

# Grid and Graph based Path Planning Methods

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Lecture 04

**B4M36UIR – Artificial Intelligence in Robotics**



# Overview of the Lecture

- Part 1 – Grid and Graph based Path Planning Methods
  - Grid-based Planning
  - DT for Path Planning
  - Graph Search Algorithms
  - D\* Lite
  - Path Planning based on Reaction-Diffusion Process *Curiosity*



# Part I

## Part 1 – Grid and Graph based Path Planning Methods



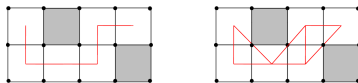
# Outline

- Grid-based Planning
- DT for Path Planning
- Graph Search Algorithms
- D\* Lite
- Path Planning based on Reaction-Diffusion Process



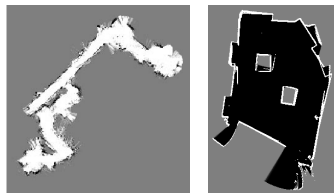
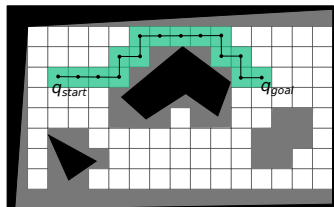
# Grid-based Planning

- A subdivision of  $\mathcal{C}_{free}$  into smaller cells
- **Grow obstacles** can be simplified by growing borders by a diameter of the robot
- Construction of the planning graph  $G = (V, E)$  for  $V$  as a set of cells and  $E$  as the **neighbor-relations**
  - 4-neighbors and 8-neighbors



- A grid map can be constructed from the so-called occupancy grid maps

*E.g., using thresholding*



# Grid-based Environment Representations

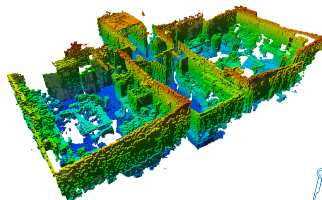
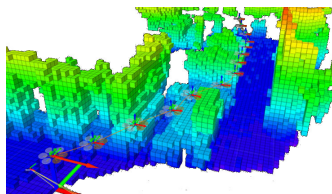
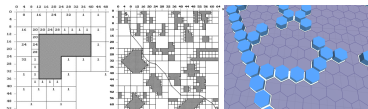
- Hierarchical planning
  - Coarse resolution and re-planning on finer resolution

Holte, R. C. et al. (1996): Hierarchical A\*: searching abstraction hierarchies efficiently. AAAI.

- Octree can be used for the map representation
- In addition to squared (or rectangular) grid a hexagonal grid can be used
- 3D grid maps – **octomap**

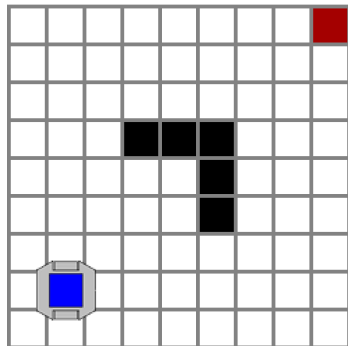
<https://octomap.github.io>

- Memory grows with the size of the environment
- Due to limited resolution it may fail in narrow passages of  $\mathcal{C}_{free}$



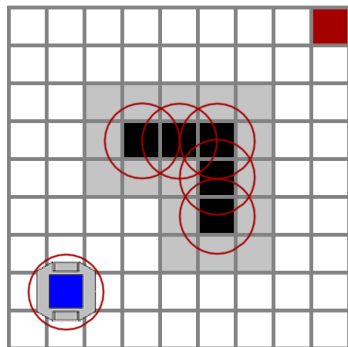
# Example of Simple Grid-based Planning

- Wave-front propagation using path simplification
- Initial map with a robot and goal
- Obstacle growing
- Wave-front propagation – “flood fill”
- Find a path using a navigation function
- Path simplification
  - “Ray-shooting” technique combined with **Bresenham’s line algorithm**
  - The path is a sequence of “key” cells for avoiding obstacles



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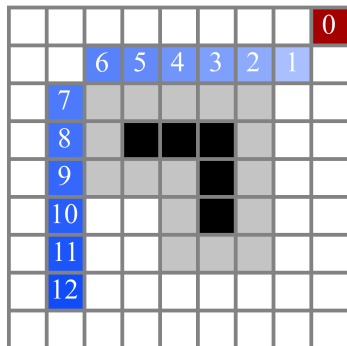
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8	7	6	5	4	3	2	1	0
8	7	6	5	4	3	2	1	1
8	7						2	2
8	8						3	3
9	9						4	4
10	10	10	10				5	5
11	11	11	10				6	6
	12	11	10	9	8	7	7	7
		11	10	9	8	8	8	8



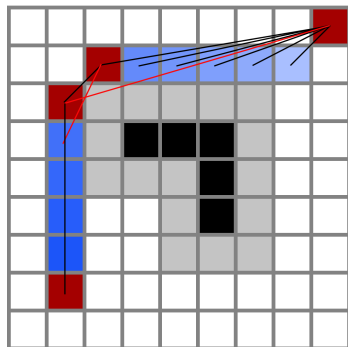
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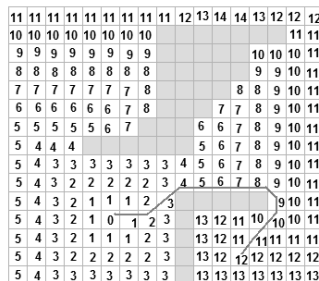
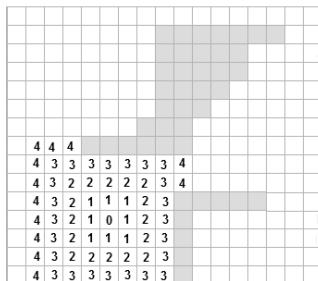
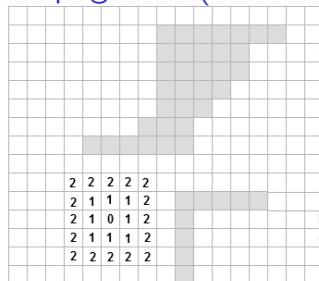
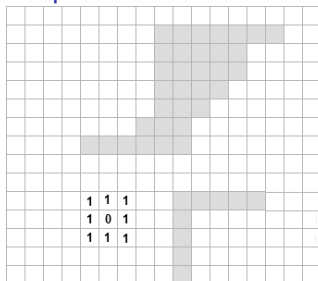


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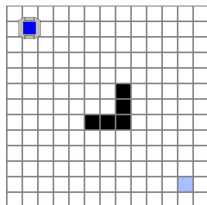


# Example – Wave-Front Propagation (Flood Fill)

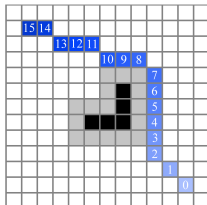
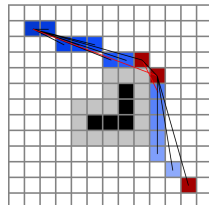


## Path Simplification

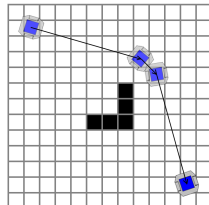
- The initial path is found in a grid using 4-neighborhood
- The rayshoot cast a line into a grid and possible collisions of the robot with obstacles are checked
- The “farthest” cells without collisions are used as “turn” points
- The final path is a sequence of straight line segments



Initial and goal locations

Obstacle growing,  
wave-front propagation

Ray-shooting



Simplified path



# Bresenham's Line Algorithm

- Filling a grid by a line with avoiding float numbers

- A line from  $(x_0, y_0)$  to  $(x_1, y_1)$  is given by  $y = \frac{y_1 - y_0}{x_1 - x_0}(x - x_0) + y_0$

```

1  CoordsVector& bresenham(const Coords& pt1, const 26      int twoDy = 2 * dy;
   Coords& pt2, CoordsVector& line)                27      int twoDyTwoDx = twoDy - 2 * dx; //2*Dy - 2*Dx
2  {                                                28      int e = twoDy - dx; //2*Dy - Dx
3      // The pt2 point is not added into line     29      int y = y0;
4      int x0 = pt1.c; int y0 = pt1.r;            30      int xDraw, yDraw;
5      int x1 = pt2.c; int y1 = pt2.r;            31      for (int x = x0; x != x1; x += xstep) {
6      Coords p;                                    32          if (steep) {
7      int dx = x1 - x0;                            33              xDraw = y;
8      int dy = y1 - y0;                            34              yDraw = x;
9      int steep = (abs(dy) >= abs(dx));            35          } else {
10     if (steep) {                                   36              xDraw = x;
11         SWAP(x0, y0);                             37              yDraw = y;
12         SWAP(x1, y1);                             38          }
13         dx = x1 - x0; // recompute Dx, Dy         39         p.c = xDraw;
14         dy = y1 - y0;                             40         p.r = yDraw;
15     }                                             41         line.push_back(p); // add to the line
16     int xstep = 1;                                42         if (e > 0) {
17         if (dx < 0) {                              43             e += twoDyTwoDx; //E += 2*Dy - 2*Dx
18             xstep = -1;                            44             y = y + ystep;
19             dx = -dx;                               45         } else {
20         }                                           46             e += twoDy; //E += 2*Dy
21         int ystep = 1;                              47         }
22         if (dy < 0) {                              48     }
23             ystep = -1;                            49     return line;
24             dy = -dy;                               50 }
25     }

```



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## Distance Transform based Path Planning

- For a given goal location and grid map compute a navigational function using *wave-front* algorithm, i.e., a kind of *potential field*
  - The value of the goal cell is set to 0 and all other free cells are set to some very high value
  - For each free cell compute a number of cells towards the goal cell
  - It uses 8-neighbors and distance is the Euclidean distance of the centers of two cells, i.e.,  $EV=1$  for orthogonal cells or  $EV = \sqrt{2}$  for diagonal cells
  - The values are iteratively computed until the values are changed
  - The value of the cell  $c$  is computed as

$$cost(c) = \min_{i=1}^8 (cost(c_i) + EV_{c_i,c}),$$

where  $c_i$  is one of the neighboring cells from 8-neighborhood of the cell  $c$

- The algorithm provides a cost map of the path distance from any free cell to the goal cell
- The path is then used following the gradient of the cell cost

Jarvis, R. (2004): Distance Transform Based Visibility Measures for Covert Path Planning in Known but Dynamic Environments





# Distance Transform Path Planning

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## Algorithm 1: Distance Transform for Path Planning

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```

for  $y := 0$  to  $yMax$  do
  for  $x := 0$  to  $xMax$  do
    if goal  $[x,y]$  then
       $cell [x,y] := 0$ ;
    else
       $cell [x,y] := xMax * yMax$ ; //initialization, e.g., pragmatically use longest distance as  $\infty$  ;

repeat
  for  $y := 1$  to  $(yMax - 1)$  do
    for  $x := 1$  to  $(xMax - 1)$  do
      if not blocked  $[x,y]$  then
         $cell [x,y] := cost(x, y)$ ;

  for  $y := (yMax-1)$  downto  $1$  do
    for  $x := (xMax-1)$  downto  $1$  do
      if not blocked  $[x,y]$  then
         $cell[x,y] := cost(x, y)$ ;

until no change;
  
```

---



# Distance Transform based Path Planning – Impl. 1/2

