** solution of the DTP
- First solution is provided very quickly – **any-time algorithm**

*The lower bound provides tight estimation of the solution quality*

## Uniform vs Informed Sampling

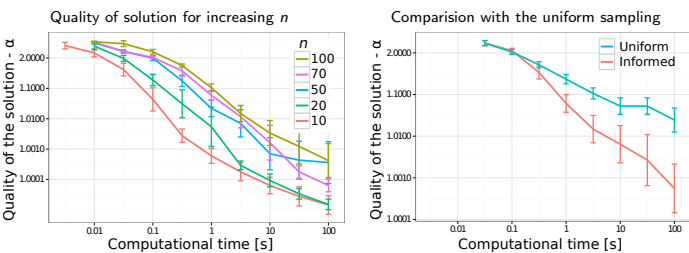


$\epsilon = 2\pi/4, N = 28, T_{CPU} = 8$  ms,  $\mathcal{L} = 27.9, \mathcal{L}_U = 13.2$

$\epsilon = 2\pi/4, N = 21, T_{CPU} = 8$  ms,  $\mathcal{L} = 29.9, \mathcal{L}_U = 13.2$

## Results and Comparison with Uniform Sampling

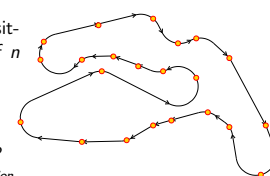
- Random instances of the DTSP with sequence determined as a solution of the Euclidean TSP
- The waypoints placed in a squared bounding box with the side  $s = (\rho\sqrt{n})/d$  for the  $\rho = 1$  and density  $d = 0.5$



- Informed sampling-based approach provides solutions up to 0.01% from the optima
- Solution of the DTP is a fundamental building block for routing problems with Dubins vehicle

## Dubins Traveling Salesman Problem (DTSP)

- Determine a closed shortest Dubins path visiting each location  $p_i \in P$  of the given set of  $n$  locations  $P = \{p_1, \dots, p_n\}$ ,  $p_i \in \mathbb{R}^2$
- Permutation  $\Sigma = (\sigma_1, \dots, \sigma_n)$  of visits  
*Sequencing part of the problem*
- Headings  $\Theta = \{\theta_{\sigma_1}, \theta_{\sigma_2}, \dots, \theta_{\sigma_n}\}$  for  $p_{\sigma_i} \in P$   
*Continuous optimization*



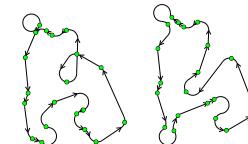
- DTSP is an optimization problem over all possible permutations  $\Sigma$  and headings  $\Theta$  in the states  $(q_{\sigma_1}, q_{\sigma_2}, \dots, q_{\sigma_n})$  such that  $q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i})$

$$\begin{aligned} \text{minimize}_{\Sigma, \Theta} \quad & \sum_{i=1}^{n-1} \mathcal{L}(q_{\sigma_i}, q_{\sigma_{i+1}}) + \mathcal{L}(q_{\sigma_n}, q_{\sigma_1}) \quad (1) \\ \text{subject to} \quad & q_i = (p_i, \theta_i) \quad i = 1, \dots, n, \quad (2) \end{aligned}$$

where  $\mathcal{L}(q_{\sigma_i}, q_{\sigma_j})$  is the length of Dubins path between  $q_{\sigma_i}$  and  $q_{\sigma_j}$ .

## Challenges of the Dubins Traveling Salesman Problem

- The key difficulty of the DTSP is that the path length mutually depends on
    - Order of the visits to the locations
    - Headings at the target locations
- We need the sequence to determine headings, but headings may influence the sequence*



Two fundamental approaches can be found in literature

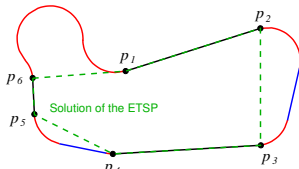
- Decoupled** approach based on a given sequence of the locations  
*E.g., found by a solution of the Euclidean TSP*
  - Sampling-based** approach with sampling the headings at the locations into discrete sets of values and considering the problem as the variant of the **Generalized TSP**
- Besides, further approaches are
- Approximation algorithms; optimal solutions for restricted variants
  - Genetic and memetic techniques (evolutionary algorithms)
  - Unsupervised learning based approaches

