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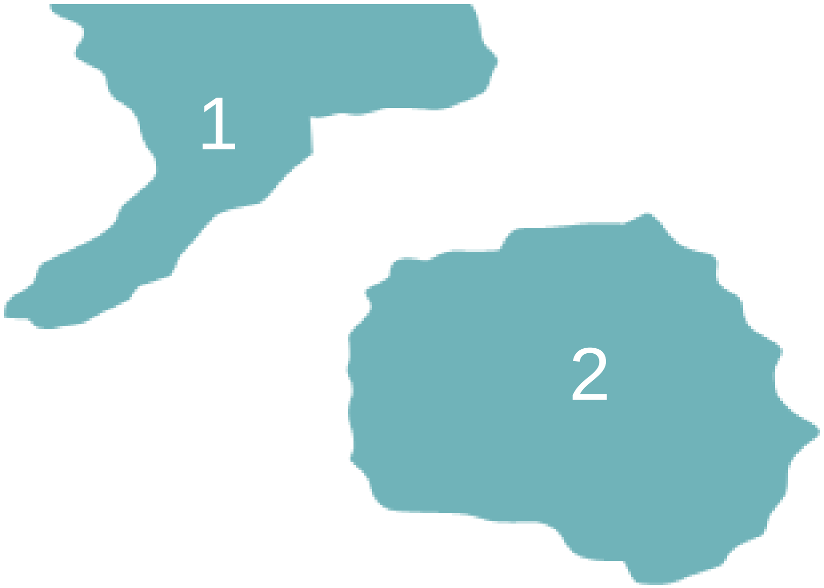
DEPARTMENT OF COMPUTER GRAPHICS AND INTERACTION

Overlay of planar subdivisions

Radek Smetana

Overlay of planar subdivisions



Overlay of planar subdivisions

Overlay of planar subdivisions

Requirements and Goals

- Representation of planar subdivision
- Computing intersections of line segments
- Merging subdivisions
- Boolean operations

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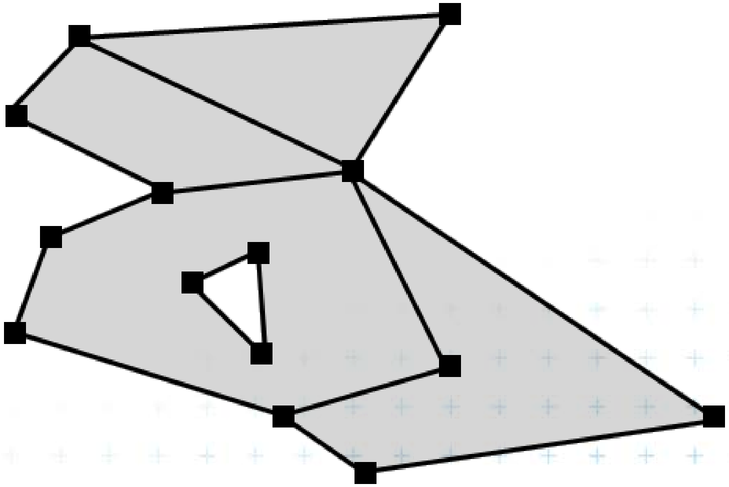
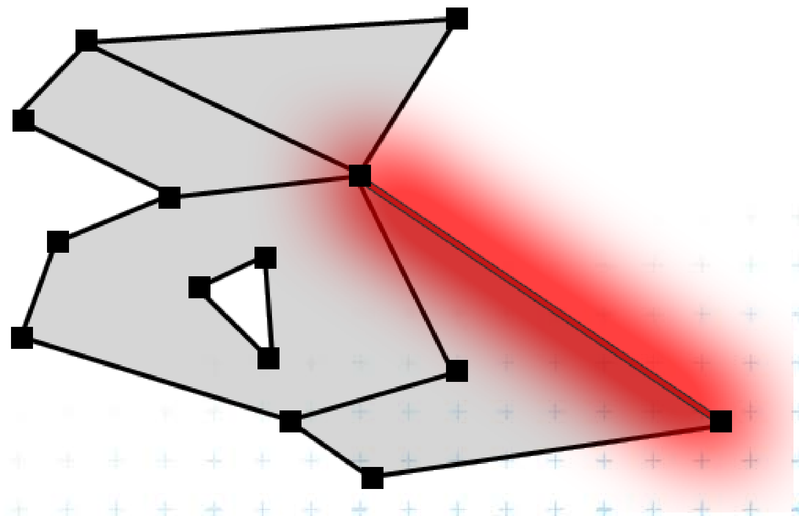
The Doubly-Connected Edge List

- Data structure with basic operations
- Edges are the straight lines, not crossing each other (only in verteces)
- Three collections: verteces, edges and faces



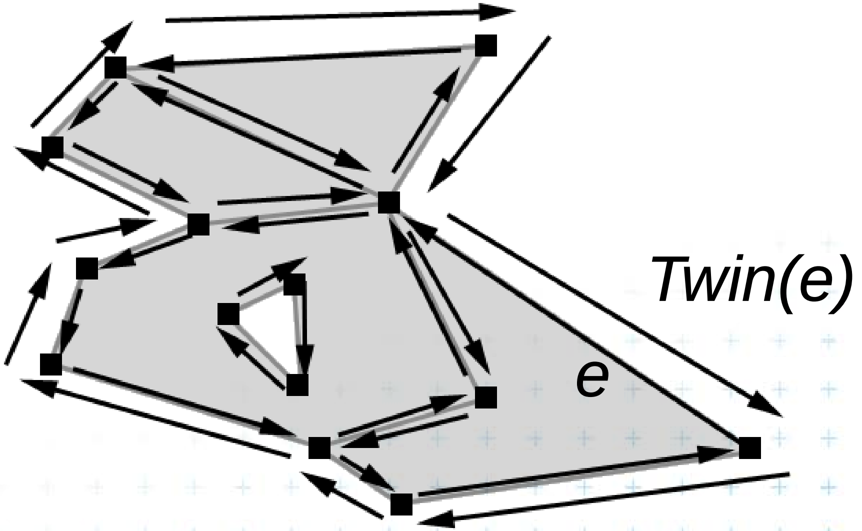
The Doubly-Connected Edge List

- Edge
  - **Open** - endpoints (= verteces) not included
  - Divided to **half-edges** - Two vectors in both directions



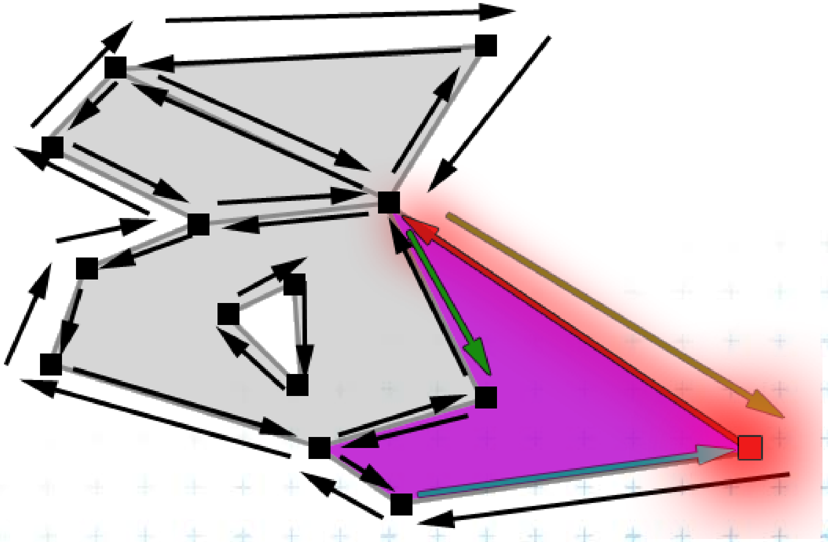
The Doubly-Connected Edge List

- Half-edge
  - Walk around a face in counterclockwise order
  - Holes have oposite direction
  - Half-edge e and Twin (e)



