

Grid and Graph based Path Planning Methods

Jan Faigl

Department of Computer Science
Faculty of Electrical Engineering
Czech Technical University in Prague

Lecture 04

B4M36UIR – Artificial Intelligence in Robotics

Overview of the Lecture

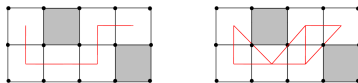
- Part 1 – Grid and Graph based Path Planning Methods
 - Grid-based Planning
 - DT for Path Planning
 - Graph Search Algorithms
 - D* Lite
 - Path Planning based on Reaction-Diffusion Process *Curiosity*

Part I

Part 1 – Grid and Graph based Path Planning Methods

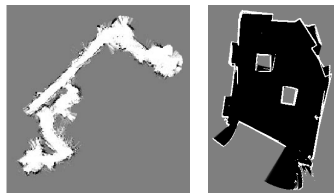
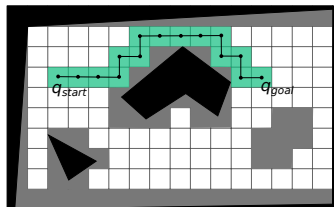
Grid-based Planning

- A subdivision of \mathcal{C}_{free} into smaller cells
- **Grow obstacles** can be simplified by growing borders by a diameter of the robot
- Construction of the planning graph $G = (V, E)$ for V as a set of cells and E as the **neighbor-relations**
 - 4-neighbors and 8-neighbors



- A grid map can be constructed from the so-called occupancy grid maps

E.g., using thresholding



Grid-based Environment Representations

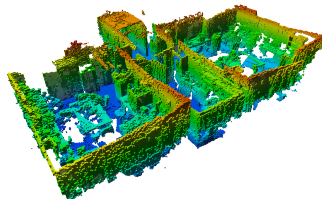
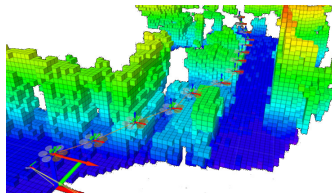
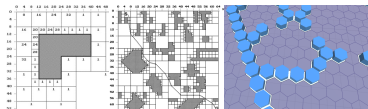
- Hierarchical planning
 - Coarse resolution and re-planning on finer resolution

Holte, R. C. et al. (1996): Hierarchical A*: searching abstraction hierarchies efficiently. AAAI.

- Octree can be used for the map representation
- In addition to squared (or rectangular) grid a hexagonal grid can be used
- 3D grid maps – **octomap**

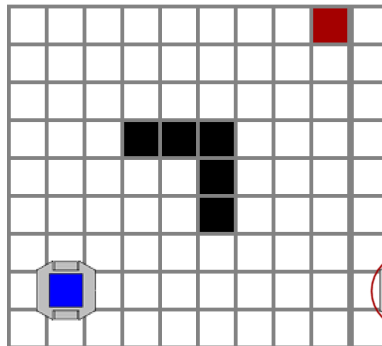
<https://octomap.github.io>

- Memory grows with the size of the environment
- Due to limited resolution it may fail in narrow passages of \mathcal{C}_{free}

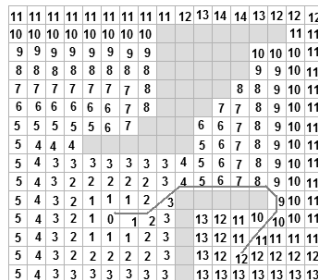
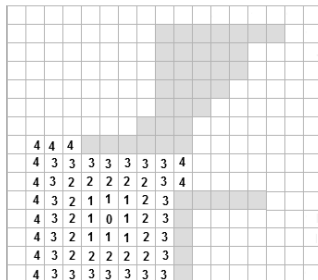
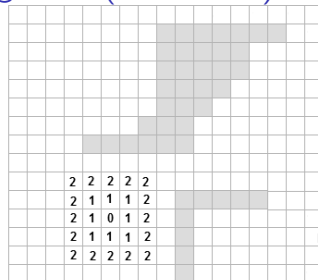
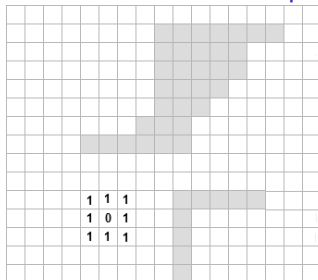


Example of Simple Grid-based Planning

- Wave-front propagation using path simplification
- Initial map with a robot and goal
- Obstacle growing
- Wave-front propagation – “flood fill”
- Find a path using a navigation function
- Path simplification
 - “Ray-shooting” technique combined with **Bresenham’s line algorithm**
 - The path is a sequence of “key” cells for avoiding obstacles

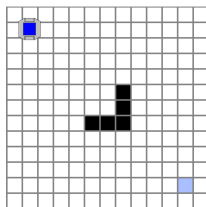


Example – Wave-Front Propagation (Flood Fill)

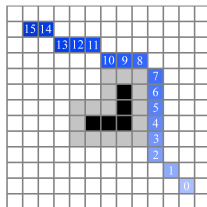
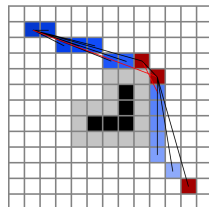


Path Simplification

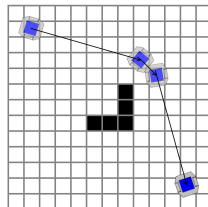
- The initial path is found in a grid using 4-neighborhood
- The rayshoot cast a line into a grid and possible collisions of the robot with obstacles are checked
- The “farthest” cells without collisions are used as “turn” points
- The final path is a sequence of straight line segments



Initial and goal locations

Obstacle growing,
wave-front propagation

Ray-shooting



Simplified path

Bresenham's Line Algorithm

- Filling a grid by a line with avoiding float numbers

- A line from (x_0, y_0) to (x_1, y_1) is given by $y = \frac{y_1 - y_0}{x_1 - x_0}(x - x_0) + y_0$

```

1  CoordsVector& bresenham(const Coords& pt1, const Coords& pt2, CoordsVector& line)
2  {
3      // The pt2 point is not added into line
4      int x0 = pt1.c; int y0 = pt1.r;
5      int x1 = pt2.c; int y1 = pt2.r;
6      Coords p;
7      int dx = x1 - x0;
8      int dy = y1 - y0;
9      int steep = (abs(dy) >= abs(dx));
10     if (steep) {
11         SWAP(x0, y0);
12         SWAP(x1, y1);
13         dx = x1 - x0; // recompute Dx, Dy
14         dy = y1 - y0;
15     }
16     int xstep = 1;
17     if (dx < 0) {
18         xstep = -1;
19         dx = -dx;
20     }
21     int ystep = 1;
22     if (dy < 0) {
23         ystep = -1;
24         dy = -dy;
25     }
26     int twoDy = 2 * dy;
27     int twoDyTwoDx = twoDy - 2 * dx; //2*Dy - 2*Dx
28     int e = twoDy - dx; //2*Dy - Dx
29     int y = y0;
30     int xDraw, yDraw;
31     for (int x = x0; x != x1; x += xstep) {
32         if (steep) {
33             xDraw = y;
34             yDraw = x;
35         } else {
36             xDraw = x;
37             yDraw = y;
38         }
39         p.c = xDraw;
40         p.r = yDraw;
41         line.push_back(p); // add to the line
42         if (e > 0) {
43             e += twoDyTwoDx; //E += 2*Dy - 2*Dx
44             y = y + ystep;
45         } else {
46             e += twoDy; //E += 2*Dy
47         }
48     }
49     return line;
50 }

```

Distance Transform based Path Planning

- For a given goal location and grid map compute a navigational function using *wave-front* algorithm, i.e., a kind of *potential field*
 - The value of the goal cell is set to 0 and all other free cells are set to some very high value
 - For each free cell compute a number of cells towards the goal cell
 - It uses 8-neighbors and distance is the Euclidean distance of the centers of two cells, i.e., $EV=1$ for orthogonal cells or $EV = \sqrt{2}$ for diagonal cells
 - The values are iteratively computed until the values are changed
 - The value of the cell c is computed as

$$cost(c) = \min_{i=1}^8 (cost(c_i) + EV_{c_i,c}),$$

where c_i is one of the neighboring cells from 8-neighborhood of the cell c

- The algorithm provides a cost map of the path distance from any free cell to the goal cell
- The path is then used following the gradient of the cell cost

Jarvis, R. (2004): Distance Transform Based Visibility Measures for Covert Path Planning in Known but Dynamic Environments

Distance Transform Path Planning

Algorithm 1: Distance Transform for Path Planning

```
for  $y := 0$  to  $yMax$  do
  for  $x := 0$  to  $xMax$  do
    if goal  $[x,y]$  then
      cell  $[x,y] := 0$ ;
    else
      cell  $[x,y] := xMax * yMax$ ; //initialization, e.g., pragmatically use longest distance as  $\infty$  ;

repeat
  for  $y := 1$  to  $(yMax - 1)$  do
    for  $x := 1$  to  $(xMax - 1)$  do
      if not blocked  $[x,y]$  then
        cell  $[x,y] := cost(x, y)$ ;

  for  $y := (yMax-1)$  downto 1 do
    for  $x := (xMax-1)$  downto 1 do
      if not blocked  $[x,y]$  then
        cell $[x,y] := cost(x, y)$ ;

until no change;
```

Distance Transform based Path Planning – Impl. 1/2

```

1  Grid& DT::compute(Grid& grid) const           35
2  {                                             36
3      static const double DIAGONAL = sqrt(2);  37
4      static const double ORTOGONAL = 1;      38
5      const int H = map.H;                    39
6      const int W = map.W;                    40
7      assert(grid.H == H and grid.W == W, "size"); 41
8      bool anyChange = true;                 42
9      int counter = 0;                        43
10     while (anyChange) {                      44
11         anyChange = false;                  45
12         for (int r = 1; r < H - 1; ++r) {   46
13             for (int c = 1; c < W - 1; ++c) { 47
14                 if (map[r][c] != FREESPACE) { 48
15                     continue;              49
16                 } //obstacle detected      50
17                 double t[4];              51
18                 t[0] = grid[r - 1][c - 1] + DIAGONAL; 52
19                 t[1] = grid[r - 1][c] + ORTOGONAL; 53
20                 t[2] = grid[r - 1][c + 1] + DIAGONAL; 54
21                 t[3] = grid[r][c - 1] + ORTOGONAL; 55
22                 double pom = grid[r][c];    56
23                 for (int i = 0; i < 4; i++) { 57
24                     if (pom > t[i]) {      58
25                         pom = t[i];      59
26                         anyChange = true; 60
27                     }                     61
28                 }                         62
29             }
30             if (anyChange) {
31                 grid[r][c] = pom;
32             }
33     }

