

# Randomized Sampling-based Motion Planning Methods

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Lecture 06

B4M36UIR – Artificial Intelligence in Robotics

## (Randomized) Sampling-based Motion Planning

- It uses an explicit representation of the obstacles in  $C$ -space
  - A “black-box” function is used to evaluate a configuration  $q$  is a collision-free, e.g.,
  - Based on geometrical models and testing collisions of the models
  - In 2D or 3D shape of the robot and environment can be represented as sets of triangles, i.e., tessellated models
  - Collision test – an intersection of triangles  
E.g., using RAPID library <http://gamma.cs.unc.edu/OBB/>
- Creates a discrete representation of  $C_{free}$
- Configurations in  $C_{free}$  are sampled randomly and connected to a roadmap (probabilistic roadmap)
- Rather than full completeness they provide probabilistic completeness or resolution completeness  
*Probabilistic complete algorithms: with increasing number of samples an admissible solution would be found (if exists)*



## Probabilistic Roadmap Strategies

- Multi-Query** – roadmap based
  - Generate a single roadmap that is then used for planning queries several times.
  - An representative technique is **Probabilistic RoadMap (PRM)**  
Kavraki, L., Svestka, P., Latombe, J.-C., Overmars, M. H.B (1996): Probabilistic Roadmaps for Path Planning in High Dimensional Configuration Spaces. T-RO.
- Single-Query** – incremental
  - For each planning problem constructs a new roadmap to characterize the subspace of  $C$ -space that is relevant to the problem.
    - Rapidly-exploring Random Tree – RRT  
LaValle, 1998
    - Expansive-Space Tree – EST  
Hsu et al., 1997
    - Sampling-based Roadmap of Trees – SRT  
(combination of multiple-query and single-query approaches)  
Plaku et al., 2005

## Overview of the Lecture

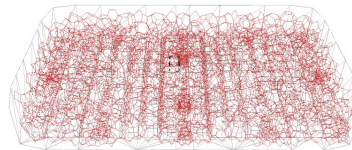
- Part 1 – Randomized Sampling-based Motion Planning Methods
  - Sampling-Based Methods
  - Probabilistic Road Map (PRM)
  - Characteristics
  - Rapidly Exploring Random Tree (RRT)
- Part 2 – Optimal Sampling-based Motion Planning Methods
  - Optimal Motion Planners
  - Rapidly-exploring Random Graph (RRG)

## Probabilistic Roadmaps

A discrete representation of the continuous  $C$ -space generated by randomly sampled configurations in  $C_{free}$  that are connected into a graph.

- Nodes** of the graph represent admissible configuration of the robot.
- Edges** represent a feasible path (trajectory) between the particular configurations.

*Probabilistic complete algorithms: with increasing number of samples an admissible solution would be found (if exists)*




Having the graph, the final path (trajectory) is found by a graph search technique.

## Multi-Query Strategy

Build a roadmap (graph) representing the environment

- Learning phase
  - Sample  $n$  points in  $C_{free}$
  - Connect the random configurations using a local planner
- Query phase
  - Connect start and goal configurations with the PRM  
E.g., using a local planner
  - Use the graph search to find the path

 Probabilistic Roadmaps for Path Planning in High Dimensional Configuration Spaces  
Lydia E. Kavraki and Petr Svestka and Jean-Claude Latombe and Mark H. Overmars,  
IEEE Transactions on Robotics and Automation, 12(4):566–580, 1996.