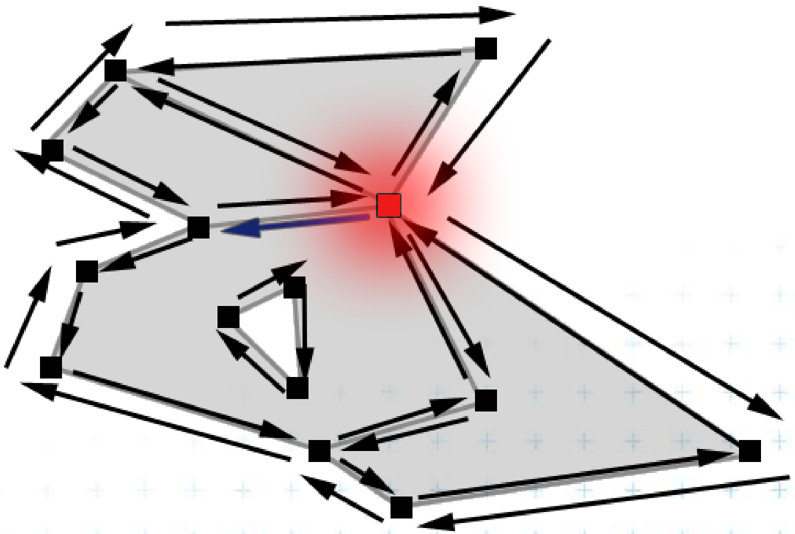
The Doubly-Connected Edge List

- Half-edge
  - e
  - Twin(e)
  - Origin(e)
  - IncidentFace(e)
  - Next(e)
  - Prev(e)



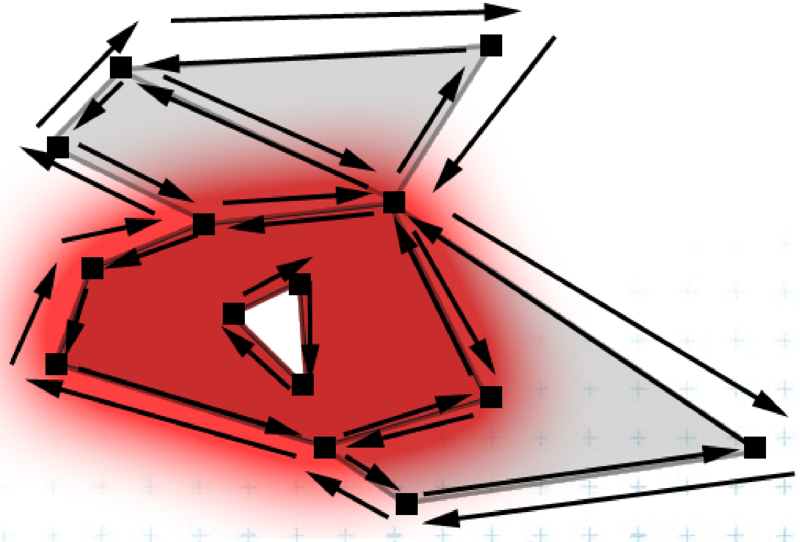
The Doubly-Connected Edge List

- Vertex
  - Coordinates(v)
  - IncidentEdge(v) – (náhodná) arbitary half-edge



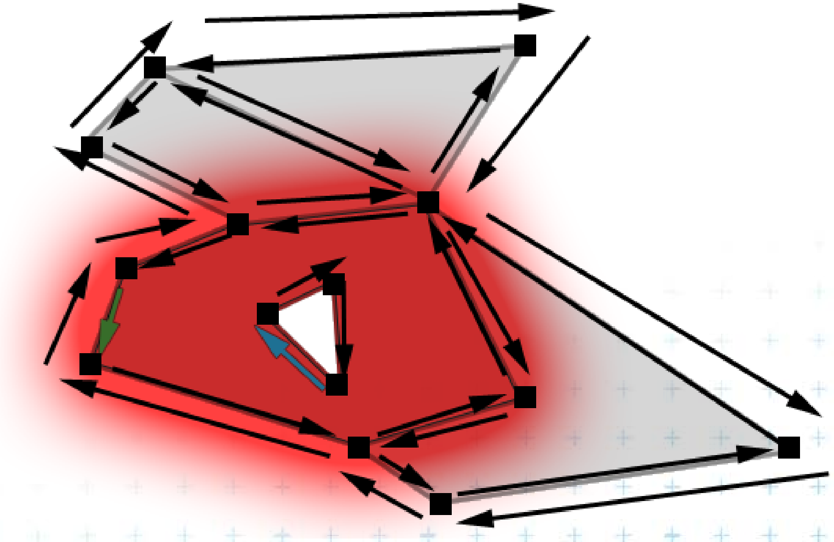
The Doubly-Connected Edge List

- Face
  - Do not contain a point on an edge or a vertex
  - On the left side of the half-edge

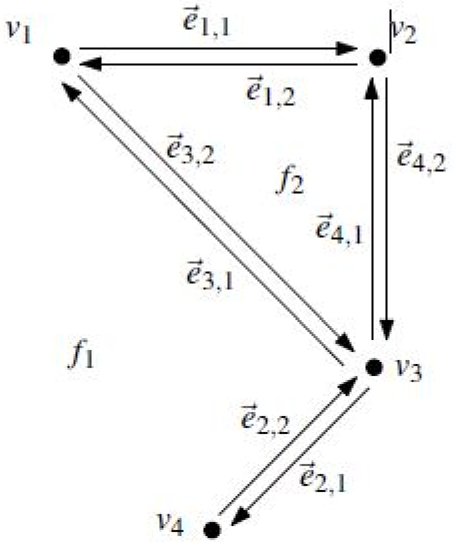


The Doubly-Connected Edge List

- Face
  - OuterComponent (f) - Unbounded has nil
  - InnerComponents (f) – Each hole – one pointer



The Doubly-Connected Edge List



Vertex	Coordinates	IncidentEdge
v_1	$(0,4)$	$\vec{e}_{1,1}$
v_2	$(2,4)$	$\vec{e}_{4,2}$
v_3	$(2,2)$	$\vec{e}_{2,1}$
v_4	$(1,1)$	$\vec{e}_{2,2}$

Face	OuterComponent	InnerComponents
f_1	**nil**	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	**nil**

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

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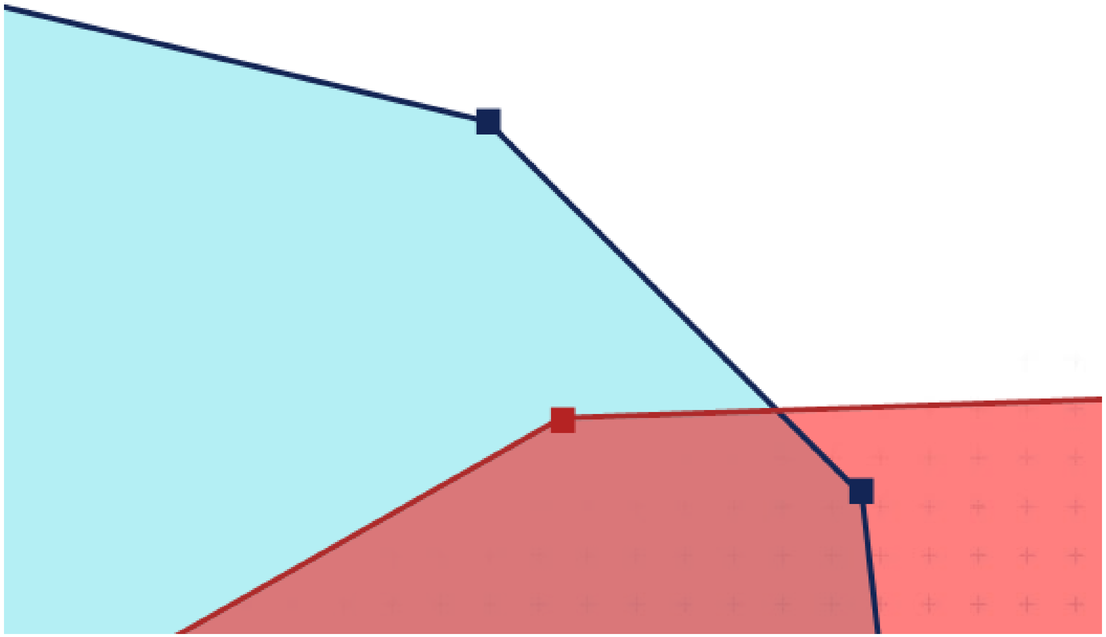
Computing intersections of line segments

