Improved Sampling-based Motion Planning Methods

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Lecture 06

B4M36UIR – Artificial Intelligence in Robotics

Overview of the Lecture

- Part 1 – Improved Sampling-based Motion Planning Methods
  - Optimal Motion Planners
  - Rapidly-exploring Random Graph (RRG)

Part I

Part 1 – Improved Sampling-based Motion Planning Methods

Efficient Sampling–Based Motion Planning

- PRM and RRT are theoretically probabilistic complete
- They provide a feasible solution without quality guarantee

  Despite that, they are successfully used in many practical applications
- In 2011, a study of the asymptotic behaviour has been published

  It shows, that in some case, they converges to a noon-optimal value with a probability 1.
- Based on the study, new algorithms have been proposed: RRG and optimal RRT (RRT^{star})

Sampling-based algorithms for optimal motion planning
Sertac Karaman, Emilio Frazzoli
International Journal of Robotic Research, 30(7):846–894, 2011.

RRT and Quality of Solution

- RRT provides a feasible solution without quality guarantee

  Despite of that, it is successfully used in many practical applications

- In 2011, a systematical study of the asymptotic behaviour of randomized sampling-based planners has been published

  It shows, that in some cases, they converge to a non-optimal value with a probability 1.

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http://sertac.scripts.mit.edu/rrtstar

RRT and Quality of Solution 1/2

- Let Y_i^{RRT} be the cost of the best path in the RRT at the end of iteration i.
- Y_i^{RRT} converges to a random variable

$$\lim_{i \to \infty} Y_i^{RRT} = Y_\infty^{RRT}.$$

- The random variable Y_∞^{RRT} is sampled from a distribution with zero mass at the optimum, and

$$Pr[Y_\infty^{RRT} > c^*] = 1.$$

Karaman and Frazzoli, 2011

- The best path in the RRT converges to a sub-optimal solution almost surely.

RRT and Quality of Solution 2/2

- RRT does not satify a necessary condition for the asymptotic optimality
  - For $0 < R < \inf_{q \in \mathcal{Q}_{goal}} ||q - q_{init}||$, the event $\{\lim_{n \to \infty} Y_n^{RTT} = c^*\}$ occurs only if the k-th branch of the RRT contains vertices outside the R-ball centered at q_{init} for infinitely many k.

See Appendix B in Karaman&Frazzoli, 2011

- It is required the root node will have infinitely many subtrees that extend at least a distance ϵ away from q_{init}

The sub-optimality is caused by disallowing new better paths to be discovered.

Rapidly-exploring Random Graph (RRG)

RRG Algorithm

Vstup: q_{init}, number of samples n
Výstup: $G = (V, E)$

```
V ← ∅; E ← ∅;
for i = 0, ..., n do
    q_rand ← SampleFree;
    q_nearest ← Nearest(G = (V, E), q_rand);
    q_new ← Steer(q_nearest, q_rand);
    if CollisionFree(q_nearest, q_new) then
        Q_near ← Near(G = (V, E), q_new, min{γ_RRG (log(card(V))/ card(V))^{1/d}, η});
        V ← V ∪ {q_new}; E ← E ∪ {(q_nearest, q_new), (q_new, q_nearest)};
        foreach q_near ∈ Q_near do
            if CollisionFree(q_near, q_new) then
                E ← E ∪ {(q_rand, u), (u, q_rand)};
return G = (V, E);
```

Proposed by Karaman and Frazzoli (2011). Theoretical results are related to properties of Random Geometric Graphs (RGG) introduced by Gilbert (1961) and further studied by Penrose (1999).