*First planner that demonstrates ability to solve general planning problems in more than 4-5 dimensions.*

## Part I

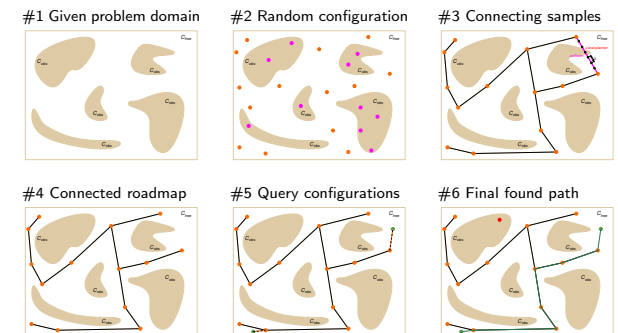
# Part 1 – Sampling-based Motion Planning

## Incremental Sampling and Searching

- Single query sampling-based algorithms incrementally created a search graph (roadmap)
  - Initialization** –  $G(V, E)$  an undirected search graph,  $V$  may contain  $q_{start}$ ,  $q_{goal}$  and/or other points in  $C_{free}$
  - Vertex selection method** – choose a vertex  $q_{cur} \in V$  for expansion
  - Local planning method** – for some  $q_{new} \in C_{free}$ , attempt to construct a path  $\tau : [0, 1] \rightarrow C_{free}$  such that  $\tau(0) = q_{cur}$  and  $\tau(1) = q_{new}$ ,  $\tau$  must be checked to ensure it is collision free
    - If  $\tau$  is not a collision-free, go to Step 2
  - Insert an edge in the graph** – Insert  $\tau$  into  $E$  as an edge from  $q_{cur}$  to  $q_{new}$  and insert  $q_{new}$  to  $V$  if  $q_{new} \notin V$
  - Check for a solution** – Determine if  $G$  encodes a solution, e.g., single search tree or graph search
  - Repeat to Step 2** – iterate unless a solution has been found or a termination condition is satisfied

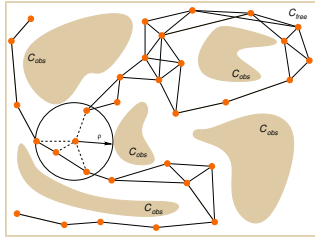
LaValle, S. M.: Planning Algorithms (2006), Chapter 5.4

## PRM Construction



## Practical PRM

- Incremental construction
- Connect nodes in a radius  $\rho$
- Local planner tests collisions up to selected resolution  $\delta$
- Path can be found by Dijkstra's algorithm



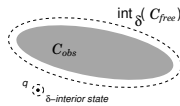
What are the properties of the PRM algorithm?

We need a couple of more formalisms.

## Probabilistic Completeness 1/2

First, we need **robustly feasible path planning problem**  $(C_{free}, q_{init}, Q_{goal})$ .

- $q \in C_{free}$  is  **$\delta$ -interior state** of  $C_{free}$  if the closed ball of radius  $\delta$  centered at  $q$  lies entirely inside  $C_{free}$ .



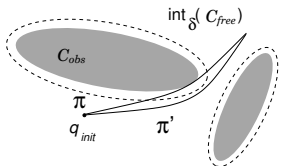
- **$\delta$ -interior** of  $C_{free}$  is  $\text{int}_\delta(C_{free}) = \{q \in C_{free} | \mathcal{B}_{\delta}(q) \subseteq C_{free}\}$ .  
A collection of all  $\delta$ -interior states.

- A collision free path  $\pi$  has **strong  $\delta$ -clearance**, if  $\pi$  lies entirely inside  $\text{int}_\delta(C_{free})$ .
- $(C_{free}, q_{init}, Q_{goal})$  is **robustly feasible** if a solution exists and it is a feasible path with **strong  $\delta$ -clearance**, for  $\delta > 0$ .

## Asymptotic Optimality 2/4

- A collision-free path  $\pi : [0, s] \rightarrow C_{free}$  has **weak  $\delta$ -clearance** if there exists a path  $\pi'$  that has **strong  $\delta$ -clearance** and homotopy  $\psi$  with  $\psi(0) = \pi$ ,  $\psi(1) = \pi'$ , and for all  $\alpha \in (0, 1]$  there exists  $\delta_\alpha > 0$  such that  $\psi(\alpha)$  has strong  $\delta$ -clearance.

