

# Robotic Information Garthering - Exploration of Unknown Environment

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Lecture 05

**B4M36UIR – Artificial Intelligence in Robotics**



# Overview of the Lecture

- Part 1 – Robotic Information Gathering - Robotic Exploration
  - Robotic Information Gathering
  - Robotic Exploration
  - TSP-based Robotic Exploration
  - Robotic Information Gathering



# Part I

## Part 1 – Robotic Exploration



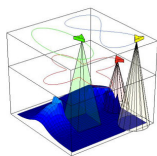
# Outline

- Robotic Information Gathering
- Robotic Exploration
- TSP-based Robotic Exploration
- Robotic Information Gathering



# Robotic Information Gathering

*Create a model of phenomena by autonomous mobile robots performing measurements in a dynamic unknown environment.*



# Challenges in Robotic Information Gathering

## ■ Where to take new measurements?

*To improve the phenomena model*

## ■ What locations visit first?

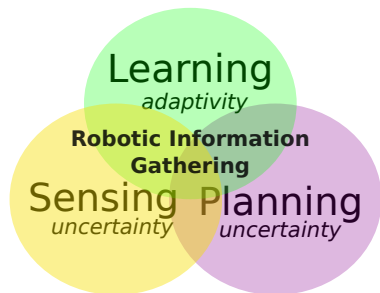
*On-line decision-making*

## ■ How to efficiently utilize more robots?

*To divide the task between the robots*

## ■ How to navigate robots to the selected locations?

*Improve Localization vs Model*



**How to address all these aspects altogether to find a cost efficient solution using in-situ decisions?**



# Robotic Exploration of Unknown Environment

- Robotic exploration is a fundamental problem of robotic information gathering
- The problem is:

## How to efficiently utilize a group of mobile robots to autonomously create a map of an unknown environment

- Performance indicators vs constraints

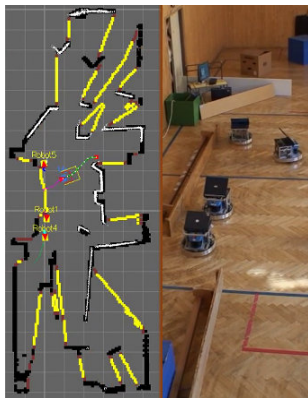
*Time, energy, map quality vs robots, communication*

- Performance in a real mission depends on the on-line **decision-making**

- It includes the problems of:

- Map building and localization
- Determination of the navigational waypoints
- Path planning and navigation to the waypoints
- Coordination of the actions (multi-robot team)

*Where to go next?*



*Courtesy of M. Kulich*



# Outline

- Robotic Information Gathering
- **Robotic Exploration**
- TSP-based Robotic Exploration
- Robotic Information Gathering





# Mobile Robot Exploration

- Create a map of the environment
- **Frontier**-based approach

*Yamauchi (1997)*

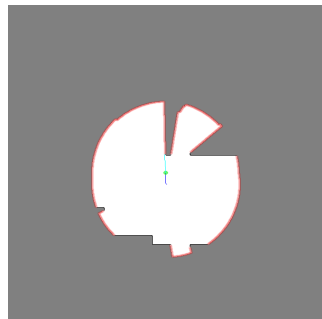
- Occupancy grid map

*Moravec and Elfes (1985)*

- Laser scanner sensor

- Next-best-view approach

*Select the next robot goal*



Performance metric:

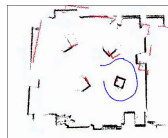
Time to create the map of the whole environment

*search and rescue mission*



# Environment Representation – Mapping and Occupancy Grid

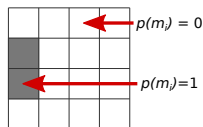
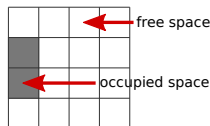
- The robot uses its sensors to build a map of the environment
- The robot should be localized to integrate new sensor measurements into a globally consistent map
- **SLAM** – Simultaneous Localization and Mapping
  - The robot uses the map being built to localize itself
  - The map is primarily to help to localize the robot
  - The map is a “side product” of SLAM
- **Grid map** – discretized world representation
  - A cell is **occupied** (an obstacle) or **free**
- **Occupancy grid map**
  - Each cell is a binary random variable modeling the occupancy of the cell