## Decoupled Solution of the DTSP – Alternating Algorithm

**Alternating Algorithm (AA)** provides a solution of the DTSP for an **even** number of targets  $n$

*Savla et al. (2005)*

- Solve the related Euclidean TSP  
*Relaxed motion constraints*
- Establish headings for even edges using straight line segments
- Determine optimal maneuvers for odd edges



*Courtesy of P. Váňa*

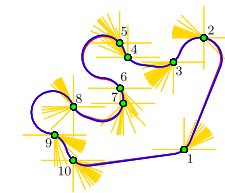
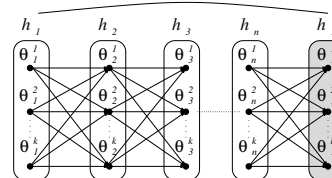
## DTSP with the Given Sequence of the Visits to the Targets

- If the sequence of the visits  $\Sigma$  to the target locations is given
- the problem is to determine the optimal heading at each location
- We call the problem as the **Dubins Touring Problem (DTP)**  
*Váňa and Faigl (2016)*

- Let for each location  $g_i \in G$  sample possible heading to  $k$  values, i.e., for each  $g_i$  the set of headings be  $h_i = \{\theta_1^i, \dots, \theta_k^i\}$ .
- Since  $\Sigma$  is given, we can construct a graph connecting two consecutive locations in the sequence by all possible headings
- For such a graph and particular headings  $\{h_1, \dots, h_n\}$ , we can find an optimal headings and thus, **the optimal solution of the DTP**.

## DTSP as a Solution of the DTP

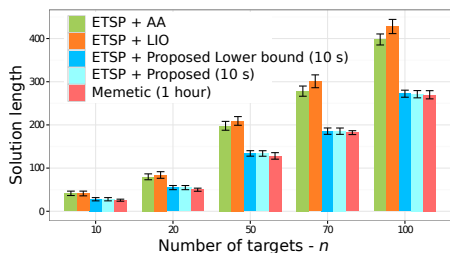
*The first layer is duplicated layer to support the forward search method*



- The edge cost corresponds to the length of Dubins maneuver
  - Better solution of the DTP can be found for more samples, which will also improve the DTSP but only for the given sequence
- Two questions arise for a practical solution of the DTP*
- How to sample the headings?** Since more samples makes finding solution more demanding
  - We need to sample the headings in a "smart" way.*
  - What is the solution quality? Is there a tight lower bound?**

## DTP Solver in Solution of the DTSP

- The solution of the DTP can be used to solve DTSP for the given sequence of the waypoints  
*E.g., determined as a solution of the Euclidean TSP as in the Alternating Algorithm*
- Comparison with the Algorithm Algorithm (AA), Local Iterative Algorithm (LIO) and Memetic algorithm  
*AA – Savla et al., 2005, LIO – Váňa & Faigl, 2015, Memetic – Zhang et al. 2014*



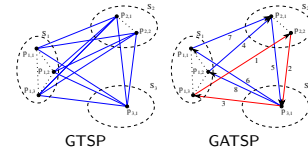
## DTSP – Sampling-based Approach

- Sampled heading values can be directly utilized to find the sequence as a solution of the **Generalized Traveling Salesman Problem (GTSP)**  
*Notice For Dubins vehicle, it is the Generalized Asymmetric TSP (GATSP)*

The problem is to determine a shortest tour in a graph that visits all specified subsets of the graph's vertices.

*The TSP is a special case of the GTSP when each subset to be visited consists just a single vertex.*

- GATSP  $\rightarrow$  ATSP  
*Noon and Bean (1991)*
- ATSP can be solved by LKH
- ATSP  $\rightarrow$  TSP, which can be solved optimally, e.g., by Concorde



