Multi-Goal Path and Motion Planning

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Lecture 07

B4M36UIR – Artificial Intelligence in Robotics

Overview of the Lecture

Part I

Part 1 – Improved Sampling-based Motion Planning

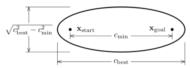
Improved Sampling-based Motion Planners

- Although Asymptotically optimal sampling-based motion planners such RRT* or RRG may provide high-quality or even optimal solutions of complex problem, their performance in simple, e.g., 2D scenarios, is relatively poor — In a comparison to the previous approaches
- They are computationally demanding and performance can be improved similarly as RRT, e.g.,
  - Goal biasing, supporting sampling in narrow passages, multi-tree grows (Bidirectional RRT)
- The general idea of improvements is based on **informing** the sampling process
- Many modifications of the algorithms exists, selected representative modifications are
  - **Informed RRT***
  - Batch Informed Trees (**BIT***)
  - Regionally Accelerated BIT* (**RABIT***)

Informed RRT*

- Focused RRT* search to increase the convergence rate
- Use Euclidean distance as an admissible heuristic
- Ellipsoidal informed subset – the current best solution c_{best}

$$X_{\hat{f}} = \{\mathbf{x} \in X \,|\, ||x_{start} - \mathbf{x}||_2 + ||\mathbf{x} - \mathbf{x}_{goal}||_2 \le c_{best}\}$$



- Directly Based on the RRT*
- Having a feasible solution
- Sampling inside the ellipse

Algorithm 2: Sample ($\mathbf{x}_{start}, \mathbf{x}_{goal}, c_{max}$)

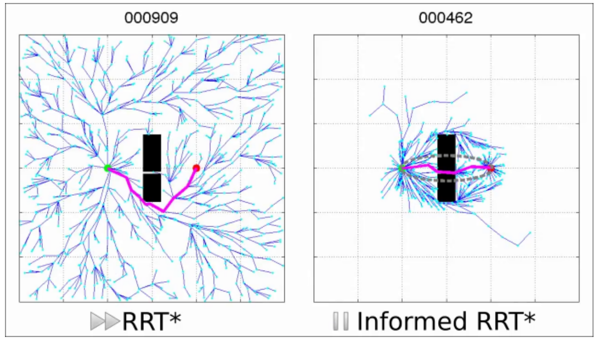
```
if c_max < ∞ then
    c_min ← ||x_goal − x_start||_2;
    x_centre ← (x_start + x_goal)/2;
    C ← RotationToWorldFrame(x_start, x_goal);
    r_1 ← c_max/2;
    {r_i}_{i=2,...,n} ← (sqrt(c_max^2 − c_min^2))/2;
    L ← diag{r_1, r_2, ..., r_n};
    x_ball ← SampleUnitNBall;
    x_rand ← (C L x_ball + x_centre) ∩ X;
else
    x_rand ∼ U(X);
return x_rand;
```

Algorithm 1: Informed RRT*($\mathbf{x}_{start}, \mathbf{x}_{goal}$)

```
V ← {x_start};
E ← ∅;
X_soln ← ∅;
T = (V, E);
for iteration = 1 ... N do
    c_best ← min_{x_soln ∈ X_soln} {Cost(x_soln)};
    x_rand ← Sample(x_start, x_goal, c_best);
    x_nearest ← Nearest(T, x_rand);
    x_new ← Steer(x_nearest, x_rand);
    if CollisionFree(x_nearest, x_new) then
        V ← ∪{x_new};
        X_near ← Near(T, x_new, r_RRT*);
        x_min ← x_nearest;
        c_min ← Cost(x_min) + c · Line(x_nearest, x_new);
        for ∀x_near ∈ X_near do
            c_new ← Cost(x_near) + c · Line(x_near, x_new);
            if c_new < c_min then
                if CollisionFree(x_near, x_new) then
                    x_min ← x_near;
                    c_min ← c_new;
        E ← E ∪ {(x_min, x_new)};
        for ∀x_near ∈ X_near do
            c_near ← Cost(x_near);
            c_new ← Cost(x_new) + c · Line(x_new, x_near);
            if c_new < c_near then
                if CollisionFree(x_new, x_near) then
                    x_parent ← Parent(x_near);
                    E ← E \ {(x_parent, x_near)};
                    E ← E ∪ {(x_new, x_near)};
        if InGoalRegion(x_new) then
            X_soln ← X_soln ∪ {x_new};
return T;
```

Gammell, J. B., Srinivasa, S. S., Barfoot, T. D. (2014): Informed RRT*: Optimal Sampling-based Path Planning Focused via Direct Sampling of an Admissible Ellipsoidal Heuristic. IROS.