# Occupancy Grid

## Assumptions

- The area of a cell is either completely free or occupied
  - Cells (random variables) are independent of each other
  - The state is **static**
- A cell is a binary random variable modeling the occupancy of the cell
  - Cell  $m_i$  is occupied  $p(m_i) = 1$
  - Cell  $m_i$  is not occupied  $p(m_i) = 0$
  - Unknown**  $p(m_i) = 0.5$
- Probability distribution of the map  $m$



$$p(m) = \prod_i p(m_i)$$

- Estimation of map from sensor data  $z_{1:t}$  and robot poses  $x_{1:t}$

$$p(m|z_{1:t}, x_{1:t}) = \prod_i p(m_i|z_{1:t}, x_{1:t})$$

*Binary Bayes filter – Bayes rule and Markov process assumption*



# Occupancy Grid

## Assumptions

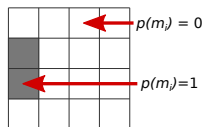
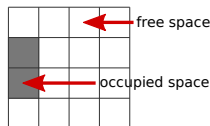
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*Binary Bayes filter – Bayes rule and Markov process assumption*



## Binary Bayes Filter 1/2

- Sensor data  $z_{1:t}$  and robot poses  $x_{1:t}$
- Binary random variables are independent and states are static

$$p(m_i | z_{1:t}, x_{1:t}) \stackrel{\text{Bayes rule}}{=} \frac{p(z_t | m_i, z_{1:t-1}, x_{1:t}) p(m_i | z_{1:t-1}, x_{1:t})}{p(z_t | z_{1:t-1}, x_{1:t})}$$

$$\stackrel{\text{Markov}}{=} \frac{p(z_t | m_i, x_t) p(m_i | z_{1:t-1}, x_{1:t-1})}{p(z_t | z_{1:t-1}, x_{1:t})}$$

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$$p(z_t | m_i, x_t) = \frac{p(m_i, z_t, x_t) p(z_t, x_t)}{p(m_i | x_t)}$$

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$$p(m_i, z_{1:t}, x_{1:t}) \stackrel{\text{Bayes rule}}{=} \frac{p(m_i | z_t, x_t) p(z_t | x_t) p(m_i | z_{1:t-1}, x_{1:t-1})}{p(m_i | x_t) p(z_t | z_{1:t-1}, x_{1:t})}$$

$$\stackrel{\text{Markov}}{=} \frac{p(m_i | z_t, x_t) p(z_t | x_t) p(m_i | z_{1:t-1}, x_{1:t-1})}{p(m_i) p(z_t | z_{1:t-1}, x_{1:t})}$$



## Binary Bayes Filter 2/2

- Probability a cell is occupied

$$p(m_i | z_{1:t}, x_{1:t}) = \frac{p(m_i | z_t, x_t) p(z_t | x_t) p(m_i | z_{1:t-1}, x_{1:t-1})}{p(m_i) p(z_t | z_{1:t-1}, x_{1:t})}$$

- Probability a cell is not occupied

$$p(\neg m_i | z_{1:t}, x_{1:t}) = \frac{p(\neg m_i | z_t, x_t) p(z_t | x_t) p(\neg m_i | z_{1:t-1}, x_{1:t-1})}{p(\neg m_i) p(z_t | z_{1:t-1}, x_{1:t})}$$

- Ratio of the probabilities

$$\begin{aligned} \frac{p(m_i | z_{1:t}, x_{1:t})}{p(\neg m_i | z_{1:t}, x_{1:t})} &= \frac{p(m_i | z_t, x_t) p(m_i | z_{1:t-1}, x_{1:t-1}) p(\neg m_i)}{p(\neg m_i | z_t, x_t) p(\neg m_i | z_{1:t-1}, x_{1:t-1}) p(m_i)} \\ &= \frac{p(m_i | z_t, x_t)}{1 - p(m_i | z_t, x_t)} \frac{p(m_i, z_{1:t-1}, x_{1:t-1})}{1 - p(m_i | z_{1:t-1}, x_{1:t-1})} \frac{1 - p(m_i)}{p(m_i)} \end{aligned}$$

sensor model  $z_t$ , recursive term, prior



## Logs Odds Notation

- Log odds ratio is defined as

$$l(x) = \log \frac{p(x)}{1 - p(x)}$$

- and the probability  $p(x)$  is

$$p(x) = 1 - \frac{1}{1 + e^{l(x)}}$$

- The product modeling the cell  $m_i$  based on  $z_{1:t}$  and  $x_{1:t}$

$$l(m_i | z_{1:t}, x_{1:t}) = \underbrace{l(m_i | z_t, x_t)}_{\text{inverse sensor model}} + \underbrace{l(m_i, | z_{1:t-1}, x_{1:t-1})}_{\text{recursive term}} - \underbrace{l(m_i)}_{\text{prior}}$$