- Have to handle intercestion of edges
- For now - Line segments



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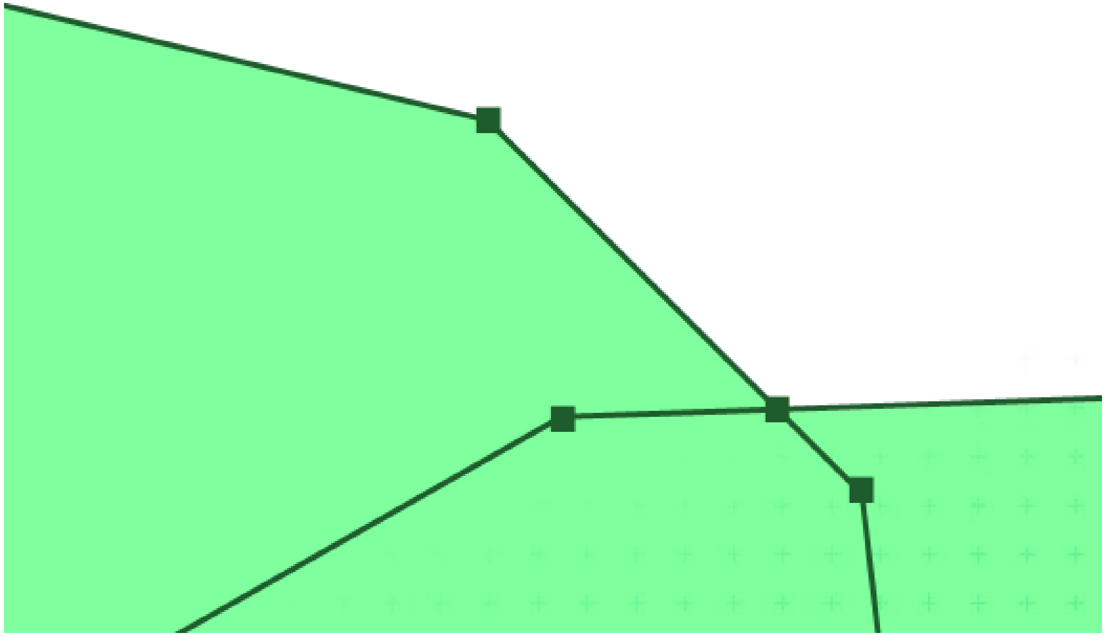

Computing intersections of line segments

- Intersection of two lines – just take equation of the line and make it equal to the second line
- Segments are not infinite
- Indicate if segments has intersection
- Brute force $O(n^2)$
- When each segment has intersection - $\Omega(n^2)$

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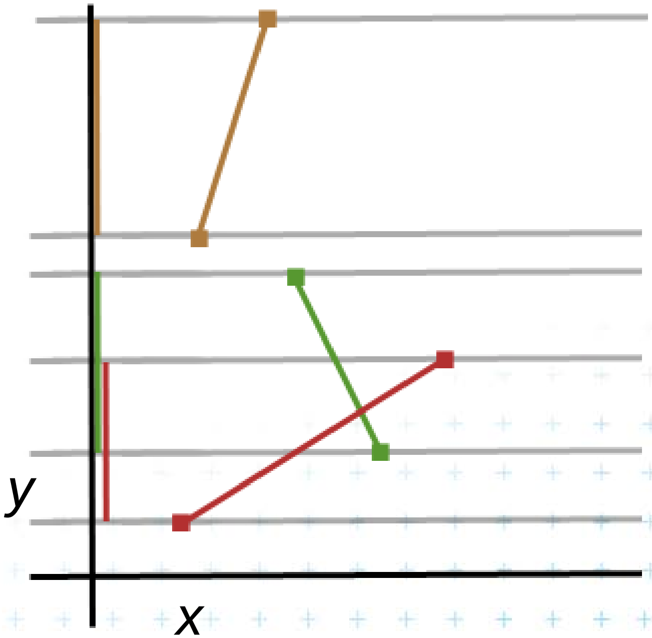
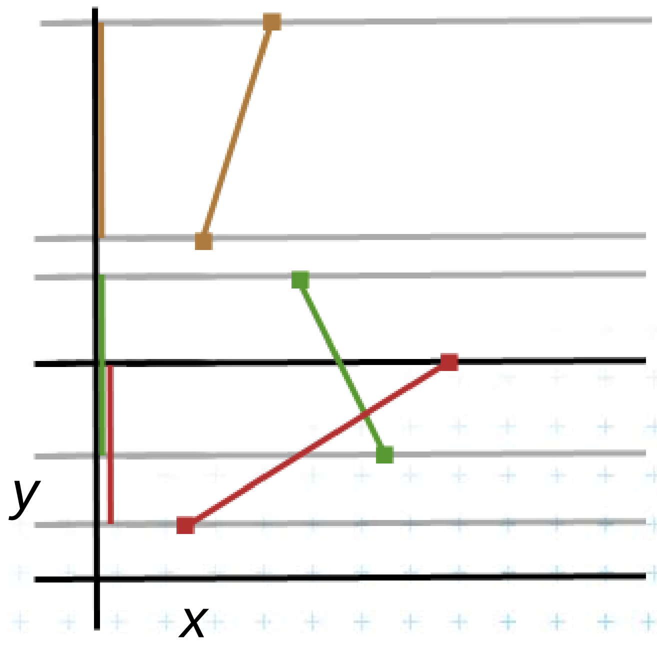
Plane sweep algorithm (to find intersections)

- Output sensitive algorithm
- Basic thoughts
  - Projection on y-axis



Plane sweep algorithm (to find intersections)

- Output sensitive algorithm
- Basic thoughts
  - Projection on y-axis
  - Divide by sweep line by x-axis
  - Test only horizontal neighbors



Plane sweep algorithm (to find intersections)

- Event queue Q
  - Event – endpoints of segments (two for each) or possible intersections
  - Balanced binary search tree implemetation
    - dynamic structure with removing and adding events on the fly
  - **Ordering:** p is first if $p.y > q.y$ holds or $py = qy$ and $p.x < q.x$
  - If event is the endpoint, where its segment starts, this **segment** is **stored** as well