```

An boundary is assumed around the rectangular map

Distance Transform based Path Planning – Impl. 2/2

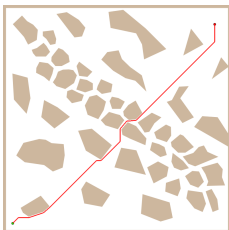
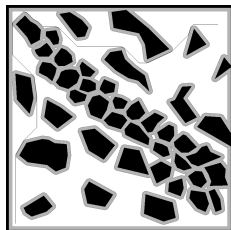
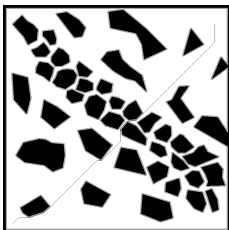
- The path is retrieved by following the minimal value towards the goal using `min8Point()`

```

1  Coords& min8Point(const Grid& grid, Coords& p)          22  CoordsVector& DT::findPath(const Coords& start,
2  {                                                       const Coords& goal, CoordsVector& path)
3  {                                                       {
4      double min = std::numeric_limits<double>::max();    23  {
5      const int H = grid.H;                               24      static const double DIAGONAL = sqrt(2);
6      const int W = grid.W;                               25      static const double ORTOGONAL = 1;
7      Coords t;                                           26      const int H = map.H;
8  }                                                       27      const int W = map.W;
9  for (int r = p.r - 1; r <= p.r + 1; r++) {            28      Grid grid(H, W, H*W); // H*W max grid value
10     if (r < 0 or r >= H) { continue; }                  29      grid[goal.r][goal.c] = 0;
11     for (int c = p.c - 1; c <= p.c + 1; c++) {          30      compute(grid);
12         if (c < 0 or c >= W) { continue; }              31
13         if (min > grid[r][c]) {                          32         if (grid[start.r][start.c] >= H*W) {
14             min = grid[r][c];                             33             WARN("Path has not been found");
15             t.r = r; t.c = c;                             34         } else {
16         }                                                 35             Coords pt = start;
17     }                                                     36             while (pt.r != goal.r or pt.c != goal.c) {
18     p = t;                                                 37                 path.push_back(pt);
19     return p;                                             38                 min8Point(grid, pt);
20 }                                                         39             }
                                                         40             path.push_back(goal);
                                                         41         }
                                                         42         return path;
                                                         43     }

```

DT Example



$\delta = 10 \text{ cm}$, $L = 27.2 \text{ m}$



$\delta = 30 \text{ cm}$, $L = 42.8 \text{ m}$

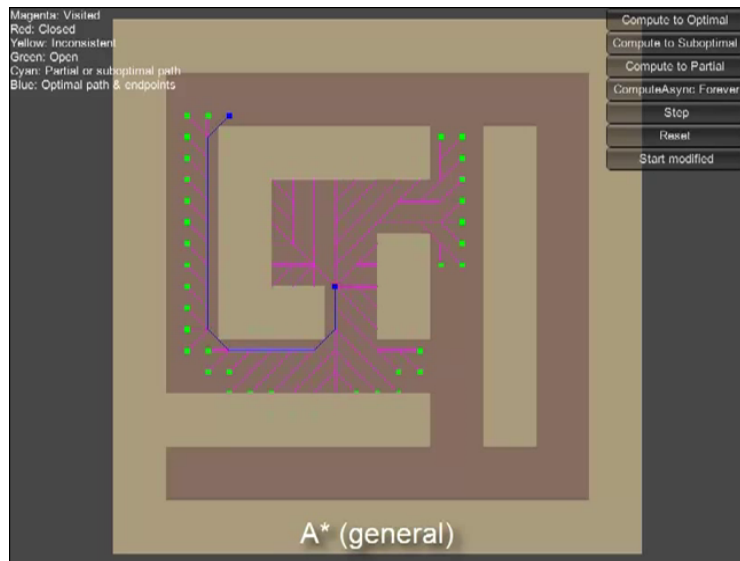
Graph Search Algorithms

- The grid can be considered as a graph and the path can be found using graph search algorithms
- The search algorithms working on a graph are of general use, e.g.
 - Breadth-first search (BSD)
 - Depth first search (DFS)
 - Dijkstra's algorithm,
 - A* algorithm and its variants
- There can be grid based speedups techniques, e.g.,
 - Jump Search Algorithm (JPS) and JPS+
- There are many search algorithm for on-line search, incremental search and with any-time and real-time properties, e.g.,
 - Lifelong Planning A* (LPA*)

Koenig, S., Likhachev, M. and Furcy, D. (2004): Lifelong Planning A*. AIJ.
 - E-Graphs – Experience graphs

Phillips, M. et al. (2012): E-Graphs: Bootstrapping Planning with Experience Graphs. RSS.

Examples of Graph/Grid Search Algorithms



<https://www.youtube.com/watch?v=X5a149nSE9s>

Dijkstra's Algorithm

- Dijkstra's algorithm determines paths as iterative update of the cost of the shortest path to the particular nodes

- Let start with the initial cell (node) with the cost set to 0 and update all successors

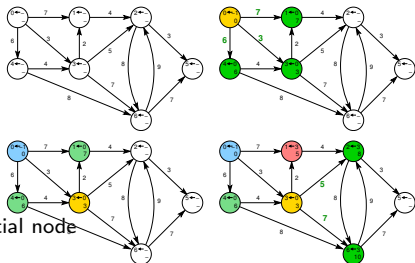
- Select the node

- with a path from the initial node
- and has a lower cost

- Repeat until there is a reachable node

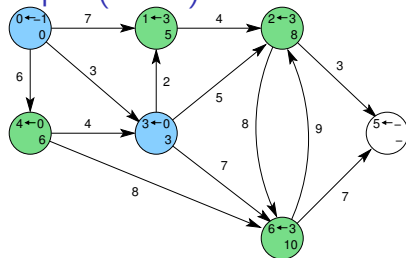
- I.e., a node with a path from the initial node
- has a cost and parent (green nodes).

Edsger W. Dijkstra, 1956

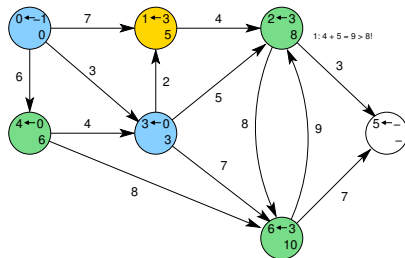


The cost of nodes can only decrease (edge cost is positive). Therefore, for a node with the currently lowest cost, there cannot be a shorter path from the initial node.

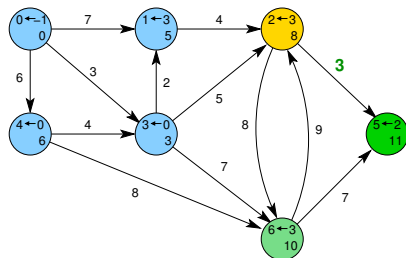
Example (cont.)



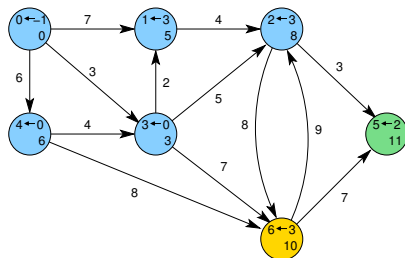
1: After the expansion, the shortest path to the node 2 is over the node 3



2: There is not shorter path to the node 2 over the node 1



3: After the expansion, there is a new path to the node 5



4: The path does not improve for further expansions

Dijkstra's Algorithm – Impl.

```
1  dij->nodes[dij->start_node].cost = 0; // init
2  void *pq = pq_alloc(dij->num_nodes); // set priority queue
3  int cur_label;
4  pq_push(pq, dij->start_node, 0);
5  while ( !pq_is_empty(pq) && pq_pop(pq, &cur_label)) {
6      node_t *cur = &(dij->nodes[cur_label]); // remember the current node
7      for (int i = 0; i < cur->edge_count; ++i) { // all edges of cur
8          edge_t *edge = &(dij->graph->edges[cur->edge_start + i]);
9          node_t *to = &(dij->nodes[edge->to]);
10         const int cost = cur->cost + edge->cost;
11         if (to->cost == -1) { // node to has not been visited
12             to->cost = cost;
13             to->parent = cur_label;
14             pq_push(pq, edge->to, cost); // put node to the queue
15         } else if (cost < to->cost) { // node already in the queue
16             to->cost = cost; // test if the cost can be reduced
17             to->parent = cur_label; // update the parent node
18             pq_update(pq, edge->to, cost); // update the priority queue
19         }
20     } // loop for all edges of the cur node
21 } // priority queue empty
22 pq_free(pq); // release memory
```

A* Algorithm

- A* uses a user-defined h -values (heuristic) to focus the search

Peter Hart, Nils Nilsson, and Bertram Raphael, 1968

- Prefer expansion of the node n with the lowest value

$$f(n) = g(n) + h(n),$$

where $g(n)$ is the cost (path length) from the start to n and $h(n)$ is the estimated cost from n to the goal

- h -values approximate the goal distance from particular nodes
- **Admissibility condition** – heuristic always underestimate the remaining cost to reach the goal
 - Let $h^*(n)$ be the true cost of the optimal path from n to the goal
 - Then $h(n)$ is **admissible** if for all n : $h(n) \leq h^*(n)$
 - E.g., Euclidean distance is admissible
 - A straight line will always be the shortest path
- Dijkstra's algorithm – $h(n) = 0$

A* Implementation Notes

- The most costly operations of A* are
 - Insert and lookup an element in the **closed list**
 - Insert element and get minimal element (according to $f()$ value) from the **open list**
- The **closed list** can be efficiently implemented as a **hash set**
- The **open list** is usually implemented as a **priority queue**, e.g.,
 - Fibonacci heap, binomial heap, k -level bucket
 - **binary heap** is usually sufficient ($O(\log n)$)
- Forward A*
 1. Create a search tree and initiate it with the start location
 2. Select generated but not yet expanded state s with the smallest f -value, $f(s) = g(s) + h(s)$
 3. Stop if s is the goal
 4. Expand the state s
 5. Goto Step 2

Similar to Dijkstra's algorithm but it used $f(s)$ with heuristic $h(s)$ instead of pure $g(s)$

Dijkstra's vs A* vs Jump Point Search (JPS)



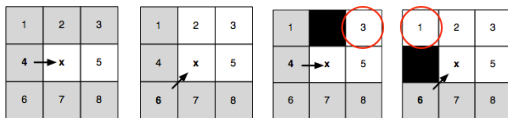
<https://www.youtube.com/watch?v=ROG4Ud081LY>

Jump Point Search Algorithm for Grid-based Path Planning

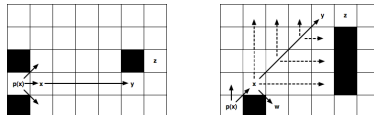
- **Jump Point Search** (JPS) algorithm is based on a macro operator that identifies and selectively expands only certain nodes (**jump points**)

Harabor, D. and Grastien, A. (2011): Online Graph Pruning for Pathfinding on Grid Maps. AAAI.