# Occupancy Mapping Algorithm

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**Algorithm 1:** OccupancyGridMapping( $\{l_{t-1,i}\}, x_t, z_t$ )

---

**foreach**  $m_i$  of the map  $m$  **do**

**if**  $m_i$  in the perceptual field of  $z_t$  **then**

$l_{t,i} := l_{t-1,i} + \text{inv\_sensor\_model}(m_i, x_t, z_t) - l_0;$

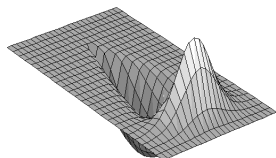
**else**

$l_{t,i} := l_{t-1,i};$

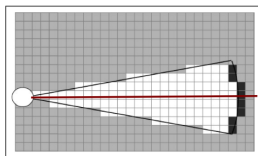
**return**  $\{l_{t,i}\}$

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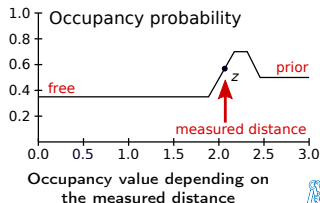
- Occupancy grid mapping developed by Moravec and Elfes in mid 80'ies for noisy sonars



Inverse sensor model for sonars range sensors



Field of view of the sonar range sensor





## Model for Laser Sensor

- The model is “sharp” with a precise detection of the obstacle
- For the range measurement  $d_i$ , update the grid cells along a sensor beam

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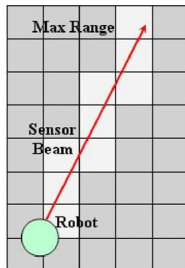
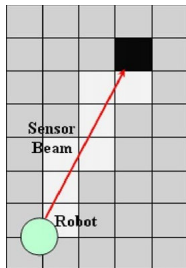
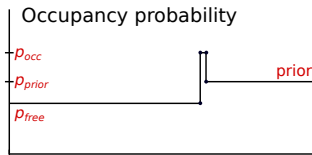
**Algorithm 2:** Update map for  $\mathcal{L} = (d_1, \dots, d_n)$

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```

foreach  $d_i \in \mathcal{L}$  do
  foreach cell  $m_i$  raycasted towards  $\min(d_i, \text{range})$  do
     $p := \text{grid}(m_i)p_{\text{free}}$ ;
     $\text{grid}(m_i) := p/2p - p_{\text{free}} - \text{grid}(m_i) + 1$ ;
   $m_d := \text{cell at } d_i$ ;
  if obstacle detected at  $m_d$  then
     $p := \text{grid}(m_d)p_{\text{occ}}$ ;
     $\text{grid}(m_i) := p/2p - p_{\text{occ}} - \text{grid}(m_i) + 1$ 
  else
     $p := \text{grid}(m_d)p_{\text{free}}$ ;
     $\text{grid}(m_i) := p/2p - p_{\text{free}} - \text{grid}(m_i) + 1$ 
    