```

1  Grid& DT::compute(Grid& grid) const          35
2  {                                             36
3      static const double DIAGONAL = sqrt(2);  37
4      static const double ORTOGONAL = 1;      38
5      const int H = map.H;                    39
6      const int W = map.W;                    40
7      assert(grid.H == H and grid.W == W, "size"); 41
8      bool anyChange = true;                 42
9      int counter = 0;                        43
10     while (anyChange) {                     44
11         anyChange = false;                  45
12         for (int r = 1; r < H - 1; ++r) {   46
13             for (int c = 1; c < W - 1; ++c) { 47
14                 if (map[r][c] != FREESPACE) { 48
15                     continue;                49
16                 } //obstacle detected       50
17                 double t[4];                51
18                 t[0] = grid[r - 1][c - 1] + DIAGONAL; 52
19                 t[1] = grid[r - 1][c] + ORTOGONAL; 53
20                 t[2] = grid[r - 1][c + 1] + DIAGONAL; 54
21                 t[3] = grid[r][c - 1] + ORTOGONAL; 55
22                 double pom = grid[r][c];     56
23                 for (int i = 0; i < 4; i++) { 57
24                     if (pom > t[i]) {         58
25                         pom = t[i];          59
26                         anyChange = true;    60
27                     }                         61
28                 }                             62
29             }
30             if (anyChange) {
31                 grid[r][c] = pom;
32             }
33     }

```

An boundary is assumed around the rectangular map



## Distance Transform based Path Planning – Impl. 2/2

- The path is retrieved by following the minimal value towards the goal using `min8Point()`

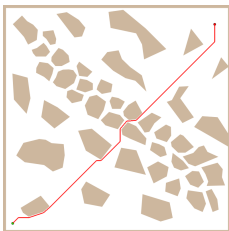
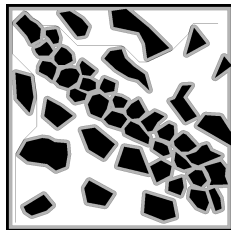
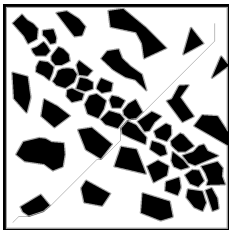
```

1  Coords& min8Point(const Grid& grid, Coords& p)          22  CoordsVector& DT::findPath(const Coords& start,
2  {                                                       const Coords& goal, CoordsVector& path)
3  {                                                       {
4      double min = std::numeric_limits<double>::max();    23  {
5      const int H = grid.H;                               24      static const double DIAGONAL = sqrt(2);
6      const int W = grid.W;                               25      static const double ORTOGONAL = 1;
7      Coords t;                                           26      const int H = map.H;
8  }                                                       27      const int W = map.W;
9  for (int r = p.r - 1; r <= p.r + 1; r++) {             28      Grid grid(H, W, H*W); // H*W max grid value
10     if (r < 0 or r >= H) { continue; }                  29      grid[goal.r][goal.c] = 0;
11     for (int c = p.c - 1; c <= p.c + 1; c++) {          30      compute(grid);
12         if (c < 0 or c >= W) { continue; }              31
13         if (min > grid[r][c]) {                          32         if (grid[start.r][start.c] >= H*W) {
14             min = grid[r][c];                             33             WARN("Path has not been found");
15             t.r = r; t.c = c;                             34         } else {
16         }                                                 35             Coords pt = start;
17     }                                                     36             while (pt.r != goal.r or pt.c != goal.c) {
18     p = t;                                                 37                 path.push_back(pt);
19     return p;                                             38                 min8Point(grid, pt);
20 }                                                         39             }
                                                         40             path.push_back(goal);
                                                         41         }
                                                         42         return path;
                                                         43     }

```



## DT Example



$\delta = 10 \text{ cm}, L = 27.2 \text{ m}$



$\delta = 30 \text{ cm}, L = 42.8 \text{ m}$



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# Graph Search Algorithms

- The grid can be considered as a graph and the path can be found using graph search algorithms
- The search algorithms working on a graph are of general use, e.g.
  - Breadth-first search (BSD)
  - Depth first search (DFS)
  - Dijkstra's algorithm,
  - A\* algorithm and its variants
- There can be grid based speedups techniques, e.g.,
  - Jump Search Algorithm (JPS) and JPS+
- There are many search algorithm for on-line search, incremental search and with any-time and real-time properties, e.g.,
  - Lifelong Planning A\* (LPA\*)

Koenig, S., Likhachev, M. and Furcy, D. (2004): Lifelong Planning A\*. AIJ.

- E-Graphs – Experience graphs

Phillips, M. et al. (2012): E-Graphs: Bootstrapping Planning with Experience Graphs. RSS.



# Examples of Graph/Grid Search Algorithms

Magenta: Visited  
Red: Closed  
Yellow: Inconsistent  
Green: Open  
Cyan: Partial or suboptimal path  
Blue: Optimal path & endpoints

Compute to Optimal  
Compute to Suboptimal  
Compute to Partial  
Compute Async: Forever  
Stop  
Reset  
Start modified

A\* (general)

The image shows a maze environment with a grid-based search algorithm. The maze is represented by a brown background with black walls. A path is shown starting from a blue square at the top center and ending at a blue square at the bottom center. The path is highlighted in blue. The search space is filled with magenta lines representing visited nodes and green squares representing open nodes. The text 'A\* (general)' is displayed at the bottom of the maze. On the right side, there is a control panel with buttons for 'Compute to Optimal', 'Compute to Suboptimal', 'Compute to Partial', 'Compute Async: Forever', 'Stop', 'Reset', and 'Start modified'. On the left side, there is a legend for the colors used in the visualization.

<https://www.youtube.com/watch?v=X5a149nSE9s>

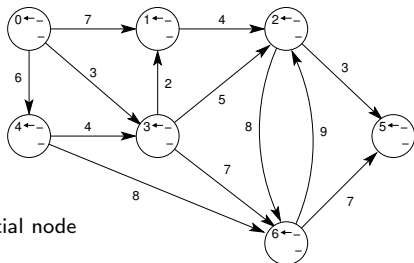


# Dijkstra's Algorithm

- Dijkstra's algorithm determines paths as iterative update of the cost of the shortest path to the particular nodes

- Let start with the initial cell (node) with the cost set to 0 and update all successors
- Select the node
  - with a path from the initial node
  - and has a lower cost
- Repeat until there is a reachable node
  - I.e., a node with a path from the initial node
  - has a cost and parent (*green nodes*).

Edsger W. Dijkstra, 1956



The cost of nodes can only decrease (edge cost is positive). Therefore, for a node with the currently lowest cost, there cannot be a shorter path from the initial node.



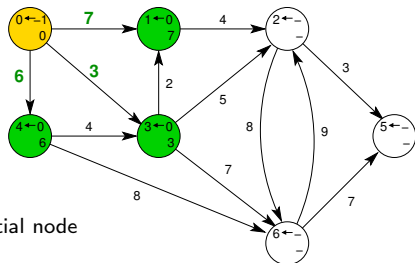


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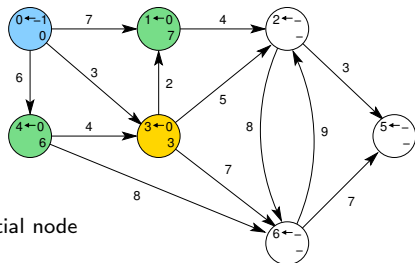


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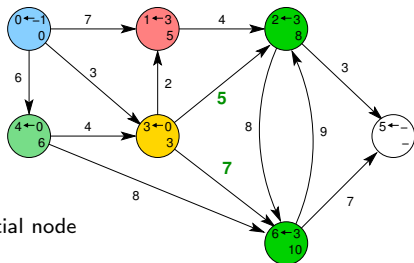


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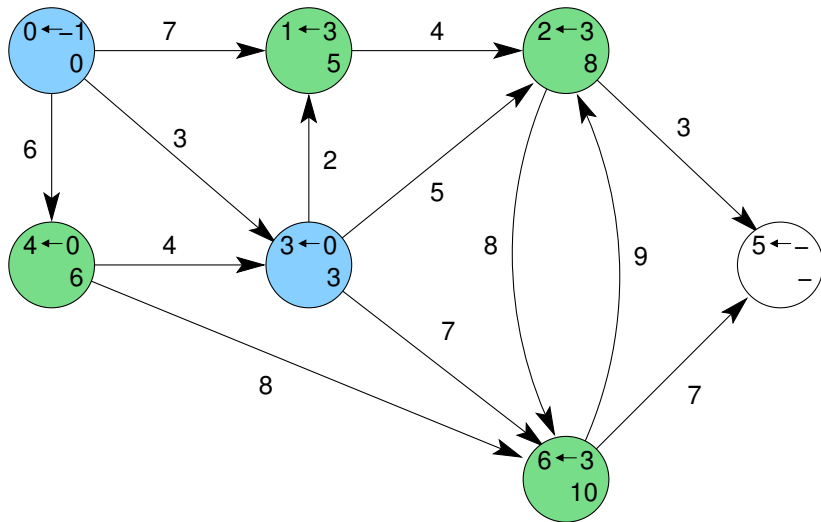


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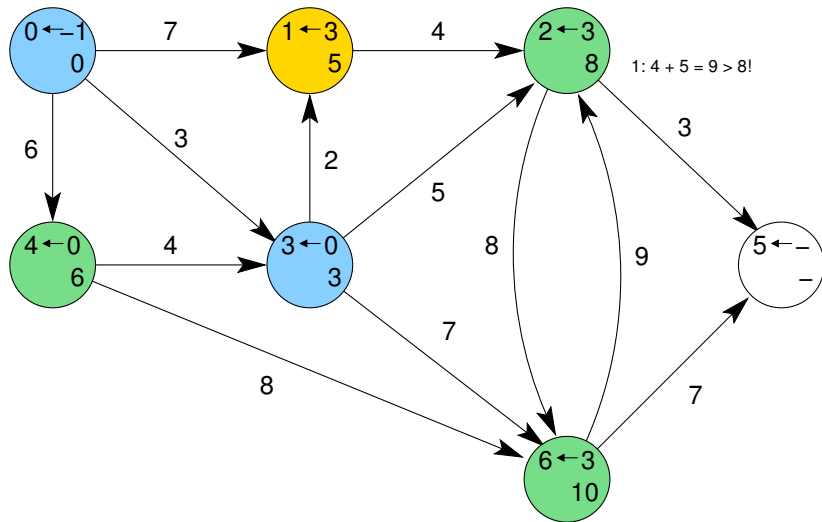
## Example (cont.)

1: After the expansion, the shortest path to the node 2 is over the node 3



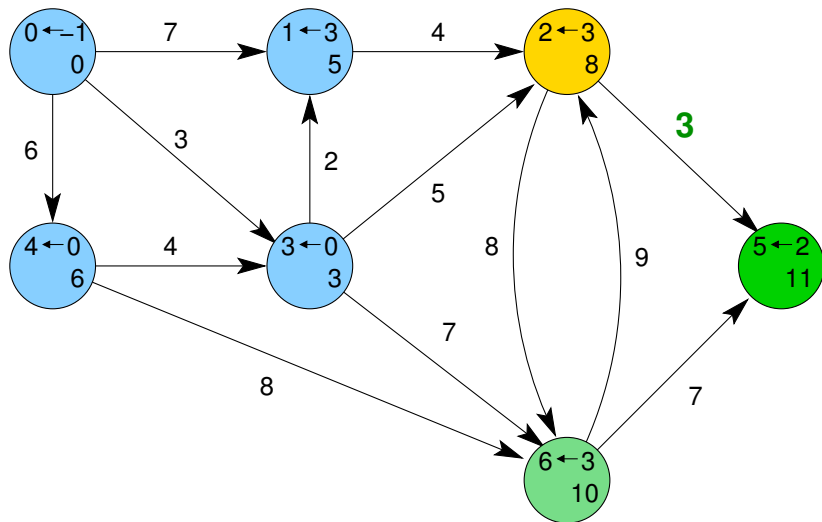
## Example (cont.)

2: There is not shorter path to the node 2 over the node 1



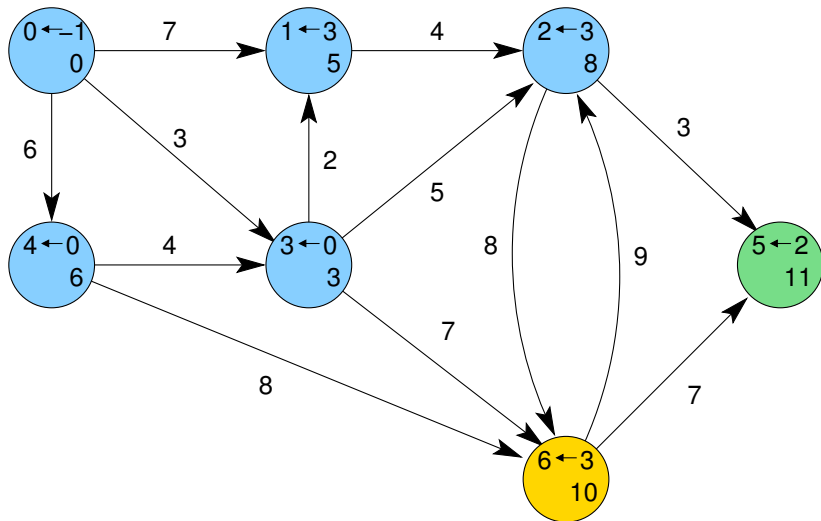
## Example (cont.)

3: After the expansion, there is a new path to the node 5



## Example (cont.)

4: The path does not improve for further expansions



## Dijkstra's Algorithm – Impl.