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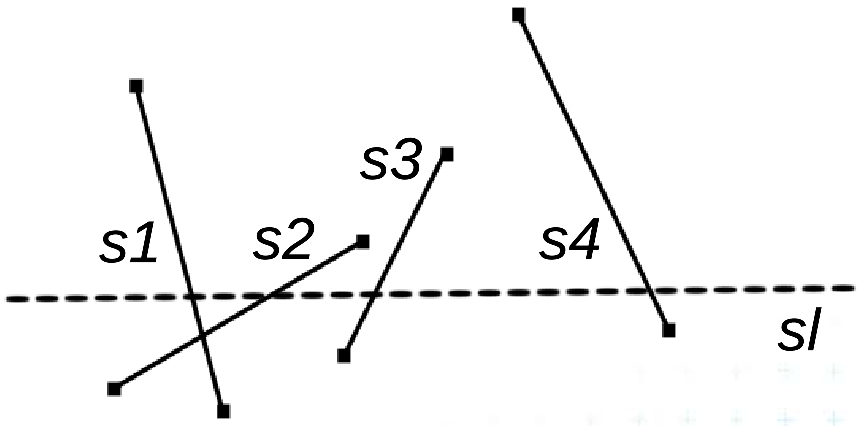
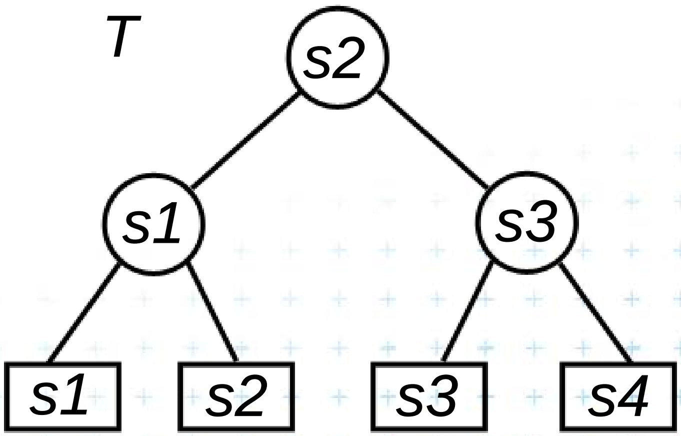
Plane sweep algorithm (to find intersections)

- Ordered sequence of segments *T*
  - To create order of segments defined by intersecting sweep line = dynamic structure of neighbors
  - Balanced binary search tree
  - Left-to-right order
  - Leaves store segments itself
  - Internal node store the segment from the **rightmost leaf** in its **left subtree** to guide search

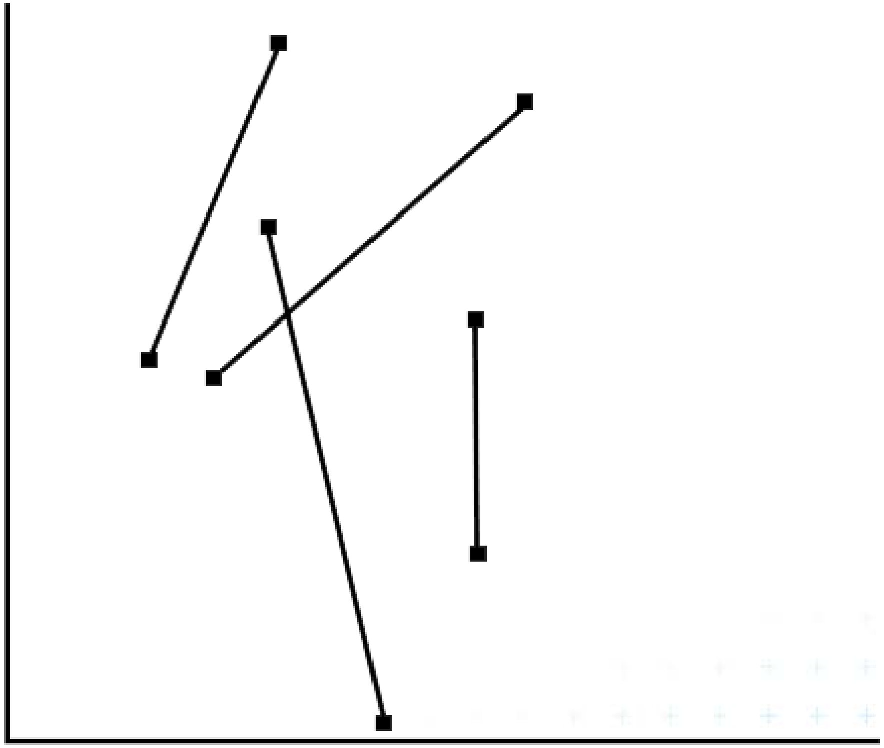
Plane sweep algorithm (to find intersections)

- Ordered sequence of segments *T*
  - Testing in each internal node position of the searched point
  - Result is the leaf or immediately to the left of it





Plane sweep algorithm (to find intersections)



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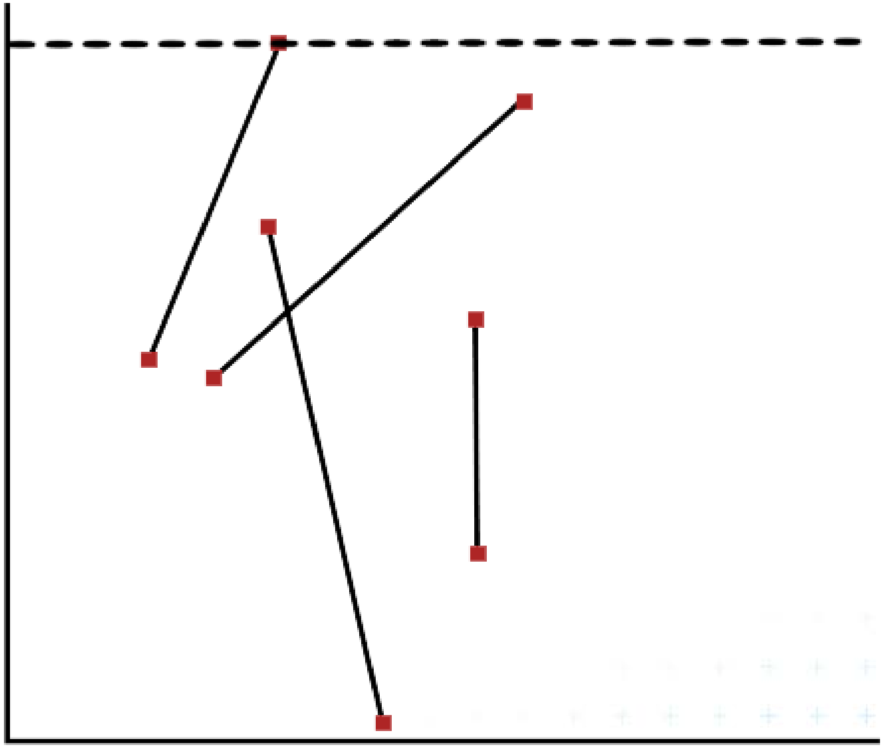

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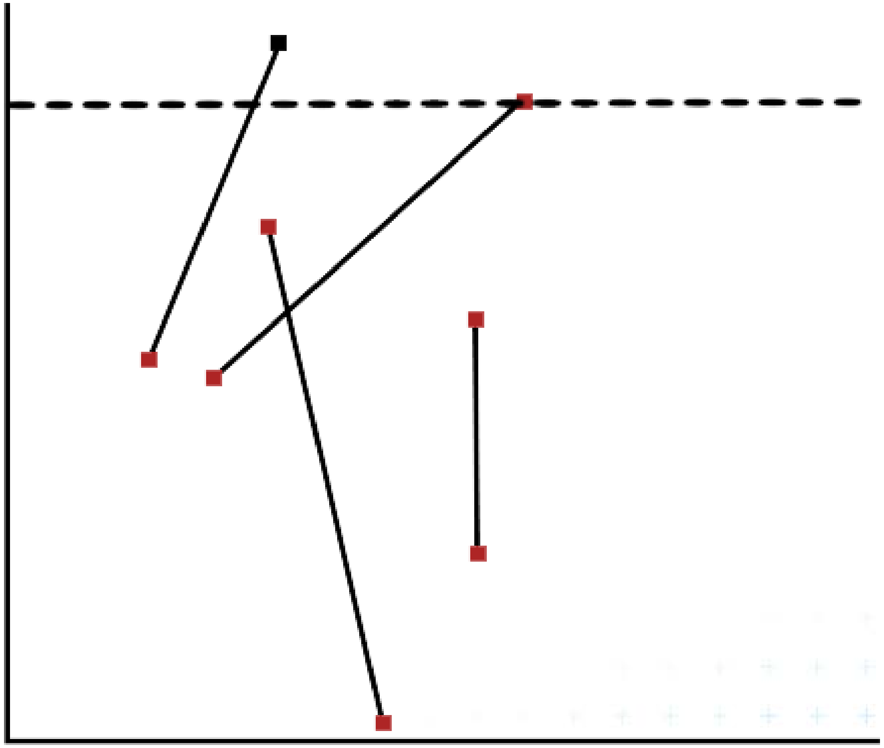