Informed RRT* – Demo



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Gammell, J. B., Srinivasa, S. S., Barfoot, T. D. (2014): Informed RRT*: Optimal Sampling-based Path Planning Focused via Direct Sampling of an Admissible Ellipsoidal Heuristic. IROS.

Batch Informed Trees (BIT*)

- Combining RGG (Random Geometric Graph) with the heuristic in incremental graph search technique, e.g., Lifelong Planning A* (LPA*)
  - The properties of the RGG are used in the RRG and RRT*
- Batches of samples – a new batch starts with denser implicit RGG
- The search tree is updated using LPA* like incremental search to reuse existing information

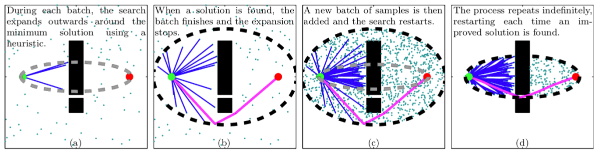
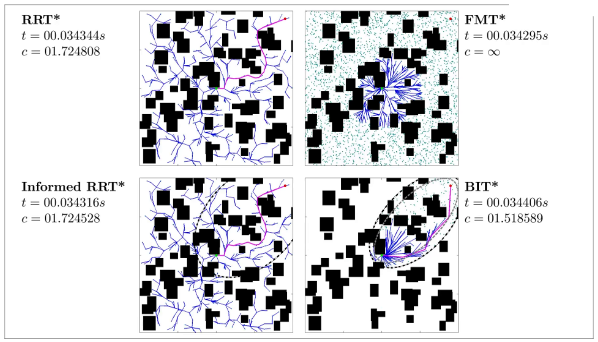


Fig. 3. An illustration of the informed search procedure used by BIT*. The start and goal states are shown as green and red, respectively. The current solution is highlighted in magenta. The subproblem that contains any better solutions is shown as a black dashed line, while the progress of the current batch is shown as a grey dashed line. Fig. (a) shows the growing search of the first batch of samples, and (b) shows the first search ending when a solution is found. After pruning and adding a second batch of samples, Fig. (c) shows the search restarting on a denser graph while (d) shows the second search ending when an improved solution is found. An animated illustration is available in the attached video.

Gammell, J. B., Srinivasa, S. S., Barfoot, T. D. (2015): Batch Informed Trees (BIT*): Sampling-based optimal planning via the heuristically guided search of implicit random geometric graphs. ICRA.

Batch Informed Trees (BIT*) – Demo



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Gammell, J. B., Srinivasa, S. S., Barfoot, T. D. (2015): Batch Informed Trees (BIT*): Sampling-based optimal planning via the heuristically guided search of implicit random geometric graphs. ICRA.

Regionally Accelerated BIT* (RABIT*)

- Use local optimizer with the BIT* to improve the convergence speed
- Local search Covariant Hamiltonian Optimization for Motion Planning (CHOMP) is utilized to connect edges in the search graphs using local information about the obstacles.

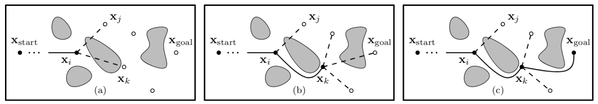
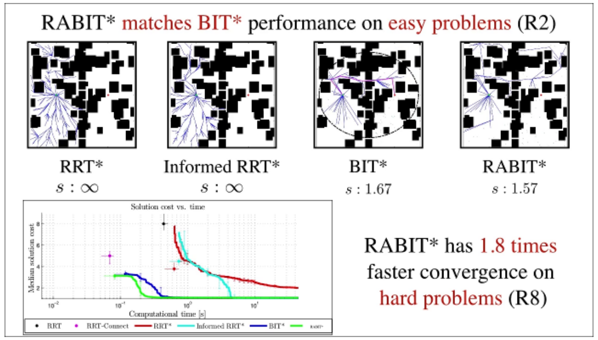