## Dubins Traveling Salesman Problem with Neighborhoods

- In surveillance planning, it may be required to visit a set of target regions  $G = \{R_1, \dots, R_n\}$  by the Dubins vehicle
- Then, for each target region  $R_i$ , we have to determine a particular point of the visit  $p_i \in R_i$  and DTSP becomes the **Dubins Traveling Salesman Problem with Neighborhoods (DTSPN)**  
*In addition to  $\Sigma$  and headings  $\Theta$ , waypoint locations  $P$  have to be determined*
- DTSPN is an optimization problem over all permutations  $\Sigma$ , headings  $\Theta = \{\theta_{\sigma_1}, \dots, \theta_{\sigma_n}\}$  and points  $P = (p_{\sigma_1}, \dots, p_{\sigma_n})$  for the states  $(q_{\sigma_1}, \dots, q_{\sigma_n})$  such that  $q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i})$  and  $p_{\sigma_i} \in R_{\sigma_i}$ :

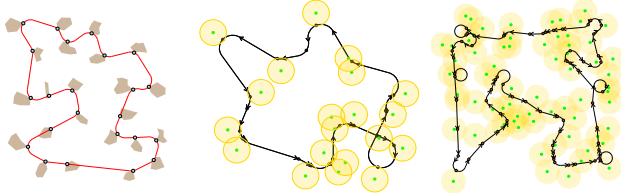
$$\begin{aligned} \text{minimize}_{\Sigma, \Theta, P} \quad & \sum_{i=1}^{n-1} \mathcal{L}(q_{\sigma_i}, q_{\sigma_{i+1}}) + \mathcal{L}(q_{\sigma_n}, q_{\sigma_1}) \quad (3) \\ \text{subject to} \quad & q_i = (p_i, \theta_i), p_i \in R_i \quad i = 1, \dots, n \quad (4) \end{aligned}$$

- $\mathcal{L}(q_{\sigma_i}, q_{\sigma_j})$  is the length of the shortest possible Dubins maneuver connecting the states  $q_{\sigma_i}$  and  $q_{\sigma_j}$



## DTSPN – Approches and Examples of Solution

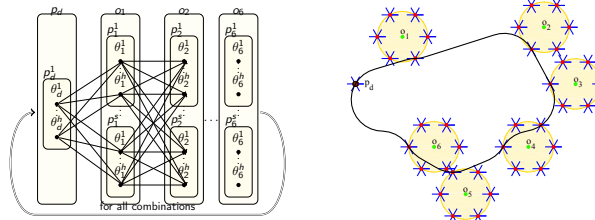
- Similarly to the DTSP, also DTSPN can be addressed by
  - Decoupled approaches** for which a sequence of visits to the regions can be found as a solution of the ETSP(N)
  - Sampling-based approaches** and transformation to the GTSP
    - Clusters of sampled waypoint locations each with sampled possible heading values
  - Soft-computing** techniques such as, memetic algorithms
  - Unsupervised learning** techniques



Váňa and Faigl (IROS 2015), Faigl and Váňa (ICANN 2016, IJCNN 2017)

## DTSPN – Decoupled Approach

- Determine a sequence of visits to the  $n$  target regions as the solution of the ETSP
- Sample possible waypoint locations and for each such a location sample possible heading values, e.g.,  $s$  locations per each region and  $h$  heading per each locations
- Construct a search graph and determine a solution in  $O(nsh)^3$
- Example of the search graph for  $n = 6$ ,  $s = 6$ , and  $h = 6$



Dubins Touring Region Problem (DTRP)

## DTSPN – Local Iterative Optimization (LIO)

- Instead of sampling into discrete set of waypoint locations and possible headings, we can perform local optimization using hill-climbing technique
- At each waypoint location  $p_i$ , the heading  $\Theta_i \in [0, 2\pi)$
- Each waypoint location  $p_i$  can be parametrized as a point on the boundary of the respective region  $R_i$ , i.e., as a parameter  $\alpha \in [0, 1)$  measuring a normalized distance on the boundary of  $R_i$
- The multi-variable optimization is treated independently for each particular variable  $\theta_i$  and  $\alpha_i$  iteratively