```
1  dij->nodes[dij->start_node].cost = 0; // init
2  void *pq = pq_alloc(dij->num_nodes); // set priority queue
3  int cur_label;
4  pq_push(pq, dij->start_node, 0);
5  while ( !pq_is_empty(pq) && pq_pop(pq, &cur_label)) {
6      node_t *cur = &(dij->nodes[cur_label]); // remember the current node
7      for (int i = 0; i < cur->edge_count; ++i) { // all edges of cur
8          edge_t *edge = &(dij->graph->edges[cur->edge_start + i]);
9          node_t *to = &(dij->nodes[edge->to]);
10         const int cost = cur->cost + edge->cost;
11         if (to->cost == -1) { // node to has not been visited
12             to->cost = cost;
13             to->parent = cur_label;
14             pq_push(pq, edge->to, cost); // put node to the queue
15         } else if (cost < to->cost) { // node already in the queue
16             to->cost = cost; // test if the cost can be reduced
17             to->parent = cur_label; // update the parent node
18             pq_update(pq, edge->to, cost); // update the priority queue
19         }
20     } // loop for all edges of the cur node
21 } // priority queue empty
22 pq_free(pq); // release memory
```





# A\* Algorithm

- A\* uses a user-defined  $h$ -values (heuristic) to focus the search

Peter Hart, Nils Nilsson, and Bertram Raphael, 1968

- Prefer expansion of the node  $n$  with the lowest value

$$f(n) = g(n) + h(n),$$

where  $g(n)$  is the cost (path length) from the start to  $n$  and  $h(n)$  is the estimated cost from  $n$  to the goal

- $h$ -values approximate the goal distance from particular nodes

- **Admissibility condition** – heuristic always underestimate the remaining cost to reach the goal

- Let  $h^*(n)$  be the true cost of the optimal path from  $n$  to the goal
- Then  $h(n)$  is **admissible** if for all  $n$ :  $h(n) \leq h^*(n)$
- E.g., Euclidean distance is admissible
  - A straight line will always be the shortest path

- Dijkstra's algorithm –  $h(n) = 0$



## A\* Implementation Notes

- The most costly operations of A\* are
  - Insert and lookup an element in the **closed list**
  - Insert element and get minimal element (according to  $f()$  value) from the **open list**
- The **closed list** can be efficiently implemented as a **hash set**
- The **open list** is usually implemented as a **priority queue**, e.g.,
  - Fibonacci heap, binomial heap,  $k$ -level bucket
  - **binary heap** is usually sufficient ( $O(\log n)$ )
- Forward A\*
  1. Create a search tree and initiate it with the start location
  2. Select generated but not yet expanded state  $s$  with the smallest  $f$ -value,  $f(s) = g(s) + h(s)$
  3. Stop if  $s$  is the goal
  4. Expand the state  $s$
  5. Goto Step 2

Similar to Dijkstra's algorithm but it used  $f(s)$  with heuristic  $h(s)$  instead of pure  $g(s)$



# Dijkstra's vs A\* vs Jump Point Search (JPS)



<https://www.youtube.com/watch?v=ROG4Ud081LY>

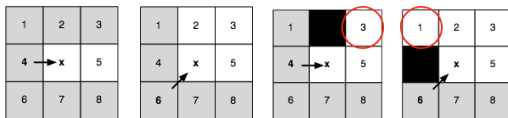


# Jump Point Search Algorithm for Grid-based Path Planning

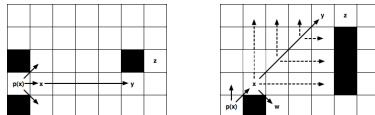
- **Jump Point Search** (JPS) algorithm is based on a macro operator that identifies and selectively expands only certain nodes (**jump points**)

Harabor, D. and Grastien, A. (2011): Online Graph Pruning for Pathfinding on Grid Maps. AAAI.

- Natural neighbors after neighbor pruning with forced neighbors because of obstacle



- Intermediate nodes on a path connecting two jump points are never expanded



- No preprocessing and no memory overheads while it speeds up A\*

<https://harablog.wordpress.com/2011/09/07/jump-point-search/>

- JPS+ – optimized preprocessed version of **JPS** with goal bounding

<https://github.com/SteveRabin/JPSPlusWithGoalBounding>

<http://www.gdcvault.com/play/1022094/JPS-Over-100x-Faster-than>



# Theta\* – Any-Angle Path Planning Algorithm

- Any-angle path planning algorithms simplify the path during the search
- Theta\* is an extension of A\* with `LineOfSight()`

Nash, A., Daniel, K., Koenig, S. and Felner, A. (2007): Theta\*: Any-Angle Path Planning on Grids. AAAI.

---

## Algorithm 2: Theta\* Any-Angle Planning

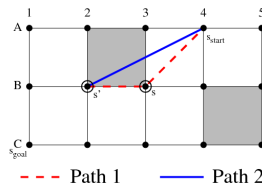
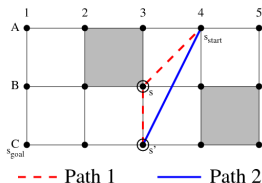
---

```

if LineOfSight(parent(s), s') then
    /* Path 2 – any-angle path */
    if g(parent(s)) + c(parent(s), s') < g(s') then
        parent(s') := parent(s);
        g(s') := g(parent(s)) + c(parent(s), s');
else
    /* Path 1 – A* path */
    if g(s) + c(s, s') < g(s') then
        parent(s') := s;
        g(s') := g(s) + c(s, s');
  
```

---

- Path 2: considers path from start to parent(s) and from parent(s) to s' if s' has line-of-sight to parent(s)

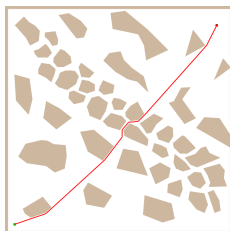


<http://aigamedev.com/open/tutorials/theta-star-any-angle-paths/>

## Theta\* Any-Angle Path Planning Examples

- Example of found paths by the Theta\* algorithm for the same problems as for the DT-based examples on Slide 16

Both algorithms implemented in C++



$\delta = 10$  cm,  $L = 26.3$  m



$\delta = 30$  cm,  $L = 40.3$  m

The same path planning problems solved by DT (without path smoothing) have  $L_{\delta=10} = 27.2$  m and  $L_{\delta=30} = 42.8$  m, while DT seems to be faster

- Lazy Theta\*** – reduces the number of line-of-sight checks

Nash, A., Koenig, S. and Tovey, C. (2010): Lazy Theta\*: Any-Angle Path Planning and Path Length Analysis in 3D. AAAI.

<http://aigamedev.com/open/tutorial/lazy-theta-star/>



## A\* Variants – Online Search

- The state space (map) may not be known exactly in advance
  - Environment can **dynamically** change
  - True travel costs are **experienced** during the path execution
- Repeated A\* searches can be computationally demanding
- **Incremental heuristic search**
  - Repeated planning of the path from the current state to the goal
  - Planning under the **free-space** assumption
  - **Reuse** information from the previous searches (**closed list** entries):
    - Focused Dynamic A\* (**D\***) –  $h^*$  is based on **traversability**, it has been used, e.g., for the Mars rover “Opportunity”
 

Stentz, A. (1995): The Focussed D\* Algorithm for Real-Time Replanning. IJCAI.
    - **D\* Lite** – similar to D\*
 

Koenig, S. and Likhachev, M. (2005): Fast Replanning for Navigation in Unknown Terrain. T-RO.
- **Real-Time Heuristic Search**
  - Repeated planning with limited **look-ahead** – suboptimal but fast
    - Learning Real-Time A\* (**LRTA\***)
 

Korf, E. (1990): Real-time heuristic search. JAI
    - Real-Time Adaptive A\* (**RTAA\***)
 

Koenig, S. and Likhachev, M. (2006): Real-time adaptive A\*. AAMAS.



## Real-Time Adaptive A\* (RTAA\*)

- Execute A\* with limited **look-ahead**
- Learns better informed **heuristic** from the experience, initially  $h(s)$ , e.g., Euclidean distance
- Look-ahead defines **trade-off** between optimality and computational cost
  - `astar(lookahead)`
 A\* expansion as far as "look-ahead" nodes and it terminates with the state  $s'$

---

```

while ( $s_{curr} \notin GOAL$ ) do
  astar(lookahead);
  if  $s' = FAILURE$  then
    _ return FAILURE;
  for all  $s \in CLOSED$  do
    _  $H(s) := g(s') + h(s') - g(s)$ ;
  _ execute(plan); // perform one step
return SUCCESS;
  
```

---

$s'$  is the last state expanded during the previous A\* search



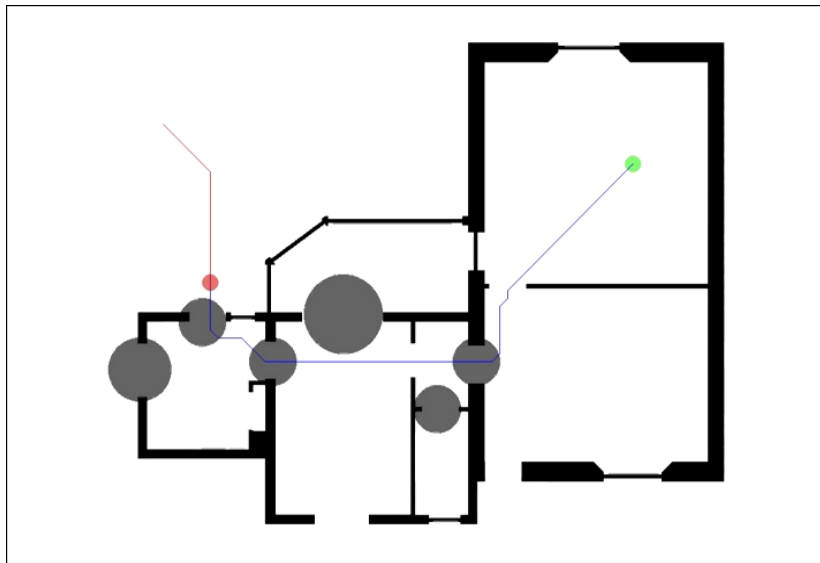


# Outline

- Grid-based Planning
- DT for Path Planning
- Graph Search Algorithms
- D\* Lite
- Path Planning based on Reaction-Diffusion Process



## D\* Lite – Demo



<https://www.youtube.com/watch?v=X5a149nSE9s>



## D\* Lite Overview

- It is similar to D\*, but it is based on **Lifelong Planning A\***

Koenig, S. and Likhachev, M. (2002): D\* Lite. AAAI.

- It searches from the goal node to the start node, i.e.,  $g$ -values estimate the goal distance
- Store pending nodes in a priority queue
- Process nodes in order of increasing objective function value
- Incrementally repair solution paths when changes occur
- Maintains two estimates of costs per node
  - $g$  – the objective function value – based on what we know
  - $rhs$  – one-step lookahead of the objective function value – based on what we know
- **Consistency**
  - Consistent –  $g = rhs$
  - Inconsistent –  $g \neq rhs$
- Inconsistent nodes are stored in the priority queue (open list) for processing



## D\* Lite: Cost Estimates

- *rhs* of the node  $u$  is computed based on  $g$  of its successors in the graph and the transition costs of the edge to those successors

$$rhs(u) = \min_{s' \in Succ(u)} (g(s') + c(u, s'))$$

- The key/priority of a node  $s$  on the open list is the minimum of  $g(s)$  and  $rhs(s)$  plus a focusing heuristic  $h$

$$[\min(g(s), rhs(s)) + h(s_{start}, s); \min(g(s), rhs(s))]$$

- The first term is used as the primary key
- The second term is used as the secondary key for tie-breaking



# D\* Lite Algorithm

- **Main** – repeat until the robot reaches the goal (or  $g(s_{start}) = \infty$  there is no path)

---