Weak  $\delta$ -clearance does not require points along a path to be at least a distance  $\delta$  away from obstacles.



- A path  $\pi$  with a weak  $\delta$ -clearance
- $\pi'$  lies in  $\text{int}_\delta(C_{free})$  and it is the same homotopy class as  $\pi$

## Path Planning Problem Formulation

- Path planning problem is defined by a triplet  $\mathcal{P} = (C_{free}, q_{init}, Q_{goal})$ ,

- $C_{free} = \text{cl}(C \setminus C_{obs})$ ,  $C = (0, 1)^d$ , for  $d \in \mathbb{N}$ ,  $d \geq 2$
- $q_{init} \in C_{free}$  is the initial configuration (condition)
- $Q_{goal}$  is the goal region defined as an open subspace of  $C_{free}$

- Function  $\pi : [0, 1] \rightarrow \mathbb{R}^d$  of **bounded variation** is called :

- **path** if it is continuous;
- **collision-free path** if it is path and  $\pi(\tau) \in C_{free}$  for  $\tau \in [0, 1]$ ;
- **feasible** if it is collision-free path, and  $\pi(0) = q_{init}$  and  $\pi(1) \in \text{cl}(Q_{goal})$ .

- A function  $\pi$  with the total variation  $\text{TV}(\pi) < \infty$  is said to have bounded variation, where  $\text{TV}(\pi)$  is the total variation

$$\text{TV}(\pi) = \sup_{\{n \in \mathbb{N}, 0 = \tau_0 < \tau_1 < \dots < \tau_n = 1\}} \sum_{i=1}^n |\pi(\tau_i) - \pi(\tau_{i-1})|$$

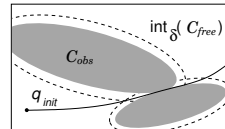
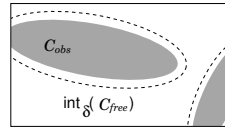
- The total variation  $\text{TV}(\pi)$  is de facto a path length.

## Probabilistic Completeness 2/2

An algorithm  $\mathcal{ALG}$  is **probabilistically complete** if, for any **robustly feasible path planning problem**  $\mathcal{P} = (C_{free}, q_{init}, Q_{goal})$

$$\lim_{n \rightarrow \infty} \Pr(\mathcal{ALG} \text{ returns a solution to } \mathcal{P}) = 1.$$

- It is a “relaxed” notion of completeness
- Applicable only to problems with a **robust solution**.



We need some space, where random configurations can be sampled

## Asymptotic Optimality 3/4

- It is applicable with a **robust optimal solution** that can be obtained as a limit of robust (non-optimal) solutions.
- A collision-free path  $\pi^*$  is **robustly optimal solution** if it has **weak  $\delta$ -clearance** and for any sequence of collision free paths  $\{\pi_n\}_{n \in \mathbb{N}}$ ,  $\pi_n \in C_{free}$  such that  $\lim_{n \rightarrow \infty} \pi_n = \pi^*$ ,

$$\lim_{n \rightarrow \infty} c(\pi_n) = c(\pi^*).$$

There exists a path with strong  $\delta$ -clearance, and  $\pi^*$  is of **the lower cost**.

- Weak  $\delta$ -clearance implies robustly feasible solution problem  
(thus, probabilistic completeness)

## Path Planning Problem

- **Feasible path planning:**

For a path planning problem  $(C_{free}, q_{init}, Q_{goal})$

- Find a feasible path  $\pi : [0, 1] \rightarrow C_{free}$  such that  $\pi(0) = q_{init}$  and  $\pi(1) \in \text{cl}(Q_{goal})$ , if such path exists.
- Report failure if no such path exists.

- **Optimal path planning:**

The optimality problem asks for a feasible path with the minimum cost.

