Grid and Graph based Path Planning Methods

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Lecture 04

B4M36UIR – Artificial Intelligence in Robotics

Overview of the Lecture

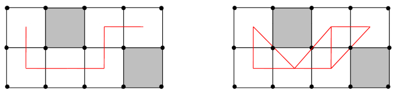
- Part 1 – Grid and Graph based Path Planning Methods
  - Grid-based Planning
  - DT for Path Planning
  - Graph Search Algorithms
  - D* Lite
  - Path Planning based on Reaction-Diffusion Process *Curiosity*

Part I

Part 1 – Grid and Graph based Path Planning Methods

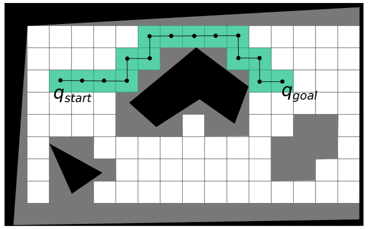
Grid-based Planning

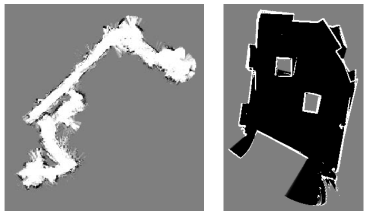
- A subdivision of $\mathcal{C}_{free}$ into smaller cells
- **Grow obstacles** can be simplified by growing borders by a diameter of the robot
- Construction of the planning graph $G = (V, E)$ for V as a set of cells and E as the **neighbor-relations**
  - 4-neighbors and 8-neighbors



- A grid map can be constructed from the so-called occupancy grid maps

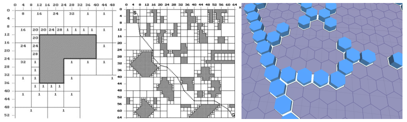
E.g., using thresholding





Grid-based Environment Representations

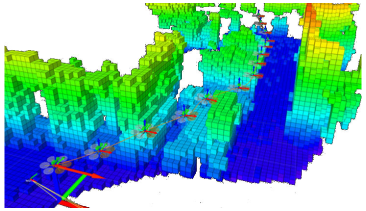
- Hiearchical planning
  - Coarse resolution and re-planning on finer resolution

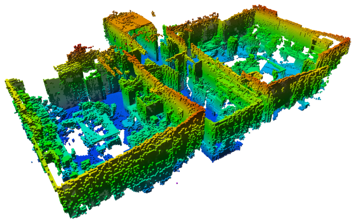


Holte, R. C. et al. (1996): Hierarchical A *: searching abstraction hierarchies efficiently. AAAI.

- Octree can be used for the map representation
- In addition to squared (or rectangular) grid a hexagonal grid can be used
- 3D grid maps – **octomap**

https://octomap.github.io



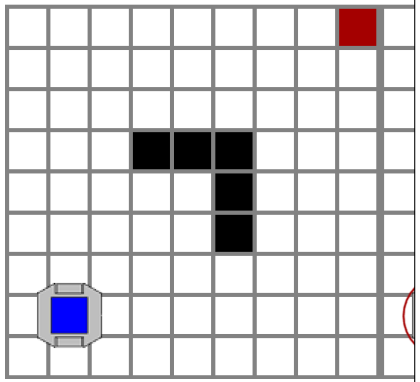


– Memory grows with the size of the environment

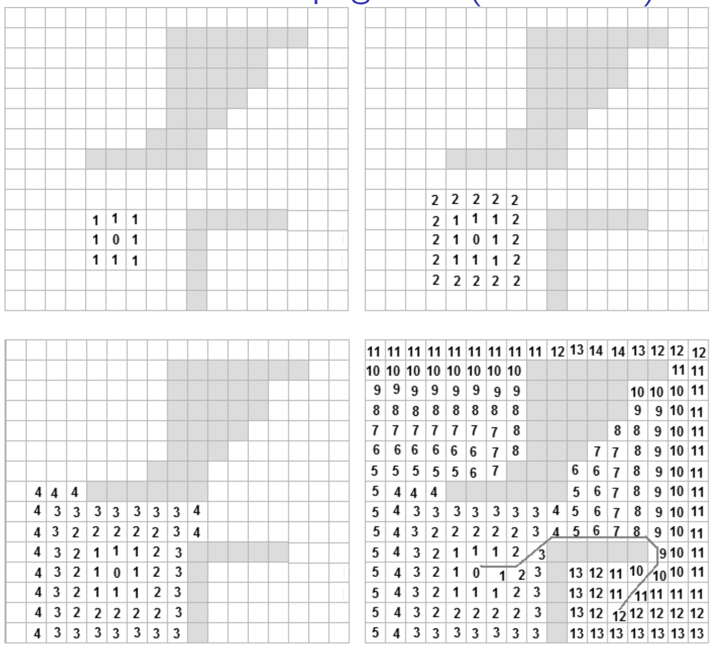
– Due to limited resolution it may fail in narrow passages of $\mathcal{C}_{free}$

Example of Simple Grid-based Planning

- Wave-front propagation using path simplification
- Initial map with a robot and goal
- Obstacle growing
- Wave-front propagation – "flood fill"
- Find a path using a navigation function
- Path simplification
  - "Ray-shooting" technique combined with **Bresenham's line algorithm**
  - The path is a sequence of "key" cells for avoiding obstacles

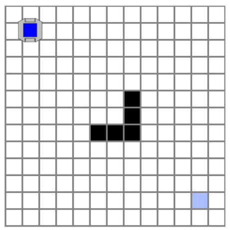


Example – Wave-Front Propagation (Flood Fill)

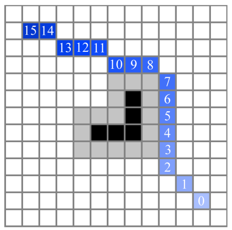


Path Simplification

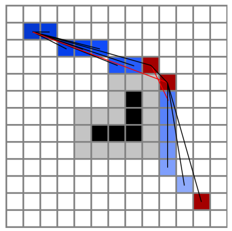
- The initial path is found in a grid using 4-neighborhood
- The rayshoot cast a line into a grid and possible collisions of the robot with obstacles are checked
- The "farthest" cells without collisions are used as "turn" points
- The final path is a sequence of straight line segments



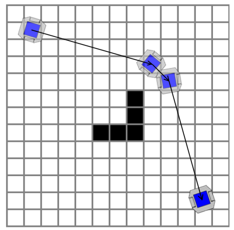
Initial and goal locations



Obtacle growing, wave-front propagation



Ray-shooting



Simplified path

Bresenham's Line Algorithm

- Filling a grid by a line with avoding float numbers
- A line from (x_0, y_0) to (x_1, y_1) is given by $y = \frac{y_1 - y_0}{x_1 - x_0}(x - x_0) + y_0$

```
CoordsVector& bresenham(const Coords& pt1, const
      Coords& pt2, CoordsVector& line)
{
   // The pt2 point is not added into line
   int x0 = pt1.c; int y0 = pt1.r;
   int x1 = pt2.c; int y1 = pt2.r;
   Coords p;
   int dx = x1 - x0;
   int dy = y1 - y0;
   int steep = (abs(dy) >= abs(dx));
   if (steep) {
      SWAP(x0, y0);
      SWAP(x1, y1);
      dx = x1 - x0; // recompute Dx, Dy
      dy = y1 - y0;
   }
   int xstep = 1;
   if (dx < 0) {
      xstep = -1;
      dx = -dx;
   }
   int ystep = 1;
   if (dy < 0) {
      ystep = -1;
      dy = -dy;
   }
   int twoDy = 2 * dy;
   int twoDyTwoDx = twoDy - 2 * dx; //2*Dy - 2*Dx
   int e = twoDy - dx; //2*Dy - Dx
   int y = y0;
   int xDraw, yDraw;
   for (int x = x0; x != x1; x += xstep) {
      if (steep) {
         xDraw = y;
         yDraw = x;
      } else {
         xDraw = x;
         yDraw = y;
      }
      p.c = xDraw;
      p.r = yDraw;
      line.push_back(p); // add to the line
      if (e > 0) {
         e += twoDyTwoDx; //E += 2*Dy - 2*Dx
         y = y + ystep;
      } else {
         e += twoDy; //E += 2*Dy
      }
   }
   return line;
}
```


Distance Transform based Path Planning

- For a given goal location and grid map compute a navigational function using *wave-front* algorithm, i.e., a kind of *potential field*
  - The value of the goal cell is set to 0 and all other free cells are set to some very high value
  - For each free cell compute a number of cells towards the goal cell
  - It uses 8-neighbors and distance is the Euclidean distance of the centers of two cells, i.e., EV=1 for orthogonal cells or $EV = \sqrt{2}$ for diagonal cells
  - The values are iteratively computed until the values are changing
  - The value of the cell c is computed as

$$cost(c) = \min_{i=1}^{8} \left(cost(c_i) + EV_{c_i,c} \right),$$

  where c_i is one of the neighboring cells from 8-neighborhood of the cell c

- The algorithm provides a cost map of the path distance from any free cell to the goal cell
- The path is then used following the gradient of the cell cost

Jarvis, R. (2004): Distance Transform Based Visibility Measures for Covert Path Planning in Known but Dynamic Environments

Distance Transform Path Planning

Algorithm 1: Distance Transform for Path Planning

```
for y := 0 to yMax do
    for x := 0 to xMax do
        if goal [x,y] then
            cell [x,y] := 0;
        else
            cell [x,y] := xMax * yMax; //initialization, e.g., pragmatic of the use longest distance as ∞ ;
repeat
    for y := 1 to (yMax - 1) do
        for x := 1 to (xMax - 1) do
            if not blocked [x,y] then
                cell [x,y] := cost(x, y);
    for y := (yMax-1) downto 1 do
        for x := (xMax-1) downto 1 do
            if not blocked [x,y] then
                cell[x,y] := cost(x, y);
until no change;
```


Distance Transform based Path Planning – Impl. 1/2

```
Grid& DT::compute(Grid& grid) const
{
   static const double DIAGONAL = sqrt(2);
   static const double ORTOGONAL = 1;
   const int H = map.H;
   const int W = map.W;
   assert(grid.H == H and grid.W == W, "size");
   bool anyChange = true;
   int counter = 0;
   while (anyChange) {
      anyChange = false;
      for (int r = 1; r < H - 1; ++r) {
         for (int c = 1; c < W - 1; ++c) {
            if (map[r][c] != FREESPACE) {
               continue;
            } //obstacle detected
            double t[4];
            t[0] = grid[r - 1][c - 1] + DIAGONAL;
            t[1] = grid[r - 1][c] + ORTOGONAL;
            t[2] = grid[r - 1][c + 1] + DIAGONAL;
            t[3] = grid[r][c - 1] + ORTOGONAL;
            double pom = grid[r][c];
            for (int i = 0; i < 4; i++) {
               if (pom > t[i]) {
                  pom = t[i];
                  anyChange = true;
               }
            }
            if (anyChange) {
               grid[r][c] = pom;
            }
         }
      }
      for (int r = H - 2; r > 0; --r) {
         for (int c = W - 2; c > 0; --c) {
            if (map[r][c] != FREESPACE) {
               continue;
            } //obstacle detected
            double t[4];
            t[1] = grid[r + 1][c] + ORTOGONAL;
            t[0] = grid[r + 1][c + 1] + DIAGONAL;
            t[3] = grid[r][c + 1] + ORTOGONAL;
            t[2] = grid[r + 1][c - 1] + DIAGONAL;
            double pom = grid[r][c];
            bool s = false;
            for (int i = 0; i < 4; i++) {
               if (pom > t[i]) {
                  pom = t[i];
                  s = true;
               }
            }
            if (s) {
               anyChange = true;
               grid[r][c] = pom;
            }
         }
      }
      counter++;
   } //end while any change
   return grid;
}
```