```

Initialize();
ComputeShortestPath();
while ( $s_{start} \neq s_{goal}$ ) do
  |  $s_{start} = \operatorname{argmin}_{s' \in \operatorname{Succ}(s_{start})} (c(s_{start}, s') + g(s'))$ ;
Move to  $s_{start}$ ;
Scan the graph for changed edge costs;
if any edge cost changed perform then
  | foreach directed edges  $(u, v)$  with changed edge costs do
    | | Update the edge cost  $c(u, v)$ ;
    | | UpdateVertex( $u$ );
    | foreach  $s \in U$  do
      | | U.Update( $s$ , CalculateKey( $s$ ));
  | ComputeShortestPath();
  
```

---

## Procedure Initialize

```

U = 0;
foreach  $s \in S$  do
  |  $rhs(s) := g(s) := \infty$ ;
 $rhs(s_{goal}) := 0$ ;
U.Insert( $s_{goal}$ , CalculateKey( $s_{goal}$ ));
  
```



# D\* Lite Algorithm – ComputeShortestPath()

## Procedure ComputeShortestPath

```

while  $U.TopKey() < CalculateKey(s_{start})$  OR  $rhs(s_{start}) \neq g(s_{start})$  do
   $u := U.Pop()$ ;
  if  $g(u) > rhs(u)$  then
     $g(u) := rhs(u)$ ;
    foreach  $s \in Pred(u)$  do UpdateVertex(s);
  else
     $g(u) := \infty$ ;
    foreach  $s \in Pred(u) \cup \{u\}$  do UpdateVertex(s);

```

## Procedure UpdateVertex

```

if  $u \neq s_{goal}$  then  $rhs(u) := \min_{s' \in Succ(u)} (c(u, s') + g(s'))$ ;
if  $u \in U$  then  $U.Remove(u)$ ;
if  $g(u) \neq rhs(u)$  then  $U.Insert(u, CalculateKey(u))$ ;

```

## Procedure CalculateKey

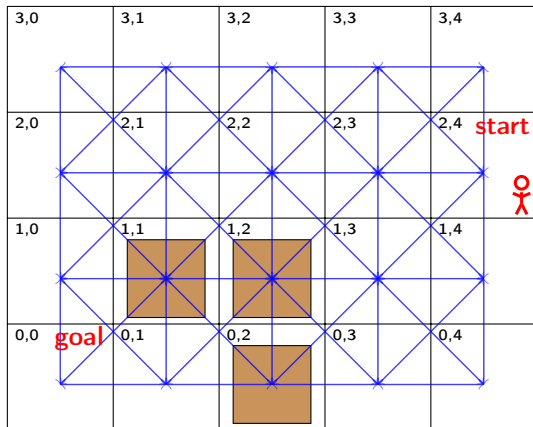
```