- Natural neighbors after neighbor pruning with forced neighbors because of obstacle



- Intermediate nodes on a path connecting two jump points are never expanded



- No preprocessing and no memory overheads while it speeds up A*

<https://harablog.wordpress.com/2011/09/07/jump-point-search/>

- JPS+ – optimized preprocessed version of **JPS** with goal bounding

<https://github.com/SteveRabin/JPSPlusWithGoalBounding>

<http://www.gdcvault.com/play/1022094/JPS-Over-100x-Faster-than>

Theta* – Any-Angle Path Planning Algorithm

- Any-angle path planning algorithms simplify the path during the search
- Theta* is an extension of A* with `LineOfSight()`

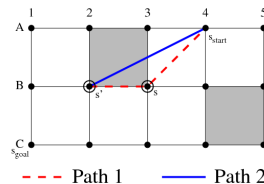
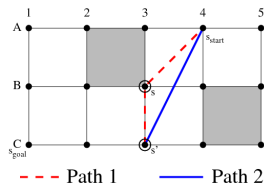
Nash, A., Daniel, K., Koenig, S. and Felner, A. (2007): Theta*: Any-Angle Path Planning on Grids. AAAI.

Algorithm 2: Theta* Any-Angle Planning

```

if LineOfSight(parent(s), s') then
    /* Path 2 – any-angle path */
    if g(parent(s)) + c(parent(s), s') < g(s') then
        parent(s') := parent(s);
        g(s') := g(parent(s)) + c(parent(s), s');
else
    /* Path 1 – A* path */
    if g(s) + c(s, s') < g(s') then
        parent(s') := s;
        g(s') := g(s) + c(s, s');
  
```

- Path 2: considers path from start to parent(s) and from parent(s) to s' if s' has line-of-sight to parent(s)

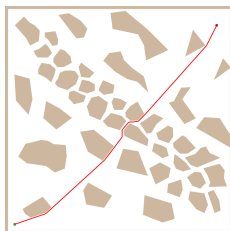


<http://aigamedev.com/open/tutorials/theta-star-any-angle-paths/>

Theta* Any-Angle Path Planning Examples

- Example of found paths by the Theta* algorithm for the same problems as for the DT-based examples on Slide 16

Both algorithms implemented in C++



$\delta = 10$ cm, $L = 26.3$ m



$\delta = 30$ cm, $L = 40.3$ m

The same path planning problems solved by DT (without path smoothing) have $L_{\delta=10} = 27.2$ m and $L_{\delta=30} = 42.8$ m, while DT seems to be faster

- **Lazy Theta*** – reduces the number of line-of-sight checks

Nash, A., Koenig, S. and Tovey, C. (2010): Lazy Theta*: Any-Angle Path Planning and Path Length Analysis in 3D. AAAI.

<http://aigamedev.com/open/tutorial/lazy-theta-star/>

A* Variants – Online Search

- The state space (map) may not be known exactly in advance
 - Environment can **dynamically** change
 - True travel costs are **experienced** during the path execution
- Repeated A* searches can be computationally demanding
- **Incremental heuristic search**
 - Repeated planning of the path from the current state to the goal
 - Planning under the **free-space** assumption
 - **Reuse** information from the previous searches (**closed list** entries):
 - Focused Dynamic A* (**D***) – h^* is based on **traversability**, it has been used, e.g., for the Mars rover “Opportunity”
Stentz, A. (1995): The Focussed D* Algorithm for Real-Time Replanning. IJCAI.
 - **D* Lite** – similar to D*
Koenig, S. and Likhachev, M. (2005): Fast Replanning for Navigation in Unknown Terrain. T-RO.
- **Real-Time Heuristic Search**
 - Repeated planning with limited **look-ahead** – suboptimal but fast
 - Learning Real-Time A* (**LRTA***)
Korf, E. (1990): Real-time heuristic search. JAI
 - Real-Time Adaptive A* (**RTAA***)
Koenig, S. and Likhachev, M. (2006): Real-time adaptive A*. AAMAS.

Real-Time Adaptive A* (RTAA*)

- Execute A* with limited **look-ahead**
- Learns better informed **heuristic** from the experience, initially $h(s)$, e.g., Euclidean distance
- Look-ahead defines **trade-off** between optimality and computational cost
 - `astar(lookahead)`

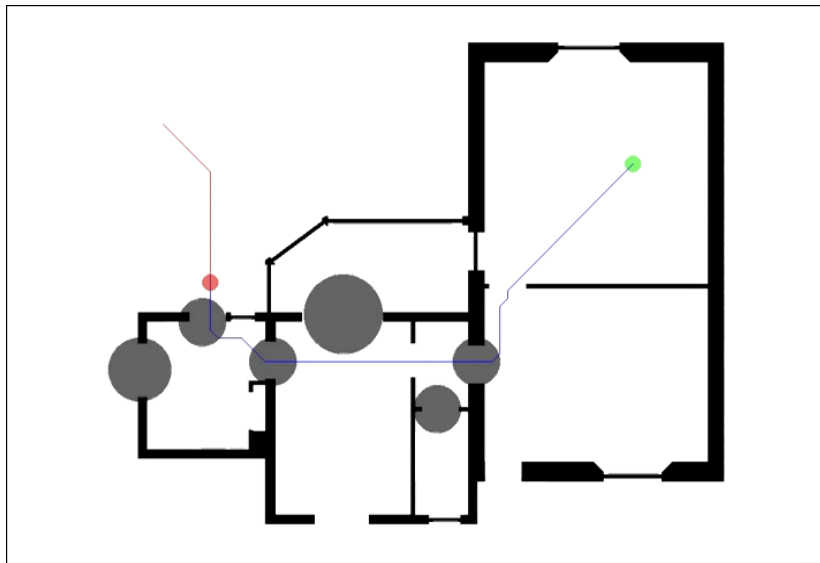
A* expansion as far as "lookahead" nodes and it terminates with the state s'

```

while ( $s_{curr} \notin GOAL$ ) do
  astar(lookahead);
  if  $s' = FAILURE$  then
    return FAILURE;
  for all  $s \in CLOSED$  do
     $H(s) := g(s') + h(s') - g(s)$ ;
  execute(plan); // perform one step
return SUCCESS;
  
```

s' is the last state expanded during the previous A* search

D* Lite – Demo



<https://www.youtube.com/watch?v=X5a149nSE9s>

D* Lite Overview

- It is similar to D*, but it is based on **Lifelong Planning A***

Koenig, S. and Likhachev, M. (2002): D* Lite. AAAI.

- It searches from the goal node to the start node, i.e., g -values estimate the goal distance
- Store pending nodes in a priority queue
- Process nodes in order of increasing objective function value
- Incrementally repair solution paths when changes occur
- Maintains two estimates of costs per node
 - g – the objective function value – based on what we know
 - rhs – one-step lookahead of the objective function value – based on what we know
- **Consistency**
 - Consistent – $g = rhs$
 - Inconsistent – $g \neq rhs$
- Inconsistent nodes are stored in the priority queue (open list) for processing

D* Lite: Cost Estimates

- *rhs* of the node u is computed based on g of its successors in the graph and the transition costs of the edge to those successors

$$rhs(u) = \min_{s' \in Succ(u)} (g(s') + c(u, s'))$$

- The key/priority of a node s on the open list is the minimum of $g(s)$ and $rhs(s)$ plus a focusing heuristic h

$$[\min(g(s), rhs(s)) + h(s_{start}, s); \min(g(s), rhs(s))]$$

- The first term is used as the primary key
- The second term is used as the secondary key for tie-breaking

D* Lite Algorithm

- **Main** – repeat until the robot reaches the goal (or $g(s_{start}) = \infty$ there is no path)

```

Initialize();
ComputeShortestPath();
while ( $s_{start} \neq s_{goal}$ ) do
  |  $s_{start} = \operatorname{argmin}_{s' \in \operatorname{Succ}(s_{start})} (c(s_{start}, s') + g(s'))$ ;
Move to  $s_{start}$ ;
Scan the graph for changed edge costs;
if any edge cost changed perform then
  | foreach directed edges  $(u, v)$  with changed edge costs do
    | | Update the edge cost  $c(u, v)$ ;
    | | UpdateVertex( $u$ );
    | foreach  $s \in U$  do
      | | U.Update( $s$ , CalculateKey( $s$ ));
  | ComputeShortestPath();

```

Procedure Initialize

```

U = 0;
foreach  $s \in S$  do
  |  $rhs(s) := g(s) := \infty$ ;
 $rhs(s_{goal}) := 0$ ;
U.Insert( $s_{goal}$ , CalculateKey( $s_{goal}$ ));

```

D* Lite Algorithm – ComputeShortestPath()

Procedure ComputeShortestPath

```

while  $U.TopKey() < CalculateKey(s_{start})$  OR  $rhs(s_{start}) \neq g(s_{start})$  do
   $u := U.Pop()$ ;
  if  $g(u) > rhs(u)$  then
     $g(u) := rhs(u)$ ;
    foreach  $s \in Pred(u)$  do UpdateVertex(s);
  else
     $g(u) := \infty$ ;
    foreach  $s \in Pred(u) \cup \{u\}$  do UpdateVertex(s);