A boundary is assumed around the rectangular map

Distance Transform based Path Planning – Impl. 2/2

- The path is retrived by following the minimal value towards the goal using min8Point()

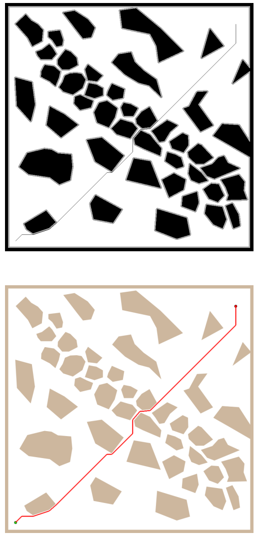
```
Coords& min8Point(const Grid& grid, Coords& p)
{
   double min = std::numeric_limits<double>::max();
   const int H = grid.H;
   const int W = grid.W;
   Coords t;

   for (int r = p.r - 1; r <= p.r + 1; r++) {
      if (r < 0 or r >= H) { continue; }
      for (int c = p.c - 1; c <= p.c + 1; c++) {
         if (c < 0 or c >= W) { continue; }
         if (min > grid[r][c]) {
            min = grid[r][c];
            t.r = r; t.c = c;
         }
      }
   }
   p = t;
   return p;
}
```

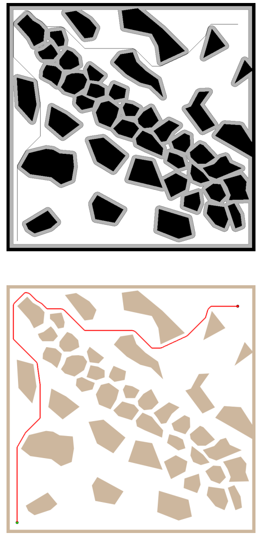
```
CoordsVector& DT::findPath(const Coords& start,
      const Coords& goal, CoordsVector& path)
{
   static const double DIAGONAL = sqrt(2);
   static const double ORTOGONAL = 1;
   const int H = map.H;
   const int W = map.W;
   Grid grid(H, W, H*W); // H*W max grid value
   grid[goal.r][goal.c] = 0;
   compute(grid);

   if (grid[start.r][start.c] >= H*W) {
      WARN("Path has not been found");
   } else {
      Coords pt = start;
      while (pt.r != goal.r or pt.c != goal.c) {
         path.push_back(pt);
         min8Point(grid, pt);
      }
      path.push_back(goal);
   }
   return path;
}
```


DT Example



$\delta = 10$ cm, $L = 27.2$ m



$\delta = 30$ cm, $L = 42.8$ m

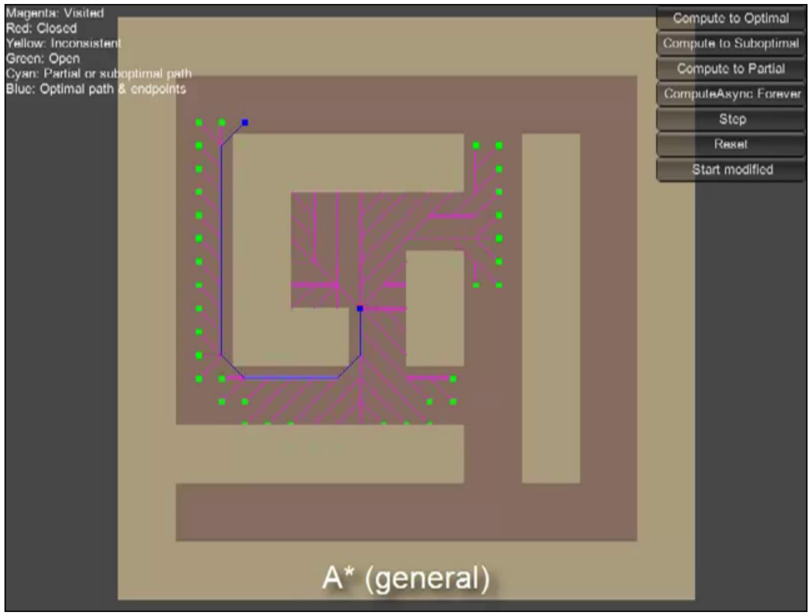
Graph Search Algorithms

- The grid can be considered as a graph and the path can be found using graph search algorithms
- The search algorithms working on a graph are of general use, e.g.
  - Breadth-first search (BSD)
  - Depth first search (DFS)
  - Dijsktra's algorithm,
  - A* algorithm and its variants
- There can be grid based speedups techniques, e.g.,
  - Jump Search Algorithm (JPS) and JPS+
- There are many search algorithm for on-line search, incremental search and with any-time and real-time properties, e.g.,
  - Lifelong Planning A* (LPA*)

    Koenig, S., Likhachev, M. and Furcy, D. (2004): Lifelong Planning A*. AIJ.
  - E-Graphs – Experience graphs

    Phillips, M. et al. (2012): E-Graphs: Bootstrapping Planning with Experience Graphs. RSS.

Examples of Graph/Grid Search Algorithms



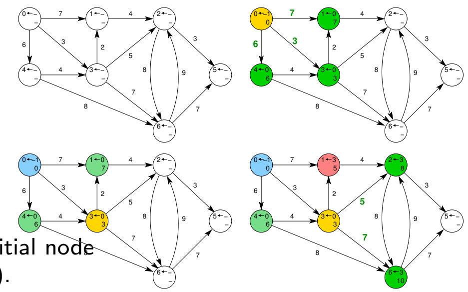
https://www.youtube.com/watch?v=U2XNjCoKZjM.mp4

Dijkstra's Algorithm

- Dijsktra's algorithm determines paths as iterative update of the cost of the shortest path to the particular nodes

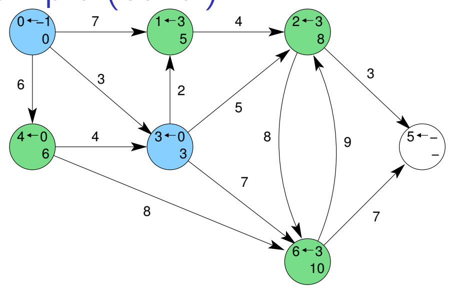
Edsger W. Dijkstra, 1956

- Let start with the initial cell (node) with the cost set to 0 and update all successors
- Select the node
  - with a path from the initial node
  - and has a lower cost
- Repeat until there is a reachable node
  - I.e., a node with a path from the initial node
  - has a cost and parent (*green nodes*).

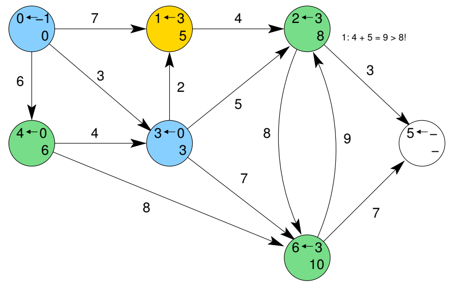


The cost of nodes can only decrease (edge cost is positive). Therefore, for a node with the currently lowest cost, there cannot be a shorter path from the initial node.

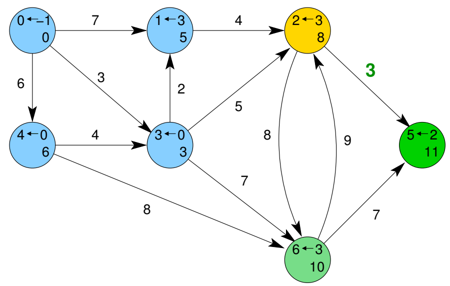
Example (cont.)



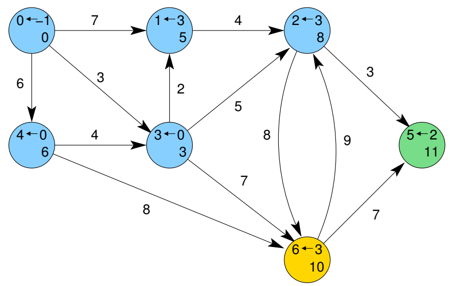
1: After the expansion, the shortest path to the node 2 is over the node 3



2: There is not shorter path to the node 2 over the node 1



3: After the expansion, there is a new path to the node 5



4: The path does not improve for further expansions

Dijkstra's Algorithm

Algorithm 2: Dijkstra's algorithm

Initialize(s_{start}); /* $g(s) := \infty$; $g(s_{start}) := 0$ */
PQ.push(s_{start}, $g(s_{start})$);
while *(not PQ.empty?)* **do**
  $s :=$ PQ.pop();
  foreach $s' \in Succ(s)$ **do**
    if s' *in PQ* **then**
      if $g(s') > g(s) + cost(s, s')$ **then**
        $g(s') := g(s) + cost(s, s')$;
        PQ.update(s', $g(s')$);
    else if $s' \notin CLOSED$ **then**
      $g(s') := g(s) + cost(s, s')$;
      PQ.push(s', $g(s')$);
  CLOSED := CLOSED $\bigcup \{s\}$;

Dijkstra's Algorithm – Impl.

```
dij->nodes[dij->start_node].cost = 0; // init
void *pq = pq_alloc(dij->num_nodes); // set priority queue
int cur_label;
pq_push(pq, dij->start_node, 0);
while ( !pq_is_empty(pq) && pq_pop(pq, &cur_label)) {
   node_t *cur = &(dij->nodes[cur_label]); // remember the current node
   for (int i = 0; i < cur->edge_count; ++i) { // all edges of cur
      edge_t *edge = &(dij->graph->edges[cur->edge_start + i]);
      node_t *to = &(dij->nodes[edge->to]);
      const int cost = cur->cost + edge->cost;
      if (to->cost == -1) { // node to has not been visited
         to->cost = cost;
         to->parent = cur_label;
         pq_push(pq, edge->to, cost); // put node to the queue
      } else if (cost < to->cost) { // node already in the queue
         to->cost = cost; // test if the cost can be reduced
         to->parent = cur_label; // update the parent node
         pq_update(pq, edge->to, cost); // update the priority queue
      }
   } // loop for all edges of the cur node
} // priority queue empty
pq_free(pq); // release memory
```

A* Algorithm

- A* uses a user-defined h-values (heuristic) to focus the search

  Peter Hart, Nils Nilsson, and Bertram Raphael, 1968

  - Prefer expansion of the node n with the lowest value

    $$f(n) = g(n) + h(n),$$

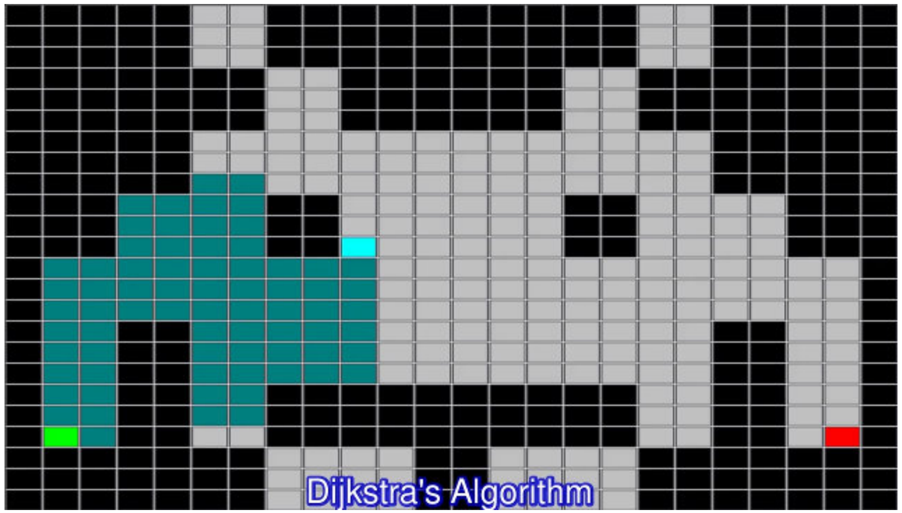
    where $g(n)$ is the cost (path length) from the start to n and $h(n)$ is the estimated cost from n to the goal
- h-values approximate the goal distance from particular nodes
- **Admissiblity condition** – heuristic always underestimate the remaining cost to reach the goal
  - Let $h^*(n)$ be the true cost of the optimal path from n to the goal
  - Then $h(n)$ is **admissible** if for all n: $h(n) \leq h^*(n)$
  - E.g., Euclidean distance is admissible
    - A straight line will always be the shortest path
- Dijkstra's algorithm – $h(n) = 0$