For  $(C_{free}, q_{init}, Q_{goal})$  and a cost function  $c : \Sigma \rightarrow \mathbb{R}_{\geq 0}$

- Find a feasible path  $\pi^*$  such that  $c(\pi^*) = \min\{c(\pi) : \pi \text{ is feasible}\}$ .
- Report failure if no such path exists.

The cost function is assumed to be monotonic and bounded, i.e., there exists  $k_c$  such that  $c(\pi) \leq k_c \text{TV}(\pi)$ .

## Asymptotic Optimality 1/4

Asymptotic optimality relies on a notion of **weak  $\delta$ -clearance**

Notice, we use strong  $\delta$ -clearance for probabilistic completeness

- Function  $\psi : [0, 1] \rightarrow C_{free}$  is called **homotopy**, if  $\psi(0) = \pi_1$  and  $\psi(1) = \pi_2$  and  $\psi(\tau)$  is collision-free path for all  $\tau \in [0, 1]$ .
- A collision-free path  $\pi_1$  is **homotopic** to  $\pi_2$  if there exists homotopy function  $\psi$ .

A path homotopic to  $\pi$  can be continuously transformed to  $\pi$  through  $C_{free}$ .

## Asymptotic Optimality 4/4

An algorithm  $\mathcal{ALG}$  is **asymptotically optimal** if, for any path planning problem  $\mathcal{P} = (C_{free}, q_{init}, Q_{goal})$  and cost function  $c$  that admit a robust optimal solution with the finite cost  $c^*$

$$\Pr\left(\left\{\lim_{i \rightarrow \infty} Y_i^{\mathcal{ALG}} = c^*\right\}\right) = 1.$$

- $Y_i^{\mathcal{ALG}}$  is the extended random variable corresponding to the minimum-cost solution included in the graph returned by  $\mathcal{ALG}$  at the end of iteration  $i$ .

## Properties of the PRM Algorithm

- Completeness for the standard PRM has not been provided when it was introduced
- A simplified version of the PRM (called sPRM) has been mostly studied
- sPRM is probabilistically complete

*What are the differences between PRM and sPRM?*

## PRM vs simplified PRM (sPRM)

### Algorithm 1: PRM

**Vstup:**  $q_{init}$ , number of samples  $n$ , radius  $\rho$   
**Výstup:** PRM –  $G = (V, E)$

```

V ← ∅; E ← ∅;
for i = 0, ..., n do
    qrand ← SampleFree;
    U ← Near(G = (V, E), qrand, ρ);
    V ← V ∪ {qrand};
    foreach u ∈ U, with increasing ||u - qrand|| do
        if qrand and u are not in the same connected component of G = (V, E) then
            if CollisionFree(qrand, u) then
                E ← E ∪ {(qrand, u), (u, qrand)};
return G = (V, E);
    
```

### Algorithm 2: sPRM

**Vstup:**  $q_{init}$ , number of samples  $n$ , radius  $\rho$   
**Výstup:** PRM –  $G = (V, E)$

```

V ← {qinit} ∪ {SampleFree};
foreach v ∈ V do
    U ← Near(G = (V, E), v, ρ) \ {v};
    foreach u ∈ U do
        if CollisionFree(v, u) then
            E ← E ∪ {(v, u), (u, v)};
return G = (V, E);
    
```

There are several ways for the set  $U$  of vertices to connect them

- $k$ -nearest neighbors to  $v$
- variable connection radius  $\rho$  as a function of  $n$

## PRM – Properties

### ■ sPRM (simplified PRM)

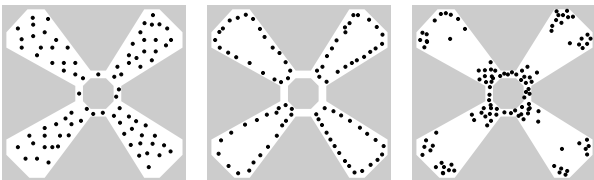
- **Probabilistically complete and asymptotically optimal**
  - Processing complexity  $O(n^2)$
  - Query complexity  $O(n^2)$
  - Space complexity  $O(n^2)$
  - Heuristics practically used are usually not probabilistic complete
    - $k$ -nearest sPRM is not probabilistically complete
    - variable radius sPRM is not probabilistically complete
- Based on analysis of Karaman and Frazzoli*