### Algorithm 2: Local Iterative Optimization (LIO) for the DTSPN

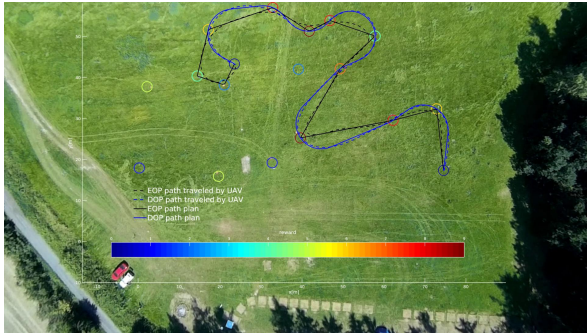
```

Data: Input sequence of the regions  $R = (R_{\sigma_1}, \dots, R_{\sigma_n})$ , for the permutation  $\Sigma$ 
Result: Waypoints  $(q_{\sigma_1}, \dots, q_{\sigma_n})$ ,  $q_i = (p_i, \theta_i)$ ,  $p_i \in \delta R_i$ 
initialization() // random assignment of  $q_i \in \delta R_i$ 
while global solution is improving do
  for every  $R_i \in R$  do
     $\theta_i := \text{optimizeHeadingLocally}(\theta_i)$ ;
     $\alpha_i := \text{optimizePositionLocally}(\alpha_i)$ ;
     $q_i := \text{checkLocalMinima}(\alpha_i, \theta_i)$ ;
  end
end
    
```

Váňa and Faigl (IROS 2015)

## Data Collection / Surveillance Planning with Travel Budget

- Visit the most important targets because of limited travel budget
- The problem can be formulated as the **Orienteering Problem** with Dubins vehicle, a.k.a. **Dubins Orienteering Problem (DOP)**  
Robert Pěnička, Jan Faigl, Petr Váňa and Martin Saska, RA-L 2017

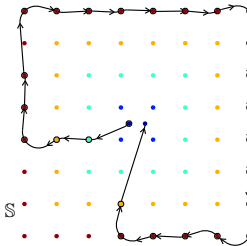


<http://mrs.felk.cvut.cz/icral7dop>

## Dubins Orienteering Problem

- Curvature-constrained data collection path respecting Dubins vehicle model with the minimal turning radius  $\rho$  and constant forward velocity  $v$
- The path is a sequence of waypoints  $q_i \in SE(2)$ ,  $q = (s, \theta)$ ,  $\theta \in S^1$ .
- In addition to  $S_k, k, \Sigma$  (OP) determine headings  $\Theta = (\theta_{\sigma_1}, \dots, \theta_{\sigma_k})$  such that

$$\begin{aligned}
 &\text{maximize}_{k, S_k, \Sigma} R = \sum_{i=1}^k r_{\sigma_i} \\
 &\text{subject to} \sum_{i=2}^k \mathcal{L}(q_{\sigma_{i-1}}, q_{\sigma_i}) \leq T_{max}, \\
 &q_{\sigma_i} = (s_{\sigma_i}, \theta_{\sigma_i}), s_{\sigma_i} \in S, \theta_{\sigma_i} \in S^1 \\
 &s_{\sigma_1} = s_1, s_{\sigma_k} = s_n
 \end{aligned}$$



The problem combines discrete combinatorial optimization (OP) with the continuous optimization for determining the vehicle headings

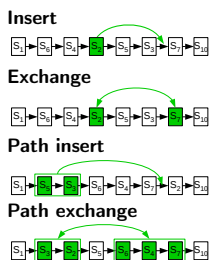
## Variable Neighborhood Search (VNS)

- Variable Neighborhood Search (VNS)** is a general metaheuristic for combinatorial optimization (routing problems)  
Hansen, P. and Mladenović, N. (2001): *Variable neighborhood search: Principles and applications*. European Journal of Operational Research.
- The VNS is based on **shake** and **local search** procedures
  - Shake** procedure aims to escape from local optima by changing the solution within the neighborhoods  $M_{1, \dots, k_{max}}$   
*The neighborhoods are particular operators*
  - Local search** procedure searches fully specific neighborhoods of the solution using  $l_{max}$  predefined operators

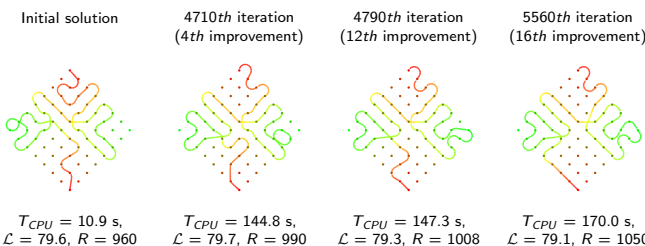
## Variable Neighborhood Search (VNS) for the DOP