```

Procedure UpdateVertex

```

if  $u \neq s_{goal}$  then  $rhs(u) := \min_{s' \in Succ(u)} (c(u, s') + g(s'))$ ;
if  $u \in U$  then  $U.Remove(u)$ ;
if  $g(u) \neq rhs(u)$  then  $U.Insert(u, CalculateKey(u))$ ;

```

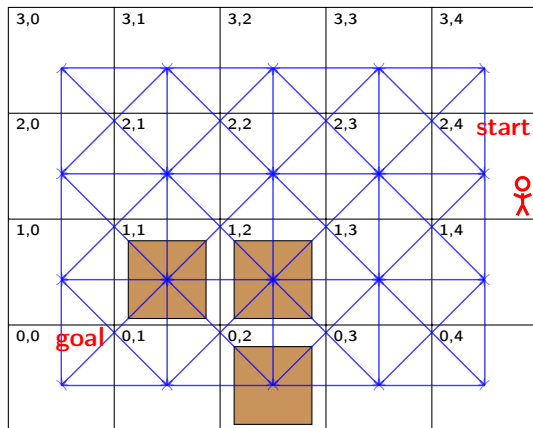
Procedure CalculateKey

```

return  $[\min(g(s), rhs(s)) + h(s_{start}, s); \min(g(s), rhs(s))]$ 

```


D* Lite – Example



Legend

Free node

Obstacle node

On open list


Active node

- A grid map of the environment (what is actually known)
- 8-connected graph superimposed on the grid (bidirectional)
- Focusing heuristic is not used ($h = 0$)

■ Transition costs

- Free space – Free space: 1.0 and 1.4 (for diagonal edge)
- From/to obstacle: ∞

D* Lite – Example Planning (1)

3,0 g: ∞ rhs: ∞	3,1 g: ∞ rhs: ∞	3,2 g: ∞ rhs: ∞	3,3 g: ∞ rhs: ∞	3,4 g: ∞ rhs: ∞
2,0 g: ∞ rhs: ∞	2,1 g: ∞ rhs: ∞	2,2 g: ∞ rhs: ∞	2,3 g: ∞ rhs: ∞	2,4 start g: ∞ rhs: ∞ 
1,0 g: ∞ rhs: ∞	1,1 g: ∞ rhs: ∞	1,2 g: ∞ rhs: ∞	1,3 g: ∞ rhs: ∞	1,4 g: ∞ rhs: ∞
0,0 goal g: ∞ rhs: 0	0,1 g: ∞ rhs: ∞	0,2 g: ∞ rhs: ∞	0,3 g: ∞ rhs: ∞	0,4 g: ∞ rhs: ∞

Legend

Free node

Obstacle node


On open list

Active node

Initialization

- Set $rhs = 0$ for the goal
- Set $rhs = g = \infty$ for all other nodes

D* Lite – Example Planning (2)

3,0 g: ∞ rhs: ∞	3,1 g: ∞ rhs: ∞	3,2 g: ∞ rhs: ∞	3,3 g: ∞ rhs: ∞	3,4 g: ∞ rhs: ∞
2,0 g: ∞ rhs: ∞	2,1 g: ∞ rhs: ∞	2,2 g: ∞ rhs: ∞	2,3 g: ∞ rhs: ∞	2,4 start g: ∞ rhs: ∞ 
1,0 g: ∞ rhs: ∞	1,1 g: ∞ rhs: ∞	1,2 g: ∞ rhs: ∞	1,3 g: ∞ rhs: ∞	1,4 g: ∞ rhs: ∞
0,0 goal g: ∞ rhs: 0	0,1 g: ∞ rhs: ∞	0,2 g: ∞ rhs: ∞	0,3 g: ∞ rhs: ∞	0,4 g: ∞ rhs: ∞

Legend

Free node

Obstacle node


On open list

Active node

Initialization

- Put the goal to the open list
It is inconsistent

D* Lite – Example Planning (3)

3,0 g: ∞ rhs: ∞	3,1 g: ∞ rhs: ∞	3,2 g: ∞ rhs: ∞	3,3 g: ∞ rhs: ∞	3,4 g: ∞ rhs: ∞
2,0 g: ∞ rhs: ∞	2,1 g: ∞ rhs: ∞	2,2 g: ∞ rhs: ∞	2,3 g: ∞ rhs: ∞	2,4 start g: ∞ rhs: ∞ 
1,0 g: ∞ rhs: ∞	1,1 g: ∞ rhs: ∞	1,2 g: ∞ rhs: ∞	1,3 g: ∞ rhs: ∞	1,4 g: ∞ rhs: ∞
0,0 goal g: 0 rhs: 0	0,1 g: ∞ rhs: ∞	0,2 g: ∞ rhs: ∞	0,3 g: ∞ rhs: ∞	0,4 g: ∞ rhs: ∞

Legend

Free node

Obstacle node


On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (goal)
- It is over-consistent ($g > rhs$), therefore set $g = rhs$

D* Lite – Example Planning (4)

3,0 g: ∞ rhs: ∞	3,1 g: ∞ rhs: ∞	3,2 g: ∞ rhs: ∞	3,3 g: ∞ rhs: ∞	3,4 g: ∞ rhs: ∞
2,0 g: ∞ rhs: ∞	2,1 g: ∞ rhs: ∞	2,2 g: ∞ rhs: ∞	2,3 g: ∞ rhs: ∞	2,4 start g: ∞ rhs: ∞ 
1,0 g: ∞ rhs: 1	1,1 g: ∞ rhs: ∞	1,2 g: ∞ rhs: ∞	1,3 g: ∞ rhs: ∞	1,4 g: ∞ rhs: ∞
0,0 goal g: 0 rhs: 0	0,1 g: ∞ rhs: 1	0,2 g: ∞ rhs: ∞	0,3 g: ∞ rhs: ∞	0,4 g: ∞ rhs: ∞

Small black arrows denote the node used for computing the *rhs* value, i.e., using the respective transition cost

- The *rhs* value of (1,1) is ∞ because the transition to obstacle has cost ∞

Legend

Free node

Obstacle node


On open list

Active node

ComputeShortestPath

- Expand popped node (UpdateVertex()) on all its predecessors)
- This computes the *rhs* values for the predecessors
- Nodes that become inconsistent are added to the open list

D^{*} Lite – Example Planning (5)

3,0 g: ∞ rhs: ∞	3,1 g: ∞ rhs: ∞	3,2 g: ∞ rhs: ∞	3,3 g: ∞ rhs: ∞	3,4 g: ∞ rhs: ∞
2,0 g: ∞ rhs: ∞	2,1 g: ∞ rhs: ∞	2,2 g: ∞ rhs: ∞	2,3 g: ∞ rhs: ∞	2,4 start g: ∞ rhs: ∞ 
1,0 g: 1 rhs: 1	1,1 g: ∞ rhs: ∞	1,2 g: ∞ rhs: ∞	1,3 g: ∞ rhs: ∞	1,4 g: ∞ rhs: ∞
0,0 goal g: 0 rhs: 0	0,1 g: ∞ rhs: 1	0,2 g: ∞ rhs: ∞	0,3 g: ∞ rhs: ∞	0,4 g: ∞ rhs: ∞

Legend

Free node

Obstacle node


On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (1,0)
- It is over-consistent ($g > rhs$)
set $g = rhs$

D* Lite – Example Planning (6)

3,0 g: ∞ rhs: ∞	3,1 g: ∞ rhs: ∞	3,2 g: ∞ rhs: ∞	3,3 g: ∞ rhs: ∞	3,4 g: ∞ rhs: ∞
2,0 g: ∞ rhs: 2	2,1 g: ∞ rhs: 2.4	2,2 g: ∞ rhs: ∞	2,3 g: ∞ rhs: ∞	2,4 start g: ∞ rhs: ∞ 
1,0 g: 1 rhs: 1	1,1 g: ∞ rhs: ∞	1,2 g: ∞ rhs: ∞	1,3 g: ∞ rhs: ∞	1,4 g: ∞ rhs: ∞
0,0 goal g: 0 rhs: 0	0,1 g: ∞ rhs: 1	0,2 g: ∞ rhs: ∞	0,3 g: ∞ rhs: ∞	0,4 g: ∞ rhs: ∞

Legend

Free node

Obstacle node

On open list


Active node

ComputeShortestPath

- Expand the popped node (UpdateVertex()) on all predecessors in the graph)
- Compute *rhs* values of the predecessors accordingly
- Put them to the open list if they become inconsistent

- The *rhs* value of (0,0), (1,1) does not change
- They do not become inconsistent and thus they are not put on the open list

D^{*} Lite – Example Planning (7)

3,0 g: ∞ rhs: ∞	3,1 g: ∞ rhs: ∞	3,2 g: ∞ rhs: ∞	3,3 g: ∞ rhs: ∞	3,4 g: ∞ rhs: ∞
2,0 g: ∞ rhs: 2	2,1 g: ∞ rhs: 2.4	2,2 g: ∞ rhs: ∞	2,3 g: ∞ rhs: ∞	2,4 start g: ∞ rhs: ∞ 
1,0 g: 1 rhs: 1	1,1 g: ∞ rhs: ∞	1,2 g: ∞ rhs: ∞	1,3 g: ∞ rhs: ∞	1,4 g: ∞ rhs: ∞
0,0 goal g: 0 rhs: 0	0,1 g: 1 rhs: 1	0,2 g: ∞ rhs: ∞	0,3 g: ∞ rhs: ∞	0,4 g: ∞ rhs: ∞

Legend

Free node

Obstacle node


On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (0,1)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$
- Expand the popped element, e.g., call UpdateVertex()

D^{*} Lite – Example Planning (8)

3,0 g: ∞ rhs: ∞	3,1 g: ∞ rhs: ∞	3,2 g: ∞ rhs: ∞	3,3 g: ∞ rhs: ∞	3,4 g: ∞ rhs: ∞
2,0 g: 2 rhs: 2	2,1 g: ∞ rhs: 2.4	2,2 g: ∞ rhs: ∞	2,3 g: ∞ rhs: ∞	2,4 start g: ∞ rhs: ∞ 
1,0 g: 1 rhs: 1	1,1 g: ∞ rhs: ∞	1,2 g: ∞ rhs: ∞	1,3 g: ∞ rhs: ∞	1,4 g: ∞ rhs: ∞
0,0 goal g: 0 rhs: 0	0,1 g: 1 rhs: 1	0,2 g: ∞ rhs: ∞	0,3 g: ∞ rhs: ∞	0,4 g: ∞ rhs: ∞