A* Implementation Notes

- The most costly operations of A* are
  - Insert and lookup an element in the **closed list**
  - Insert element and get minimal element (according to $f()$ value) from the **open list**
- The **closed list** can be efficiently implemented as a **hash set**
- The **open list** is usually implemented as a **priority queue**, e.g.,
  - Fibonacii heap, binomial heap, k-level bucket
  - **binary heap** is usually sufficient ($O(logn)$)
- Forward A*
  1. Create a search tree and initiate it with the start location
  2. Select generated but not yet expanded state s with the smallest f-value, $f(s) = g(s) + h(s)$
  3. `Stop` if s is the goal
  4. Expand the state s
  5. `Goto` Step 2

Similar to Dijkstra's algorithm but it used $f(s)$ with heuristic $h(s)$ instead of pure $g(s)$

Dijsktra's vs A* vs Jump Point Search (JPS)



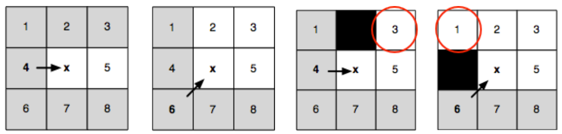
https://www.youtube.com/watch?v=ROG4Ud08lLY

Jump Point Search Algorithm for Grid-based Path Planning

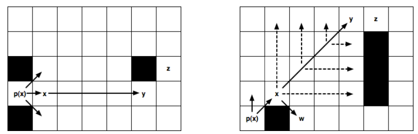
- **Jump Point Search** (JPS) algorithm is based on a macro operator that identifies and selectively expands only certain nodes (**jump points**)

  Harabor, D. and Grastien, A. (2011): Online Graph Pruning for Pathfinding on Grid Maps. AAAI.

- Natural neighbors after neighbor prunning with forced neighbors because of obstacle



- Intermediate nodes on a path connecting two jump points are never expanded



- No preprocessing and no memory overheads while it speeds up A*

  https://harablog.wordpress.com/2011/09/07/jump-point-search/

- JPS+ – optimized preprocessed version of **JPS** with goal bounding

  https://github.com/SteveRabin/JPSPlusWithGoalBounding

  http://www.gdcvault.com/play/1022094/JPS-Over-100x-Faster-than

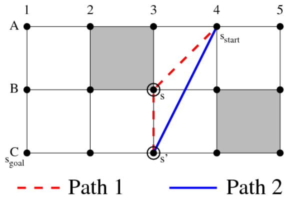
Theta* – Any-Angle Path Planning Algorithm

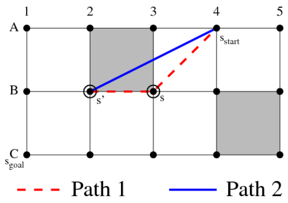
- **Any-angle path planning algorithms** simplify the path during the search
- **Theta*** is an extension of A* with `LineOfSight()`

Nash, A., Daniel, K, Koenig, S. and Felner, A. (2007): Theta*: Any-Angle Path Planning on Grids. AAAI.

Algorithm 3: Theta* Any-Angle Planning

```
if LineOfSight(parent(s), s') then
    /* Path 2 – any-angle path */
    if g(parent(s))+ c(parent(s), s') < g(s') then
        parent(s') := parent(s);
        g(s') := g(parent(s)) + c(parent(s), s');
else
    /* Path 1 – A* path */
    if g(s) + c(s,s') < g(s') then
        parent(s'):= s;
        g(s') := g(s) + c(s,s');
```





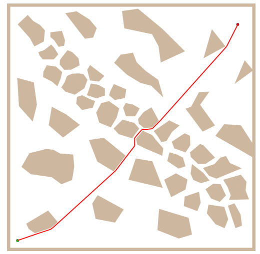
- Path 2: considers path from start to parent(s) and from parent(s) to s' if s' has line-of-sight to parent(s)

http://aigamedev.com/open/tutorials/theta-star-any-angle-paths/

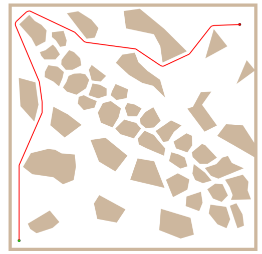
Theta* Any-Angle Path Planning Examples

- Example of found paths by the Theta* algorithm for the same problems as for the DT-based examples on Slide 16

Both algorithms implemented in C++



$\delta = 10$ cm, $L = 26.3$ m



$\delta = 30$ cm, $L = 40.3$ m

The same path planning problems solved by DT (without path smoothing) have $L_{\delta=10} = 27.2$ m and $L_{\delta=30} = 42.8$ m, while DT seems to be significantly faster

- **Lazy Theta*** – reduces the number of line-of-sight checks

Nash, A., Koenig, S. and Tovey, C. (2010): Lazy Theta*: Any-Angle Path Planning and Path Length Analysis in 3D. AAAI.

http://aigamedev.com/open/tutorial/lazy-theta-star/

A* Variants – Online Search

- The state space (map) may not be known exactly in advance
  - Environment can **dynamically** change
  - True travel costs are **experienced** during the path execution
- Repeated A* searches can be computationally demanding
- **Incremental heuristic search**
  - Repeated planning of the path from the current state to the goal
  - Planning under the **free-space** assumption
  - **Reuse** information from the previous searches (**closed list** entries):
    - Focused Dynamic A* (**D***) – h^* is based on **traversability**, it has been used, e.g., for the Mars rover "Opportunity"

      Stentz, A. (1995): The Focussed D* Algorithm for Real-Time Replanning. IJCAI.
    - **D* Lite** – similar to D*

      Koenig, S. and Likhachev, M. (2005): Fast Replanning for Navigation in Unknown Terrain. T-RO.
- **Real-Time Heuristic Search**
  - Repeated planning with limited **look-ahead** – suboptimal but fast
    - Learning Real-Time A* (**LRTA***)

      Korf, E. (1990): Real-time heuristic search. JAI
    - Real-Time Adaptive A* (**RTAA***)

      Koenig, S. and Likhachev, M. (2006): Real-time adaptive A*. AAMAS.

Real-Time Adaptive A* (RTAA*)

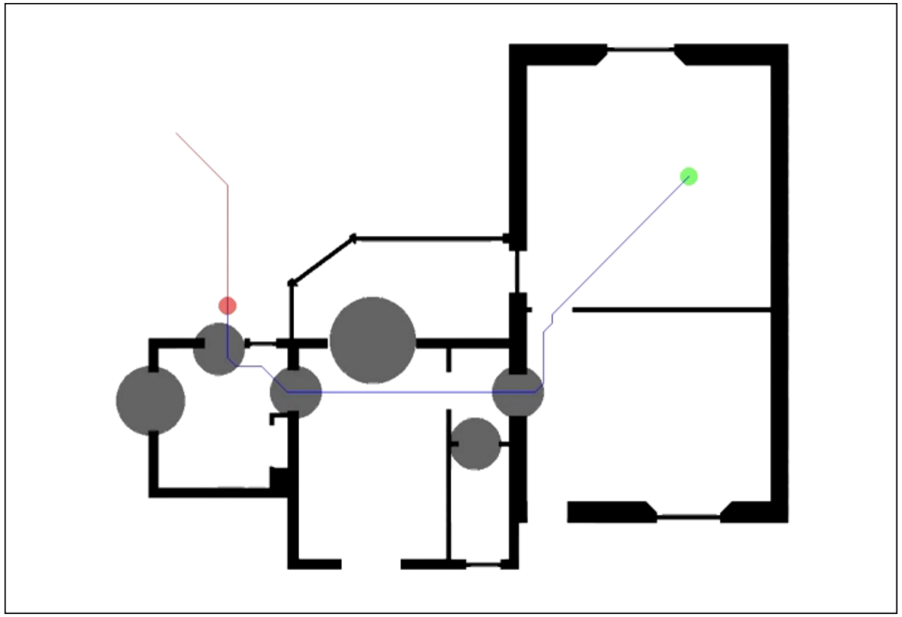
- Execute A* with limited **look-ahead**
- Learns better informed **heuristic** from the experience, initially $h(s)$, e.g., Euclidean distance
- Look-ahead defines **trade-off** between optimality and computational cost
  - `astar(lookahead)`

  A* expansion as far as "lookahead" nodes and it terminates with the state s'

```
while (s_curr ∉ GOAL) do
    astar(lookahead);
    if s' = FAILURE then
        return FAILURE;
    for all s ∈ CLOSED do
        H(s) := g(s') + h(s') - g(s);
    execute(plan); // perform one step
return SUCCESS;
```

s' is the last state expanded during the previous A* search

D* Lite – Demo



https://www.youtube.com/watch?v=X5a149nSE9s

D* Lite Overview

- It is similar to D*, but it is based on **Lifelong Planning A***

  Koenig, S. and Likhachev, M. (2002): D* Lite. AAAI.
- It searches from the goal node to the start node, i.e., g-values estimate the goal distance
- Store pending nodes in a priority queue
- Process nodes in order of increasing objective function value
- Incrementally repair solution paths when changes occur
- Maintains two estimates of costs per node
  - g – the objective function value – based on what we know
  - rhs – one-step lookahead of the objective function value – based on what we know
- **Consistency**
  - Consistent – $g = rhs$
  - Inconsistent – $g \neq rhs$
- Inconsistent nodes are stored in the priority queue (open list) for processing

D* Lite: Cost Estimates

- rhs of the node u is computed based on g of its successors in the graph and the transition costs of the edge to those successors

$$rhs(u) = \min_{s' \in Succ(u)} (g(s') + c(u, s'))$$

- The key/priority of a node s on the open list is the minimum of $g(s)$ and $rhs(s)$ plus a focusing heuristic h

$$[\min(g(s), rhs(s)) + h(s_{start}, s); \min(g(s), rhs(s))]$$

  - The first term is used as the primary key
  - The second term is used as the secondary key for tie-breaking

D* Lite Algorithm

- **Main** – repeat until the robot reaches the goal (or $g(s_{start}) = \infty$ there is no path)

Initialize();
ComputeShortestPath();
while ($s_{start} \neq s_{goal}$) **do**
  $s_{start} = \text{argmin}_{s' \in Succ(s_{start})}(c(s_{start}, s') + g(s'))$;
  Move to s_{start};
  Scan the graph for changed edge costs;
  if *any edge cost changed perform* **then**
    foreach *directed edges* (u, v) *with changed edge costs* **do**
      Update the edge cost $c(u, v)$;
      UpdateVertex(u);
    foreach $s \in U$ **do**
      U.Update(s, CalculateKey(s));
    ComputeShortestPath();

Procedure Initialize
U = 0;
foreach $s \in S$ **do**
  $rhs(s) := g(s) := \infty$;
$rhs(s_{goal}) := 0$;
U.Insert(s_{goal}, CalculateKey(s_{goal}));