### PRM algorithm:

- + Has very simple implementation
- + Completeness (for sPRM)
- Differential constraints (car-like vehicles) are not straightforward

## Comments about Random Sampling 1/2

- Different sampling strategies (distributions) may be applied



- Notice, one of the main issue of the randomized sampling-based approaches is the narrow passage
- Several modifications of sampling based strategies have been proposed in the last decades

## Comments about Random Sampling 2/2

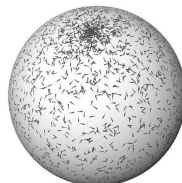
- A solution can be found using only a few samples.

*Do you know the Oracleum? (from Alice in Wonderland)*

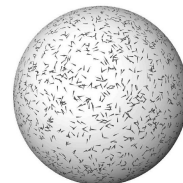
- Sampling strategies are important

- Near obstacles
- Narrow passages
- Grid-based
- Uniform sampling must be carefully considered.

*James J. Kuffner (2004): Effective Sampling and Distance Metrics for 3D Rigid Body Path Planning. ICRA.*



Naïve sampling



Uniform sampling of SO(3) using Euler angles

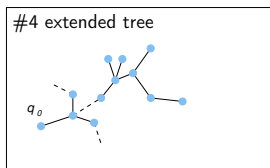
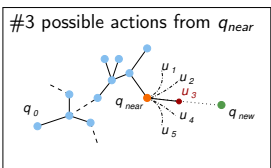
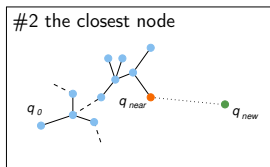
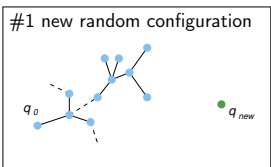
## Rapidly Exploring Random Tree (RRT)

### Single-Query algorithm

- It incrementally builds a graph (tree) towards the goal area.  
*It does not guarantee precise path to the goal configuration.*
- 1. Start with the initial configuration  $q_0$ , which is a root of the constructed graph (tree)
- 2. Generate a new random configuration  $q_{new}$  in  $C_{free}$
- 3. Find the closest node  $q_{near}$  to  $q_{new}$  in the tree  
*E.g., using KD-tree implementation like ANN or FLANN libraries*
- 4. Extend  $q_{near}$  towards  $q_{new}$   
*Extend the tree by a small step, but often a direct control  $u \in U$  that will move robot the position closest to  $q_{new}$  is selected (applied for  $\delta t$ ).*
- 5. Go to Step 2, until the tree is within a sufficient distance from the goal configuration

*Or terminates after dedicated running time.*

## RRT Construction



## RRT Algorithm

- Motivation is a single query and *control-based* path finding
- It incrementally builds a graph (tree) towards the goal area.

### Algorithm 3: Rapidly Exploring Random Tree (RRT)

**Vstup:**  $q_{init}$ , number of samples  $n$   
**Výstup:** Roadmap  $G = (V, E)$

```

V ← {qinit}; E ← ∅;
for i = 1, ..., n do
    qrand ← SampleFree;
    qnearest ← Nearest(G = (V, E), qrand);
    qnew ← Steer(qnearest, qrand);
    if CollisionFree(qnearest, qnew) then
        V ← V ∪ {qnew}; E ← E ∪ {(qnearest, qnew)};
return G = (V, E);
    
```

*Extend the tree by a small step, but often a direct control  $u \in U$  that will move robot to the position closest to  $q_{new}$  is selected (applied for  $\delta t$ ).*