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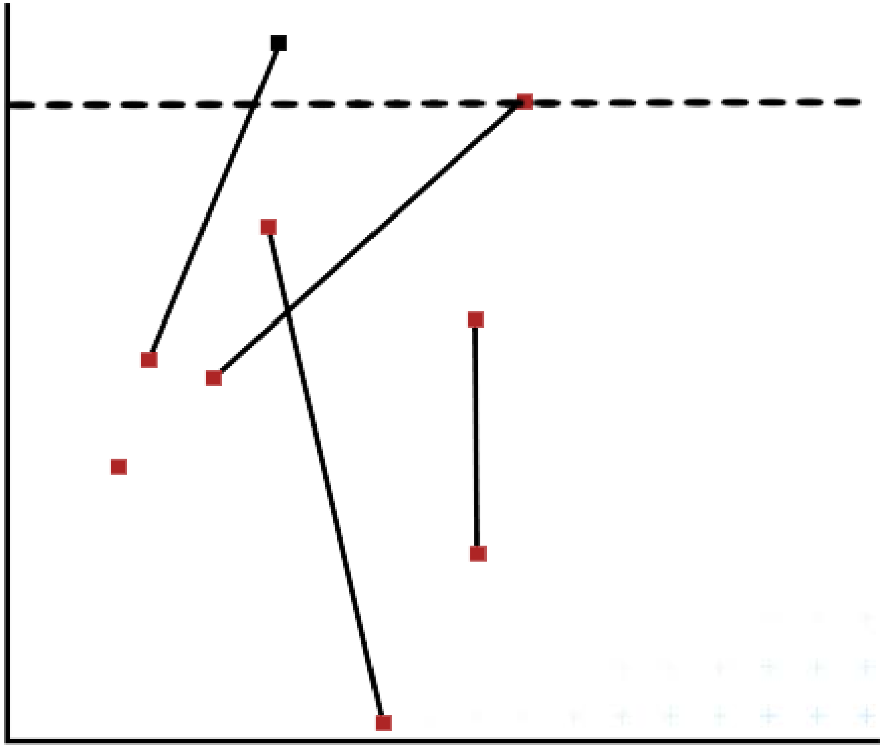

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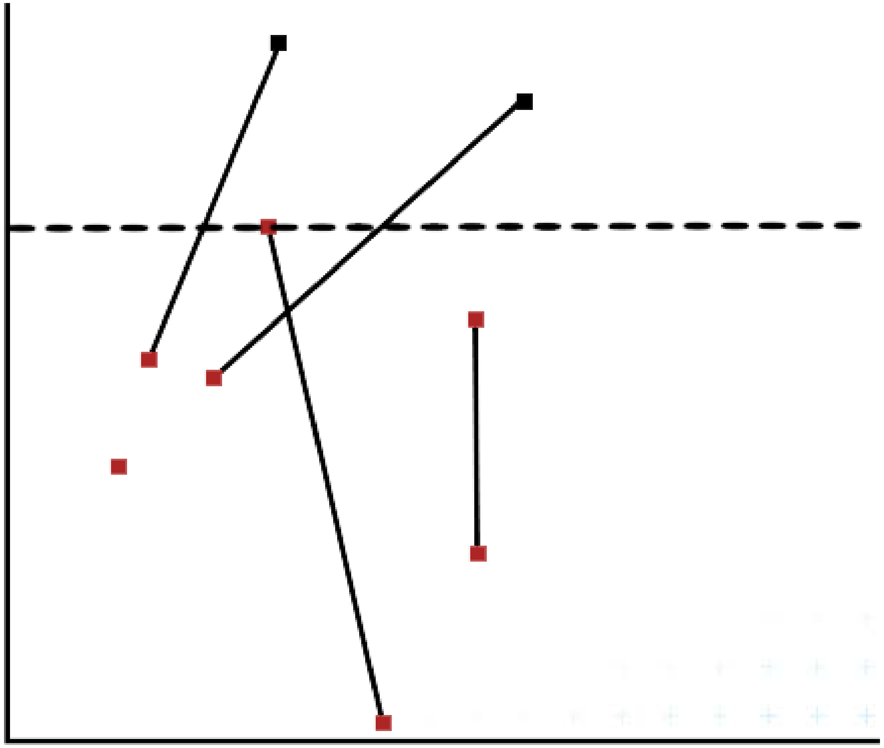

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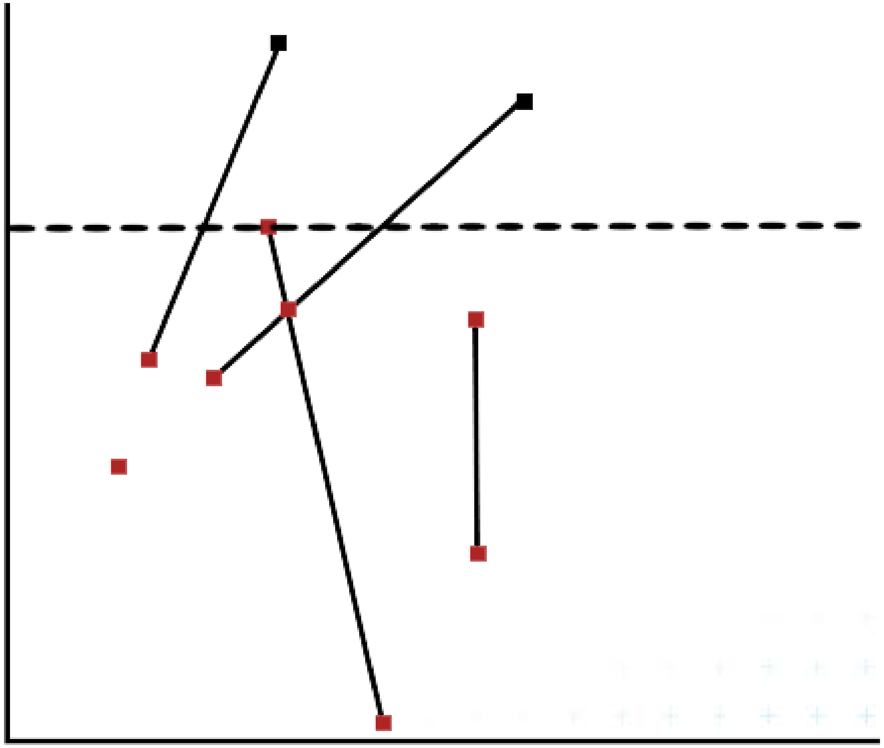