```

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## Frontier-based Exploration

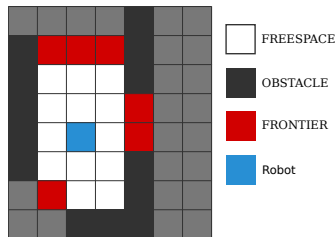
- The basic idea of the **frontier** based exploration is navigation of the mobile robot towards unknown regions

*Yamauchi (1997)*

- **Frontier** – a border of the known and unknown regions of the environment
- Based on the probability of individual cells in the occupancy grid, cells are classified into:

- **FREESPACE** –  $p(m_i) < 0.5$
- **OBSTACLE** –  $p(m_i) > 0.5$
- **UNKNOWN** –  $p(m_i) = 0.5$

- **Frontier cell** is a **FREESPACE** cell that is incident with an **UNKNOWN** cell
- Frontier cells as the navigation way-points have to be reachable, e.g., after obstacle growing



Use grid-based path planning



## Frontier-based Exploration Strategy

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### Algorithm 3: Frontier-based Exploration

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$map := \text{init}(\text{robot}, \text{scan});$

**while** *there are some reachable frontiers* **do**

    Update occupancy  $map$  using new sensor data and Bayes rule;

$\mathcal{M} :=$  Created grid map from  $map$  using thresholding;

$\mathcal{M} :=$  Grow obstacle according to the dimension of the robot;

$\mathcal{F} :=$  Determine frontier cells from  $\mathcal{M}$ ;

$\mathcal{F} :=$  Filter out unreachable frontiers from  $\mathcal{F}$ ;

$f :=$  Select the closest frontier from  $\mathcal{F}$ , e.g. using shortest path;

$path :=$  Plan a path from the current robot position to  $f$ ;

    Navigate robot towards  $f$  along  $path$  (for a while);

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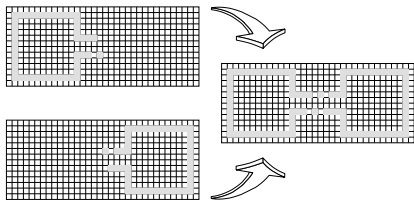
## Multi-Robot Exploration – Map Merge

- The individual maps can be merged in a similar way as integration of new sensor measurements

$$P(occ_{x,y}) = \frac{odds_{x,y}}{1 + odds_{x,y}},$$

$$odds_{x,y} = \prod_{i=1}^n odds_{x,y}^i,$$

$$odds_{x,y}^i = \frac{P(occ_{x,y}^i)}{1 - P(occ_{x,y}^i)}.$$



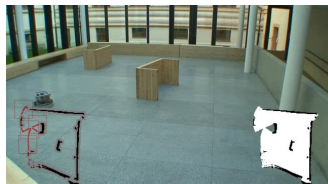
$P(occ_{x,y}^i)$  is the probability that grid cell on the global coordinate is occupied in the map of the robot.

We need the same global reference frame (localization).



## Multi-Robot Exploration – Overview

- We need to assign navigation waypoint to each robot, which can be formulated as the **task-allocation problem**
- Exploration can be considered as an **iterative procedure**
  1. Initialize the occupancy grid  $Occ$
  2.  $\mathcal{M} \leftarrow \text{create\_navigation\_grid}(Occ)$   
*cells of  $\mathcal{M}$  have values {freespace, obstacle, unknown}*
  3.  $\mathbf{F} \leftarrow \text{detect\_frontiers}(\mathcal{M})$
  4. Goal candidates  $\mathbf{G} \leftarrow \text{generate}(\mathbf{F})$
  5. **Assign next goals to each robot  $r \in \mathbf{R}$ ,**  
 $((r_1, g_{r_1}), \dots, (r_m, g_{r_m})) = \text{assign}(\mathbf{R}, \mathbf{G}, \mathcal{M})$
  6. **Create a plan  $\mathbf{P}_i$  for each pair  $\langle r_i, g_{r_i} \rangle$**   
*consisting of simple operations*
  7. **Perform each plan up to  $s_{max}$  operations**  
*At each step, update  $Occ$  using new sensor measurements*
  8. If  $|G| == 0$  exploration finished, otherwise go to Step 2

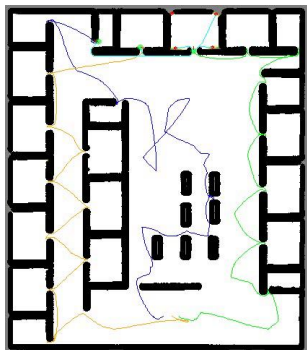