**Rapidly-exploring random trees: A new tool for path planning**

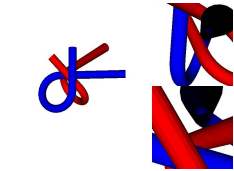
S. M. LaValle,

Technical Report 98-11, Computer Science Dept., Iowa State University, 1998

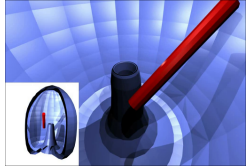
## Properties of RRT Algorithms

- Rapidly explores the space  
 *$q_{new}$  will more likely be generated in large not yet covered parts.*
- Allows considering kinodynamic/dynamic constraints (during the expansion).
- Can provide trajectory or a sequence of direct control commands for robot controllers.
- A collision detection test is usually used as a “black-box”.  
*E.g., RAPID, Bullet libraries.*
- Similarly to PRM, RRT algorithms have poor performance in narrow passage problems.
- RRT algorithms provides feasible paths.  
*It can be relatively far from optimal solution, e.g., according to the length of the path.*
- Many variants of RRT have been proposed.

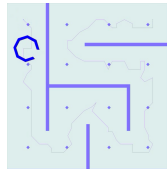
## RRT – Examples 1/2



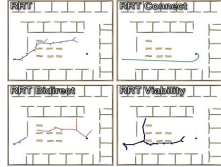
Alpha puzzle benchmark



Bugtrap benchmark



Apply rotations to reach the goal



Variants of RRT algorithms

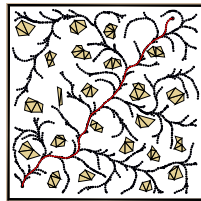
Courtesy of V. Vonásek and P. Vaněk

## Control-Based Sampling

- Select a configuration  $q$  from the tree  $T$  of the current configurations

- Pick a control input  $\vec{u} = (v, \varphi)$  and integrate system (motion) equation over a short period

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta \varphi \end{pmatrix} = \int_t^{t+\Delta t} \begin{pmatrix} v \cos \phi \\ v \sin \phi \\ \frac{v}{L} \tan \varphi \end{pmatrix} dt$$



- If the motion is collision-free, add the endpoint to the tree

E.g., considering  $k$  configurations for  $k\delta t = dt$ .

## RRT and Quality of Solution 1/2

- Let  $Y_i^{RRT}$  be the cost of the best path in the RRT at the end of iteration  $i$ .
- $Y_i^{RRT}$  converges to a random variable

$$\lim_{i \rightarrow \infty} Y_i^{RRT} = Y_\infty^{RRT}$$

- The random variable  $Y_\infty^{RRT}$  is sampled from a distribution with zero mass at the optimum, and

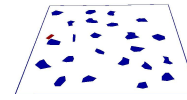
$$Pr\{Y_\infty^{RRT} > c^*\} = 1.$$

Karaman and Frazzoli, 2011

- The best path in the RRT converges to a sub-optimal solution almost surely.

## RRT – Examples 2/2

- Planning for a car-like robot

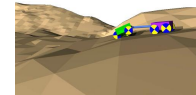


- Planning on a 3D surface



- Planning with dynamics

(friction forces)



Courtesy of V. Vonásek and P. Vaněk

## Car-Like Robot

- Configuration

$$\vec{x} = \begin{pmatrix} x \\ y \\ \phi \end{pmatrix}$$

position and orientation

- Controls

$$\vec{u} = \begin{pmatrix} v \\ \varphi \end{pmatrix}$$

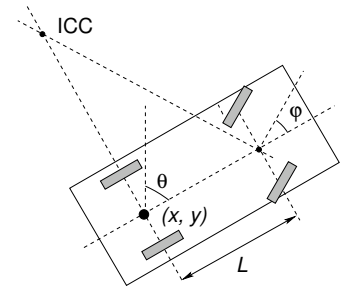
forward velocity, steering angle

- System equation

$$\dot{x} = v \cos \phi$$

$$\dot{y} = v \sin \phi$$

$$\dot{\phi} = \frac{v}{L} \tan \varphi$$



Kinematic constraints  $\dim(\vec{u}) < \dim(\vec{x})$

Differential constraints on possible  $\dot{q}$ :

$$\dot{x} \sin(\phi) - \dot{y} \cos(\phi) = 0$$

## Sampling-Based Motion Planning

- PRM and RRT are theoretically probabilistic complete
- They provide a feasible solution without quality guarantee