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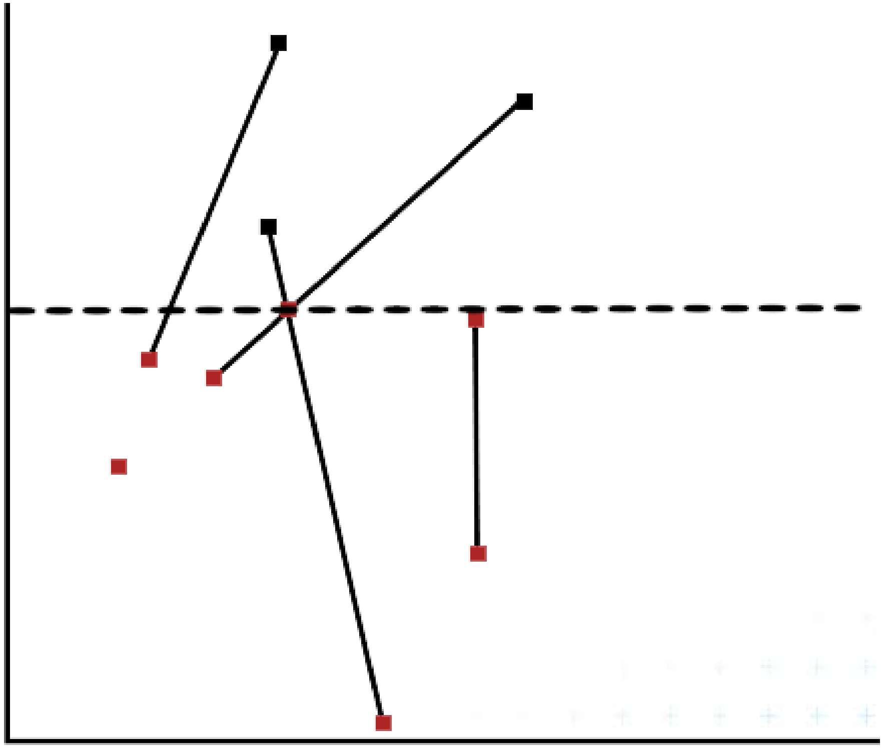

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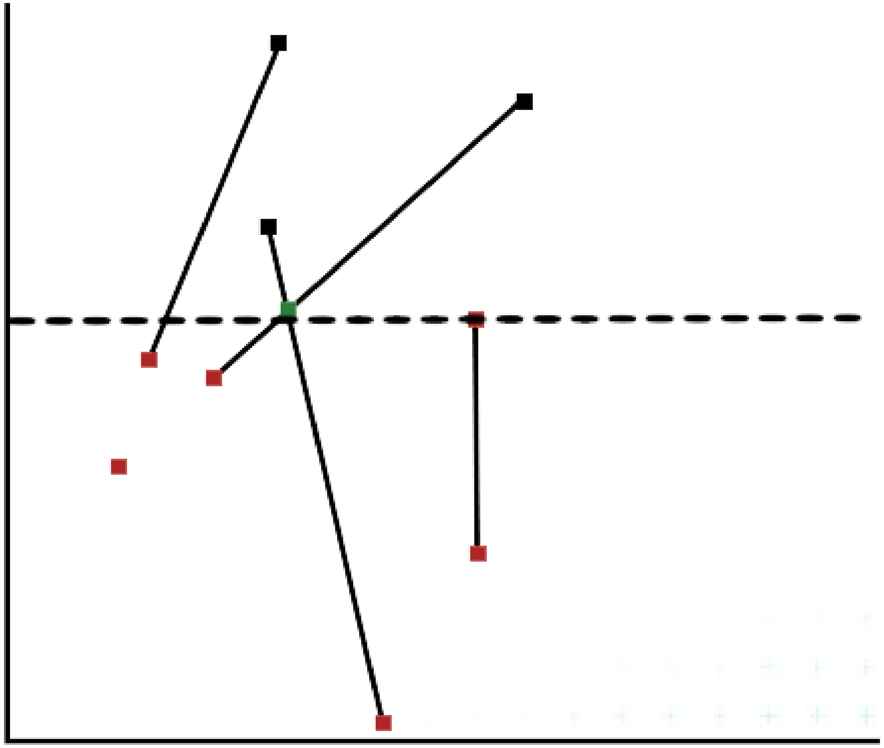

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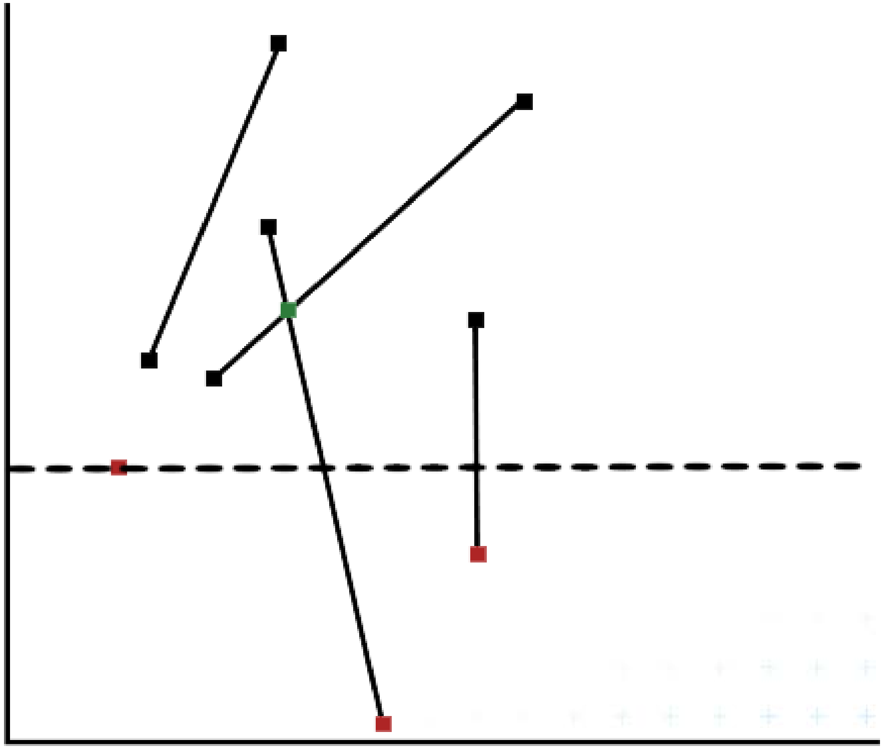

Plane sweep algorithm (to find intersections)

Create event queue Q with endpoints of all segments

(when an upper endpoint is inserted, the corresponding segment should be stored with it)

Initialize an empty status structure T.

while Q is not empty

Determine the next event point **p** in Q and delete it.

Let U(**p**) be the set of segments whose upper endpoint is **p**

Find all segments stored in **T** that contain **p**; they are adjacent in **T**

Let L(**p**) denote the subset of segments found whose lower endpoint is **p**

Let C(**p**) denote the subset of segments found that contain **p** in their interior

if L(**p**) ∪ U(**p**) ∪ C(**p**) contains more than one segment

then Report **p** as an intersection, together with L(**p**), U(**p**), and C(**p**)

Delete the segments in L(p) ∪ C(p) from T

Insert the segments in U(p) ∪ C(p) into T (below sweep, =reversing order of C(p))

if U(p) ∪ C(p) = empty

then Let sl and sr be the left and right neighbors of p in T

FindEvent(sl , sr, p) - (see below)

else Let s be the leftmost segment of U(p)∪C(p) in T

Let sl be the left neighbor of s in T

FindEvent (sl , s, p) – (see below)

Let s be the rightmost segment of U(p)∪C(p) in T

Let sr be the right neighbor of s in T

FindEvent (s, sr, p) – if intersect is below the sweep line and was not added yet- add event point to T

Plane sweep algorithm (to find intersections)

- Event queue *O(n log n)*
- Deletions, insertions and neighbor finding on Q take *O(log n)* time each
- The running time is O(n log n + I log n), where I is the number of intersections

Merging subdivisions

- Two subdivisions – *S1* and *S2*
- Looking for *O (S1, S2)*
- Using plane sweep algorithm
- Closed edges – like segments
- Preserve names

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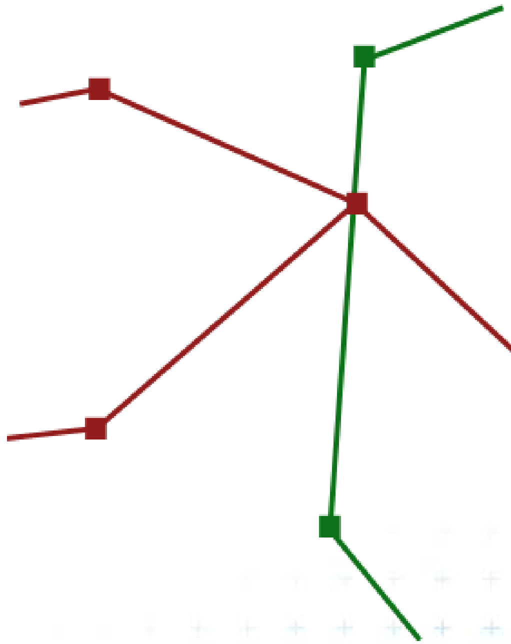
Merging subdivisions - edges