D* Lite Algorithm – ComputeShortestPath()

Procedure ComputeShortestPath

```
while U.TopKey() < CalculateKey(s_start) OR rhs(s_start) ≠ g(s_start) do
    u := U.Pop();
    if g(u) > rhs(u) then
        g(u) := rhs(u);
        foreach s ∈ Pred(u) do UpdateVertex(s);
    else
        g(u) := ∞;
        foreach s ∈ Pred(u) ⋃ {u} do UpdateVertex(s);
```

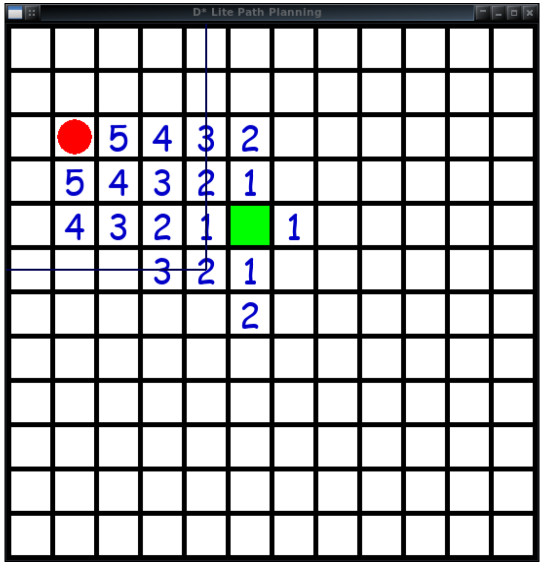
Procedure UpdateVertex

```
if u ≠ s_goal then  rhs(u) := min_{s'∈Succ(u)}(c(u, s') + g(s'));
if u ∈ U then  U.Remove(u);
if g(u) ≠ rhs(u) then  U.Insert(u, CalculateKey(u));
```

Procedure CalculateKey

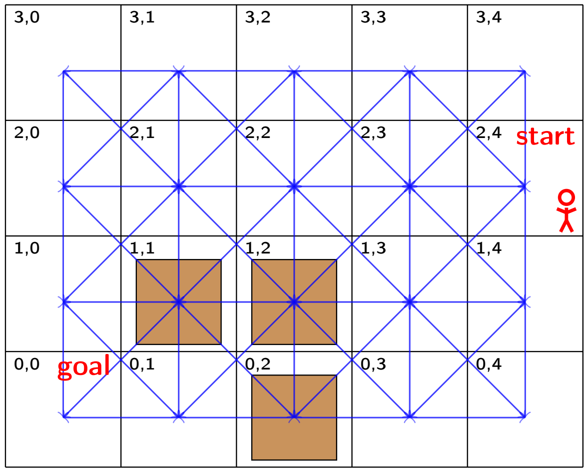
```
return [min(g(s), rhs(s)) + h(s_start, s); min(g(s), rhs(s))]
```


D* Lite – Demo



https://github.com/mdeyo/d-star-lite

D* Lite – Example



Legend

Free node	Obstacle node
On open list	Active node

- A grid map of the environment (what is actually known)
- 8-connected graph superimposed on the grid (bidirectional)
- Focusing heuristic is not used ($h = 0$)

- Transition costs
  - Free space – Free space: 1.0 and 1.4 (for diagonal edge)
  - From/to obstacle: ∞

D* Lite – Example Planning (1)

3,0 g: ∞ rhs: ∞	3,1 g: ∞ rhs: ∞	3,2 g: ∞ rhs: ∞	3,3 g: ∞ rhs: ∞	3,4 g: ∞ rhs: ∞
2,0 g: ∞ rhs: ∞	2,1 g: ∞ rhs: ∞	2,2 g: ∞ rhs: ∞	2,3 g: ∞ rhs: ∞	2,4 start g: ∞ rhs: ∞
1,0 g: ∞ rhs: ∞	1,1 g: ∞ rhs: ∞	1,2 g: ∞ rhs: ∞	1,3 g: ∞ rhs: ∞	1,4 g: ∞ rhs: ∞
0,0 goal g: ∞ rhs: 0	0,1 g: ∞ rhs: ∞	0,2 g: ∞ rhs: ∞	0,3 g: ∞ rhs: ∞	0,4 g: ∞ rhs: ∞

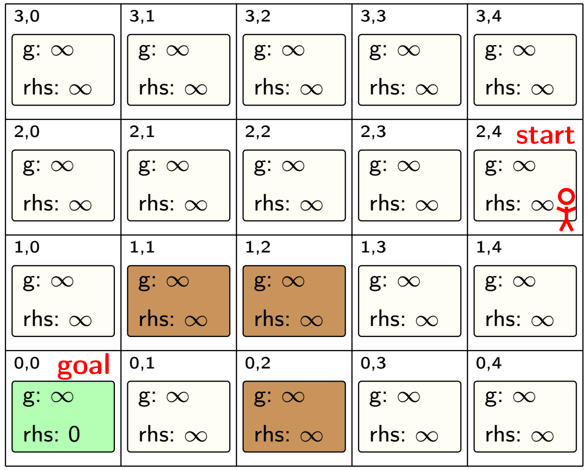
Legend

Free node	Obstacle node
On open list	Active node

Initialization

- Set $rhs = 0$ for the goal
- Set $rhs = g = \infty$ for all other nodes

D* Lite – Example Planning (2)



Legend

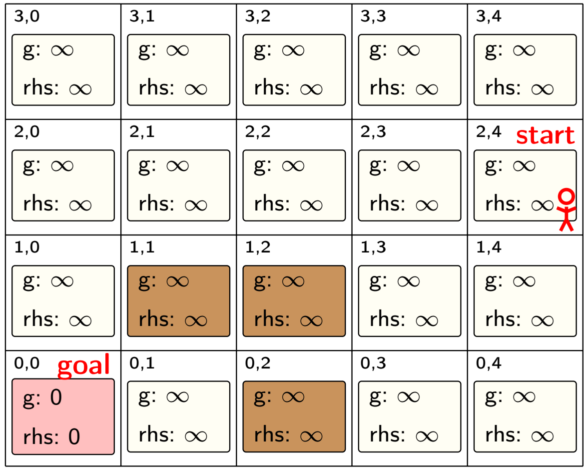
Free node | Obstacle node | On open list | Active node

Initialization

- Put the goal to the open list

  It is inconsistent

D* Lite – Example Planning (3)



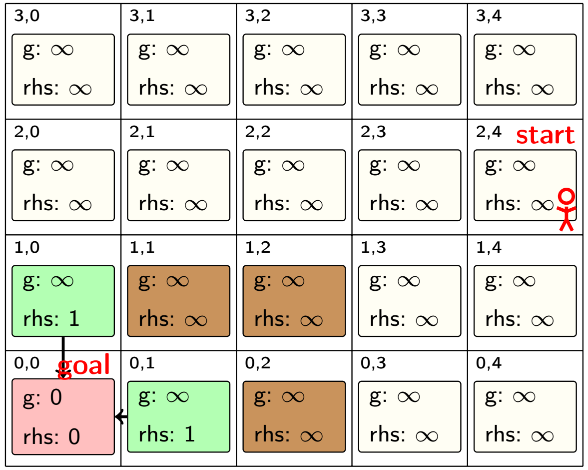
Legend

Free node | Obstacle node | On open list | Active node

ComputeShortestPath

- Pop the minimum element from the open list (goal)
- It is over-consistent ($g > rhs$), therefore set $g = rhs$

D* Lite – Example Planning (4)



Legend

Free node | Obstacle node | On open list | Active node

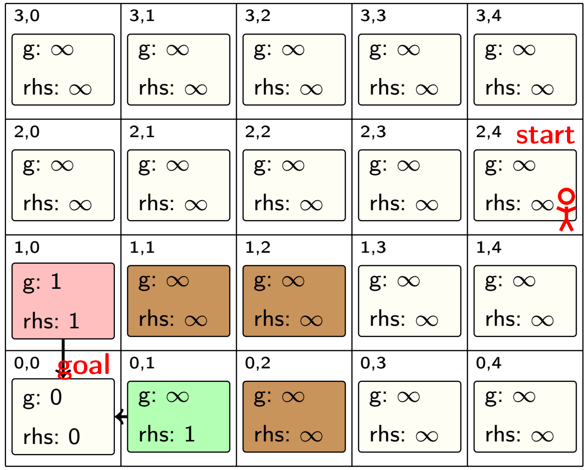
ComputeShortestPath

- Expand popped node (`UpdateVertex()` on all its predecessors)
- This computes the *rhs* values for the predecessors
- Nodes that become inconsistent are added to the open list

Small black arrows denote the node used for computing the *rhs* value, i.e., using the respective transition cost

- The *rhs* value of (1,1) is ∞ because the transition to obstacle has cost ∞

D* Lite – Example Planning (5)



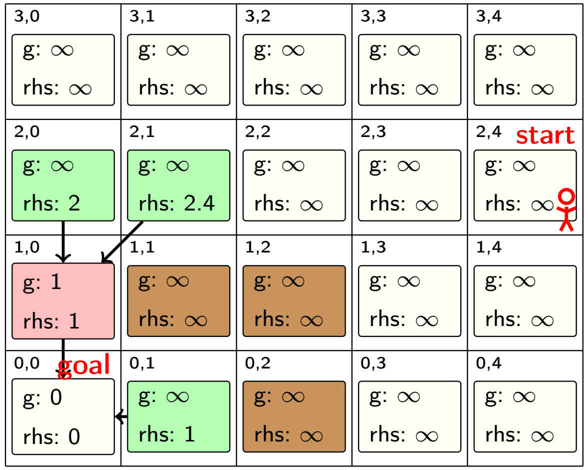
Legend

Free node | Obstacle node | On open list | Active node

ComputeShortestPath

- Pop the minimum element from the open list (1,0)
- It is over-consistent ($g > rhs$) set $g = rhs$

D* Lite – Example Planning (6)



Legend

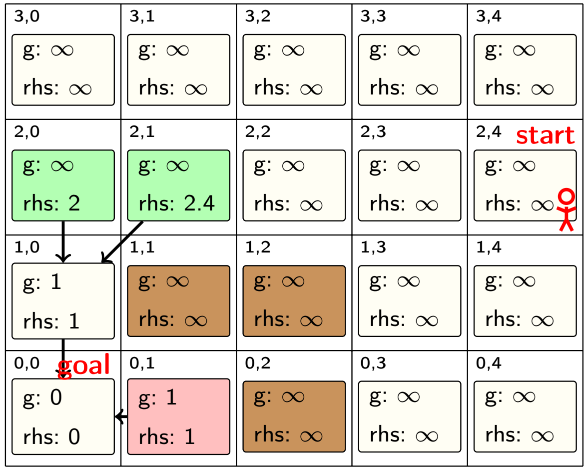
Free node	Obstacle node
On open list	Active node

ComputeShortestPath

- Expand the popped node (`UpdateVertex()` on all predecessors in the graph)
- Compute *rhs* values of the predecessors accordingly
- Put them to the open list if they become inconsistent

- The *rhs* value of (0,0), (1,1) does not change
- They do not become inconsistent and thus they are not put on the open list

D* Lite – Example Planning (7)