*Despite that, they are successfully used in many practical applications*

- In 2011, a systematical study of the asymptotic behavior of randomized sampling-based planners has been published

*It shows, that in some cases, they converge to a non-optimal value with a probability 1.*

- Based on the study, new algorithms have been proposed: **RRG** and optimal RRT (**RRT\***)

Karaman, S., Frazzoli, E. (2011): Sampling-based algorithms for optimal motion planning. IJRR.



<http://seerac.scripts.mit.edu/rtrtstar>

## Part II

## Part 2 – Optimal Sampling-based Motion Planning Methods

## RRT and Quality of Solution 2/2

- RRT does not satisfy a necessary condition for the asymptotic optimality

- For  $0 < R < \inf_{q \in Q_{goal}} \|q - q_{init}\|$ , the event  $\{\lim_{n \rightarrow \infty} Y_n^{RRT} = c^*\}$  occurs only if the  $k$ -th branch of the RRT contains vertices outside the  $R$ -ball centered at  $q_{init}$  for infinitely many  $k$ .

See Appendix B in Karaman&Frazzoli, 2011

- It is required the root node will have infinitely many subtrees that extend at least a distance  $\epsilon$  away from  $q_{init}$

*The sub-optimality is caused by disallowing new better paths to be discovered.*

## Rapidly-exploring Random Graph (RRG)

**Algorithm 4:** Rapidly-exploring Random Graph (RRG)

**Vstup:**  $q_{init}$ , number of samples  $n$

**Výstup:**  $G = (V, E)$

```

V ← ∅; E ← ∅;
for i = 0, ..., n do
    qrand ← SampleFree;
    qnearest ← Nearest(G = (V, E), qrand);
    qnew ← Steer(qnearest, qrand);
    if CollisionFree(qnearest, qnew) then
        Qnear ← Near(G = (V, E), qnew, min{γRRG(log(card(V))/card(V))1/d, η});
        V ← V ∪ {qnew};
        E ← E ∪ {(qnearest, qnew), (qnew, qnearest)};
        foreach qnear ∈ Qnear do
            if CollisionFree(qnear, qnew) then
                E ← E ∪ {(qrand, u), (u, qrand)};
return G = (V, E);
    
```

*Proposed by Karaman and Frazzoli (2011). Theoretical results are related to properties of Random Geometric Graphs (RGG) introduced by Gilbert (1961) and further studied by Penrose (1999).*



## RRG Expansions

- At each iteration, RRG tries to connect new sample to all vertices in the  $r_n$  ball centered at it.

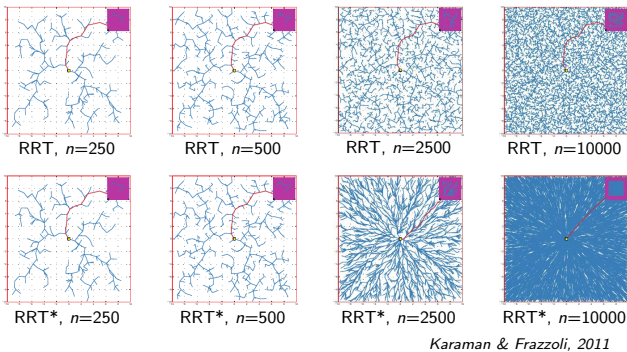
- The ball of radius

$$r(\text{card}(V)) = \min \left\{ \gamma_{RRG} \left( \frac{\log(\text{card}(V))}{\text{card}(V)} \right)^{1/d}, \eta \right\}$$

where

- $\eta$  is the constant of the local steering function
- $\gamma_{RRG} > \gamma_{RRG}^* = 2(1 + 1/d)^{1/d} (\mu(C_{free})/\xi_d)^{1/d}$ 
  - $d$  – dimension of the space;
  - $\mu(C_{free})$  – Lebesgue measure of the obstacle-free space;
  - $\xi_d$  – volume of the unit ball in  $d$ -dimensional Euclidean space.
- The connection radius decreases with  $n$
- The rate of decay  $\approx$  the average number of connections attempted is proportional to  $\log(n)$