Fig. 2. An illustration of how the RABIT* algorithm uses a local optimizer to exploit obstacle information and improve a global search. The global search is performed, as in BIT*, by incrementally processing an edge queue (dashed lines) into a tree (a). Using heuristics, the potential edge from $\mathbf{x}_i$ to $\mathbf{x}_k$ is processed first as it could provide a better solution than an edge from $\mathbf{x}_i$ to $\mathbf{x}_j$. The initial straight-line edge is given to a local optimizer which uses information about obstacles to find a local optima between the specified states (b). If this edge is collision free, it is added to the tree and its potential outgoing edges are added to the queue. The next-best edge in the queue is then processed in the same fashion, using the local optimizer to once again propose a better edge than a straight-line (c).

Choudhury, S., Gammell, J. D., Barfoot, T. D., Srinivasa, S. S., Scherer, S. (2016): Regionally Accelerated Batch Informed Trees (RABIT*): A Framework to Integrate Local Information into Optimal Path Planning. ICRA.

Regionally Accelerated BIT* (RABIT*) – Demo



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Choudhury, S., Gammell, J. D., Barfoot, T. D., Srinivasa, S. S., Scherer, S. (2016): Regionally Accelerated Batch Informed Trees (RABIT*): A Framework to Integrate Local Information into Optimal Path Planning. ICRA.

Overview of Improved Algorithm

- Optimal motion planning is an active research field

Approaches	Constraints	Planning Mode	Kinematic Model	Sampling Strategy	Metric
1. RRT* [7]	Holonomic	Offline	Point	Uniform	Euclidean
2. Anytime RRT* [4]	Non-holonomic	Online	Dubin Car	Uniform	Euclidean + Velocity
3. B-RRT* [58]	Holonomic	Offline	Rigid Body	Local bias	Goal biased
4. RRT*FN [33]	Holonomic	Offline	Robotic Arm	Uniform	Cumulative Euclidean
5. RRT*-Smart [35]	Holonomic	Offline	Point	Intelligent	Euclidean
6. Optimal B-RRT* [36]	Holonomic	Offline	Point	Uniform	Euclidean
7. RRT# [50]	Holonomic	Offline	Point	Uniform	Euclidean
8. Adapted RRT* [64], [49]	Non-holonomic	Offline	Car-like and UAV	Uniform	A* Heuristic
9. SRRT* [44]	Non-holonomic	Offline	UAV	Uniform	Geometric + dynamic constraint
10. Informed RRT* [34]	Holonomic	Offline	Point	Direct Sampling	Euclidean
11. IB-RRT* [37]	Holonomic	Offline	Point	Intelligent	Greedy + Euclidean
12. DT-RRT [39]	Non-holonomic	Offline	Car-like	Hybrid	Angular + Euclidean
13. RRT*i [3]	Non-holonomic	Online	UAV	Local Sampling	A* Heuristic
14. RTR+CS* [43]	Non-holonomic	Offline	Car-like	Uniform + Local Planning	Angular + Euclidean
15. Mitsubishi RRT* [2]	Non-holonomic	Online	Autonomous Car	Two-stage sampling	Weighted Euclidean
16. CARRT* [65]	Non-holonomic	Online	Humanoid	Uniform	MW Energy Cost
17. PRRT* [48]	Non-holonomic	Offline	P3-DX	Uniform	Euclidean

Noreen, I., Khan, A., Habib, Z. (2016): Optimal path planning using RRT* based approaches: a survey and future directions. IJACSA.

Part II

Part 2 – Multi-Goal Path and Motion Planning

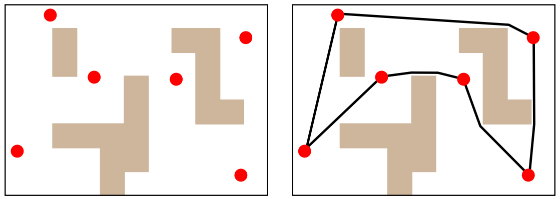
Multi-Goal Path Planning

Motivation

Having a set of locations (goals) to be visited, determine the cost-efficient path to visit them and return to a starting location.