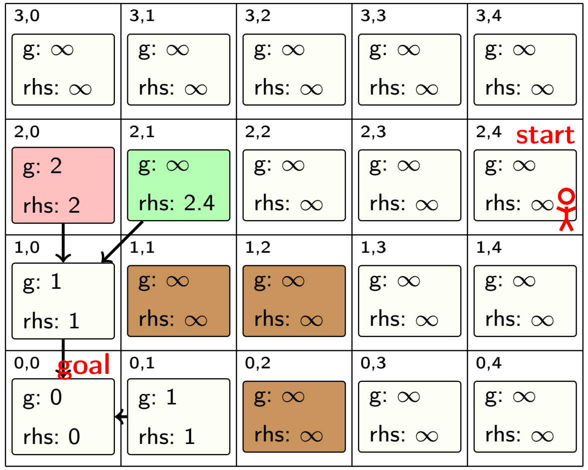
Legend

Free node	Obstacle node
On open list	Active node

ComputeShortestPath

- Pop the minimum element from the open list (0,1)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$
- Expand the popped element, e.g., call `UpdateVertex()`

D* Lite – Example Planning (8)



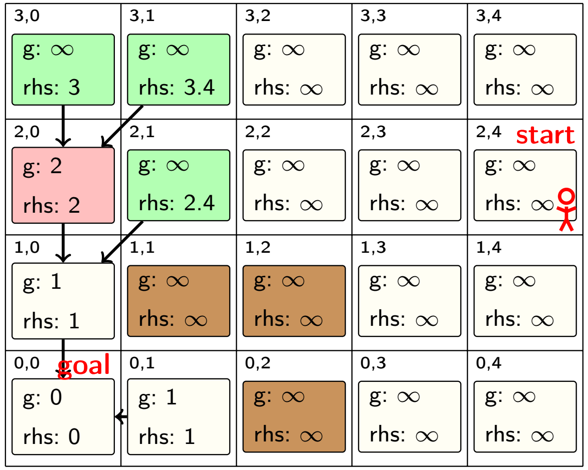
Legend

Free node	Obstacle node
On open list	Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,0)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$

D* Lite – Example Planning (9)



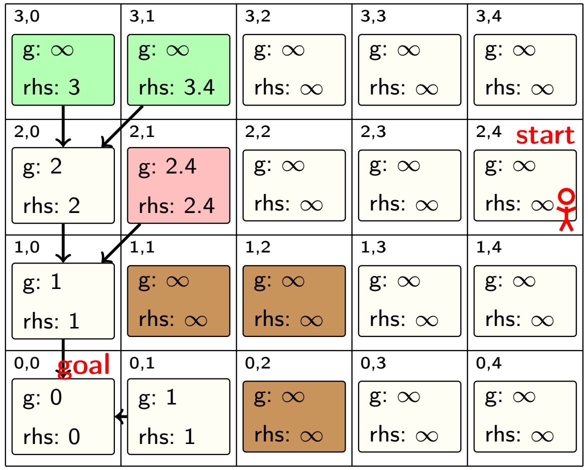
Legend

Free node	Obstacle node
On open list	Active node

ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list

D* Lite – Example Planning (10)



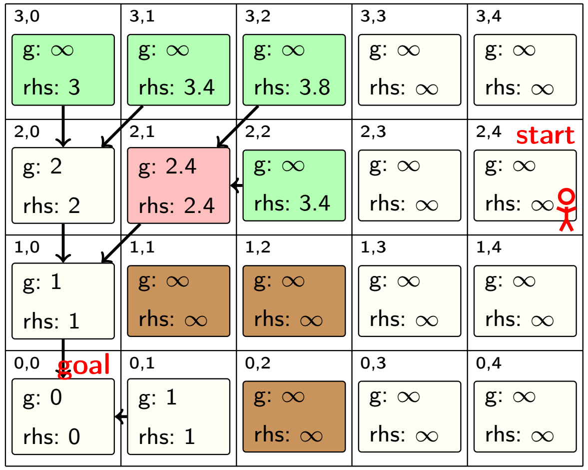
Legend

Free node | Obstacle node | On open list | Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,1)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$

D* Lite – Example Planning (11)



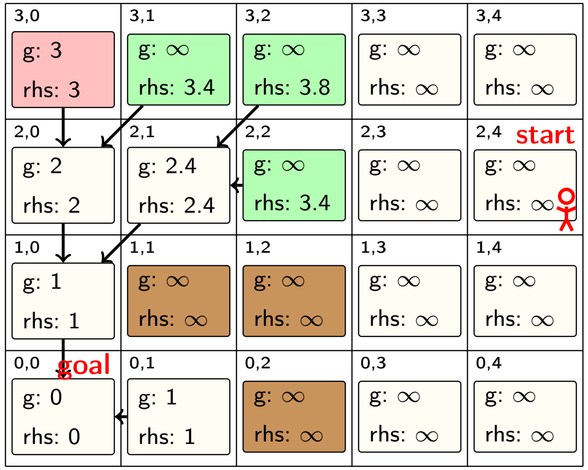
Legend

Free node | Obstacle node | On open list | Active node

ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list

D* Lite – Example Planning (12)



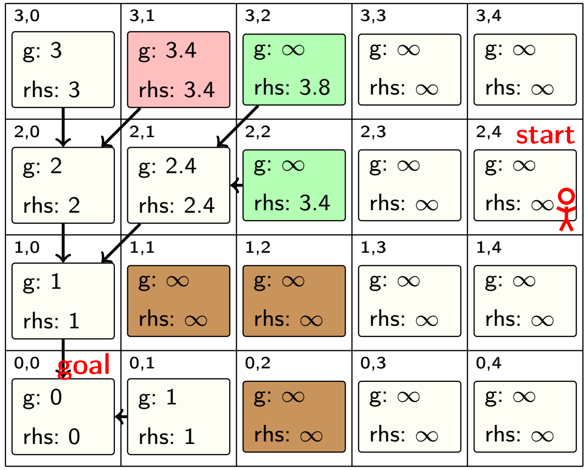
Legend

Free node | Obstacle node | On open list | Active node

ComputeShortestPath

- Pop the minimum element from the open list (3,0)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent onto the open list
- In this cases, none of the predecessors become inconsistent

D* Lite – Example Planning (13)



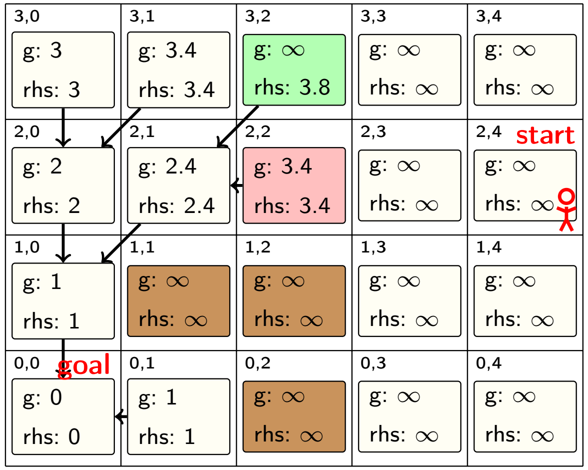
Legend

Free node | Obstacle node | On open list | Active node

ComputeShortestPath

- Pop the minimum element from the open list (3,0)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent onto the open list
- In this cases, none of the predecessors become inconsistent

D* Lite – Example Planning (14)



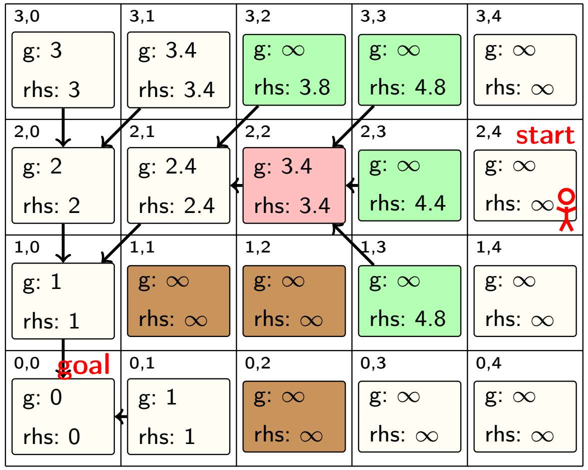
Legend

- Free node
- Obstacle node
- On open list
- Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,2)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$

D* Lite – Example Planning (15)



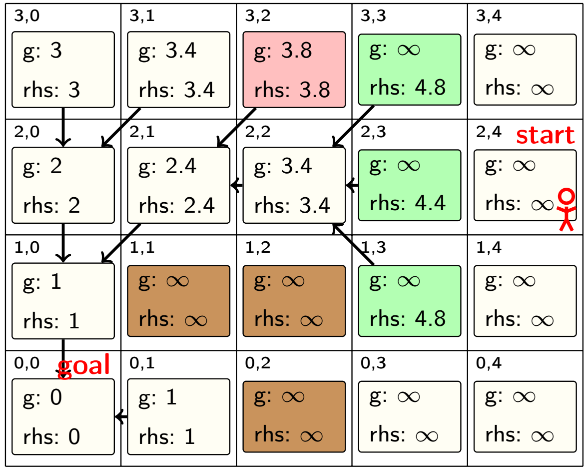
Legend

- Free node
- Obstacle node
- On open list
- Active node

ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list, i.e., (3,2), (3,3), (2,3)

D* Lite – Example Planning (16)



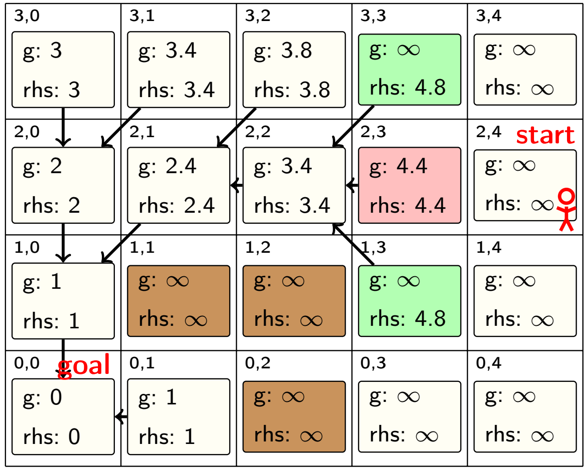
Legend

- Free node
- Obstacle node
- On open list
- Active node

ComputeShortestPath

- Pop the minimum element from the open list (3,2)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent onto the open list
- In this cases, none of the predecessors become inconsistent

D* Lite – Example Planning (17)



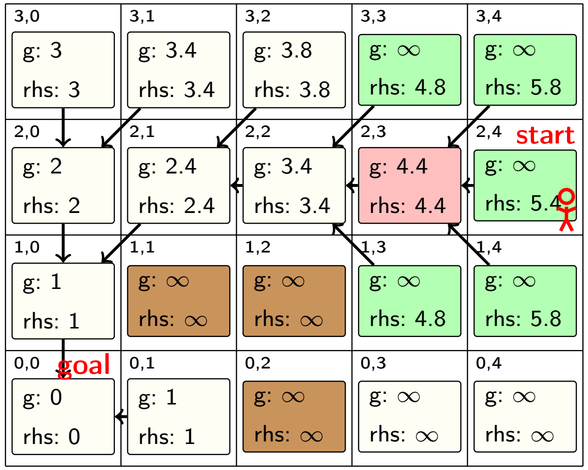
Legend

- Free node
- Obstacle node
- On open list
- Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,3)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$

D* Lite – Example Planning (18)



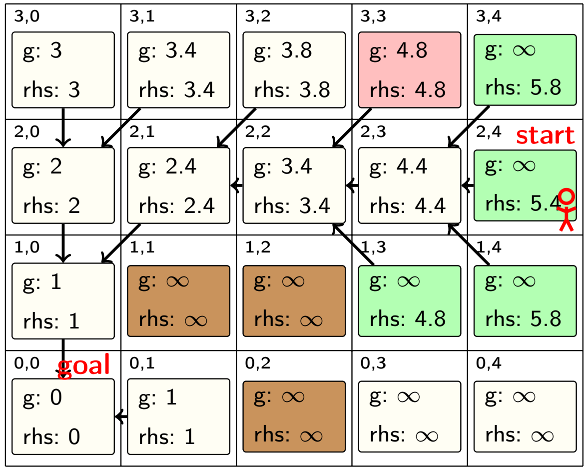
Legend

Free node	Obstacle node
On open list	Active node

ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list, i.e., (3,4), (2,4), (1,4)
- The start node is on the open list
- However, the search does not finish at this stage
- There are still inconsistent nodes (on the open list) with a lower value of *rhs*