return  $[\min(g(s), rhs(s)) + h(s_{start}, s); \min(g(s), rhs(s))]$ 

```



# D<sup>\*</sup> Lite – Example



## Legend

Free node

Obstacle node

On open list

Active node


- A grid map of the environment (what is actually known)
- 8-connected graph superimposed on the grid (bidirectional)
- Focusing heuristic is not used ( $h = 0$ )

## ■ Transition costs

- Free space – Free space: 1.0 and 1.4 (for diagonal edge)
- From/to obstacle:  $\infty$



## D\* Lite – Example Planning (1)

3,0 g: $\infty$ rhs: $\infty$	3,1 g: $\infty$ rhs: $\infty$	3,2 g: $\infty$ rhs: $\infty$	3,3 g: $\infty$ rhs: $\infty$	3,4 g: $\infty$ rhs: $\infty$
2,0 g: $\infty$ rhs: $\infty$	2,1 g: $\infty$ rhs: $\infty$	2,2 g: $\infty$ rhs: $\infty$	2,3 g: $\infty$ rhs: $\infty$	2,4 <b>start</b> g: $\infty$ rhs: $\infty$ 
1,0 g: $\infty$ rhs: $\infty$	1,1 g: $\infty$ rhs: $\infty$	1,2 g: $\infty$ rhs: $\infty$	1,3 g: $\infty$ rhs: $\infty$	1,4 g: $\infty$ rhs: $\infty$
0,0 <b>goal</b> g: $\infty$ rhs: 0	0,1 g: $\infty$ rhs: $\infty$	0,2 g: $\infty$ rhs: $\infty$	0,3 g: $\infty$ rhs: $\infty$	0,4 g: $\infty$ rhs: $\infty$

## Legend

Free node

Obstacle node

On open list

Active node


## Initialization

- Set  $rhs = 0$  for the goal
- Set  $rhs = g = \infty$  for all other nodes





# D\* Lite – Example Planning (2)

3,0 g: $\infty$ rhs: $\infty$	3,1 g: $\infty$ rhs: $\infty$	3,2 g: $\infty$ rhs: $\infty$	3,3 g: $\infty$ rhs: $\infty$	3,4 g: $\infty$ rhs: $\infty$
2,0 g: $\infty$ rhs: $\infty$	2,1 g: $\infty$ rhs: $\infty$	2,2 g: $\infty$ rhs: $\infty$	2,3 g: $\infty$ rhs: $\infty$	2,4 <b>start</b> g: $\infty$ rhs: $\infty$ 
1,0 g: $\infty$ rhs: $\infty$	1,1 g: $\infty$ rhs: $\infty$	1,2 g: $\infty$ rhs: $\infty$	1,3 g: $\infty$ rhs: $\infty$	1,4 g: $\infty$ rhs: $\infty$
0,0 <b>goal</b> g: $\infty$ rhs: 0	0,1 g: $\infty$ rhs: $\infty$	0,2 g: $\infty$ rhs: $\infty$	0,3 g: $\infty$ rhs: $\infty$	0,4 g: $\infty$ rhs: $\infty$

## Legend

Free node

Obstacle node

On open list


Active node

## Initialization

- Put the goal to the open list  
It is inconsistent



## D\* Lite – Example Planning (3)

3,0 g: $\infty$ rhs: $\infty$	3,1 g: $\infty$ rhs: $\infty$	3,2 g: $\infty$ rhs: $\infty$	3,3 g: $\infty$ rhs: $\infty$	3,4 g: $\infty$ rhs: $\infty$
2,0 g: $\infty$ rhs: $\infty$	2,1 g: $\infty$ rhs: $\infty$	2,2 g: $\infty$ rhs: $\infty$	2,3 g: $\infty$ rhs: $\infty$	2,4 <b>start</b> g: $\infty$ rhs: $\infty$ 
1,0 g: $\infty$ rhs: $\infty$	1,1 g: $\infty$ rhs: $\infty$	1,2 g: $\infty$ rhs: $\infty$	1,3 g: $\infty$ rhs: $\infty$	1,4 g: $\infty$ rhs: $\infty$
0,0 <b>goal</b> g: 0 rhs: 0	0,1 g: $\infty$ rhs: $\infty$	0,2 g: $\infty$ rhs: $\infty$	0,3 g: $\infty$ rhs: $\infty$	0,4 g: $\infty$ rhs: $\infty$

## Legend

Free node

Obstacle node

On open list


Active node

## ComputeShortestPath

- Pop the minimum element from the open list (goal)
- It is over-consistent ( $g > rhs$ ), therefore set  $g = rhs$



# D\* Lite – Example Planning (4)

3,0 g: $\infty$ rhs: $\infty$	3,1 g: $\infty$ rhs: $\infty$	3,2 g: $\infty$ rhs: $\infty$	3,3 g: $\infty$ rhs: $\infty$	3,4 g: $\infty$ rhs: $\infty$
2,0 g: $\infty$ rhs: $\infty$	2,1 g: $\infty$ rhs: $\infty$	2,2 g: $\infty$ rhs: $\infty$	2,3 g: $\infty$ rhs: $\infty$	2,4 <b>start</b> g: $\infty$ rhs: $\infty$ 
1,0 g: $\infty$ rhs: 1	1,1 g: $\infty$ rhs: $\infty$	1,2 g: $\infty$ rhs: $\infty$	1,3 g: $\infty$ rhs: $\infty$	1,4 g: $\infty$ rhs: $\infty$
0,0 <b>goal</b> g: 0 rhs: 0	0,1 g: $\infty$ rhs: 1	0,2 g: $\infty$ rhs: $\infty$	0,3 g: $\infty$ rhs: $\infty$	0,4 g: $\infty$ rhs: $\infty$

## Legend

Free node

Obstacle node

On open list

Active node

## ComputeShortestPath


- Expand popped node (UpdateVertex()) on all its predecessors)
- This computes the *rhs* values for the predecessors
- Nodes that become inconsistent are added to the open list

Small black arrows denote the node used for computing the *rhs* value, i.e., using the respective transition cost

- The *rhs* value of (1,1) is  $\infty$  because the transition to obstacle has cost  $\infty$



## D\* Lite – Example Planning (5)

3,0 g: $\infty$ rhs: $\infty$	3,1 g: $\infty$ rhs: $\infty$	3,2 g: $\infty$ rhs: $\infty$	3,3 g: $\infty$ rhs: $\infty$	3,4 g: $\infty$ rhs: $\infty$
2,0 g: $\infty$ rhs: $\infty$	2,1 g: $\infty$ rhs: $\infty$	2,2 g: $\infty$ rhs: $\infty$	2,3 g: $\infty$ rhs: $\infty$	2,4 <b>start</b> g: $\infty$ rhs: $\infty$ 
1,0 g: 1 rhs: 1	1,1 g: $\infty$ rhs: $\infty$	1,2 g: $\infty$ rhs: $\infty$	1,3 g: $\infty$ rhs: $\infty$	1,4 g: $\infty$ rhs: $\infty$
0,0 <b>goal</b> g: 0 rhs: 0	0,1 g: $\infty$ rhs: 1	0,2 g: $\infty$ rhs: $\infty$	0,3 g: $\infty$ rhs: $\infty$	0,4 g: $\infty$ rhs: $\infty$

## Legend

Free node

Obstacle node

On open list


Active node

## ComputeShortestPath

- Pop the minimum element from the open list (1,0)
- It is over-consistent ( $g > rhs$ )  
set  $g = rhs$



## D\* Lite – Example Planning (6)

3,0 g: $\infty$ rhs: $\infty$	3,1 g: $\infty$ rhs: $\infty$	3,2 g: $\infty$ rhs: $\infty$	3,3 g: $\infty$ rhs: $\infty$	3,4 g: $\infty$ rhs: $\infty$
2,0 g: $\infty$ rhs: 2	2,1 g: $\infty$ rhs: 2.4	2,2 g: $\infty$ rhs: $\infty$	2,3 g: $\infty$ rhs: $\infty$	2,4 <b>start</b> g: $\infty$ rhs: $\infty$ 
1,0 g: 1 rhs: 1	1,1 g: $\infty$ rhs: $\infty$	1,2 g: $\infty$ rhs: $\infty$	1,3 g: $\infty$ rhs: $\infty$	1,4 g: $\infty$ rhs: $\infty$
0,0 <b>goal</b> g: 0 rhs: 0	0,1 g: $\infty$ rhs: 1	0,2 g: $\infty$ rhs: $\infty$	0,3 g: $\infty$ rhs: $\infty$	0,4 g: $\infty$ rhs: $\infty$

## Legend

Free node

Obstacle node

On open list

Active node


## ComputeShortestPath

- Expand the popped node (UpdateVertex()) on all predecessors in the graph)
- Compute *rhs* values of the predecessors accordingly
- Put them to the open list if they become inconsistent

- The *rhs* value of (0,0), (1,1) does not change
- They do not become inconsistent and thus they are not put on the open list



## D\* Lite – Example Planning (7)

3,0 g: $\infty$ rhs: $\infty$	3,1 g: $\infty$ rhs: $\infty$	3,2 g: $\infty$ rhs: $\infty$	3,3 g: $\infty$ rhs: $\infty$	3,4 g: $\infty$ rhs: $\infty$
2,0 g: $\infty$ rhs: 2	2,1 g: $\infty$ rhs: 2.4	2,2 g: $\infty$ rhs: $\infty$	2,3 g: $\infty$ rhs: $\infty$	2,4 <b>start</b> g: $\infty$ rhs: $\infty$ 