- Locations where a robotic arm performs some task
- Locations where a mobile robot has to be navigated

To perform measurements such as scan the environment or read data from sensors.



Alatartsev et al. (2015) – Robotic Task Sequencing Problem: A Survey

Traveling Salesman Problem (TSP)

Given a set of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city.

- The TSP can be formulated for a graph $\boldsymbol{G}(V, E)$, where V denotes a set of locations (cities) and E represents edges connecting two cities with the associated travel cost c (distance), i.e., for each $v_i, v_j \in V$ there is an edge $e_{ij} \in E$, $e_{ij} = (v_i, v_j)$ with the cost c_{ij}.
- If the associated cost of the edge (v_i, v_j) is the Euclidean distance $c_{ij} = |(v_i, v_j)|$, the problem is called the Euclidean TSP (ETSP).

  In our case, $v \in V$ represents a point in $\mathbb{R}^2$ and solution of the ETSP is a path in the plane.
- It is known, the TSP is NP-hard (its decision variant) and several algorithms can be found in literature.

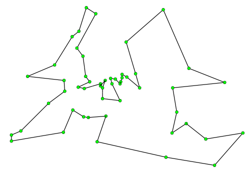
  William J. Cook (2012) – In Pursuit of the Traveling Salesman: Mathematics at the Limits of Computation

Solutions of the TSP

- Efficient heuristics from the Operational Research have been proposed
- LKH – K. Helsgaun efficient implementation of the Lin-Kernighan heuristic (1998)

  http://www.akira.ruc.dk/~keld/research/LKH/
- Concorde – Solver with several heuristics and also optimal solver

  http://www.math.uwaterloo.ca/tsp/concorde.html



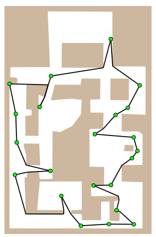
Problem Berlin52 from the TSPLIB

Beside the heuristic and approximations algorithms (such as Christofides 3/2-approximation algorithm), other („soft-computing") approaches have been proposed, e.g., based on genetic algorithms, and memetic approaches, ant colony optimization (ACO), and neural networks.

Multi-Goal Path Planning (MTP) Problem

Given a map of the environment $\mathcal{W}$, mobile robot $\mathcal{R}$, and a set of locations, what is the shortest possible collision free path that visits each location exactly once and returns to the origin location.

- MTP problem is de facto the TSP with the cost associated to the edges as the length of the *shortest* path connecting the locations
- For n locations, we need to compute up to n^2 shortest paths (solve n^2 motion planning problems)
- The paths can be found as the shortest path in a graph (roadmap), from which the $G(V, E)$ for the TSP can be constructed



Visibility graph as the roadmap for a point robot provides a straight forward solution, but such a shortest path may not be necessarily feasible for more complex robots

Multi-Goal Motion Planning

- In the previous cases, we consider existing roadmap or relatively "simple" collision free (shortest) paths in the polygonal domain
- However, determination of the collision-free path in a high dimensional configuration space ($\mathcal{C}$-space) can be a challenging problem itself
- Therefore, we can generalize the MTP to multi-goal motion planning (MGMP) considering motion (trajectory) planners in $\mathcal{C}$-space.
- An example of MGMP can be

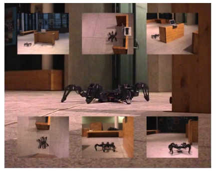
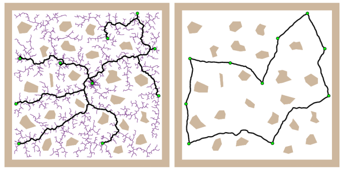
Plan a cost efficient trajectory for hexapod walking robot to visit a set of target locations.