- There are several parts of the exploration procedure where important decisions are made regarding the exploration performance, e.g.
  - How to determined goal candidates from the the frontiers?
  - How to plan a paths and assign the goals to the robots?
  - How to navigate the robots towards the goal?
  - When to replan?
  - etc.



## Exploration Procedure – Decision-Making Parts

1. Initialize – set plans for  $m$  robots,  $\mathcal{P} = (P_1, \dots, P_m)$ ,  $P_i = \emptyset$ .
2. Repeat
  - 2.1 **Navigate robots** using the plans  $\mathcal{P}$ ;
  - 2.2 Collect new measurements;
  - 2.3 Update the navigation map  $\mathcal{M}$ ;
 Until **replanning condition** is met.
3. **Determine goal candidates  $\mathbf{G}$**  from  $\mathcal{M}$ .
4. If  $|\mathbf{G}| > 0$  **assign goals to the robots**
  - $(\langle r_1, g_{r_1} \rangle, \dots, \langle r_m, g_{r_m} \rangle) = \text{assign}(\mathbf{R}, \mathbf{G}, \mathcal{M})$ ,  
 $r_i \in \mathbf{R}, g_{r_i} \in \mathbf{G}$ ;
  - **Plan paths** to the assigned goals  
 $\mathcal{P} = \text{plan}(\langle r_1, g_{r_1} \rangle, \dots, \langle r_m, g_{r_m} \rangle, \mathcal{M})$ ;
  - Go to Step 2.
5. Stop all robots or navigate them to the depot



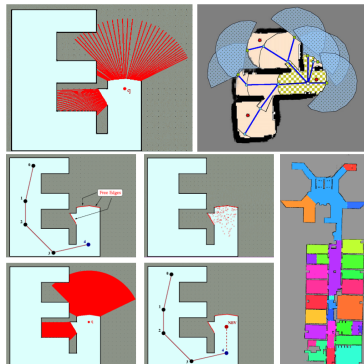
*All reachable parts of the environment are explored.*



# Improvements of the basic Frontier-based Exploration

Several improvements have been proposed in the literature

- Introducing utility as a computation of expected covered area from a frontier  
 González-Baños, Latombe (2002)
- Map segmentation for identification of rooms and exploration of the whole room by a single robot  
 Holz, Basilico, Amigoni, Behnke (2010)
- Consider longer planning horizon (as a solution of the Traveling Salesman Problem (TSP))  
 Zlot, Stentz (2006), Kulich, Faigl (2011, 2012)
- Representatives of free edges  
 Faigl, Kulich (2015)



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- **TSP-based Robotic Exploration**
- Robotic Information Gathering

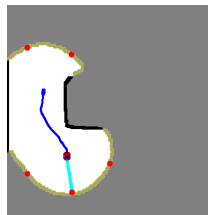




## Distance Cost Variants

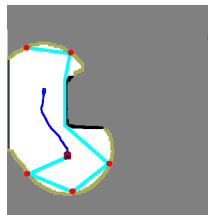
### ■ Simple robot–goal distance

- Evaluate all goals using the robot–goal distance  
*A length of the path from the robot position to the goal candidate*
- Greedy goal selection – the closest one
- Using frontier representatives improves the performance a bit



### ■ TSP distance cost

- Consider visitations of all goals  
*Solve the associated traveling salesman problem (TSP)*
- A length of the tour visiting all goals
- Use frontier representatives
- the TSP distance cost improves performance about 10-30% without any further heuristics, e.g., expected coverage (utility)



Kulich, M., Faigl, J, Přeučil, L. (2011): On Distance Utility in the Exploration Task. ICRA.



## Multi-Robot Exploration Strategy

- A set of  $m$  robots at positions  $\mathbf{R} = \{r_1, r_2, \dots, r_m\}$
- At time  $t$ , let a set of  $n$  goal candidates be  $\mathbf{G}(t) = \{g_1, \dots, g_n\}$

*I.e., frontiers*



- The exploration strategy (at the planning step  $t$ ):

*Select a goal  $g \in \mathbf{G}(t)$  for each robot  $r \in \mathbf{R}$  that will minimize the required time to explore the environment.*

The problem is formulated as the **task-allocation problem**

$$(\langle r_1, g_{r_1} \rangle, \dots, \langle r_m, g_{r_m} \rangle) = \text{assign}(\mathbf{R}, \mathbf{G}(t), \mathcal{M}),$$