- Copy S1 and S2 into doubly-connected edge list D
  - Not valid, transform into O (S1, S2)
- T has edges; D has half-edges
- Intercesion points are counted when event involves edges of both subdivisions
- By dividing – the directions are preserved

Merging subdivisions - edges

- Two new edges; four new half-edges but

  two new records
- Set *Twin(e)*, *Next(e)*, *Last(e)*
- Linear time depending on degree of spliting point



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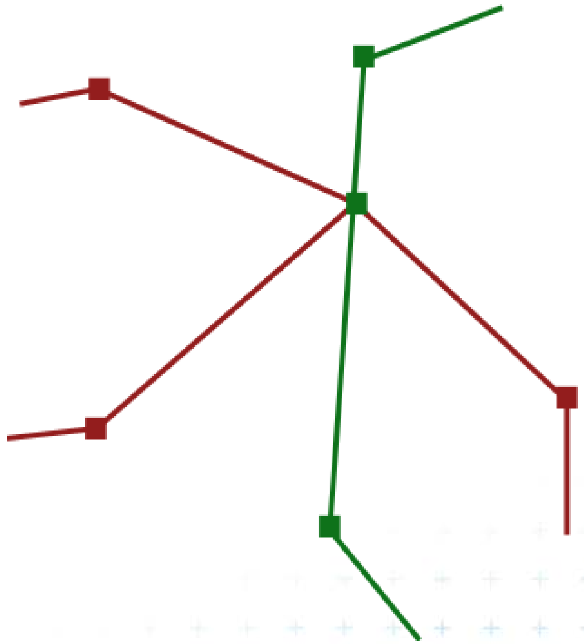

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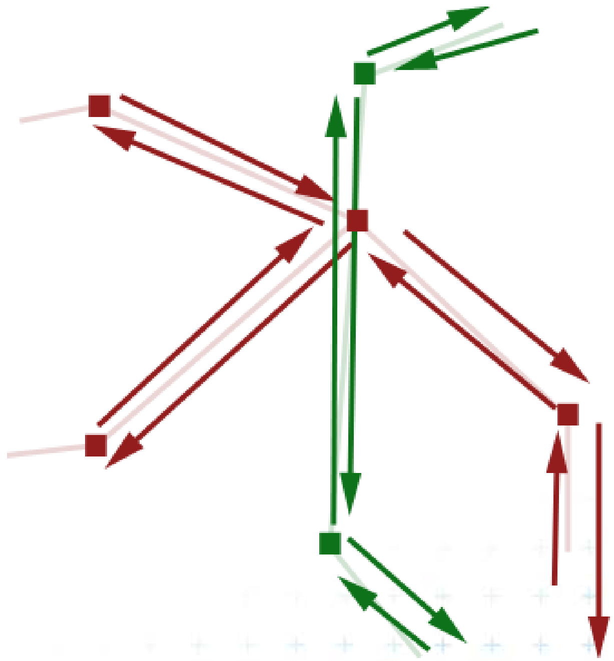

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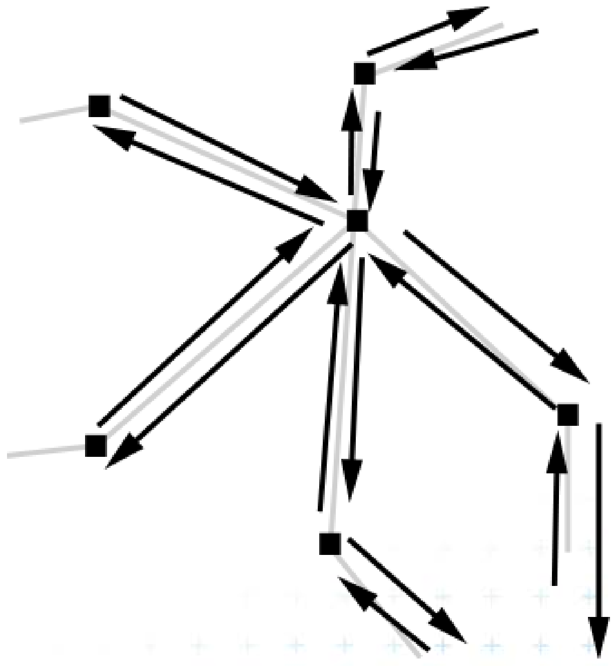
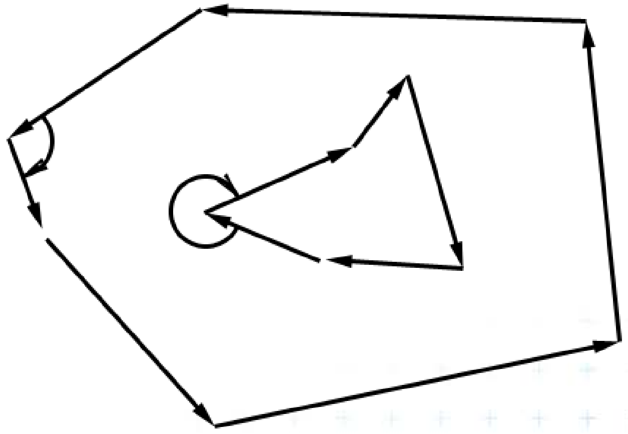


Merging subdivisions - faces

- New face records
- *OuterComponent(f),* set of *InnerComponents(f)* for new faces
- *IncidentFace()* for half-edges in their boundaries
- Label with the names of the faces in the old subdivisions that contain it.