Problem Statement – MGMP Problem

- The working environment $\mathcal{W} \subset \mathbb{R}^3$ is represented as a set of obstacles $\mathcal{O} \subset \mathcal{W}$ and the robot configuration space $\mathcal{C}$ describes all possible configurations of the robot in $\mathcal{W}$
- For $q \in \mathcal{C}$, the robot body $\mathcal{A}(q)$ at q is collision free if $\mathcal{A}(q) \cap \mathcal{O} = \emptyset$ and all collision free configurations are denoted as $\mathcal{C}_{free}$
- Set of n goal locations is $\mathcal{G} = (g_1, \ldots, g_n)$, $g_i \in \mathcal{C}_{free}$
- Collision free path from q_{start} to q_{goal} is $\kappa : [0, 1] \rightarrow \mathcal{C}_{free}$ with $\kappa(0) = q_{start}$ and $d(\kappa(1), q_{end}) < \epsilon$, for an admissible distance ϵ
- Multi–goal path τ is admissible if $\tau : [0, 1] \rightarrow \mathcal{C}_{free}$, $\tau(0) = \tau(1)$ and there are n points such that $0 \leq t_1 \leq t_2 \leq \ldots \leq t_n$, $d(\tau(t_i), v_i) < \epsilon$, and $\bigcup_{1<i\leq n} v_i = \mathcal{G}$
- **The problem is to find path τ^* for a cost function c such that $c(\tau^*) = \min\{c(\tau) \mid \tau$ *is admissible multi–goal path*$\}$**

MGMP – Examples of Solutions

- Determination of all paths connecting any two locations $g_i, g_j \in \mathcal{G}$ is usually very computationally demanding
- Several approaches can be found in literature, e.g.,
  - Considering Euclidean distance as approximation in solution of the TSP as the Minimum Spanning Tree (MST) – Edges in the MST are iteratively refined using optimal motion planner until all edges represent a feasible solution
    Saha, M., Roughgarden, T., Latombe, J.-C., Sánchez-Ante, G. (2006): Planning Tours of Robotic Arms among Partitioned Goals. IJRR.
  - **Synergistic Combination of Layers of Planning (SyCLoP)** – A combination of route and trajectory planning
    Plaku, E., Kavraki, L.E., Vardi, M.Y. (2010): Motion Planning With Dynamics by a Synergistic Combination of Layers of Planning. T-RO.
  - Steering RRG roadmap expansion by unsupervised learning for the TSP

Faigl (2016), WSOM

Multi-Goal Path Planning in Robotic Missions

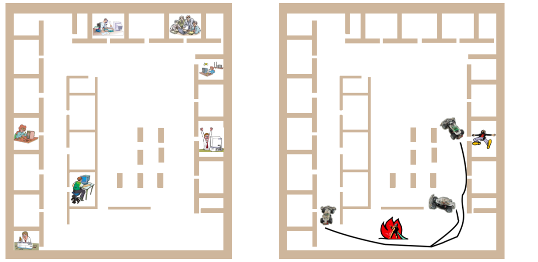
Multi-goal path planning

- It builds on a simple path and trajectory planning
- It is a combinatorial optimization problem to determine the sequence to visit the given locations
- It allows to solve (or improve performance of) more complex problems such as
  - **Inspection planning** - Find the shortest tour to see (inspect) the whole environment
  - **Data collection planning** – Determine a cost efficient path to collect data from the sensor stations (locations)
  - *Robotic exploration* - Create a map of unknown environment as quickly as possible

Inspection Planning

Motivations (examples)

- Periodically visit particular locations of the environment to check, e.g., for intruders, and return to the starting locations
- Based on available plans, provide a guideline how to search a building to find possible victims as quickly as possible (search and rescue scenario)



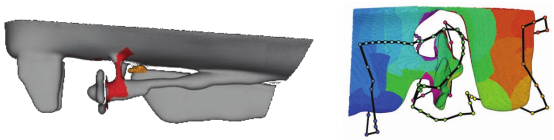
Inspection Planning – Decoupled Approach

1. Determine sensing locations such that the whole environment would be inspected (seen) by visiting them
   A solution of the Art Gallery Problem
2. Create a roadmap connecting the sensing location
   E.g., using visibility graph or randomized sampling based approaches
3. Find the inspection path visiting all the sensing locations as a solution of the multi-goal path planning
   De facto solution of the TSP

Inspection planning is also called coverage path planning in literature.