Legend

Free node

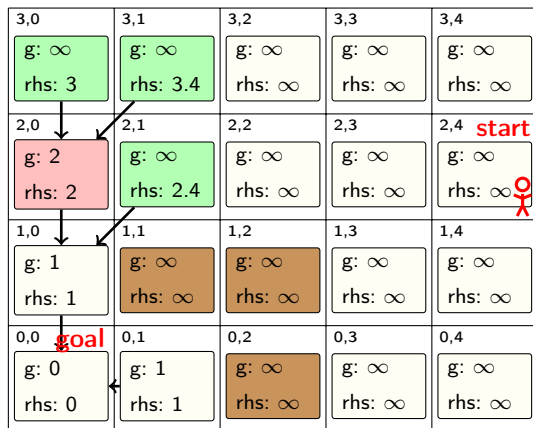
Obstacle node

On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,0)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$

D^{*} Lite – Example Planning (9)

Legend

Free node

Obstacle node

On open list

Active node

ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list

D* Lite – Example Planning (10)

3,0 g: ∞ rhs: 3	3,1 g: ∞ rhs: 3.4	3,2 g: ∞ rhs: ∞	3,3 g: ∞ rhs: ∞	3,4 g: ∞ rhs: ∞
2,0 g: 2 rhs: 2	2,1 g: 2.4 rhs: 2.4	2,2 g: ∞ rhs: ∞	2,3 g: ∞ rhs: ∞	2,4 start g: ∞ rhs: ∞
1,0 g: 1 rhs: 1	1,1 g: ∞ rhs: ∞	1,2 g: ∞ rhs: ∞	1,3 g: ∞ rhs: ∞	1,4 g: ∞ rhs: ∞
0,0 goal g: 0 rhs: 0	0,1 g: 1 rhs: 1	0,2 g: ∞ rhs: ∞	0,3 g: ∞ rhs: ∞	0,4 g: ∞ rhs: ∞

Legend

Free node

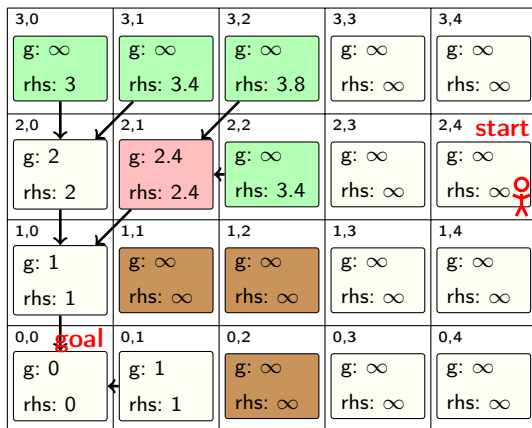
Obstacle node

On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,1)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$

D^{*} Lite – Example Planning (11)

Legend

Free node

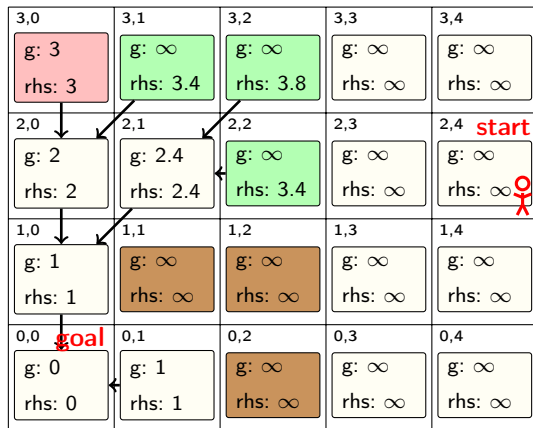
Obstacle node

On open list

Active node

ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list

D^{*} Lite – Example Planning (12)

Legend

Free node

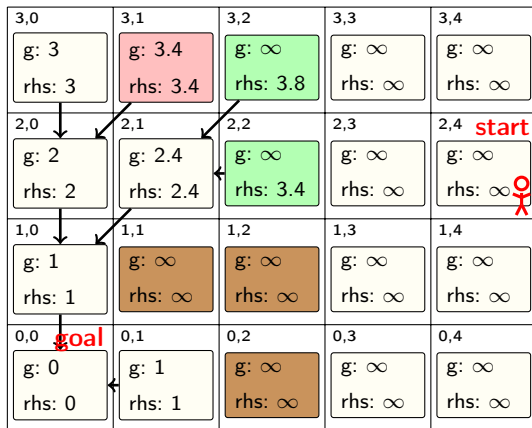
Obstacle node

On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (3,0)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent onto the open list
- In this cases, none of the predecessors become inconsistent

D^{*} Lite – Example Planning (13)

Legend

Free node

Obstacle node

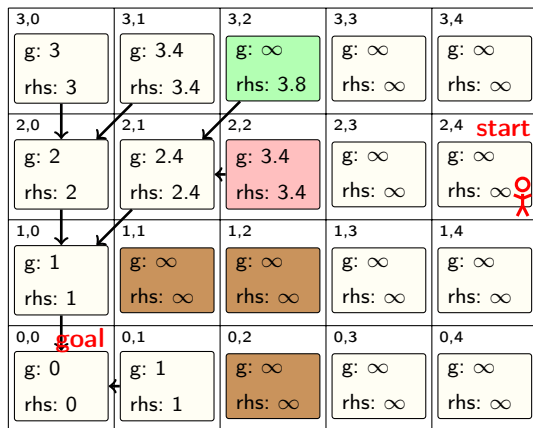
On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (3,0)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent onto the open list
- In this cases, none of the predecessors become inconsistent

D* Lite – Example Planning (14)



Legend

Free node

Obstacle node

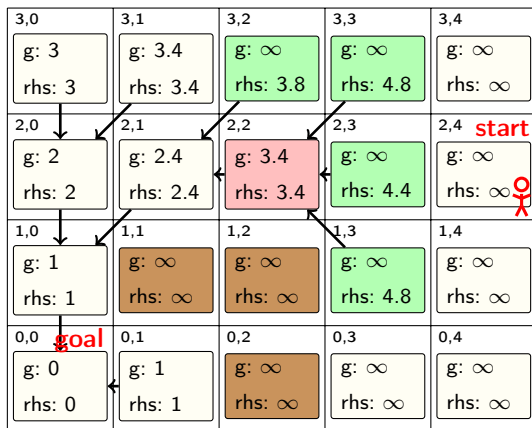
On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,2)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$

D* Lite – Example Planning (15)



Legend

Free node

Obstacle node

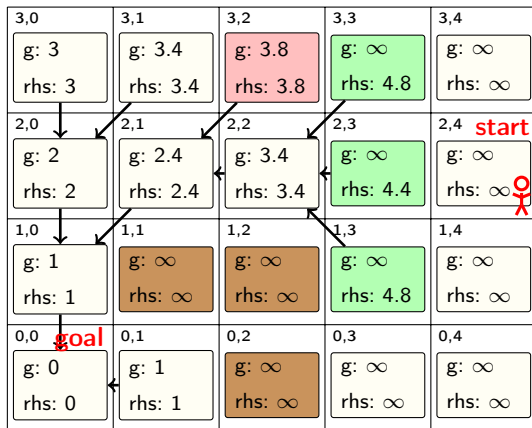
On open list

Active node

ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list, i.e., (3,2), (3,3), (2,3)

D* Lite – Example Planning (16)



Legend

Free node

Obstacle node

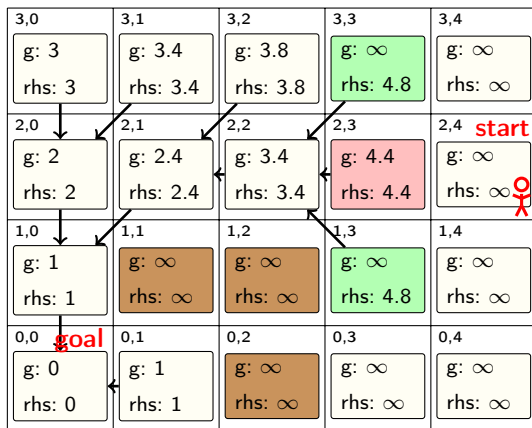
On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (3,2)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent onto the open list
- In this cases, none of the predecessors become inconsistent

D* Lite – Example Planning (17)



Legend

Free node

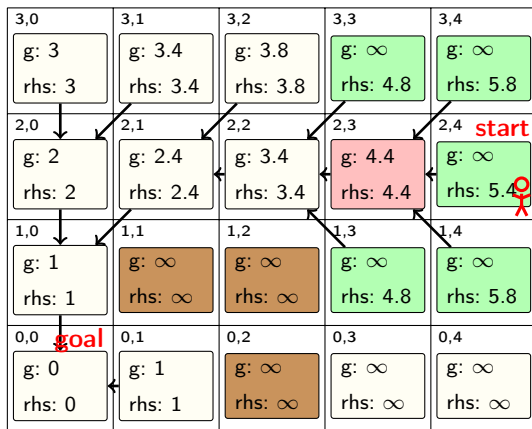
Obstacle node

On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,3)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$

D^{*} Lite – Example Planning (18)

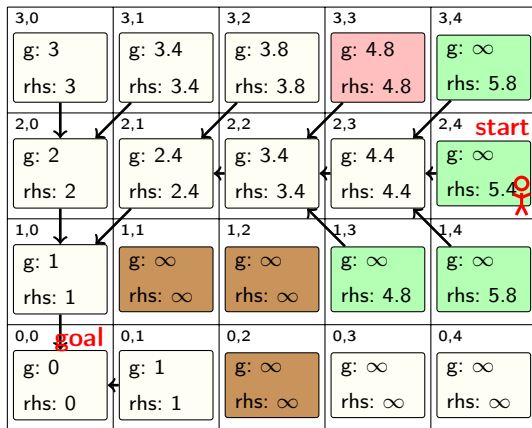
Legend

Free node	Obstacle node
On open list	Active node

ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list, i.e., (3,4), (2,4), (1,4)
- The start node is on the open list
- However, the search does not finish at this stage
- There are still inconsistent nodes (on the open list) with a lower value of *rhs*