RRG Expansions

- At each iteration, RRG tries to connect new sample to the all vertices in the r_n ball centered at it.
- The ball of radius

$$r(\mathrm{card}(V)) = \min\left\{\gamma_{RRG}\left(\frac{\log\left(\mathrm{card}(V)\right)}{\mathrm{card}(V)}\right)^{1/d}, \eta\right\}$$

  where
  - η is the constant of the local steering function
  - $\gamma_{RRG} > \gamma^*_{RRG} = 2(1 + 1/d)^{1/d}(\mu(\mathcal{C}_{free})/\xi_d)^{1/d}$
    - d – dimension of the space;
    - $\mu(\mathcal{C}_{free})$ – Lebesgue measure of the obstacle–free space;
    - ξ_d – volume of the unit ball in d-dimensional Euclidean space.
- The connection radius decreases with n
- The rate of decay ≈ the average number of connections attempted is proportional to $\log(n)$

RRG Properties

- Probabilistically complete
- Asymptotically optimal
- Complexity is $O(\log n)$

(per one sample)

- Computational efficiency and optimality
  - Attempt connection to $\Theta(\log n)$ nodes at each iteration;

    in average

    - Reduce volume of the "connection" ball as $\log(n)/n$;
    - Increase the number of connections as $\log(n)$.

Other Variants of the Optimal Motion Planning

- **PRM*** – it follows standard PRM algorithm where connections are attempted between roadmap vertices that are within connection radius r as a function of n

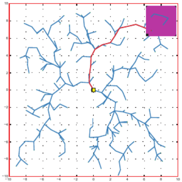
$$r(n) = \gamma_{PRM}(\log(n)/n)^{1/d}$$

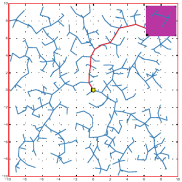
- **RRT*** – a modification of the RRG, where cycles are avoided

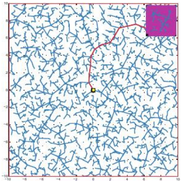
A tree version of the RRG

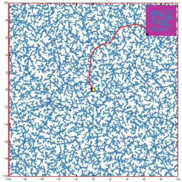
  - A tree roadmap allows to consider non-holonomic dynamics and kinodynamic constraints.
  - It is basically RRG with "rerouting" the tree when a better path is discovered.

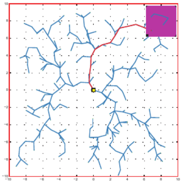
Example of Solution 1/2

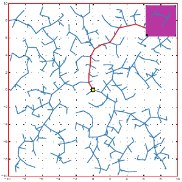

RRT, n=250

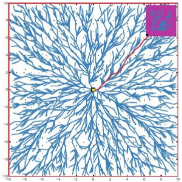

RRT, n=500

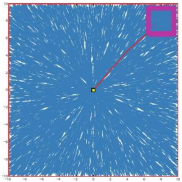

RRT, n=2500


RRT, n=10000


RRT*, n=250

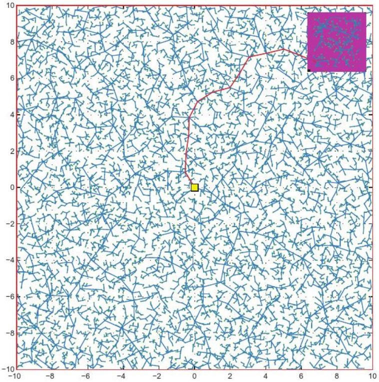

RRT*, n=500


RRT*, n=2500

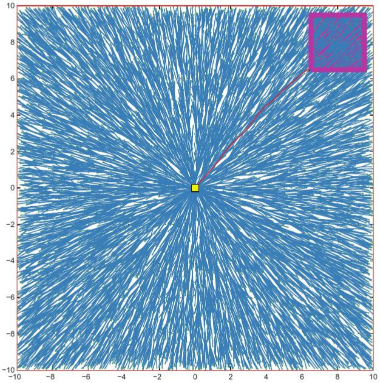

RRT*, n=10000

Karaman & Frazzoli, 2011

Example of Solution 2/2



RRT, n=20000



RRT*, n=20000

Overview of Randomized Sampling-based Algorithms

Algorithm	Probabilistic Completeness	Asymptotic Optimality
sPRM	✔	✘
k-nearest sPRM	✘	✘
RRT	✔	✘
RRG	✔	✔
PRM*	✔	✔
RRT*	✔	✔

Notice, k-nearest variants of RRG, PRM, and RRT* are complete and optimal as well.*

Summary of the Lecture

Topics Discussed

- Optimal sampling-based motion planning
- Rapidly-exploring Random Graph (RRG)

- Next: Robotic information gathering and Data collection planning