*where  $\mathcal{M}$  is the current map*



## Multi-Robot Exploration – Problem Definition

A problem of creating a grid map of the unknown environment by a set of  $m$  robots  $\mathbf{R} = \{r_1, r_2, \dots, r_m\}$ .

Exploration is an iterative procedure:

1. Collect new sensor measurements
2. Determine a set of goal candidates

$$\mathbf{G}(t) = \{g_1, g_2, \dots, g_n\}$$

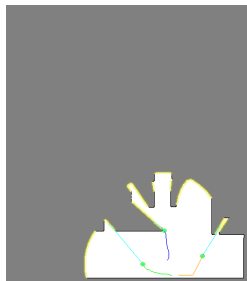
*e.g., frontiers*

3. At time step  $t$ , select next goal for each robot as the **task-allocation problem**

$$(\langle r_1, g_{r_1} \rangle, \dots, \langle r_m, g_{r_m} \rangle) = \text{assign}(\mathbf{R}, \mathbf{G}(t), \mathcal{M}(t))$$

*using the distance cost function*

4. Navigate robots towards goal
5. If  $|\mathbf{G}(t)| > 0$  go to Step 1; otherwise terminate



# Goal Assignment Strategies – Task Allocation Algorithms

## 1. Greedy Assignment

*Yamauchi B, Robotics and Autonomous Systems 29, 1999*

- Randomized greedy selection of the closest goal candidate

## 2. Iterative Assignment

*Werger B, Mataric M, Distributed Autonomous Robotic Systems 4, 2001*

- Centralized variant of the broadcast of local eligibility algorithm (BLE)

## 3. Hungarian Assignment

- Optimal solution of the task-allocation problem for assignment of  $n$  goals and  $m$  robots in  $O(n^3)$

*Stachniss C, C implementation of the Hungarian method, 2004*

## 4. Multiple Traveling Salesman Problem – MTSP Assignment

- ⟨cluster–first, route–second⟩, the TSP distance cost

*Faigl et al. 2012*



## MTSP-based Task-Allocation Approach

- Consider the task-allocation problem as the **Multiple Traveling Salesman Problem (MTSP)**

- MTSP heuristic *(cluster-first, route-second)*

1. Cluster the goal candidates  $\mathbf{G}$  to  $m$  clusters

$$\mathbf{C} = \{C_1, \dots, C_m\}, C_i \subseteq \mathbf{G}$$

*using K-means*

2. For each robot  $r_i \in \mathbf{R}, i \in \{1, \dots, m\}$  select the next goal  $g_i$  from  $C_i$  using the TSP distance cost

*Kulich et al., ICRA (2011)*

- Solve the TSP on the set  $C_i \cup \{r_i\}$

*the tour starts at  $r_i$*

- The next robot goal  $g_i$  is the first goal of the found TSP tour

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Faigl, J., Kulich, M., Přeučil, L. (2012): Goal Assignment using Distance Cost in Multi-Robot Exploration . IROS.



# Statistical Evaluation of the Exploration Strategies

- Evaluation for the number of robots  $m$  and sensor range  $\rho$

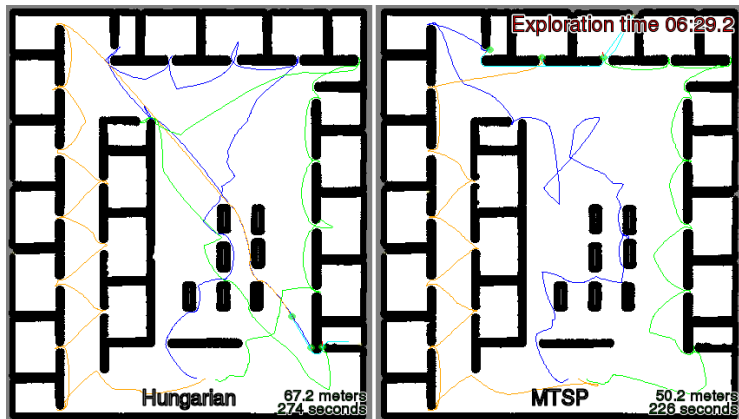
$\rho$	$m$	Iterative vs Greedy	Hungarian vs Iterative	MTSP vs Hungarian
3.0	3	+	=	+
3.0	5	+	=	+
3.0	7	+	=	+
3.0	10	+	+	-
4.0	3	+	=	+
4.0	5	+	=	=
4.0	7	+	=	+
4.0	10	+	+	-
5.0	3	+	=	+
5.0	5	+	=	+
5.0	7	+	=	+
5.0	10	+	+	-

Total number of trials 14 280



# Performance of the MTSP vs Hungarian Algorithm

- Replanning as quickly as possible;  $m = 3, \rho = 3 m$



*The MTSP assignment provides better performance*



# Outline

- Robotic Information Gathering
- Robotic Exploration
- TSP-based Robotic Exploration
- Robotic Information Gathering





# Information Theory in Robotic Information Gathering

- Employ information theory in control policy for robotic exploration
  - **Entropy** – uncertainty of  $x$ :  $H[x] = - \int p(x) \log p(x) dx$
  - **Conditional Entropy** – expected uncertainty of  $x$  after learning unknown  $z$ ;  $H[x|z]$