D* Lite – Example Planning (19)



Legend

Free node

Obstacle node

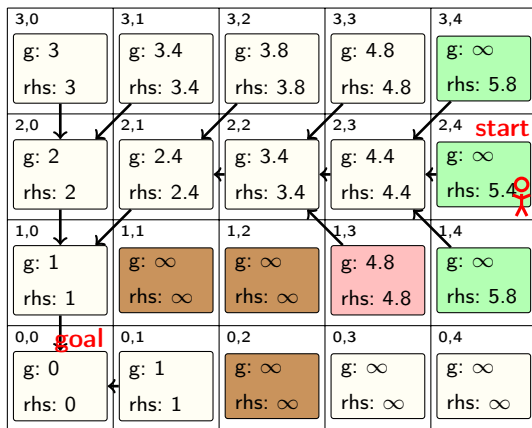
On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (3,2)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent onto the open list
- In this cases, none of the predecessors become inconsistent

D* Lite – Example Planning (20)



Legend

Free node

Obstacle node

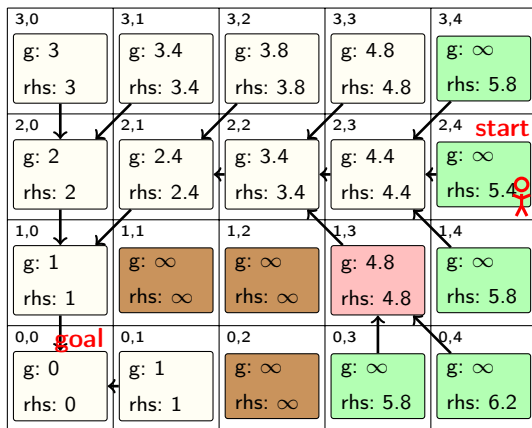
On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (1,3)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$

D* Lite – Example Planning (21)



Legend

Free node

Obstacle node

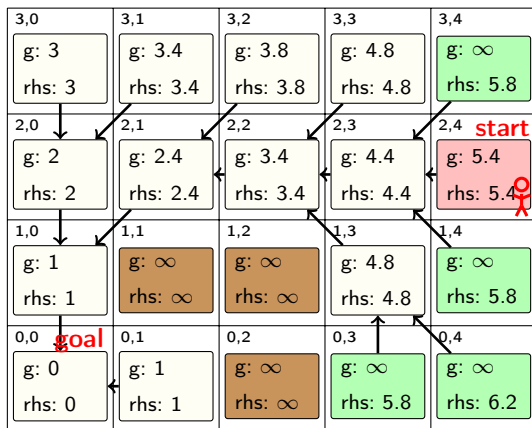
On open list

Active node

ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list, i.e., (0,3) and (0,4)

D* Lite – Example Planning (22)



Legend

Free node

Obstacle node

On open list

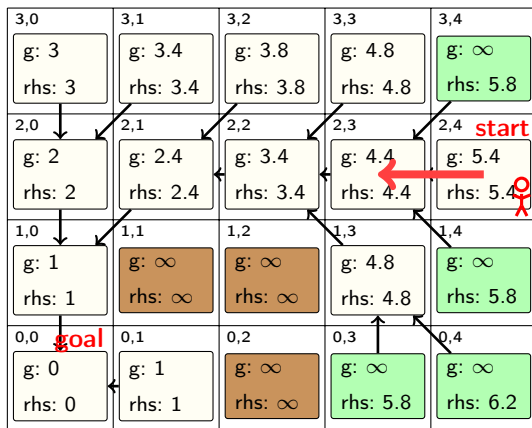
Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,4)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent (none in this case) onto the open list

- The **start** node becomes consistent and the top key on the open list is not less than the key of the start node
- An optimal path is found and the loop of the ComputeShortestPath is broken

D* Lite – Example Planning (23)



Legend

Free node

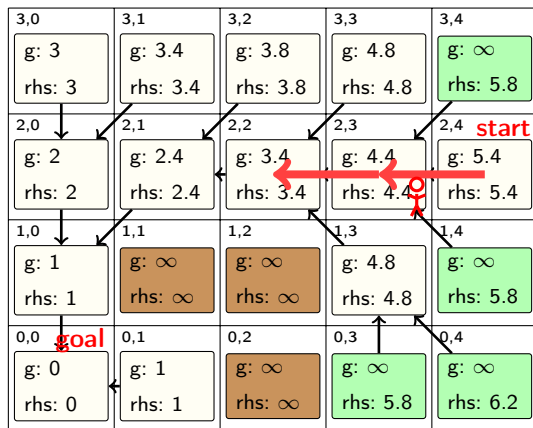
Obstacle node

On open list

Active node

- Follow the gradient of g values from the start node

D* Lite – Example Planning (24)



Legend

Free node

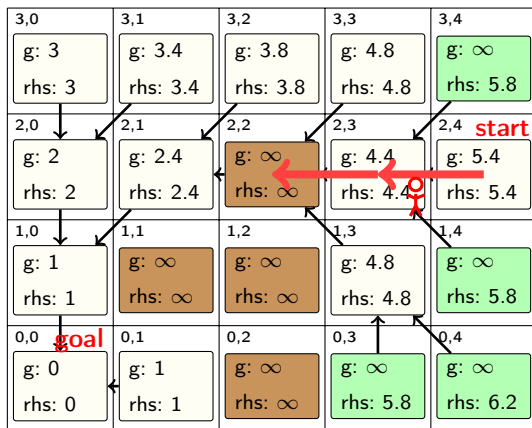
Obstacle node

On open list

Active node

- Follow the gradient of g values from the start node

D* Lite – Example Planning (25)



Legend

Free node

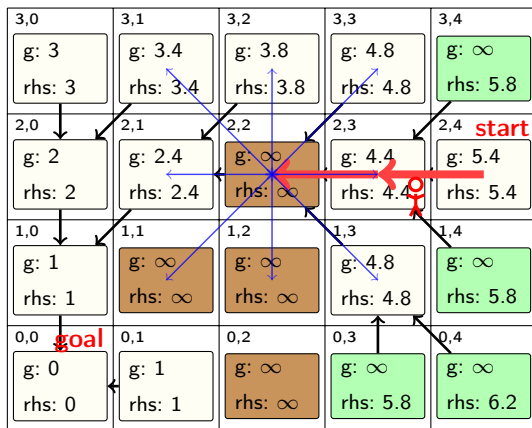
Obstacle node

On open list

Active node

- A new obstacle is detected during the movement from (2,3) to (2,2)
- **Replanning** is needed!

D* Lite – Example Planning (25 update)



Legend

Free node

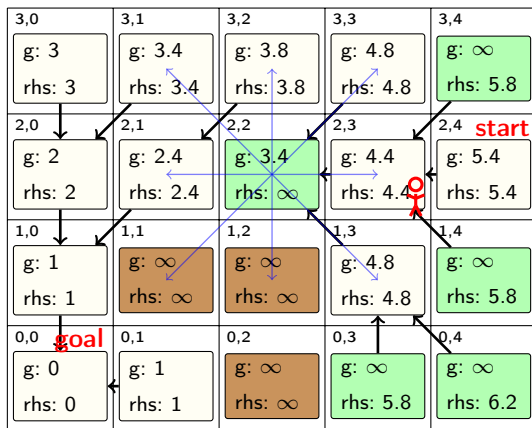
Obstacle node

On open list

Active node

- All directed edges with changed edge, we need to call the `UpdateVertex()`
- All edges into and out of (2,2) have to be considered

D* Lite – Example Planning (26 update 1/2)



Legend

Free node

Obstacle node

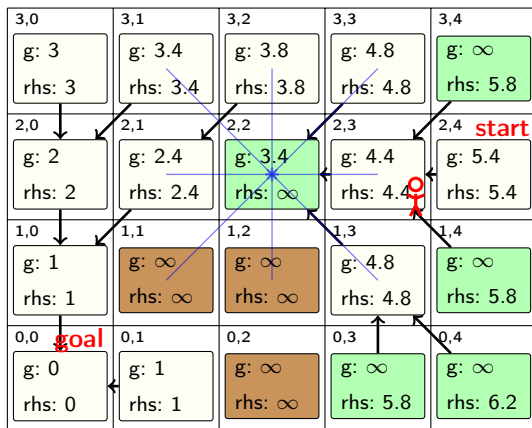
On open list

Active node

Update Vertex

- Outgoing edges from (2,2)
- Call UpdateVertex() on (2,2)
- The transition costs are now ∞ because of obstacle
- Therefore the $rhs = \infty$ and (2,2) becomes inconsistent and it is put on the open list

D* Lite – Example Planning (26 update 2/2)



Legend

Free node

Obstacle node

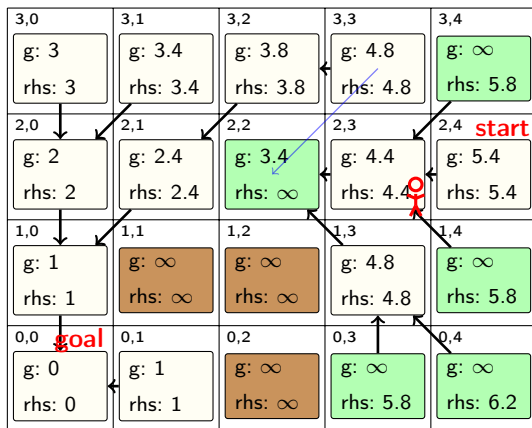
On open list

Active node

Update Vertex

- Incoming edges to (2,2)
- Call UpdateVertex() on the neighbors (2,2)
- The transition cost is ∞ , and therefore, the *rhs* value previously computed using (2,2) is changed

D* Lite – Example Planning (27)



Legend

Free node

Obstacle node

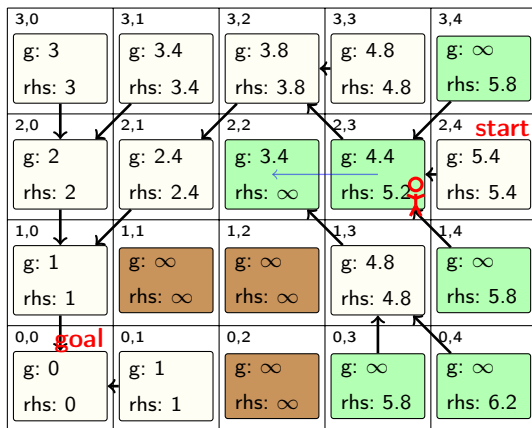
On open list

Active node

Update Vertex

- The neighbor of (2,2) is (3,3)
- The minimum possible rhs value of (3,3) is 4.8 but it is based on the g value of (3,2) and not (2,2), which is the detected obstacle
- The node (3,3) is still consistent and thus it is not put on the open list