D* Lite – Example Planning (19)



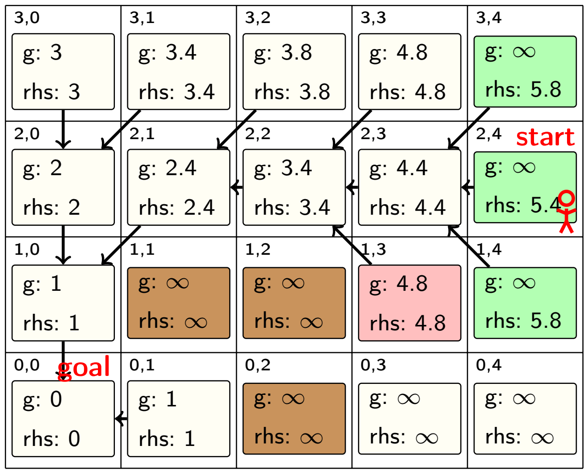
Legend

Free node	Obstacle node
On open list	Active node

ComputeShortestPath

- Pop the minimum element from the open list (3,2)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent onto the open list
- In this cases, none of the predecessors become inconsistent

D* Lite – Example Planning (20)



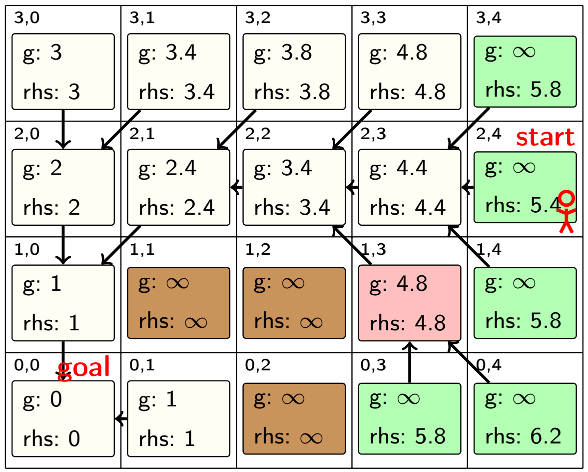
Legend

Free node	Obstacle node
On open list	Active node

ComputeShortestPath

- Pop the minimum element from the open list (1,3)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$

D* Lite – Example Planning (21)



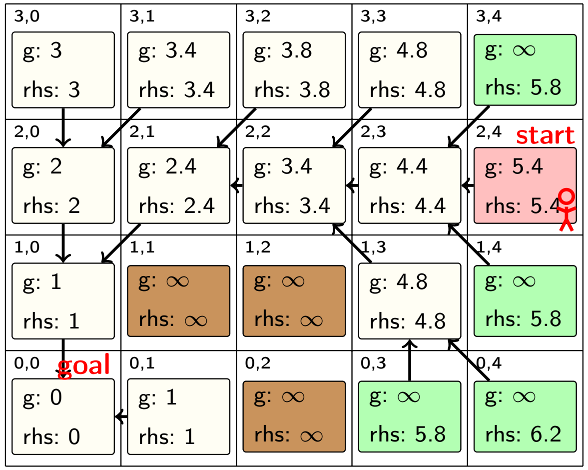
Legend

Free node	Obstacle node
On open list	Active node

ComputeShortestPath

- Expand the popped element and put the predecessors that become inconsistent onto the open list, i.e., (0,3) and (0,4)

D* Lite – Example Planning (22)



Legend

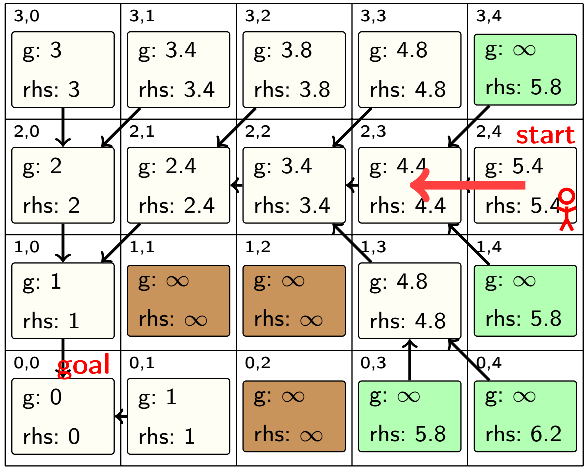
Free node | Obstacle node | On open list | Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,4)
- It is over-consistent ($g > rhs$) and thus set $g = rhs$
- Expand the popped element and put the predecessors that become inconsistent (none in this case) onto the open list

- The start node becomes consistent and the top key on the open list is not less than the key of the start node
- An optimal path is found and the loop of the `ComputeShortestPath` is breaked

D* Lite – Example Planning (23)

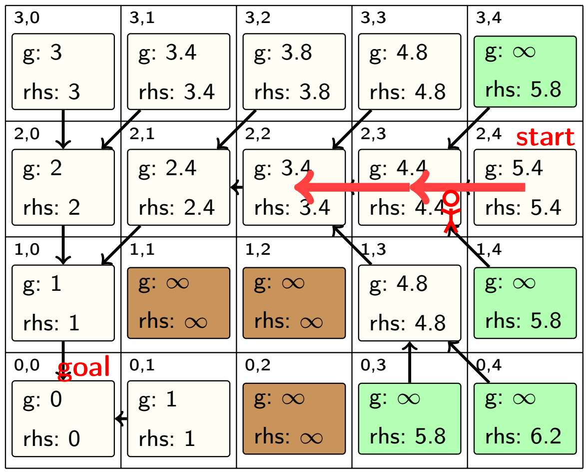


Legend

Free node | Obstacle node | On open list | Active node

- Follow the gradient of g values from the start node

D* Lite – Example Planning (24)

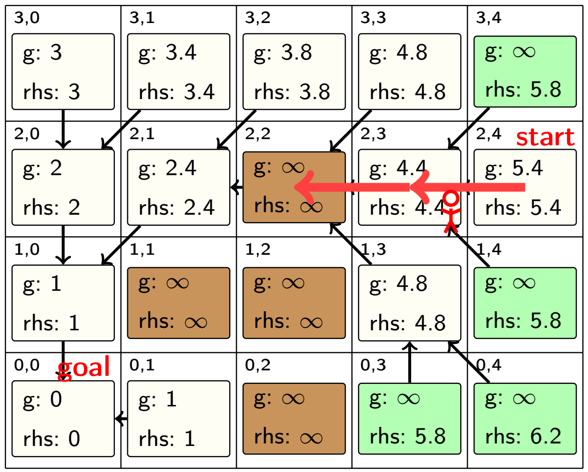


Legend

Free node | Obstacle node | On open list | Active node

- Follow the gradient of g values from the start node

D* Lite – Example Planning (25)

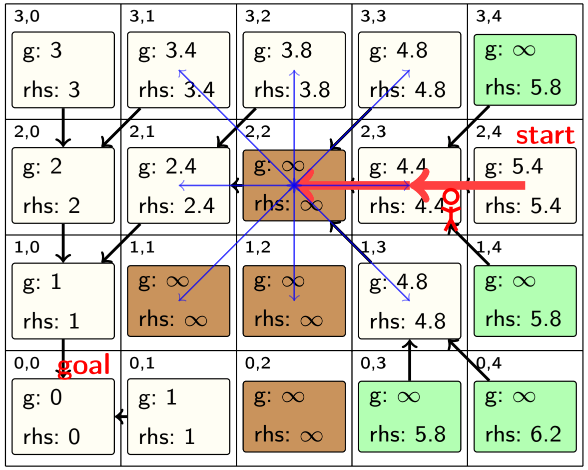


Legend

Free node | Obstacle node | On open list | Active node

- A new obstacle is detected during the movement from (2,3) to (2,2)
- **Replanning** is needed!

D* Lite – Example Planning (25 update)

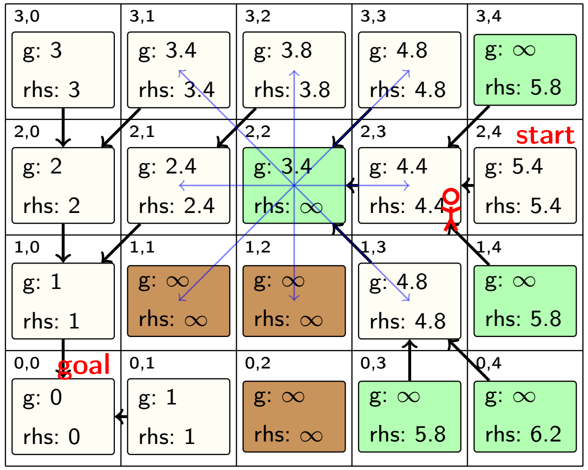


Legend

Free node	Obstacle node
On open list	Active node

- All directed edges with changed edge, we need to call the `UpdateVertex()`
- All edges into and out of (2,2) have to be considered

D* Lite – Example Planning (26 update 1/2)



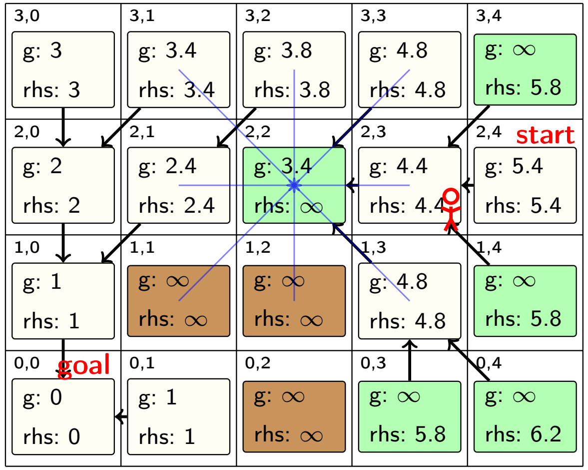
Legend

Free node	Obstacle node
On open list	Active node

Update Vertex

- Outgoing edges from (2,2)
- Call `UpdateVertex()` on (2,2)
- The transition costs are now ∞ because of obstacle
- Therefore the $rhs = \infty$ and (2,2) becomes inconsistent and it is put on the open list

D* Lite – Example Planning (26 update 2/2)



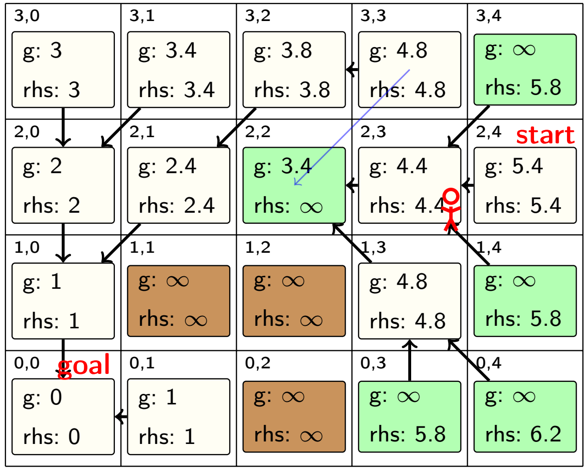
Legend

Free node	Obstacle node
On open list	Active node

Update Vertex

- Incomming edges to (2,2)
- Call `UpdateVertex()` on the neighbors (2,2)
- The transition cost is ∞, and therefore, the *rhs* value previously computed using (2,2) is changed