- The solution is the first  $k$  locations of the sequence of all target locations satisfying  $T_{max}$   
VNS for the OP – Sevkli, Z. et al. (2006)
- It is an improving heuristics, i.e., an initial solution has to be provided
- A set of predefined neighborhoods are explored to find a better solution

- Shake** – explores the configuration space and escape from a local minima using
  - Insert** – moves one random element
  - Exchange** – exchanges two random elements
- Local Search** – optimizes the solution
  - Path insert** – moves a random sub-sequence
  - Path exchange** – exchanges two random sub-sequence
- Randomized VNS** – examines only  $n^2$  changes in the *Local Search* procedure in each iteration



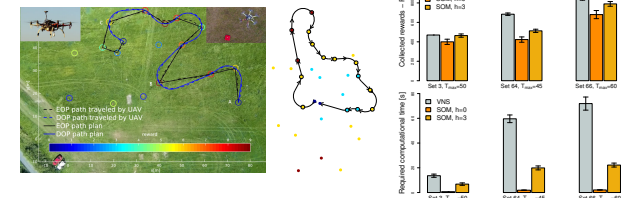
## Evolution of the VNS Solution to the DOP



| Iteration        | Improvement      | $T_{CPU}$ | $R$  |
|------------------|------------------|-----------|------|
| Initial solution |                  | 10.9 s    | 960  |
| 4710th iteration | 4th improvement  | 144.8 s   | 990  |
| 4790th iteration | 12th improvement | 147.3 s   | 1008 |
| 5560th iteration | 16th improvement | 170.0 s   | 1050 |

## Solution of the Dubins Orienteering Problem

- Solve the Euclidean OP (EOP) and then determine Dubins path  
*The final path may exceed the budget and the vehicle can miss the locations because of motion control*
- Directly solve the **Dubins Orienteering Problem (DOP)**, e.g.,
  - Sample possible heading values and use Variable Neighborhood Search (VNS)  
Pěnička, Faigl, Váňa, Saska (RA-L 2017)
  - Unsupervised learning based on Self-Organizing Maps (SOM)  
Faigl, (WSOM+ 2017)

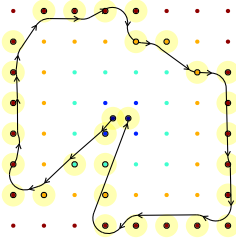


VNS-based approach provides better solutions than SOM, but it tends to be more computationally demanding

## Dubins Orienteering Problem with Neighborhoods

- Curvature-constrained path respecting Dubins vehicle model
- Each waypoint consists of location  $p \in \mathbb{R}^2$  and the heading  $\theta \in \mathbb{S}^1$
- In addition to  $S_k, k, \Sigma$  determine locations  $P_k = (p_{\sigma_1}, \dots, p_{\sigma_k})$  and headings  $\Theta = (\theta_{\sigma_1}, \dots, \theta_{\sigma_k})$  such that

$$\begin{aligned} & \text{maximize}_{k, S_k, \Sigma} R = \sum_{i=1}^k r_{\sigma_i} \\ & \text{subject to} \sum_{i=2}^k \mathcal{L}(q_{\sigma_{i-1}}, q_{\sigma_i}) \leq T_{max}, \\ & q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i}), p_{\sigma_i} \in \mathbb{R}^2, \theta_{\sigma_i} \in \mathbb{S}^1 \\ & \|p_{\sigma_i}, s_{\sigma_i}\| \leq \delta, s_{\sigma_i} \in S_k \\ & p_{\sigma_1} = s_1, p_{\sigma_k} = s_n \end{aligned}$$

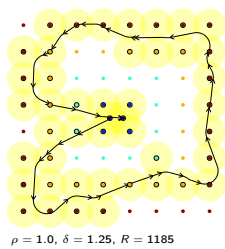


We need to solve the continuous optimization for determining the vehicle heading at each waypoint and the waypoint locations  $P_k = \{p_{\sigma_1}, \dots, p_{\sigma_k}\}, p_{\sigma_i} \in \mathbb{R}^2$