D* Lite – Example Planning (28)



Legend

Free node

Obstacle node

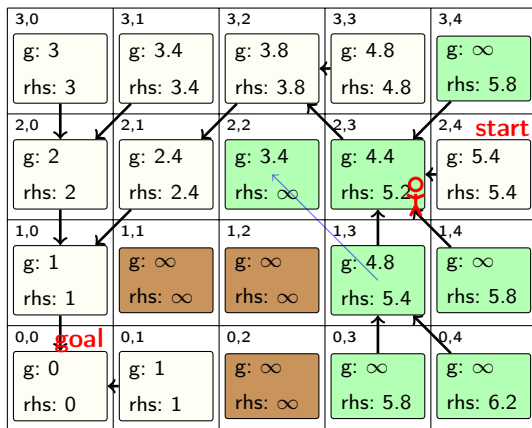
On open list

Active node

Update Vertex

- (2,3) is also a neighbor of (2,2)
- The minimum possible *rhs* value of (2,3) is 5.2 because of (2,2) is obstacle (using (3,2) with 3.8 + 1.4)
- The *rhs* value of (2,3) is different than *g* thus (2,3) is put on the open list

D* Lite – Example Planning (29)



Legend

Free node

Obstacle node

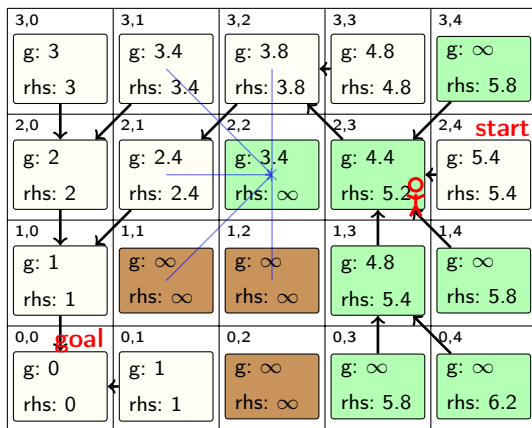
On open list

Active node

Update Vertex

- Another neighbor of (2,2) is (1,3)
- The minimum possible *rhs* value of (1,3) is 5.4 computed based on *g* of (2,3) with $4.4 + 1 = 5.4$
- The *rhs* value is always computed using the *g* values of its successors

D* Lite – Example Planning (29 update)



Legend

Free node

Obstacle node

On open list

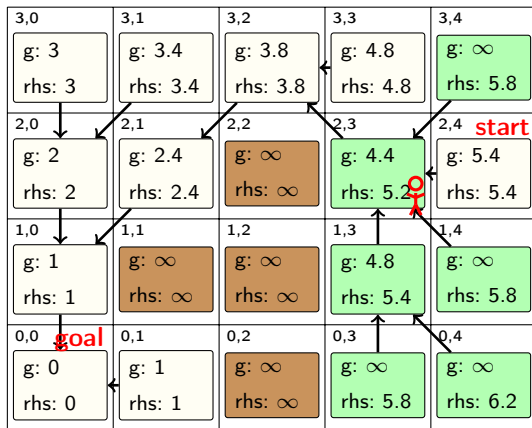
Active node

Update Vertex

- None of the other neighbors of (2,2) end up being inconsistent
- We go back to calling `ComputeShortestPath()` until an optimal path is determined

- The node corresponding to the robot's current position is inconsistent and its key is greater than the minimum key on the open list
- Thus, the optimal path is not found yet

D* Lite – Example Planning (30)



Legend

Free node

Obstacle node

On open list

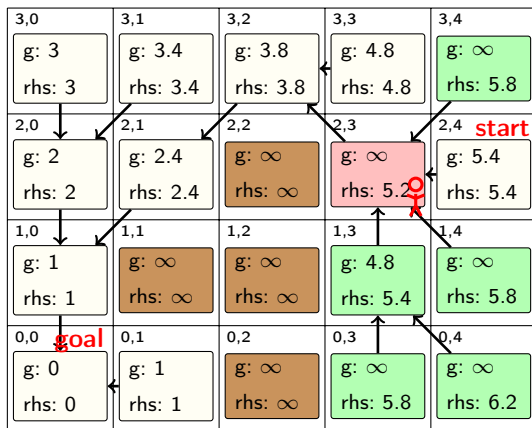
Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,2), which is obstacle
- It is under-consistent ($g < rhs$), therefore set $g = \infty$
- Expand the popped element and put the predecessors that become inconsistent (none in this case) onto the open list

- Because (2,2) was under-consistent (when popped), `UpdateVertex()` has to be called on it
- However, it has no effect as its *rhs* value is up to date and consistent

D* Lite – Example Planning (31)



Legend

Free node

Obstacle node

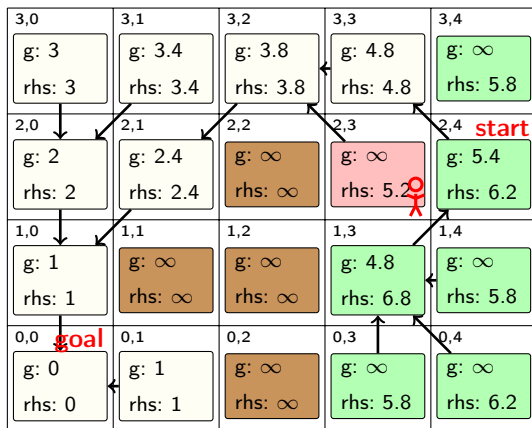
On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,3)
- It is under-consistent ($g < rhs$), therefore set $g = \infty$

D* Lite – Example Planning (32)



Legend

Free node

Obstacle node

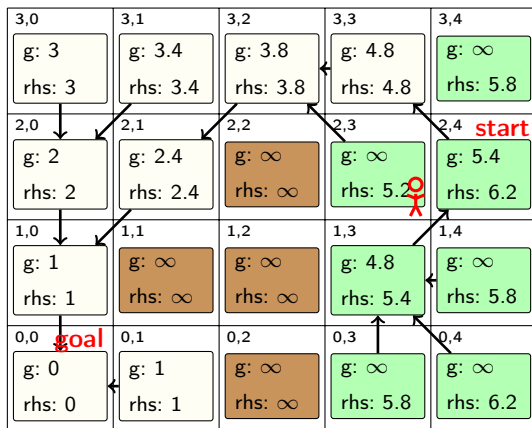
On open list

Active node

ComputeShortestPath

- Expand the popped element and update the predecessors
- (2,4) becomes inconsistent
- (1,3) gets updated and still inconsistent
- The *rhs* value (1,4) does not change, but it is now computed from the *g* value of (1,3)

D* Lite – Example Planning (33)



Legend

Free node

Obstacle node

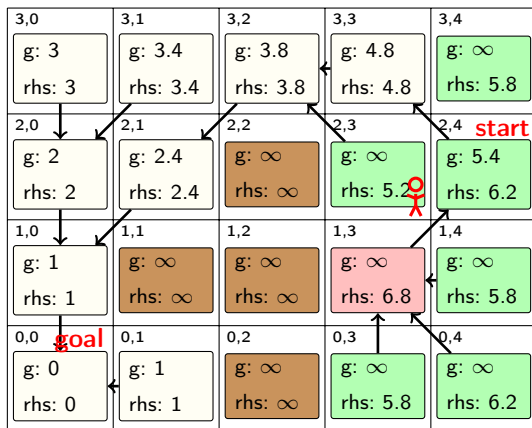
On open list

Active node

ComputeShortestPath

- Because (2,3) was under-consistent (when popped), call UpdateVertex() on it is needed
- As it is still inconsistent it is put back onto the open list

D* Lite – Example Planning (34)



Legend

Free node

Obstacle node

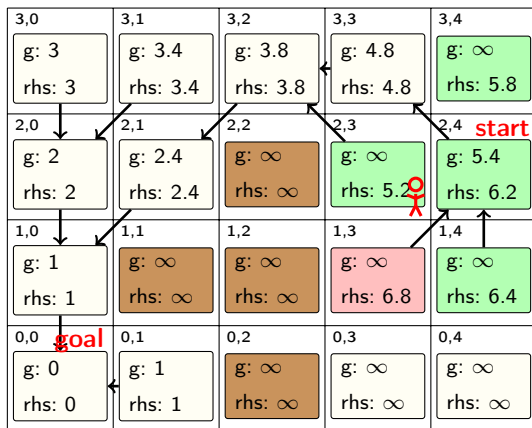
On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (1,3)
- It is under-consistent ($g < rhs$), therefore set $g = \infty$

D* Lite – Example Planning (35)



Legend

Free node

Obstacle node

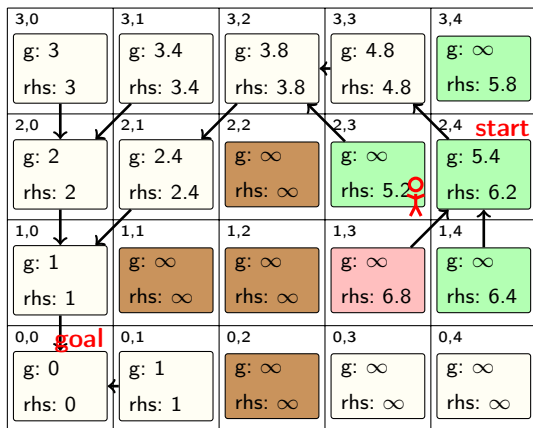
On open list

Active node

ComputeShortestPath

- Expand the popped element and update the predecessors
- (1,4) gets updated and still inconsistent
- (0,3) and (0,4) get updated and now consistent (both g and rhs are ∞)

D* Lite – Example Planning (36)