D* Lite – Example Planning (27)



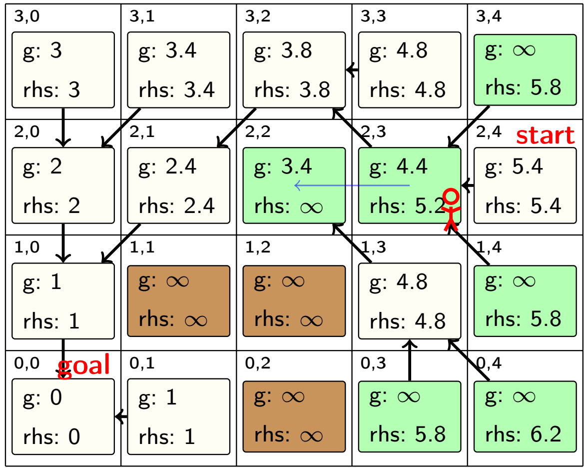
Legend

Free node	Obstacle node
On open list	Active node

Update Vertex

- The neighbor of (2,2) is (3,3)
- The minimum possible *rhs* value of (3,3) is 4.8 but it is based on the *g* value of (3,2) and not (2,2), which is the detected obstacle
- The node (3,3) is still consistent and thus it is not put on the open list

D* Lite – Example Planning (28)



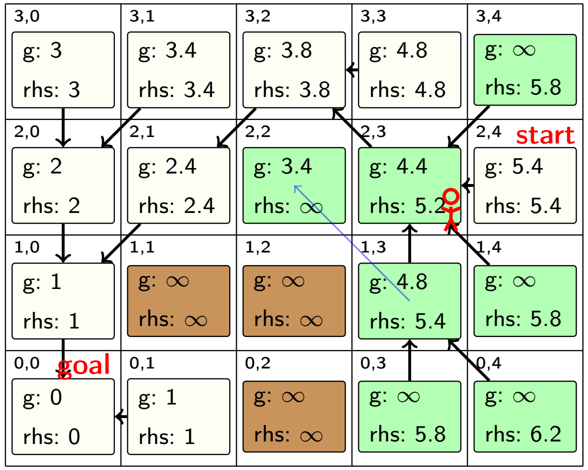
Legend

- Free node
- Obstacle node
- On open list
- Active node

Update Vertex

- (2,3) is also a neighbor of (2,2)
- The minimum possible *rhs* value of (2,3) is 5.2 because of (2,2) is obstacle (using (3,2) with 3.8 + 1.4)
- The *rhs* value of (2,3) is different than *g* thus (2,3) is put on the open list

D* Lite – Example Planning (29)



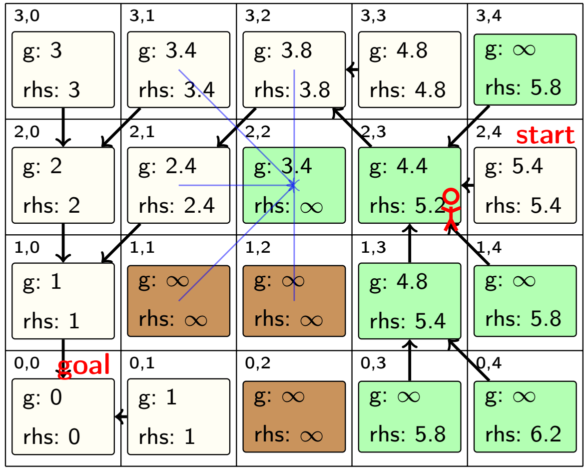
Legend

- Free node
- Obstacle node
- On open list
- Active node

Update Vertex

- Another neighbor of (2,2) is (1,3)
- The minimum possible *rhs* value of (1,3) is 5.4 computed based on *g* of (2,3) with 4.4 + 1 = 5.4
- The *rhs* value is always computed using the *g* values of its successors

D* Lite – Example Planning (29 update)



Legend

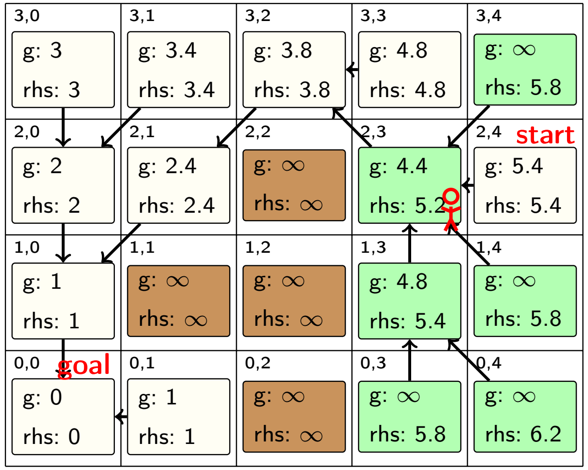
- Free node
- Obstacle node
- On open list
- Active node

Update Vertex

- None of the other neighbor of (2,2) end up being inconsistent
- We go back to calling `ComputeShortestPath()` until an optimal path is determined

- The node corresponding to the robot's current position is inconsistent and its key is greater than the minimum key on the open list
- Thus, the optimal path is not found yet

D* Lite – Example Planning (30)



Legend

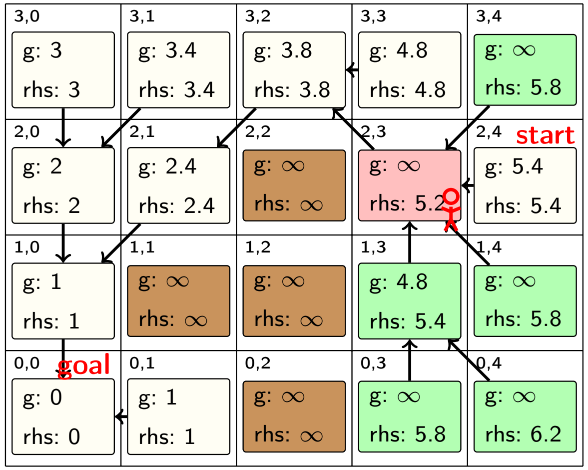
- Free node
- Obstacle node
- On open list
- Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,2), which is obstacle
- It is under-consistent ($g < rhs$), therefore set $g = \infty$
- Expand the popped element and put the predecessors that become inconsistent (none in this case) onto the open list

- Because (2,2) was under-consistent (when popped), `UpdateVertex()` has to be called on it
- However, it has no effect as its *rhs* value is up to date and consistent

D* Lite – Example Planning (31)



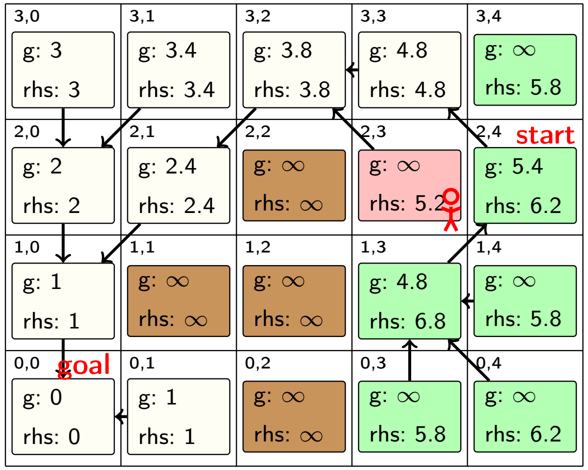
Legend

Free node | Obstacle node | On open list | Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,3)
- It is under-consistent ($g < rhs$), therefore set $g = \infty$

D* Lite – Example Planning (32)



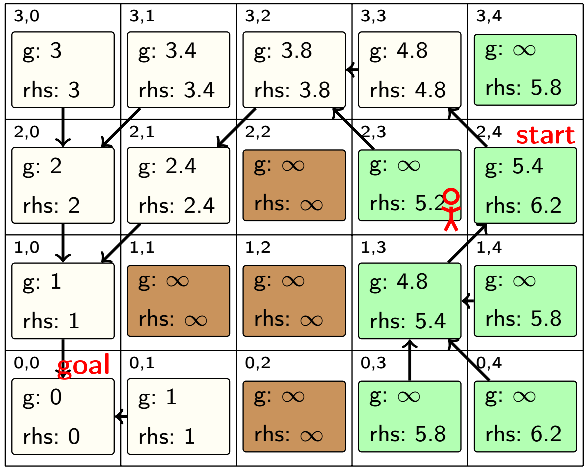
Legend

Free node | Obstacle node | On open list | Active node

ComputeShortestPath

- Expand the popped element and update the predecessors
- (2,4) becomes inconsistent
- (1,3) gets updated and still inconsistent
- The *rhs* value (1,4) does not changed, but it is now computed from the g value of (1,3)

D* Lite – Example Planning (33)



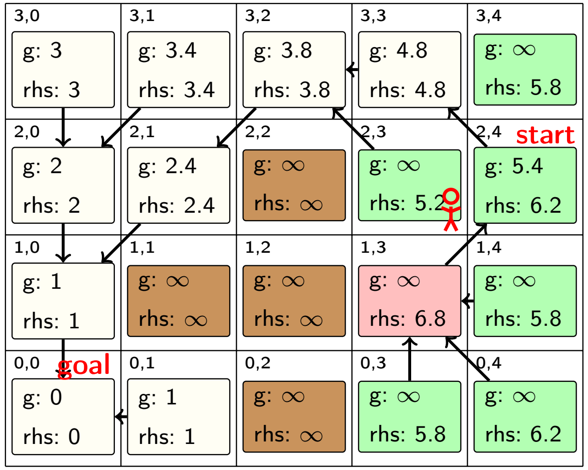
Legend

Free node | Obstacle node | On open list | Active node

ComputeShortestPath

- Because (2,3) was under-consistent (when popped), call `UpdateVertex()` on it is needed
- As it is still inconsistent it is put back onto the open list

D* Lite – Example Planning (34)



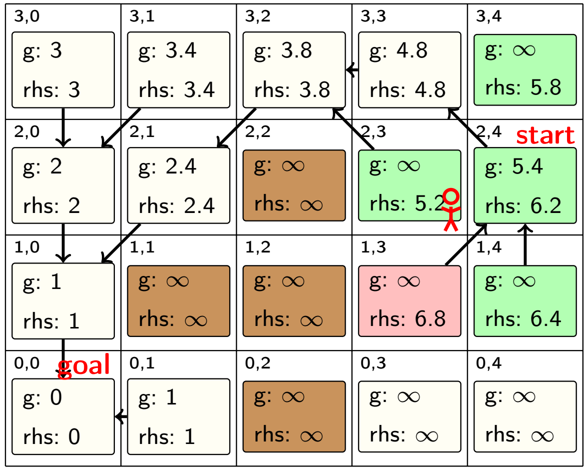
Legend

Free node | Obstacle node | On open list | Active node

ComputeShortestPath

- Pop the minimum element from the open list (1,3)
- It is under-consistent ($g < rhs$), therefore set $g = \infty$

D* Lite – Example Planning (35)



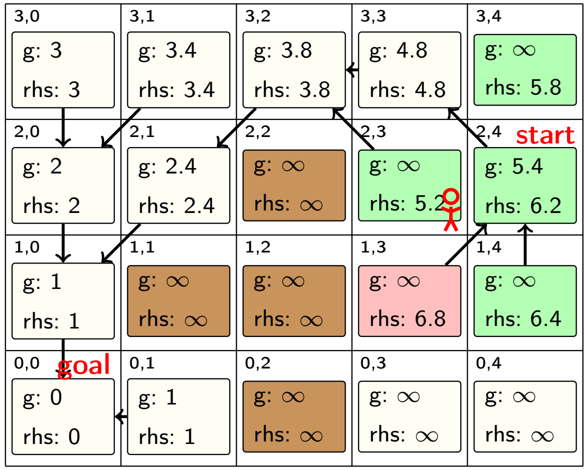
Legend

Free node	Obstacle node
On open list	Active node

ComputeShortestPath

- Expand the popped element and update the predecessors
- (1,4) gets updated and still inconsistent
- (0,3) and (0,4) get updated and now consistent (both g and rhs are ∞)