  - **Mutual information** – how much uncertainty of  $x$  will be reduced by learning  $z$ ;  
 $I_{MI}[x; z] = H[x] - H[x|z]$
- Control policy is a rule how to select the robot action that reduces the uncertainty of estimate by learning measurements:

$$\operatorname{argmax}_{a \in A} I_{MI}[x; z|a],$$

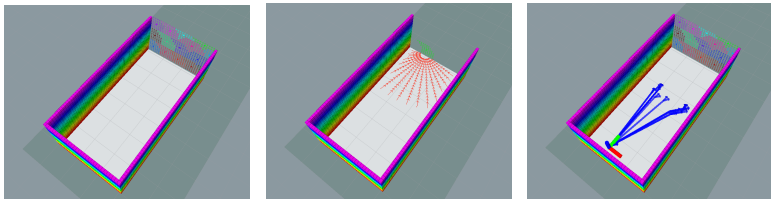
where  $A$  is a set of possible actions,  $x$  is a future estimate, and  $z$  is future measurement

- Computation of the mutual information is computationally demanding
- **Cauchy-Schwarz Quadratic Mutual Information (CSQMI)** defined similarly to mutual information
  - A linear time approximations for CSQMI  
 Charrow, B. et al., (2015): Information-theoretic mapping using Cauchy-Schwarz Quadratic Mutual Information. ICRA.
- Compute CSQMI as Cauchy-Schwarz divergence  $I_{CS}[m; z]$  – raycast of the sensor beam and determine distribution over the range returns



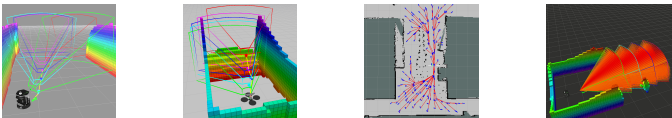
# Actions

- Actions are shortest path to cover the frontiers

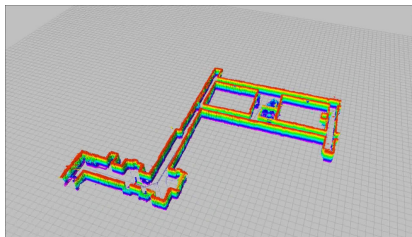


Detect and cluster frontiers   Sampled poses to cover a cluster   Paths to the sampled poses

- Select an action (a path) that maximizes the rate of Cauchy-Schwarz Quadratic Mutual Information



## Example of Autonomous Exploration using CSQMI



Ground vehicle



Aerial vehicle

- Planning with trajectory optimization – determine trajectory maximizing  $I_{CS}$   
 Charrow, B. et al., (2015): Information-Theoretic Planning with Trajectory Optimization for Dense 3D Mapping. RSS.

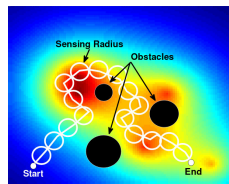


# Robotic Information Gathering

- Robotic information gathering can be considered as the **informative motion planning** problem to determine trajectory  $\mathcal{P}^*$  such that

$$\mathcal{P}^* = \operatorname{argmax}_{\mathcal{P} \in \Psi} I(\mathcal{P}), \text{ such that } c(\mathcal{P}) \leq B, \text{ where}$$

- $\Psi$  is the space of all possible robot trajectories,
  - $I(\mathcal{P})$  is the information gathered along the trajectory  $\mathcal{P}$
  - $c(\mathcal{P})$  is the cost of  $\mathcal{P}$  and  $B$  is the allowed budget
- Searching the space of all possible trajectories is complex and demanding problem
- A discretized problem can be solved by combinatorial optimization techniques
  - Usually scale poorly with the size of the problem*
- A trajectory is from a continuous domain
- Sampling-based motion planning techniques** can be employed for finding maximally informative trajectories



Hollinger, G., Sukhatme, G. (2014): Sampling-based robotic information gathering algorithms. IJRR.

# Summary of the Lecture



# Topics Discussed

- Robotic information gathering
- Robotic exploration of unknown environment
- Occupancy grid map
- Frontier based exploration
- Exploration procedure and decision-making
- TSP-based distance cost in frontier-based exploration
- Multi-robot exploration and task-allocation
- Mutual information and informative path planning *informative and motivational*
  
- Next: Randomized sampling-based motion planning methods



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- Mutual information and informative path planning *informative and motivational*
  
- **Next: Randomized sampling-based motion planning methods**