## Example of Solution 1/3



## Overview of Randomized Sampling-based Algorithms

Algorithm	Probabilistic Completeness	Asymptotic Optimality
sPRM	✓	✗
k-nearest sPRM	✗	✗
RRT	✓	✗
RRG	✓	✓
PRM*	✓	✓
RRT*	✓	✓

Notice,  $k$ -nearest variants of RRG, PRM\*, and RRT\* are complete and optimal as well.

## RRG Properties

- Probabilistically complete
- Asymptotically optimal
- Complexity is  $O(\log n)$

(per one sample)

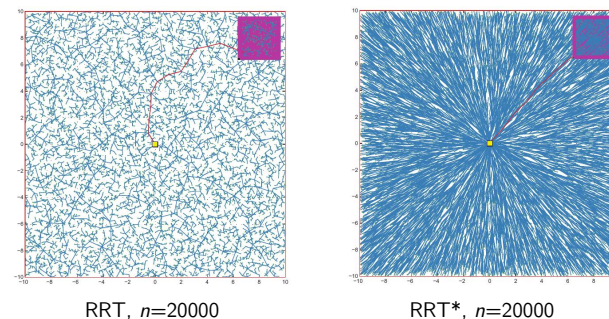
- Computational efficiency and optimality

- Attempt connection to  $\Theta(\log n)$  nodes at each iteration;

in average

- Reduce volume of the “connection” ball as  $\log(n)/n$ ;
- Increase the number of connections as  $\log(n)$ .

## Example of Solution 2/3



## Summary of the Lecture

## Other Variants of the Optimal Motion Planning

- PRM\*** – it follows standard PRM algorithm where connections are attempted between roadmap vertices that are within connection radius  $r$  as a function of  $n$

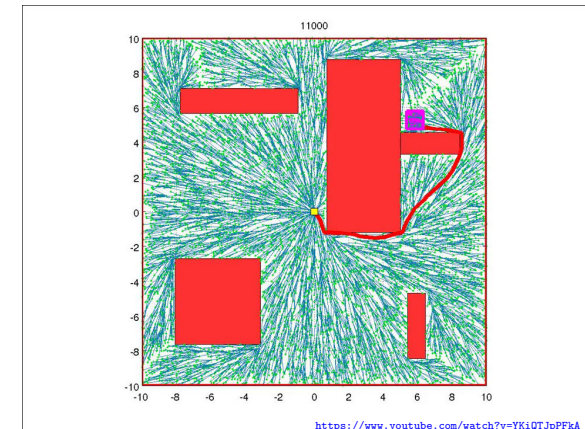
$$r(n) = \gamma_{PRM}(\log(n)/n)^{1/d}$$

- RRT\*** – a modification of the RRG, where cycles are avoided

A tree version of the RRG

- A tree roadmap allows to consider non-holonomic dynamics and kinodynamic constraints.
- It is basically RRG with “rerouting” the tree when a better path is discovered.

## Example of Solution 3/3



## Topics Discussed

- Randomized Sampling-based Methods
- Probabilistic Road Map (PRM)
- Characteristics of path planning problems
- Random sampling
- Rapidly Exploring Random Tree (RRT)
- Optimal sampling-based motion planning
- Rapidly-exploring Random Graph (RRG)

- Next: Multi-Goal Motion Planning and Multi-Goal Path Planning**