## Comparison of the DOPN Solvers

- VNS-based DOPN solver with  $s = 16$  heading samples and  $h = 16$  waypoint location samples per sensor Pěnička, Faigl, et al. (ICUAS 2017)
- SOM-based DOPN solver with  $h = 3$  Faigl, Pěnička (IROS 2017)
- Aggregate results using average relative percentage error (ARPE) and relative percentage error (RPE)

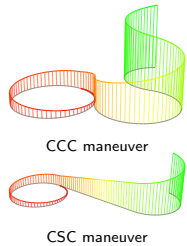
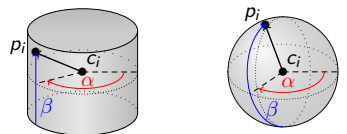
| Problem set            | VNS-based |                 | SOM-based ( $h = 3$ ) |      |               |
|------------------------|-----------|-----------------|-----------------------|------|---------------|
|                        | ARPE      | $T_{cpu}^*$ [s] | RPE                   | ARPE | $T_{cpu}$ [s] |
| Set 3, $\delta = 0.0$  | 1.0       | 1,178.9         | 3.6                   | 7.4  | 7.0           |
| Set 3, $\delta = 0.5$  | 0.9       | 13,273.3        | 6.6                   | 10.6 | 7.9           |
| Set 3, $\delta = 1.0$  | 0.5       | 13,304.4        | 5.5                   | 9.2  | 8.3           |
| Set 64, $\delta = 0.0$ | 1.9       | 5,272.2         | 17.4                  | 23.8 | 17.9          |
| Set 64, $\delta = 0.5$ | 2.8       | 13,595.6        | 18.7                  | 24.2 | 20.2          |
| Set 64, $\delta = 1.0$ | 1.3       | 13,792.3        | 9.9                   | 15.2 | 22.2          |
| Set 66, $\delta = 0.0$ | 1.5       | 6,546.6         | 3.6                   | 9.1  | 22.9          |
| Set 66, $\delta = 0.5$ | 1.4       | 13,650.1        | 6.7                   | 11.8 | 25.5          |
| Set 66, $\delta = 1.0$ | 3.2       | 13,824.5        | 16.1                  | 21.3 | 26.7          |



\*The results have been obtained with a grid Xeon CPUs running at 2.2 GHz to 3.4 GHz due to computational requirements.

## DTSPN in 3D

- Using the same principles as for the DTSPN in 2D, we can generalize the approaches for 3D planning using the Dubins Airplane model instead of simple Dubins vehicle
- The regions can be generalized to 3D and the problem can be addressed by decoupled or sampling-based approaches, i.e., using GATSP formulation
- In the case of LIO, we need a parametrization of the possible waypoint location, e.g.,



## Variable Neighborhoods Search (VNS) for the DOPN

```

Algorithm 3: VNS based method for the DOPN
Input : S – Set of the target locations
Input : T_max – Maximal allowed budget
Input : o – Initial number of position waypoints for each target
Input : m – Initial number of heading values for each waypoints
Input : r_l – Local waypoint improvement ratio
Input : l_max – Maximal neighborhood number
Output : P – Found data collecting path
S_r ← getReachableLocations(S, T_max)
P ← createInitialPath(S, T_max) // greedy
while Stopping condition is not met do
  l ← 1
  while l ≤ l_max do
    P' ← shake(P, l)
    P'' ← localSearch(P', l, r_l)
    if C_d(P'') ≤ T_max and
       [(R(P'') > R(P)) or (R(P'') == R(P) and
         C_d(P'') < C_d(P))] then
      P ← P''
    else
      l ← l + 1
  end
end
end
    
```

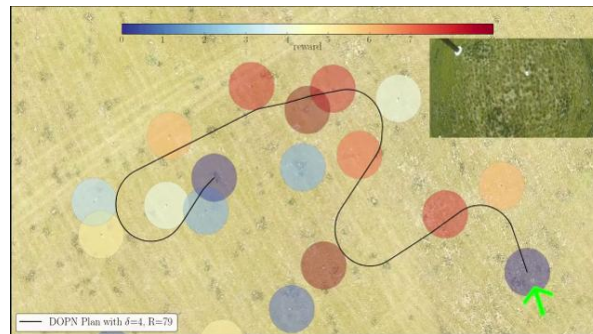
The particular  $l$  for the individual operators of the **shake** procedure are:

- Waypoint Shake ( $l = 1$ );
  - Path Move ( $l = 2$ );
  - Path Exchange ( $l = 3$ ).
- The **local search** procedure consists of three operators and the particular  $l$  for the individual operators of the **local search** procedure are:
- Waypoint Improvement ( $l = 1$ );
  - One Point Move ( $l = 2$ );
  - One Point Exchange ( $l = 3$ )

Pěnička, R., Faigl, J., Saska, M., Váňa, P. (2017)

## DOPN – Example of Solution and Practical Deployment

- VNS-based solution of the DOPN Robert Pěnička, Jan Faigl, Martin Saska and Petr Váňa, ICUAS 2017



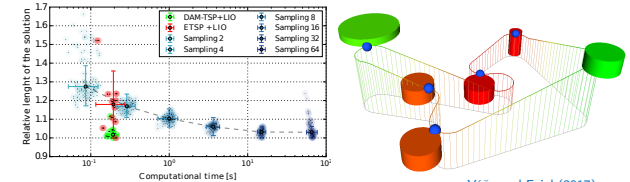
<http://mra.felk.cvut.cz/jint17dopn>

## Solutions of the 3D-DTSPN

**Algorithm 4: LIO-based Solver for 3D-DTSPN**

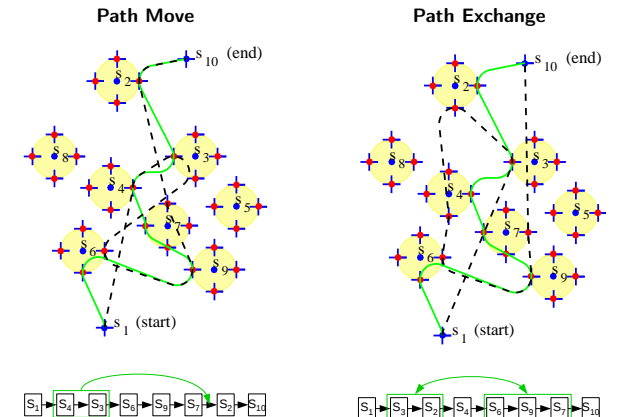
Data: Regions  $\mathcal{R}$   
 Result: Solution represented by  $Q$  and  $\Sigma$   
 $\Sigma \leftarrow \text{InitialSequence}(\mathcal{R});$   
 $Q \leftarrow \text{getInitialSolution}(\mathcal{R}, \Sigma);$   
 while terminal condition do  
    $Q \leftarrow \text{optimizeHeadings}(Q, \mathcal{R}, \Sigma);$   
    $Q \leftarrow \text{optimizeAlpha}(Q, \mathcal{R}, \Sigma);$   
    $Q \leftarrow \text{optimizeBeta}(Q, \mathcal{R}, \Sigma);$   
 end  
 return  $Q, \Sigma;$

- Solutions based on LIP (ETSP+LIO), TSP with the travel cost according to Dubins Airplane Model (DAM-TSP+LIO) and sampling-based approach with transformation of the GTSP to the ATSP using LKH



Váňa and Faigl (2017)

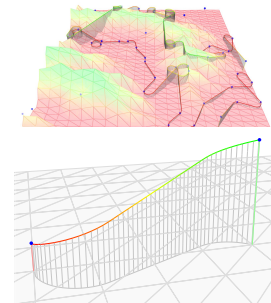
## VNS for DOPN – Example of the Shake Operators



## 3D Data Collection Planning with Dubins Airplane Model

- Dubins Airplane model describes the vehicle state  $q = (p, \theta, \psi), p \in \mathbb{R}^3$  and  $\theta, \psi \in \mathbb{S}^1$  as Chitsaz, H., LaValle, S.M. (2017)