1,0 g: 1 rhs: 1	1,1 g: $\infty$ rhs: $\infty$	1,2 g: $\infty$ rhs: $\infty$	1,3 g: $\infty$ rhs: $\infty$	1,4 g: $\infty$ rhs: $\infty$
0,0 <b>goal</b> g: 0 rhs: 0	0,1 g: 1 rhs: 1	0,2 g: $\infty$ rhs: $\infty$	0,3 g: $\infty$ rhs: $\infty$	0,4 g: $\infty$ rhs: $\infty$

## Legend

Free node

Obstacle node

On open list


Active node

## ComputeShortestPath

- Pop the minimum element from the open list (0,1)
- It is over-consistent ( $g > rhs$ ) and thus set  $g = rhs$
- Expand the popped element, e.g., call `UpdateVertex()`



## D\* Lite – Example Planning (8)

3,0 g: $\infty$ rhs: $\infty$	3,1 g: $\infty$ rhs: $\infty$	3,2 g: $\infty$ rhs: $\infty$	3,3 g: $\infty$ rhs: $\infty$	3,4 g: $\infty$ rhs: $\infty$
2,0 g: 2 rhs: 2	2,1 g: $\infty$ rhs: 2.4	2,2 g: $\infty$ rhs: $\infty$	2,3 g: $\infty$ rhs: $\infty$	2,4 g: $\infty$ rhs: $\infty$ 
1,0 g: 1 rhs: 1	1,1 g: $\infty$ rhs: $\infty$	1,2 g: $\infty$ rhs: $\infty$	1,3 g: $\infty$ rhs: $\infty$	1,4 g: $\infty$ rhs: $\infty$
0,0 g: 0 rhs: 0 <b>goal</b>	0,1 g: 1 rhs: 1	0,2 g: $\infty$ rhs: $\infty$	0,3 g: $\infty$ rhs: $\infty$	0,4 g: $\infty$ rhs: $\infty$

## Legend

Free node

Obstacle node

On open list


Active node

## ComputeShortestPath

- Pop the minimum element from the open list (2,0)
- It is over-consistent ( $g > rhs$ ) and thus set  $g = rhs$



## D\* Lite – Example Planning (9)

3,0 g: $\infty$ rhs: 3	3,1 g: $\infty$ rhs: 3.4	3,2 g: $\infty$ rhs: $\infty$	3,3 g: $\infty$ rhs: $\infty$	3,4 g: $\infty$ rhs: $\infty$
2,0 g: 2 rhs: 2	2,1 g: $\infty$ rhs: 2.4	2,2 g: $\infty$ rhs: $\infty$	2,3 g: $\infty$ rhs: $\infty$	2,4 <b>start</b> g: $\infty$ rhs: $\infty$ 
1,0 g: 1 rhs: 1	1,1 g: $\infty$ rhs: $\infty$	1,2 g: $\infty$ rhs: $\infty$	1,3 g: $\infty$ rhs: $\infty$	1,4 g: $\infty$ rhs: $\infty$
0,0 <b>goal</b> g: 0 rhs: 0	0,1 g: 1 rhs: 1	0,2 g: $\infty$ rhs: $\infty$	0,3 g: $\infty$ rhs: $\infty$	0,4 g: $\infty$ rhs: $\infty$

## Legend

Free node

Obstacle node

On open list

Active node

## ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list





## D\* Lite – Example Planning (10)

3,0 g: $\infty$ rhs: 3	3,1 g: $\infty$ rhs: 3.4	3,2 g: $\infty$ rhs: $\infty$	3,3 g: $\infty$ rhs: $\infty$	3,4 g: $\infty$ rhs: $\infty$
2,0 g: 2 rhs: 2	2,1 g: 2.4 rhs: 2.4	2,2 g: $\infty$ rhs: $\infty$	2,3 g: $\infty$ rhs: $\infty$	2,4 start g: $\infty$ rhs: $\infty$
1,0 g: 1 rhs: 1	1,1 g: $\infty$ rhs: $\infty$	1,2 g: $\infty$ rhs: $\infty$	1,3 g: $\infty$ rhs: $\infty$	1,4 g: $\infty$ rhs: $\infty$
0,0 goal g: 0 rhs: 0	0,1 g: 1 rhs: 1	0,2 g: $\infty$ rhs: $\infty$	0,3 g: $\infty$ rhs: $\infty$	0,4 g: $\infty$ rhs: $\infty$

## Legend

Free node

Obstacle node

On open list

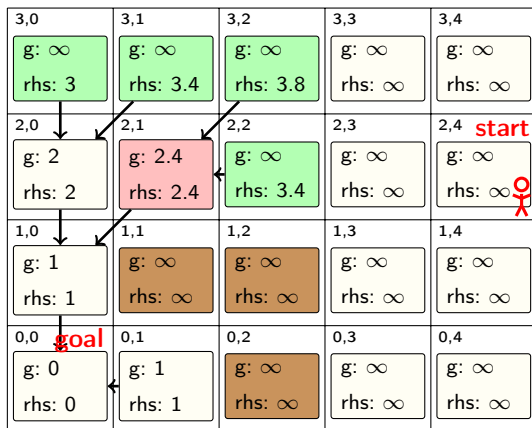
Active node

## ComputeShortestPath

- Pop the minimum element from the open list (2,1)
- It is over-consistent ( $g > rhs$ ) and thus set  $g = rhs$



## D\* Lite – Example Planning (11)



## Legend

Free node

Obstacle node

On open list

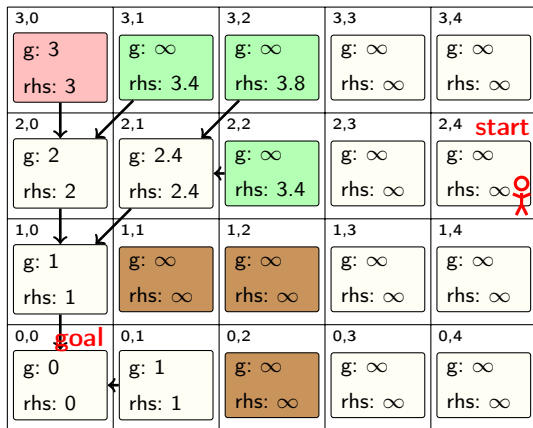
Active node

## ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list



# D\* Lite – Example Planning (12)



## Legend

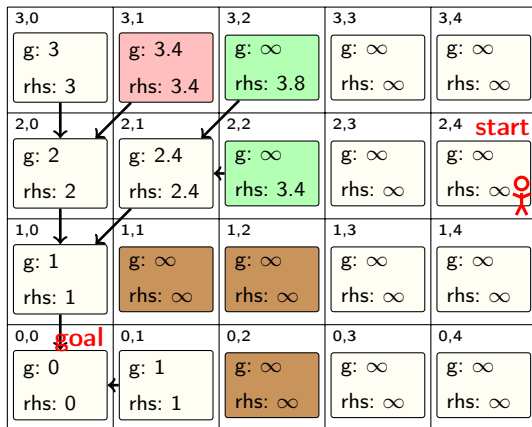
Free node	Obstacle node
On open list	Active node

## ComputeShortestPath

- Pop the minimum element from the open list (3,0)
- It is over-consistent ( $g > rhs$ ) and thus set  $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent onto the open list
- In this cases, none of the predecessors become inconsistent



## D\* Lite – Example Planning (13)



## Legend

Free node

Obstacle node

On open list

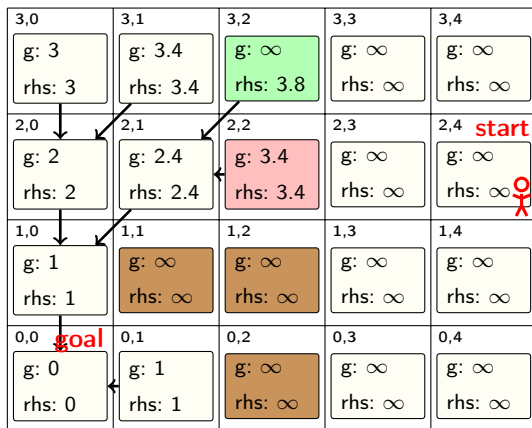
Active node

## ComputeShortestPath

- Pop the minimum element from the open list (3,0)
- It is over-consistent ( $g > rhs$ ) and thus set  $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent onto the open list
- In this cases, none of the predecessors become inconsistent



## D\* Lite – Example Planning (14)



## Legend

Free node

Obstacle node

On open list

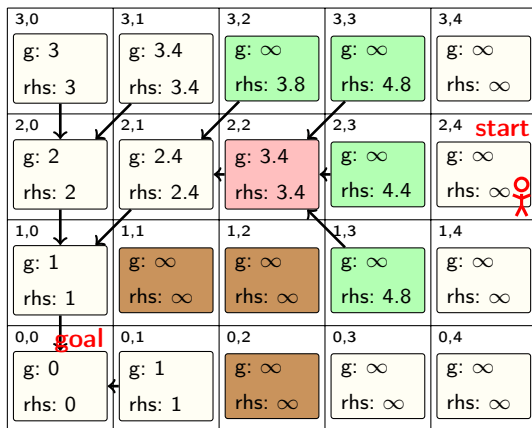
Active node

## ComputeShortestPath

- Pop the minimum element from the open list (2,2)
- It is over-consistent ( $g > rhs$ ) and thus set  $g = rhs$



## D\* Lite – Example Planning (15)



## Legend

Free node

Obstacle node

On open list

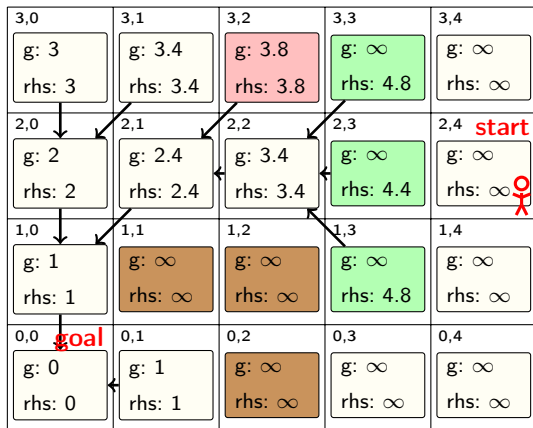
Active node

## ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list, i.e., (3,2), (3,3), (2,3)



## D\* Lite – Example Planning (16)



## Legend

Free node

Obstacle node

On open list

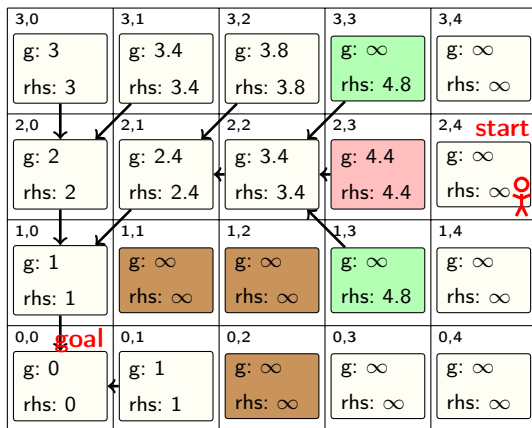
Active node

## ComputeShortestPath

- Pop the minimum element from the open list (3,2)
- It is over-consistent ( $g > rhs$ ) and thus set  $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent onto the open list
- In this cases, none of the predecessors become inconsistent



## D\* Lite – Example Planning (17)



## Legend

Free node

Obstacle node

On open list

Active node

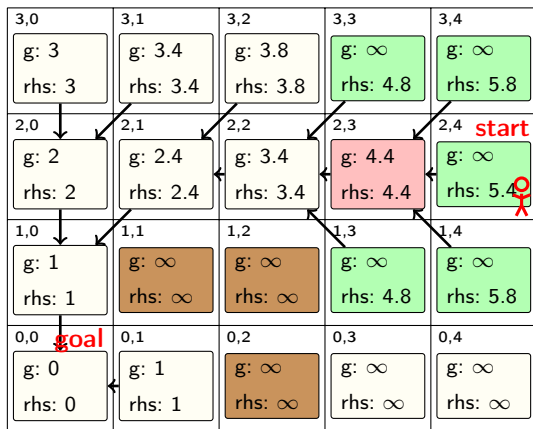
## ComputeShortestPath

- Pop the minimum element from the open list (2,3)
- It is over-consistent ( $g > rhs$ ) and thus set  $g = rhs$





# D\* Lite – Example Planning (18)



## Legend

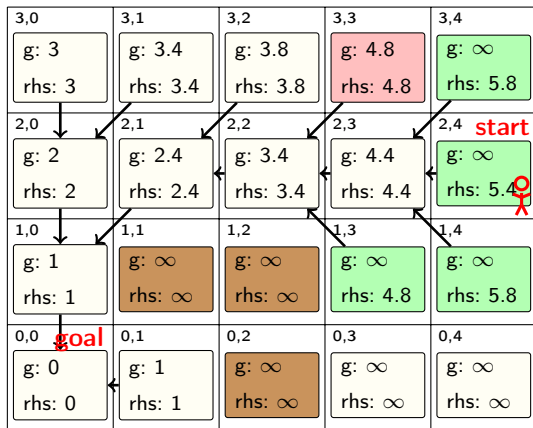
Free node	Obstacle node
On open list	Active node

## ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list, i.e., (3,4), (2,4), (1,4)
- The start node is on the open list
- However, the search does not finish at this stage
- There are still inconsistent nodes (on the open list) with a lower value of  $rhs$



## D\* Lite – Example Planning (19)



## Legend

Free node

Obstacle node

On open list

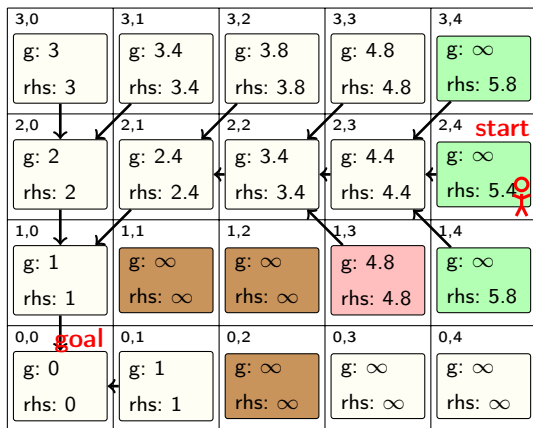
Active node

## ComputeShortestPath

- Pop the minimum element from the open list (3,2)
- It is over-consistent ( $g > rhs$ ) and thus set  $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent onto the open list
- In this cases, none of the predecessors become inconsistent



## D\* Lite – Example Planning (20)



## Legend

Free node

Obstacle node

On open list

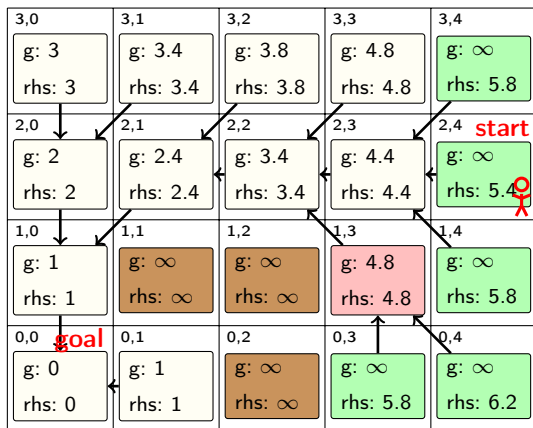
Active node

## ComputeShortestPath

- Pop the minimum element from the open list (1,3)