Example – Inspection Planning with AUV

- Determine shortest inspection path for Autonomous Underwater Vehicle (AUV) to inspect a propeller of the vessel

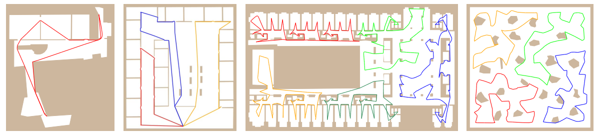


Three-dimensional coverage planning for an underwater inspection robot
Brendan Englot and Franz S. Hover
International Journal of Robotic Research, 32(9-10):1048–1073, 2013.

Inspection Planning – "*Continuous Sensing*"

- If we do not prescribe a discrete set of sensing locations, we can formulate the problem as the **Watchman route problem**

> Given a map of the environment $\mathcal{W}$ determine the shortest, closed, and collision-free path, from which the whole environment is covered by an omnidirectional sensor with the radius ρ.



Approximate Solution of the Multiple Watchman Routes Problem with Restricted Visibility Range
Jan Faigl
IEEE Transactions on Neural Networks, 21(10):1668–1679, 2010.

Self-Organizing Maps based Solution of the TSP

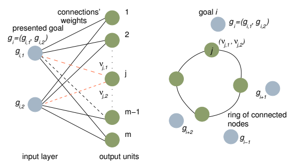
Kohonen's type of unsupervised two-layered neural network

- Neurons' weights represent nodes $\mathcal{N} = \{\nu_1, \ldots, \nu_m\}$ in a plane.
- Nodes are organized into a ring.
- Sensing locations $\boldsymbol{S} = \{s_1, \ldots s_n\}$ are presented to the network in a random order.
- Nodes compete to be winner according to their distance to the presented goal s

$$\nu^* = \mathrm{argmin}_{\nu \in \mathcal{N}} |\mathcal{D}(\nu, s)|$$

- The winner and its neighbouring nodes are adapted (moved) towards the city according to the neighbouring function

$$f(\sigma, d) = \begin{cases} e^{-\frac{d^2}{\sigma^2}} & \text{for } d < m/n_f, \\ 0 & \text{otherwise,} \end{cases}$$



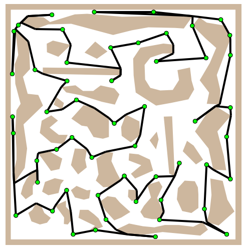
- Best matching unit ν to the presented prototype s is determined according to distance function $|\mathcal{D}(\nu, s)|$
- For the Euclidean TSP, $\mathcal{D}$ is the Euclidean distance
- However, for problems with obstacles, the multi-goal path planning, $\mathcal{D}$ should correspond to the length of the shortest, collision free path.

SOM for the Multi-Goal Path Planning

Unsupervised learning procedure

Algorithm 1: SOM-based MTP solver

$\mathcal{N} \leftarrow$ initialization$(\nu_1, \ldots, \nu_m)$;
repeat
  $error \leftarrow 0$;
  foreach $g \in \Pi(\boldsymbol{S})$ **do**
    $\nu^* \leftarrow$ selectWinner $\mathrm{argmin}_{\nu \in \mathcal{N}} |S(g, \nu)|$;
    adapt$(S(g, \nu), \mu f(\sigma, l)|S(g, \nu)|)$;
    $error \leftarrow \max\{error, |S(g, \nu^*)|\}$;
  $\sigma \leftarrow (1 - \alpha) \cdot \sigma$;
until $error \leq \delta$;



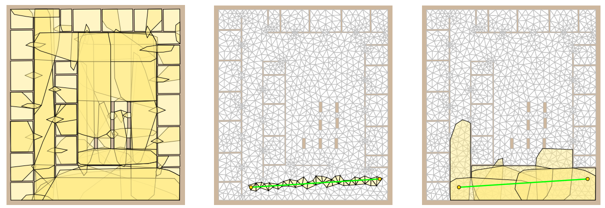
- For multi-goal path planning – the selectWinner and adapt procedures are based on the solution of the path planning problem

An Application of Self-Organizing Map in the non-Euclidean Traveling Salesman Problem
Jan Faigl, Miroslav Kulich, Vojtěch Vonásek and Libor Přeučil
Neurocomputing, 74(5):671–679, 2011.