D* Lite – Example Planning (36)



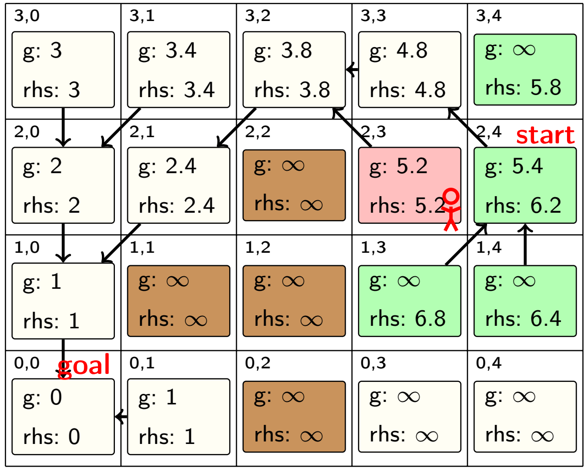
Legend

Free node	Obstacle node
On open list	Active node

ComputeShortestPath

- Because (1,3) was under-consistent (when popped), call `UpdateVertex()` on it is needed
- As it is still inconsistent it is put back onto the open list

D* Lite – Example Planning (37)



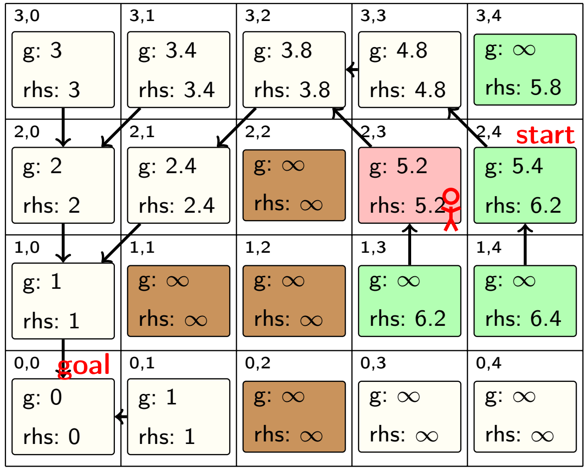
Legend

Free node	Obstacle node
On open list	Active node

ComputeShortestPath

- Pop the minimum element from the open list (2,3)
- It is over-consistent ($g > rhs$), therefore set $g = rhs$

D* Lite – Example Planning (38)



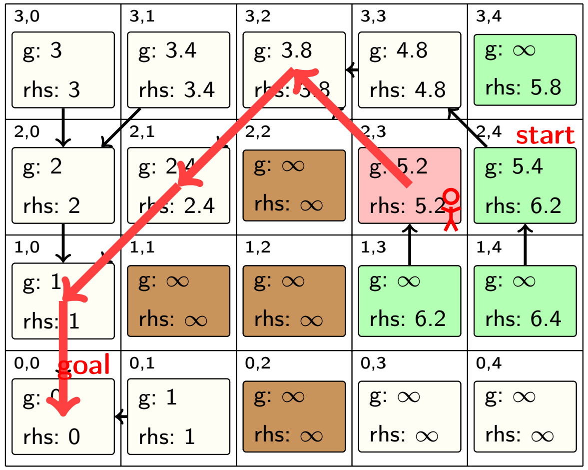
Legend

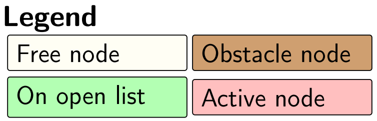
Free node	Obstacle node
On open list	Active node

ComputeShortestPath

- Expand the popped element and update the predecessors
- (1,3) gets updated and still inconsistent
- The node (2,3) corresponding to the robot's position is consistent
- Besides, top of the key on the open list is not less than the key of (2,3)
- The optimal path has been found and we can break out of the loop

D* Lite – Example Planning (39)





- Follow the gradient of g values from the robot's current position (node)

D* Lite – Comments

- D* Lite works with real valued costs, not only with binary costs (free/obstacle)
- The search can be focused with an admissible heuristic that would be added to the g and *rhs* values
- The final version of D* Lite includes further optimization (not shown in the example)
  - Updating the *rhs* value without considering all successors every time
  - Re-focusing the search as the robot moves without reordering the entire open list

Reaction-Diffusion Processes Background

- *Reaction-Diffusion* (RD) models – dynamical systems capable to reproduce the autowaves
- *Autowaves* - a class of nonlinear waves that propagate through an active media

  At the expense of the energy stored in the medium, e.g., grass combustion.
- RD model describes spatio-temporal evolution of two state variables $u = u(\vec{x}, t)$ and $v = v(\vec{x}, t)$ in space $\vec{x}$ and time t

$$\begin{aligned}\dot{u} &= f(u, v) + D_u \triangle u \\ \dot{v} &= g(u, v) + D_v \triangle v\end{aligned},$$

where $\triangle$ is the Laplacian.

This RD-based path planning is informative, just for curiosity

Reaction-Diffusion Background

- FitzHugh-Nagumo (FHN) model

  FitzHugh R, Biophysical Journal (1961)

$$\begin{aligned}\dot{u} &= \varepsilon\left(u - u^3 - v + \phi\right) + D_u \triangle u \\ \dot{v} &= \left(u - \alpha v + \beta\right) + D_v \triangle u\end{aligned},$$

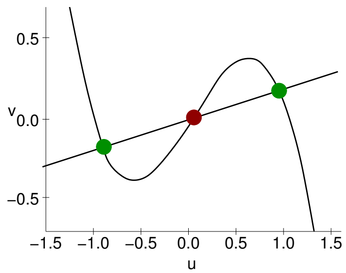
where α, β, ϵ, and ϕ are parameters of the model.

- Dynamics of RD system is determined by the associated *nullcline configurations* for $\dot{u}$=0 and $\dot{v}$=0 in the absence of diffusion, i.e.,

$$\begin{aligned}\varepsilon\left(u - u^3 - v + \phi\right) &= 0, \\ \left(u - \alpha v + \beta\right) &= 0,\end{aligned}$$

which have associated geometrical shapes

Nullcline Configurations and Steady States



- Nullclines intersections represent
  - Stable States (*SSs*)
  - Unstable States
- Bistable regime

  The system (concentration levels of (u, v) for each grid cell) tends to be in SSs.

- We can modulate relative stability of both SS

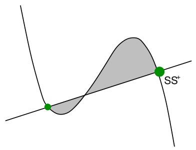
  "preference" of SS^+ over SS^-

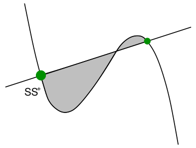
- System moves from SS^- to SS^+,

  if a small perturbation is introduced.

- The *SS*s are separated by a mobile frontier

  a kind of traveling frontwave (autowaves)





RD-based Path Planning – Computational Model

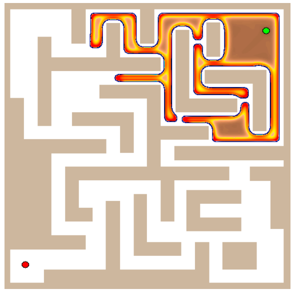
- Finite difference method on a Cartesian grid with Dirichlet boundary conditions (FTCS) *discretization → grid based computation → grid map*
- *External forcing* – introducing additional information

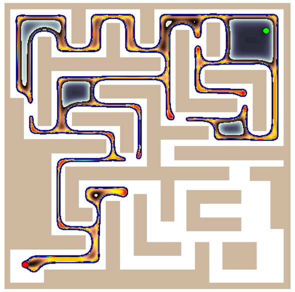
  i.e., constraining concentration levels to some specific values

- Two-phase evolution of the underlying RD model
  1. **Propagation phase**
     - Freespace is set to SS^- and the start location SS^+
     - Parallel propagation of the frontwave with *non-annihilation property*

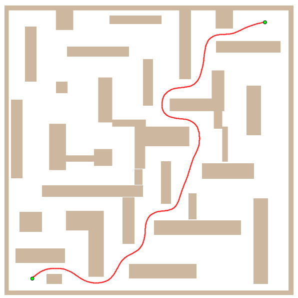
       Vázquez-Otero and Muñuzuri, CNNA (2010)

     - Terminate when the frontwave reaches the goal
  2. **Contraction phase**
     - Different nullclines configuration
     - Start and goal positions are forced towards SS^+
     - SS^- shrinks until only the path linking the forced points remains

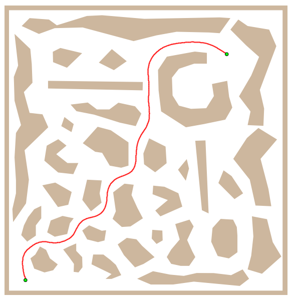




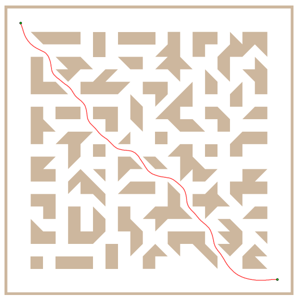
Example of Found Paths



700 × 700



700 × 700



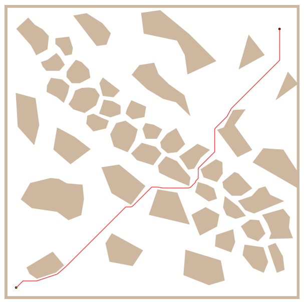
1200 × 1200

- The path clearance maybe adjusted by the wavelength and size of the computational grid.

  Control of the path distance from the obstacles (path safety)

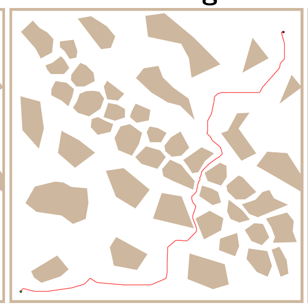
Comparison with Standard Approaches

Distance Transform



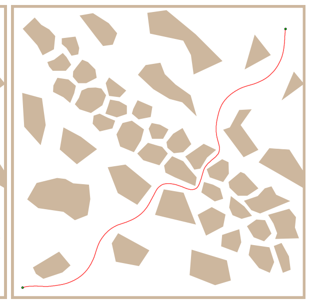
Jarvis R
Advanced Mobile Robots (1994)

Voronoi Diagram



Beeson P, Jong N, Kuipers B
ICRA (2005)

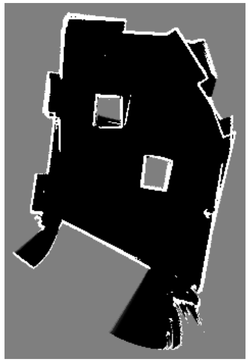
Reaction-Diffusion



Otero A, Faigl J, Muñuzuri A
IROS (2012)

- RD-based approach provides competitive paths regarding path length and clearance, while they seem to be smooth

Robustness to Noisy Data



Vázquez-Otero, A., Faigl, J., Duro, N. and Dormido, R. (2014): Reaction-Diffusion based Computational Model for Autonomous Mobile Robot Exploration of Unknown Environments. International Journal of Unconventional Computing (IJUC).

Summary of the Lecture

Topics Discussed

- Front-Wave propagation and path simplification
- Distance Transform based planning
- Graph based planning methods: Dijsktra's, A*, JPS, Theta*
- D* Lite
- Reaction-Diffusion based planning (*informative*)

- Next: Randomized Sampling-based Motion Planning Methods