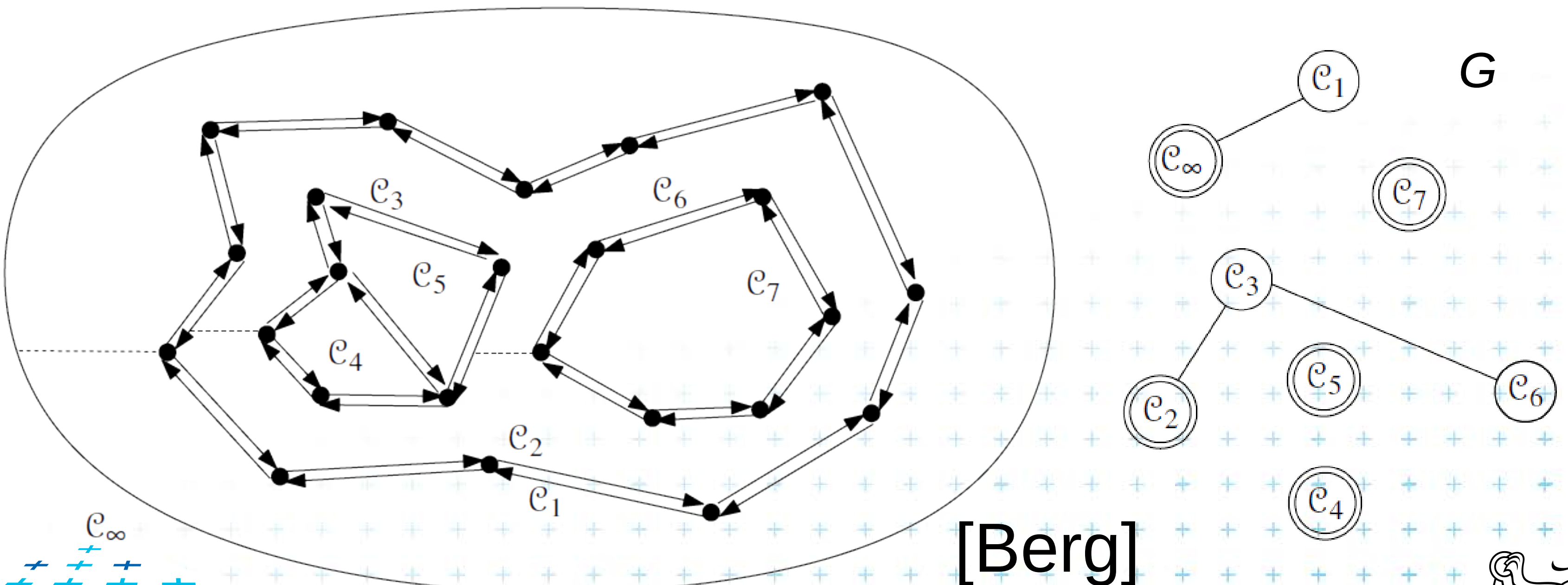
Merging subdivisions - faces

- Need to know holes of faces
- Cycle is hole, when on its leftmost vertex lies angle bigger than 180° (or the lowest when there are more of them)



Merging subdivisions - faces

- Graph *G*
  - Node is the boundary cycle
  - Connection if one of the cycles is the boundary of a hole and the other cycle has a half-edge immediately to the left of the leftmost vertex of that hole cycle



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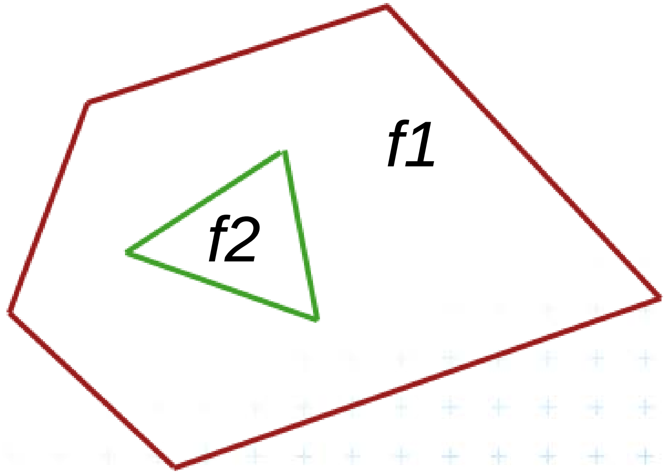
Merging subdivisions - faces

- Labeling faces
  - Need to know in which face from both merged subdivisions new face lies
  - Sweep line algorithm again

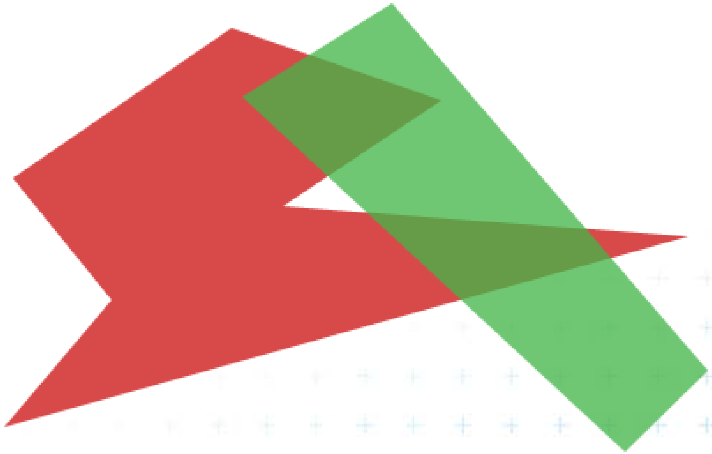


Merging subdivisions

- Copying list $O\ (n)$
- The plane sweep takes $O(n \log n + k \log n)$
- Fill in the face records takes time linear in the complexity of $O(S1,S2)$
- Labeling takes $O(n \log n + k \log n)$
- Construction in $O(n \log n + k \log n)$, where k is the complexity of the overlay

Boolean operations

- Labels of faces
  - Which face did belonge which subdivision of the operation
- Operations: union, difference, intersection



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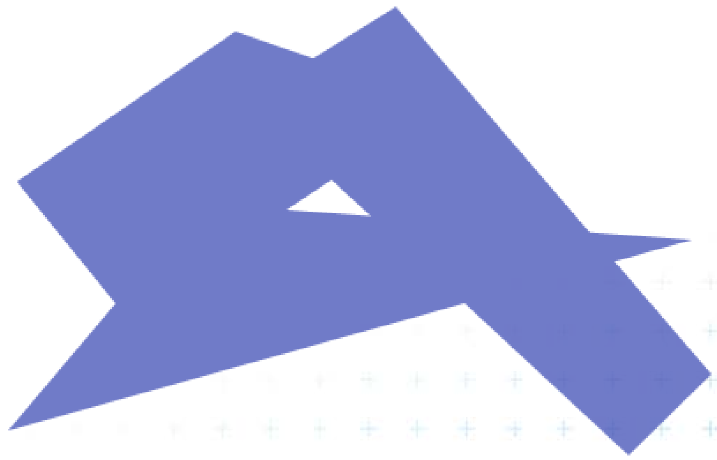

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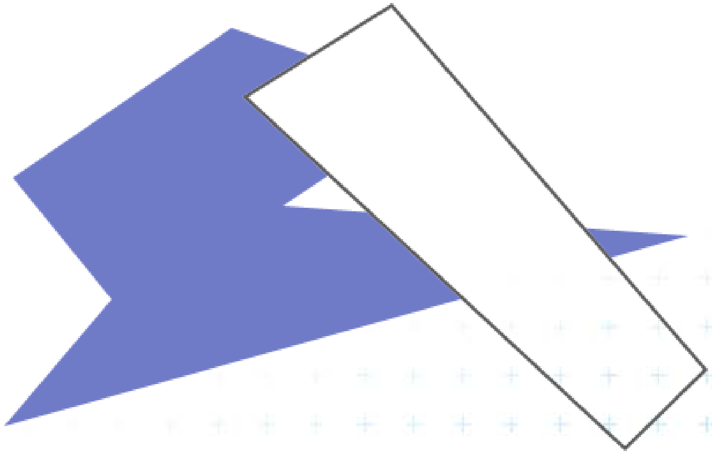

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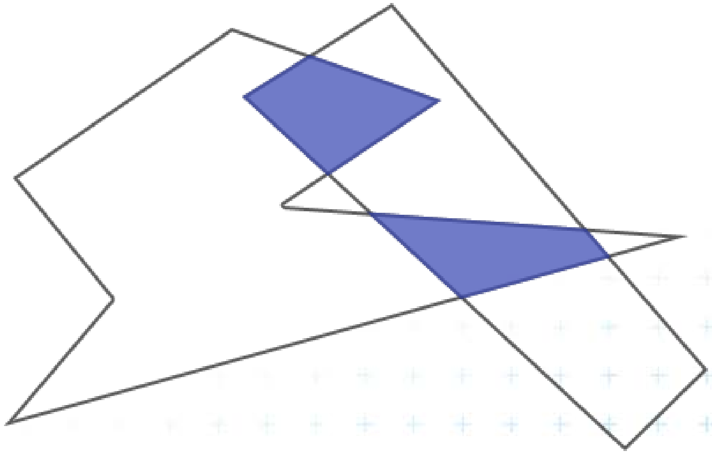

Thank for your attention

Radek Smetana, 7. 11. 2012

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Sources

- **[Berg] Mark de Berg**, Otfried Cheong, Marc van Kreveld, Mark Overmars: Computational Geometry: Algorithms and Applications, Springer- Verlag, 3rd rev. ed. 2008. 386 pages, 370 fig. ISBN: 978-3-540-77973-5, Chapter 2
- **Jiří Žára**, slide template

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