SOM for the TSP in the Watchman Route Problem

During the unsupervised learning, we can compute coverage of $\mathcal{W}$ from the current ring (solution represented by the neurons) and adapt the network towards uncovered parts of $\mathcal{W}$

- Convex cover set of $\mathcal{W}$ created on top of a triangular mesh
- Incident convex polygons with a straight line segment are found by walking in a triangular mesh technique



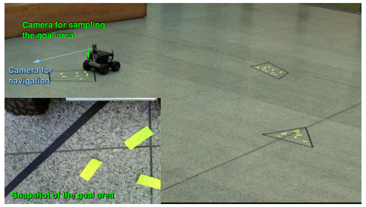
Jan Faigl (2010), TNN

The

Multi-Goal Path Planning with Goal Areas

- It may be sufficient to visit a goal region instead of the particular point location

E.g., to take a sample measurement at each goal



Not only a sequence of goals visit has to be determined, but also an appropriate sensing location for each goal need to be found.

The problem with goal regions can be considered as a variant of the **Traveling Salesman Problem with Neighborhoods** (**TSPN**).

Traveling Salesman Problem with Neighborhoods

Given a set of n regions (neighbourhoods), what is the shortest closed path that visits each region.

- The problem is NP-hard and APX-hard, it cannot be approximated to within factor $2 - \epsilon$, where $\epsilon > 0$

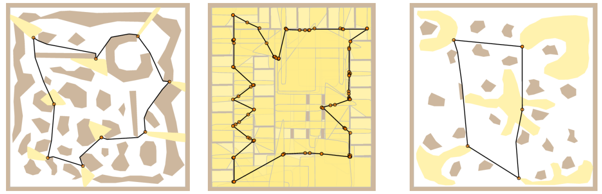
Safra and Schwartz (2006) – Computational Complexity

- Approximate algorithms exists for particular problem variants

E.g., Disjoint unit disk neighborhoods

- Flexibility of SOM for the TSP allows generalizing the unsupervised learning procedure to address the TSPN
- **TSPN provides a suitable problem formulation for planning various inspection and data collection missions**

SOM-based Solution of the Traveling Salesman Problem with Neighborhoods (TSPN)

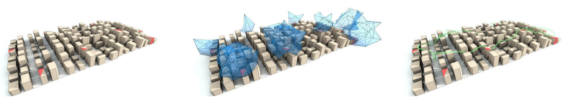


Polygonal Goals n=9, T= 0.32 s | Convex Cover Set n=106, T=5.1 s | Non-Convex Goals n=5, T=0.1 s

Visiting Convex Regions in a Polygonal Map, *Jan Faigl, Vojěch Vonásek and Libor Přeučil* Robotics and Autonomous Systems, 61(10):1070–1083, 2013.

Example – TSPN for Inspection Planning with UAV

- Determine a cost-efficient trajectory from which a given set of target regions is covered
- For each target region a subspace $S \subset \mathbb{R}^3$ from which the target can be covered is determined *S represents the neighbourhood*
- The PRM motion planning algorithm is utilized to construct a motion planning roadmap (a graph)
- SOM based solution of the TSP with a graph input is generalized to the TSPN



Janoušek and Faigl, (2013) – ICRA

Example – TSPN for Planning with Localization Uncertainty

- Selection of waypoints from the neighborhood of each location
- P3AT ground mobile robot in an outdoor environment

TSP: L=184 m, E_{avg}=0.57 m | TSPN: L=202 m, E_{avg}=0.35 m

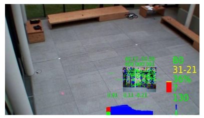
Real overall error at the goals decreased from 0.89 m → 0.58 m (about 35%)

- Decrease localization error at the target locations (indoor)

Small UGV - MMP5

Error decreased from 16.6 cm → 12.8 cm

Small UAV - Parrot AR.Drone



Improved success of the locations' visits 83%→95%

Faigl et al., (2012) – ICRA

Summary of the Lecture

Topics Discussed

- Improved sampling-based motion planners
- Multi-goal planning
  - Robotic variant of the Traveling Salesman Problem (TSP)
  - Multi-Goal Path Planning (MTP) problem
  - Multi-Goal Motion Planning (MGMP) problem
- Multi-goal planning in robotic missions
  - Traveling Salesman Problem with Neighborhoods (TSPN)
  - Inspection planning
- Next: Data collection planning