Legend

Free node

Obstacle node

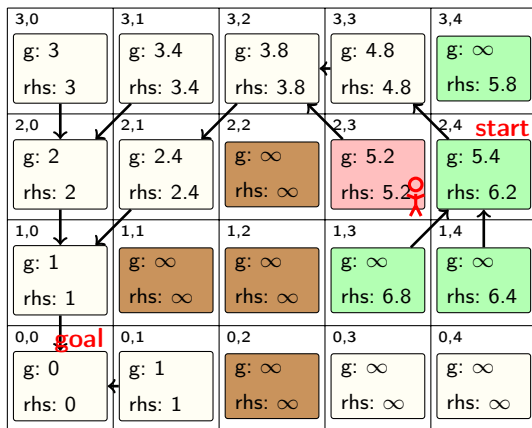
On open list

Active node

ComputeShortestPath

- Because (1,3) was under-consistent (when popped), call UpdateVertex() on it is needed
- As it is still inconsistent it is put back onto the open list

D* Lite – Example Planning (37)



Legend

Free node

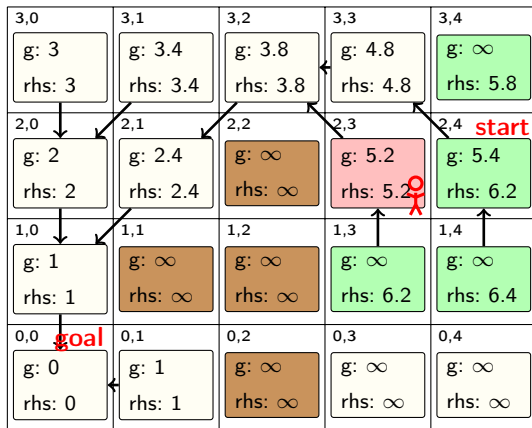
Obstacle node

On open list

Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,3)
- It is over-consistent ($g > rhs$), therefore set $g = rhs$

D^{*} Lite – Example Planning (38)

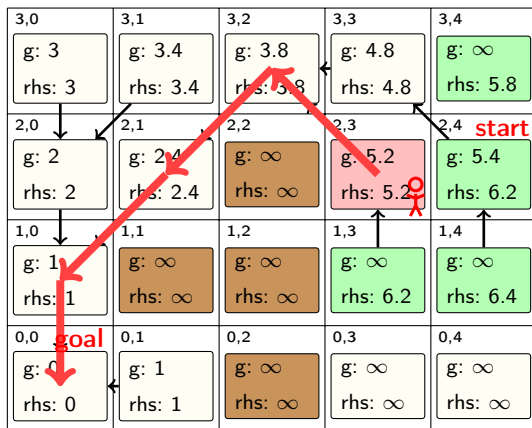
Legend

Free node	Obstacle node
On open list	Active node

ComputeShortestPath

- Expand the popped element and update the predecessors
- (1,3) gets updated and still inconsistent
- The node (2,3) corresponding to the robot's position is consistent
- Besides, top of the key on the open list is not less than the key of (2,3)
- The optimal path has been found and we can break out of the loop

D* Lite – Example Planning (39)



Legend

Free node

Obstacle node

On open list

Active node

- Follow the gradient of g values from the robot's current position (node)

D* Lite – Comments

- D* Lite works with real valued costs, not only with binary costs (free/obstacle)
- The search can be focused with an admissible heuristic that would be added to the g and rhs values
- The final version of D* Lite includes further optimization (not shown in the example)
 - Updating the rhs value without considering all successors every time
 - Re-focusing the search as the robot moves without reordering the entire open list

Reaction-Diffusion Processes Background

- *Reaction-Diffusion* (RD) models – dynamical systems capable to reproduce the autowaves
- *Autowaves* - a class of nonlinear waves that propagate through an active media

At the expense of the energy stored in the medium, e.g., grass combustion.

- RD model describes spatio-temporal evolution of two state variables $u = u(\vec{x}, t)$ and $v = v(\vec{x}, t)$ in space \vec{x} and time t

$$\begin{aligned}\dot{u} &= f(u, v) + D_u \Delta u \\ \dot{v} &= g(u, v) + D_v \Delta v \end{aligned}$$

where Δ is the Laplacian.

This RD-based path planning is informative, just for curiosity

Reaction-Diffusion Background

- FitzHugh-Nagumo (FHN) model

FitzHugh R, Biophysical Journal (1961)

$$\begin{aligned}\dot{u} &= \varepsilon (u - u^3 - v + \phi) + D_u \Delta u \\ \dot{v} &= (u - \alpha v + \beta) + D_v \Delta u\end{aligned},$$

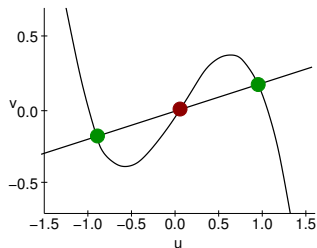
where $\alpha, \beta, \varepsilon$, and ϕ are parameters of the model.

- Dynamics of RD system is determined by the associated *nullcline configurations* for $\dot{u}=0$ and $\dot{v}=0$ in the absence of diffusion, i.e.,

$$\begin{aligned}\varepsilon (u - u^3 - v + \phi) &= 0, \\ (u - \alpha v + \beta) &= 0,\end{aligned}$$

which have associated geometrical shapes

Nullcline Configurations and Steady States



- Nullclines intersections represent

- Stable States (SSs)
- Unstable States

- Bistable regime

The system (concentration levels of (u, v) for each grid cell) tends to be in SSs.

- We can modulate relative stability of both SS

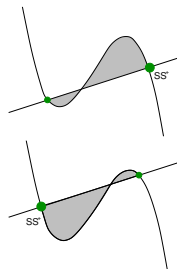
“preference” of SS^+ over SS^-

- System moves from SS^- to SS^+ ,

if a small perturbation is introduced.

- The SSs are separated by a mobile frontier

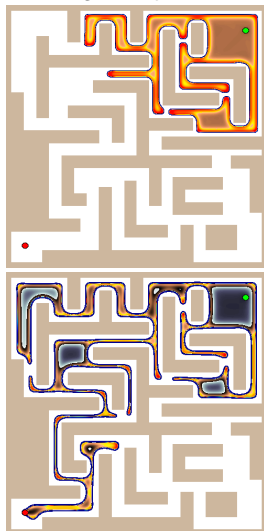
a kind of traveling frontwave (autowaves)



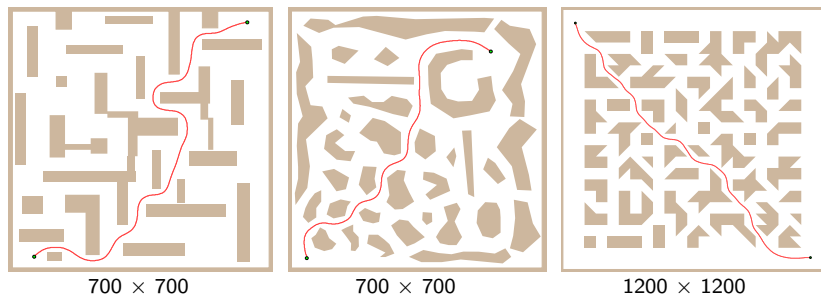
RD-based Path Planning – Computational Model

- Finite difference method on a Cartesian grid with Dirichlet boundary conditions (FTCS) *discretization* → *grid based computation* → *grid map*
- *External forcing* – introducing additional information *i.e., constraining concentration levels to some specific values*
- Two-phase evolution of the underlying RD model
 1. **Propagation phase**
 - Freespace is set to SS^- and the start location SS^+
 - Parallel propagation of the frontwave with *non-annihilation property*

Vázquez-Otero and Muñozuri, CNNA (2010)
 - Terminate when the frontwave reaches the goal
 2. **Contraction phase**
 - Different nullclines configuration
 - Start and goal positions are forced towards SS^+
 - SS^- shrinks until only the path linking the forced points remains



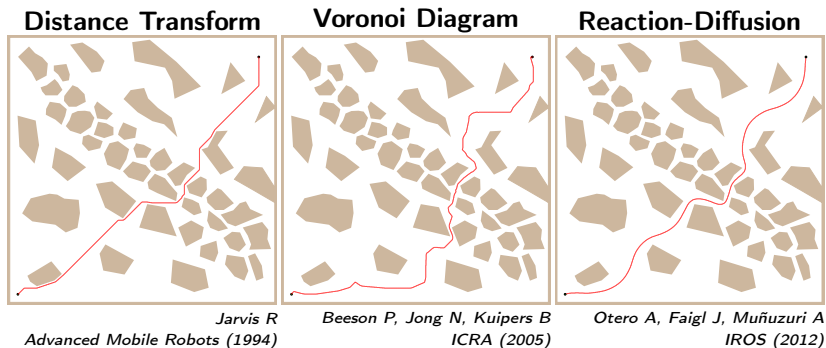
Example of Found Paths



- The path clearance may be adjusted by the **wavelength** and size of the computational grid.

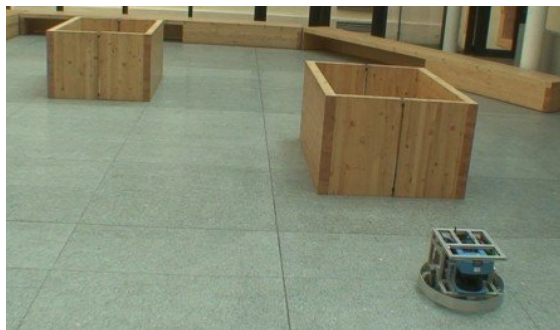
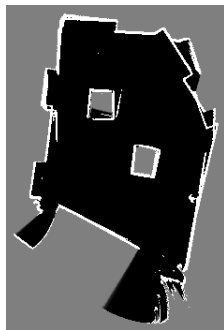
Control of the path distance from the obstacles (path safety)

Comparison with Standard Approaches



- RD-based approach provides competitive paths regarding path length and clearance, while they seem to be smooth

Robustness to Noisy Data



Vázquez-Otero, A., Faigl, J., Duro, N. and Dormido, R. (2014): Reaction-Diffusion based Computational Model for Autonomous Mobile Robot Exploration of Unknown Environments. *International Journal of Unconventional Computing (IJUC)*.

Summary of the Lecture

Topics Discussed

- Front-Wave propagation and path simplification
- Distance Transform based planning
- Graph based planning methods: Dijkstra's, A*, JPS, Theta*
- D* Lite
- Reaction-Diffusion based planning (*informative*)

- **Next: Randomized Sampling-based Motion Planning Methods**