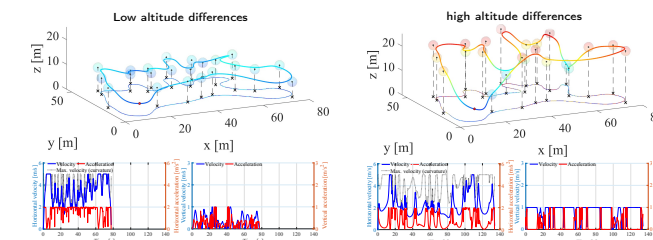
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos \theta \cdot \cos \psi \\ \sin \theta \cdot \cos \psi \\ \sin \psi \\ u_{\theta} \cdot \rho^{-1} \end{bmatrix} \quad (5)$$



- Constant forward velocity  $v$ , the minimal turning radius  $\rho$ , and limited pitch angle, i.e.,  $\psi \in [\psi_{min}, \psi_{max}]$
- $u_{\theta}$  controls the vehicle heading,  $|u_{\theta}| \leq 1$ , and  $v$  is the forward velocity
- Generation of the 3D trajectory is based on the 2D Dubins maneuver
- If altitude changes are too high, additional helix segments are inserted

## 3D Surveillance Planning

- Parametrization of smooth 3D multi-goal trajectory a sequence of Bézier curves
- Unsupervised learning for the TSPN can be generalized for such trajectories
- During the solution of the sequencing, we can determine a velocity profile along the curve and compute the so-called *Travel Time Estimation* (TTE)
- It better fits limits of the multi-rotor UAVs that are limited by maximal acceleration and velocities rather than minimal turning radius as for Dubins vehicle



Faigl and Váňa (2017)

- Low differences in altitude saturate horizontal velocity while high altitudes changes saturate vertical velocity

## Part II

## Part 2 – Data Collection Planning for Surveillance Missions

## Motivation

- There is a framework for testing and evaluation of UAVs control strategy developed and maintained by the winners of the Mohamed Bin Zayed International Robotics Challenge (MBZIRC) 2017

**Multi-robot Systems (MRS) group**  
<http://mrs.felk.cvut.cz>



<https://www.youtube.com/watch?v=ju3YbCtXpEw>

- The framework allows a direct evaluation of the planned trajectories, i.e., Dubins trajectories, in simulator and real vehicles
- It provides a unique opportunity to become familiar more with multi-rotor unmanned aerial vehicles and practical deployment of the planned trajectories to UAVs
- Full support of the evaluation environment is provided together with setuped computers at the dedicated computer lab of MRS
- Practical deployment on real UAVs would be possible during the first campaigns in spring 2018

## Assignment – HW03b

**Topics: Data Collection Planning for Surveillance Missions**

**Goal:** Solve data collection planning problem formulated as the Dubins Traveling Salesman Problem (with Neighborhoods) and deploy the planned path to the model of UAVs and eventually experimentally verify using real UAV

**Assignment:** <https://cw.fel.cvut.cz/wiki/courses/b4m36uir/hw/hw03b>

Up to additional **15 points** can be gained for implementation solution of DTSP, DTSPN and execution of the trajectories in a MRS simulation framework

- Implement a solution of the DTSP, e.g., one of the following methods
  - (2 points) for simple ETSP and Alternating Algorithm (AA), a.k.a ETSP+AA;
  - (6 points) Become familiar with the MRS simulation framework and deploy the planned trajectories within the simulator
- **Voluntary** implementation of the DTSP and DTSPN sampling-based solvers
  - (4 points) ETSP+DTP (forward search graph) or GATSP→ATSP and solution using LKH
  - (3 points) Extension of the DTSP to DTSPN, e.g., forward search graph for DTP generalized for the DTRP or GATSP based approach

## Summary of the Lecture

## Topics Discussed

- Dubins vehicles and planning – Dubins maneuvers
- Dubins Interval Problem (DIP)
- Dubins Touring Problem (DTP)
- Dubins Traveling Salesman Problem (DTSP) and Dubins Traveling Salesman with Neighborhoods (DTSPN)
  - Decoupled approaches – Alternating Algorithm
  - Sampling-based approaches – GATSP
- Dubins Orienteering Problem (OP) and Dubins Orienteering Problem with Neighborhoods (DOPN)
- Data collection and surveillance planning in 3D
- **Next: Multirobot Path Planning (MPP)**