- It is over-consistent ( $g > rhs$ ) and thus set  $g = rhs$



## D\* Lite – Example Planning (21)



## Legend

Free node

Obstacle node

On open list

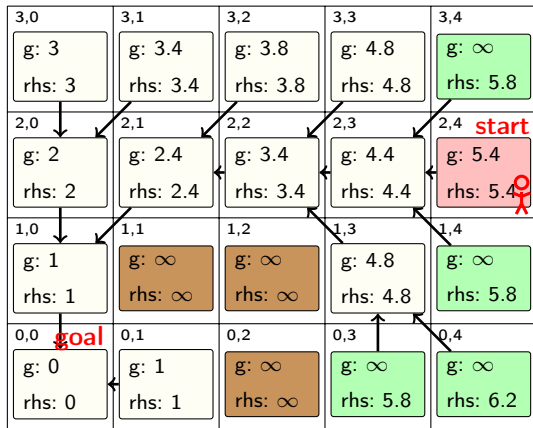
Active node

## ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list, i.e., (0,3) and (0,4)



# D\* Lite – Example Planning (22)



## Legend

Free node

Obstacle node

On open list

Active node

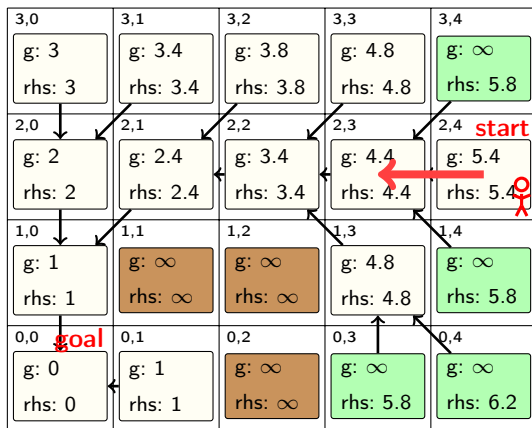
## ComputeShortestPath

- Pop the minimum element from the open list (2,4)
- It is over-consistent ( $g > rhs$ ) and thus set  $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent (none in this case) onto the open list

- The **start** node becomes consistent and the top key on the open list is not less than the key of the start node
- An optimal path is found and the loop of the ComputeShortestPath is broken



## D\* Lite – Example Planning (23)



## Legend

Free node

Obstacle node

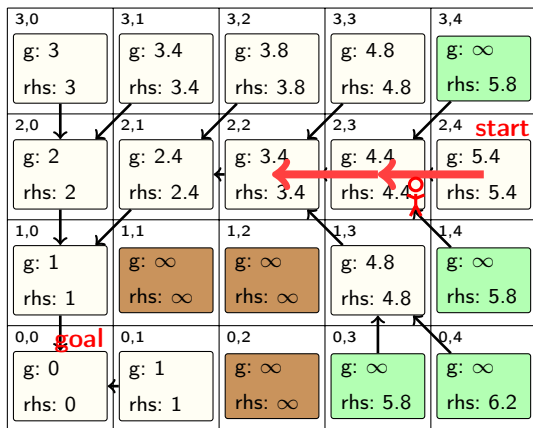
On open list

Active node

- Follow the gradient of  $g$  values from the start node



## D\* Lite – Example Planning (24)



## Legend

Free node

Obstacle node

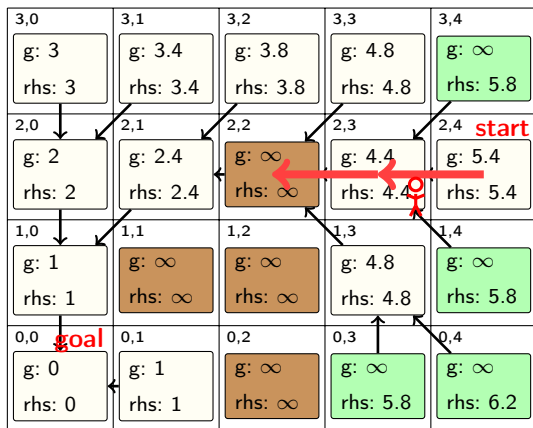
On open list

Active node

- Follow the gradient of  $g$  values from the start node



## D\* Lite – Example Planning (25)



## Legend

Free node

Obstacle node

On open list

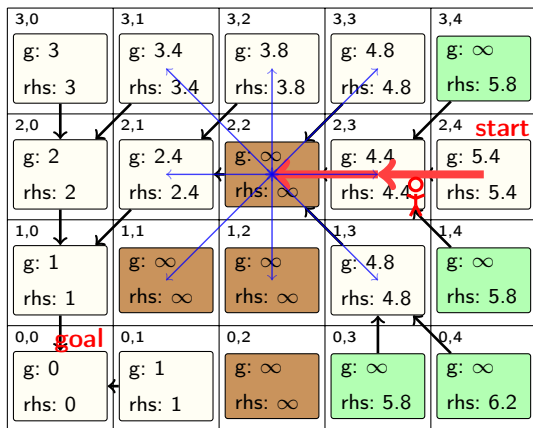
Active node

- A new obstacle is detected during the movement from (2,3) to (2,2)
- **Replanning** is needed!





# D\* Lite – Example Planning (25 update)



## Legend

Free node

Obstacle node

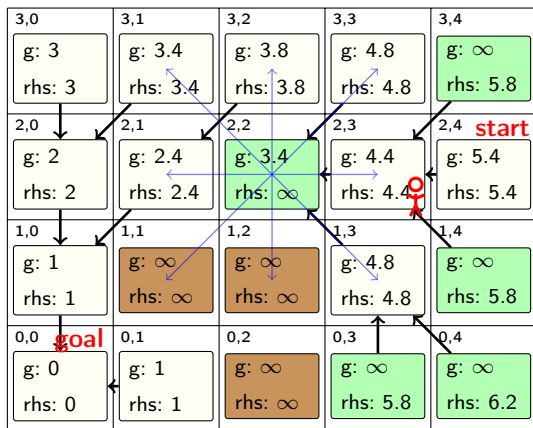
On open list

Active node

- All directed edges with changed edge, we need to call the UpdateVertex()
- All edges into and out of (2,2) have to be considered



# D\* Lite – Example Planning (26 update 1/2)



## Legend

Free node

Obstacle node

On open list

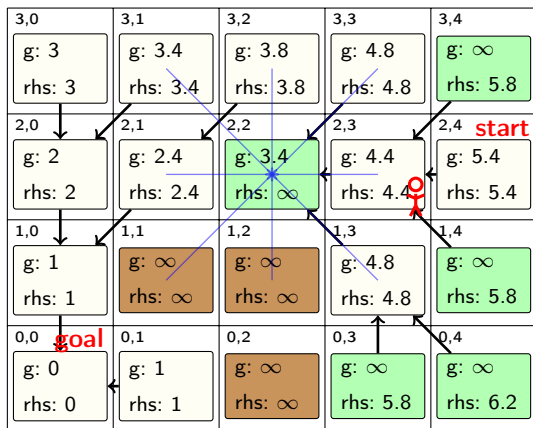
Active node

## Update Vertex

- Outgoing edges from (2,2)
- Call UpdateVertex() on (2,2)
- The transition costs are now  $\infty$  because of obstacle
- Therefore the  $rhs = \infty$  and (2,2) becomes inconsistent and it is put on the open list



# D\* Lite – Example Planning (26 update 2/2)



## Legend

Free node

Obstacle node

On open list

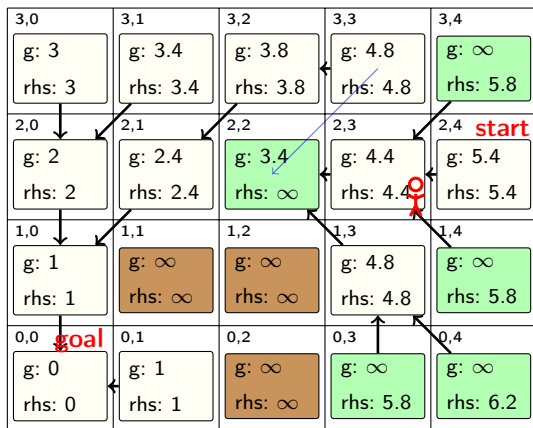
Active node

## Update Vertex

- Incoming edges to (2,2)
- Call UpdateVertex() on the neighbors (2,2)
- The transition cost is  $\infty$ , and therefore, the *rhs* value previously computed using (2,2) is changed



# D\* Lite – Example Planning (27)



## Legend

Free node

Obstacle node

On open list

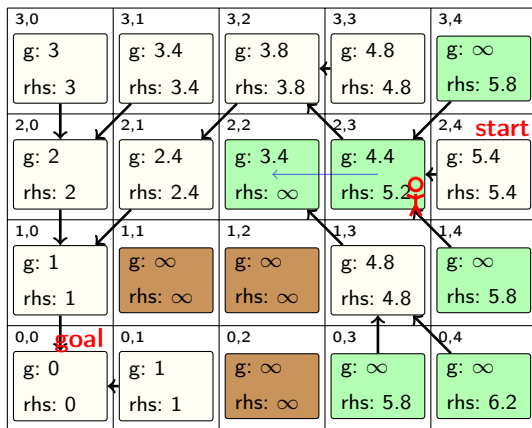
Active node

## Update Vertex

- The neighbor of (2,2) is (3,3)
- The minimum possible  $rhs$  value of (3,3) is 4.8 but it is based on the  $g$  value of (3,2) and not (2,2), which is the detected obstacle
- The node (3,3) is still consistent and thus it is not put on the open list



# D\* Lite – Example Planning (28)



## Legend

Free node

Obstacle node

On open list

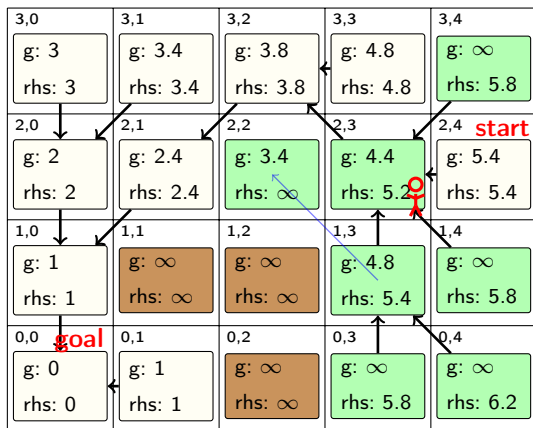
Active node

## Update Vertex

- (2,3) is also a neighbor of (2,2)
- The minimum possible *rhs* value of (2,3) is 5.2 because of (2,2) is obstacle (using (3,2) with 3.8 + 1.4)
- The *rhs* value of (2,3) is different than *g* thus (2,3) is put on the open list



## D\* Lite – Example Planning (29)



## Legend

Free node

Obstacle node

On open list

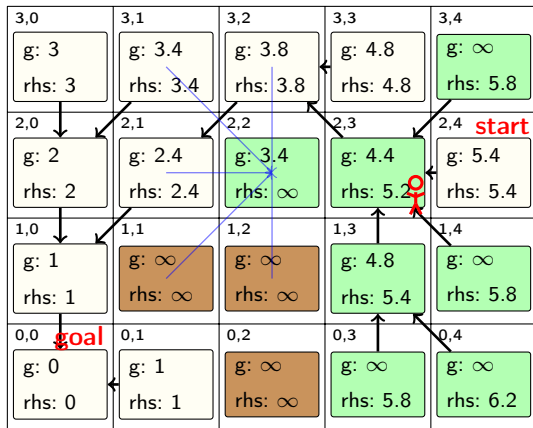
Active node

## Update Vertex

- Another neighbor of (2,2) is (1,3)
- The minimum possible *rhs* value of (1,3) is 5.4 computed based on *g* of (2,3) with  $4.4 + 1 = 5.4$
- The *rhs* value is always computed using the *g* values of its successors



# D\* Lite – Example Planning (29 update)



## Legend

Free node

Obstacle node

On open list

Active node

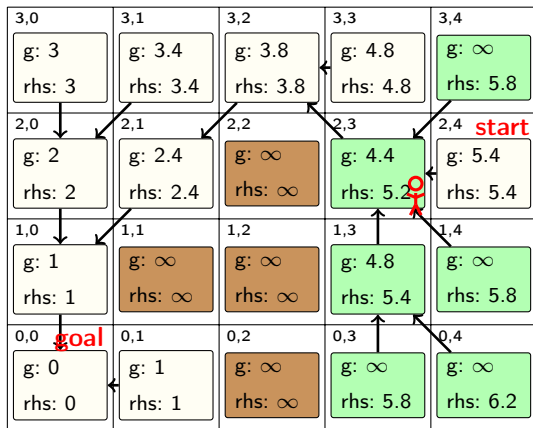
## Update Vertex

- None of the other neighbors of (2,2) end up being inconsistent
- We go back to calling `ComputeShortestPath()` until an optimal path is determined

- The node corresponding to the robot's current position is inconsistent and its key is greater than the minimum key on the open list
- Thus, the optimal path is not found yet



# D\* Lite – Example Planning (30)



## Legend

Free node

Obstacle node

On open list

Active node

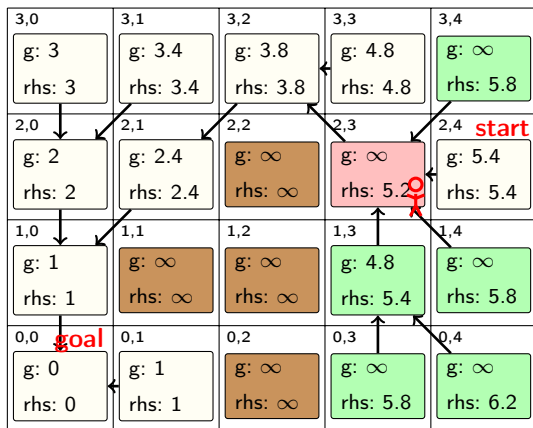
## ComputeShortestPath

- Pop the minimum element from the open list (2,2), which is obstacle
  - It is under-consistent ( $g < rhs$ ), therefore set  $g = \infty$
  - Expand the popped element and put the predecessors that become inconsistent (none in this case) onto the open list
- Because (2,2) was under-consistent (when popped), UpdateVertex() has to be called on it
  - However, it has no effect as its rhs value is up to date and consistent





## D\* Lite – Example Planning (31)



## Legend

Free node

Obstacle node

On open list

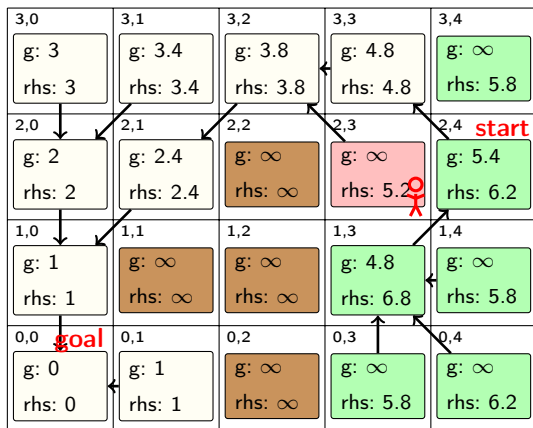
Active node

## ComputeShortestPath

- Pop the minimum element from the open list (2,3)
- It is under-consistent ( $g < rhs$ ), therefore set  $g = rhs$



## D\* Lite – Example Planning (32)



## Legend

Free node

Obstacle node

On open list

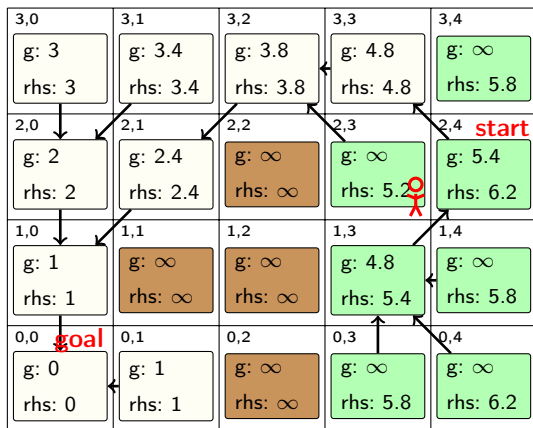
Active node

## ComputeShortestPath

- Expand the popped element and update the predecessors
- (2,4) becomes inconsistent
- (1,3) gets updated and still inconsistent
- The *rhs* value (1,4) does not change, but it is now computed from the *g* value of (1,3)



## D\* Lite – Example Planning (33)



## Legend

Free node

Obstacle node

On open list

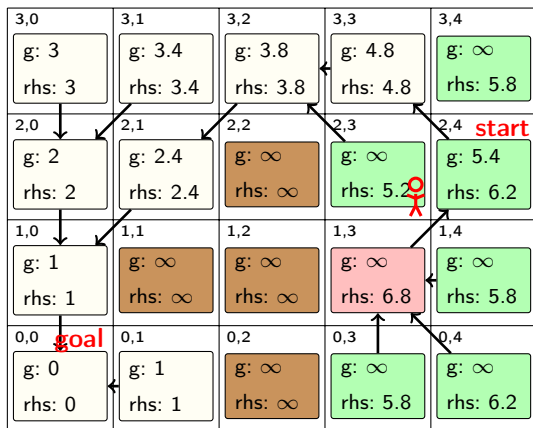
Active node

## ComputeShortestPath

- Because (2,3) was under-consistent (when popped), call UpdateVertex() on it is needed
- As it is still inconsistent it is put back onto the open list



## D\* Lite – Example Planning (34)



## Legend

Free node

Obstacle node

On open list

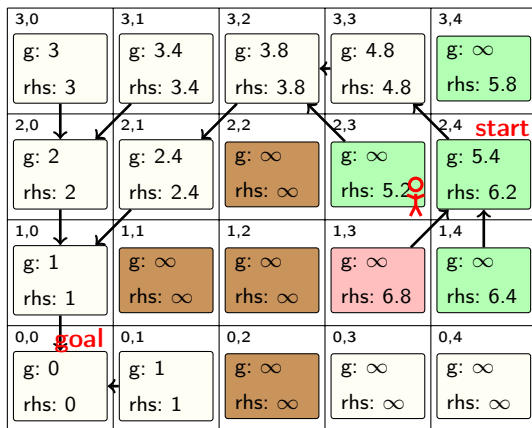
Active node

## ComputeShortestPath

- Pop the minimum element from the open list (1,3)
- It is under-consistent ( $g < rhs$ ), therefore set  $g = \infty$



## D\* Lite – Example Planning (35)



## Legend

Free node

Obstacle node

On open list

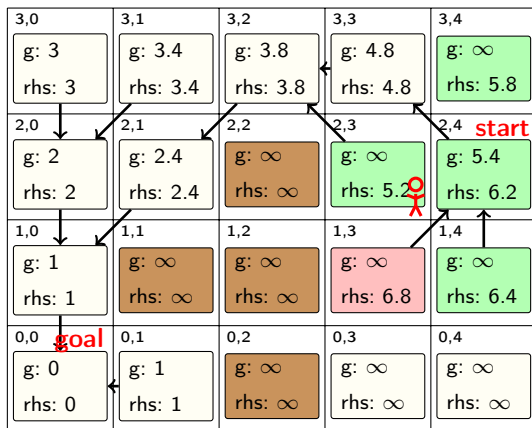
Active node

## ComputeShortestPath

- Expand the popped element and update the predecessors
- (1,4) gets updated and still inconsistent
- (0,3) and (0,4) get updated and now consistent (both  $g$  and  $rhs$  are  $\infty$ )



## D\* Lite – Example Planning (36)



## Legend

Free node

Obstacle node

On open list

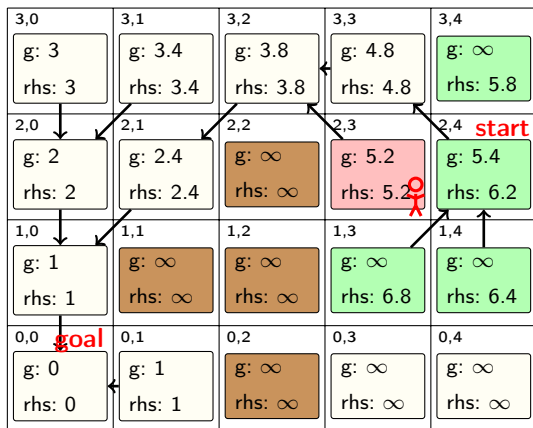
Active node

## ComputeShortestPath

- Because (1,3) was under-consistent (when popped), call UpdateVertex() on it is needed
- As it is still inconsistent it is put back onto the open list



## D\* Lite – Example Planning (37)



## Legend

Free node

Obstacle node

On open list

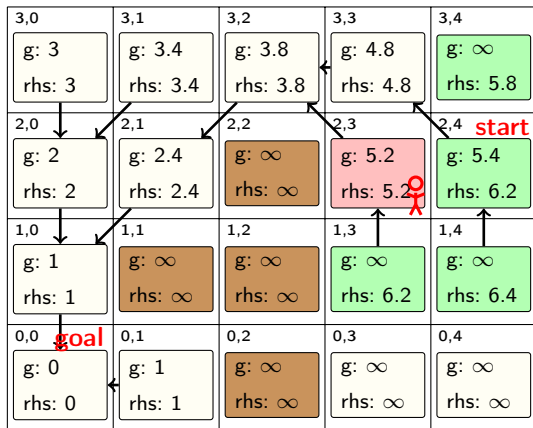
Active node

## ComputeShortestPath

- Pop the minimum element from the open list (2,3)
- It is over-consistent ( $g > rhs$ ), therefore set  $g = rhs$



# D\* Lite – Example Planning (38)



## Legend

Free node	Obstacle node
On open list	Active node

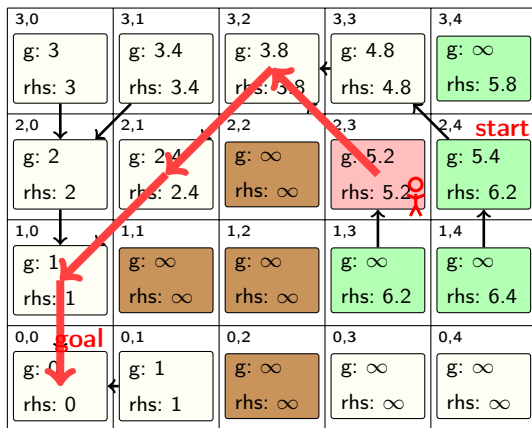
## ComputeShortestPath

- Expand the popped element and update the predecessors
- (1,3) gets updated and still inconsistent
- The node (2,3) corresponding to the robot's position is consistent
- Besides, top of the key on the open list is not less than the key of (2,3)
- The optimal path has been found and we can break out of the loop





## D\* Lite – Example Planning (39)



## Legend

Free node

Obstacle node

On open list

Active node

- Follow the gradient of  $g$  values from the robot's current position (node)



## D\* Lite – Comments

- D\* Lite works with real valued costs, not only with binary costs (free/obstacle)
- The search can be focused with an admissible heuristic that would be added to the  $g$  and  $rhs$  values
- The final version of D\* Lite includes further optimization (not shown in the example)
  - Updating the  $rhs$  value without considering all successors every time
  - Re-focusing the search as the robot moves without reordering the entire open list



# Outline

- Grid-based Planning
- DT for Path Planning
- Graph Search Algorithms
- D\* Lite
- Path Planning based on Reaction-Diffusion Process



## Reaction-Diffusion Processes Background

- *Reaction-Diffusion* (RD) models – dynamical systems capable to reproduce the autowaves
- *Autowaves* - a class of nonlinear waves that propagate through an active media

*At the expense of the energy stored in the medium, e.g., grass combustion.*

- RD model describes spatio-temporal evolution of two state variables  $u = u(\vec{x}, t)$  and  $v = v(\vec{x}, t)$  in space  $\vec{x}$  and time  $t$

$$\begin{aligned}\dot{u} &= f(u, v) + D_u \Delta u \\ \dot{v} &= g(u, v) + D_v \Delta v \end{aligned}$$

where  $\Delta$  is the Laplacian.

This RD-based path planning is informative, just for curiosity



# Reaction-Diffusion Background

- FitzHugh-Nagumo (FHN) model

*FitzHugh R, Biophysical Journal (1961)*

$$\begin{aligned}\dot{u} &= \varepsilon (u - u^3 - v + \phi) + D_u \Delta u \\ \dot{v} &= (u - \alpha v + \beta) + D_v \Delta v\end{aligned},$$

*where  $\alpha, \beta, \varepsilon$ , and  $\phi$  are parameters of the model.*

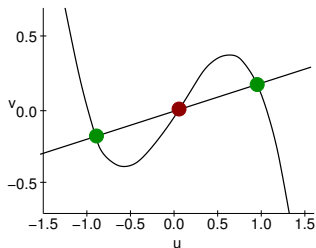
- Dynamics of RD system is determined by the associated *nullcline configurations* for  $\dot{u}=0$  and  $\dot{v}=0$  in the absence of diffusion, i.e.,

$$\begin{aligned}\varepsilon (u - u^3 - v + \phi) &= 0, \\ (u - \alpha v + \beta) &= 0,\end{aligned}$$

*which have associated geometrical shapes*



# Nullcline Configurations and Steady States



- Nullclines intersections represent

- Stable States (SSs)
- Unstable States

- Bistable regime

*The system (concentration levels of  $(u, v)$  for each grid cell) tends to be in SSs.*

- We can modulate relative stability of both SS

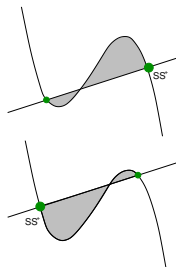
*“preference” of  $SS^+$  over  $SS^-$*

- System moves from  $SS^-$  to  $SS^+$ ,

*if a small perturbation is introduced.*

- The SSs are separated by a mobile frontier

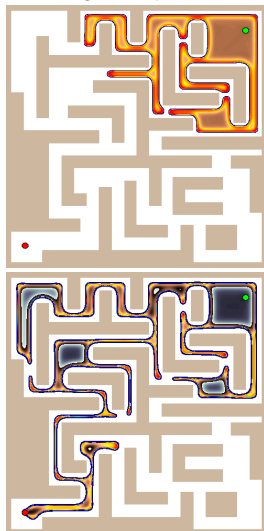
*a kind of traveling frontwave (autowaves)*



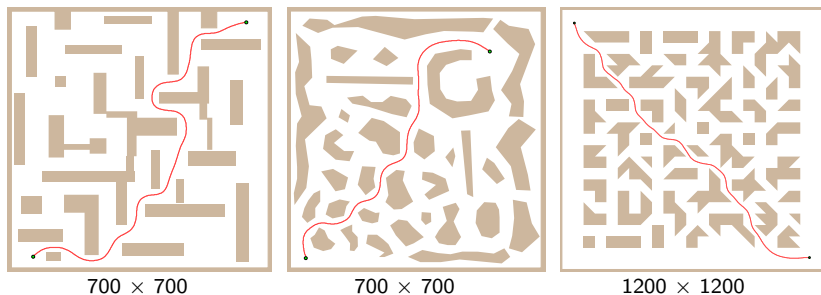
# RD-based Path Planning – Computational Model

- Finite difference method on a Cartesian grid with Dirichlet boundary conditions (FTCS) *discretization* → *grid based computation* → *grid map*
- *External forcing* – introducing additional information *i.e., constraining concentration levels to some specific values*
- Two-phase evolution of the underlying RD model
  1. **Propagation phase**
    - Freespace is set to  $SS^-$  and the start location  $SS^+$
    - Parallel propagation of the frontwave with *non-annihilation property*

Vázquez-Otero and Muñuzuri, CNNA (2010)
    - Terminate when the frontwave reaches the goal
  2. **Contraction phase**
    - Different nullclines configuration
    - Start and goal positions are forced towards  $SS^+$
    - $SS^-$  shrinks until only the path linking the forced points remains



## Example of Found Paths



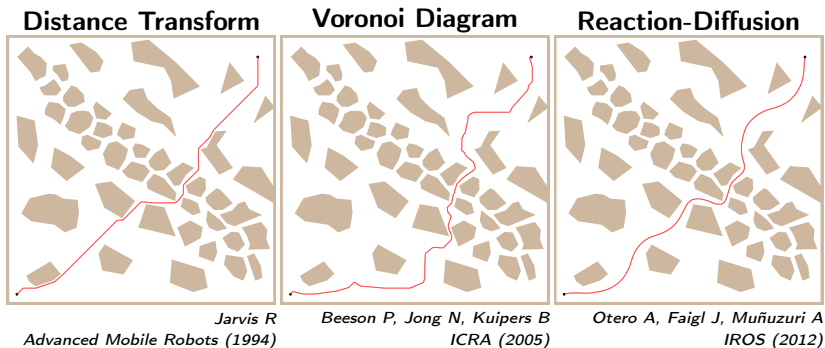
- The path clearance may be adjusted by the **wavelength** and size of the computational grid.

*Control of the path distance from the obstacles (path safety)*





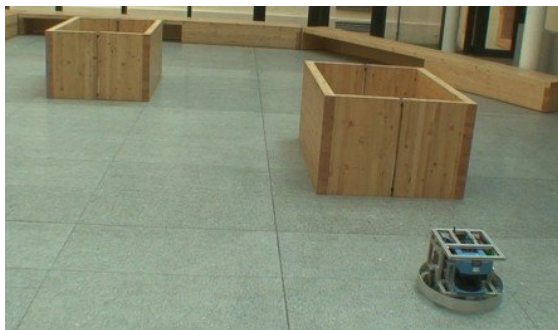
# Comparison with Standard Approaches



- RD-based approach provides competitive paths regarding path length and clearance, while they seem to be smooth



## Robustness to Noisy Data



Vázquez-Otero, A., Faigl, J., Duro, N. and Dormido, R. (2014): Reaction-Diffusion based Computational Model for Autonomous Mobile Robot Exploration of Unknown Environments. *International Journal of Unconventional Computing (IJUC)*.



# Summary of the Lecture



# Topics Discussed

- Front-Wave propagation and path simplification
- Distance Transform based planning
- Graph based planning methods: Dijkstra's, A\*, JPS, Theta\*
- D\* Lite
- Reaction-Diffusion based planning (*informative*)
  
- Next: Randomized Sampling-based Motion Planning Methods



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- **Next: Randomized Sampling-based Motion Planning Methods**

