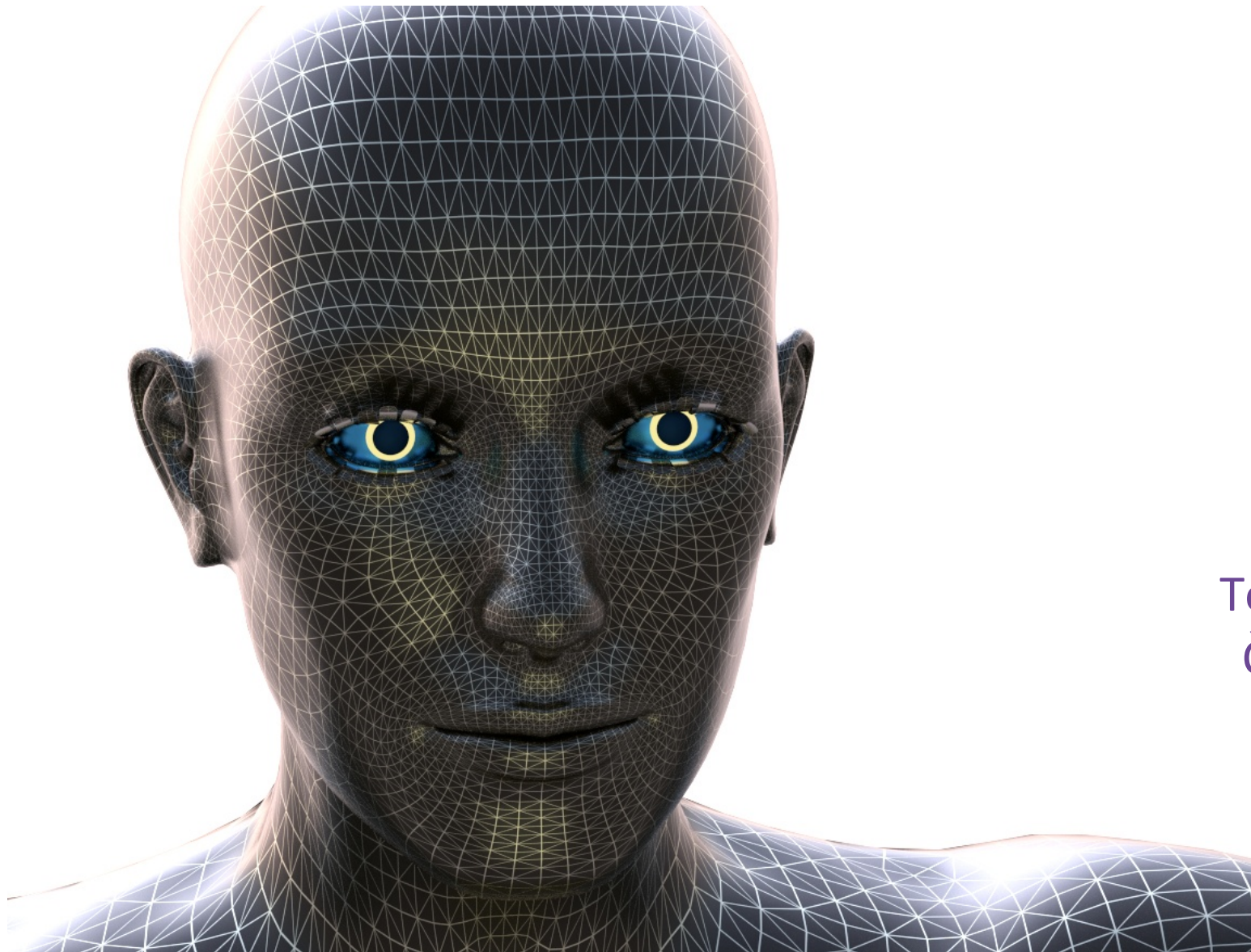




**OI-OPPA. European Social Fund
Prague & EU: We invest in your future.**

DeWall algorithm



Tomáš Buk,
ČVUT FEL

Table of contents

- Terminology
- Delauney triangulation
- DeWall algorithm
- Step by step tutorial
- InCoDe algorithm
- Remarks

Vocabulary

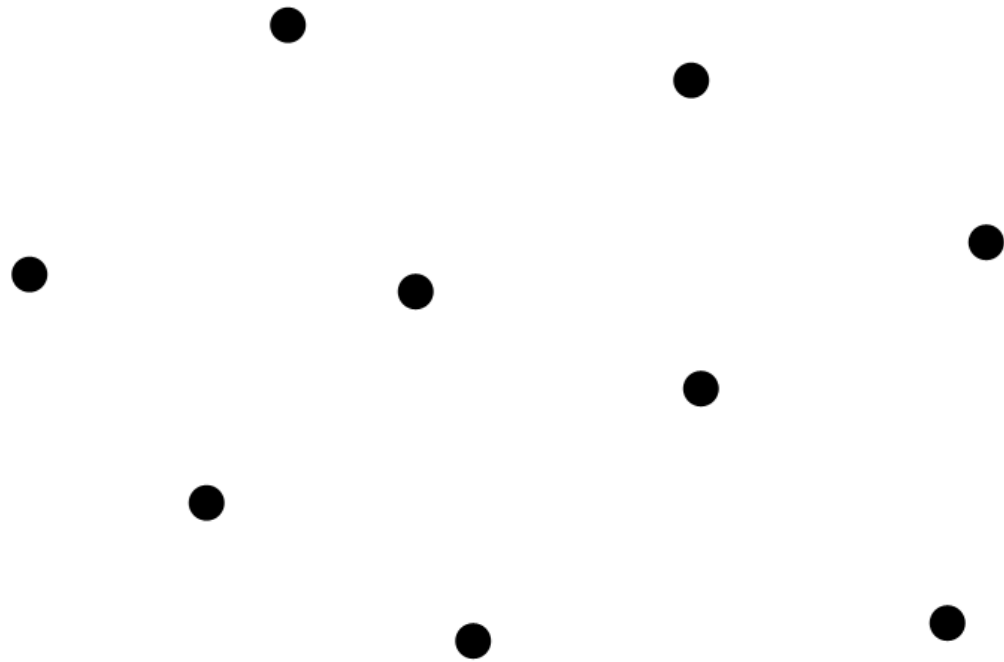
- Simplex, triangle
- 1-face, 2-face...
- Circumscribed sphere

Delaunay triangulation

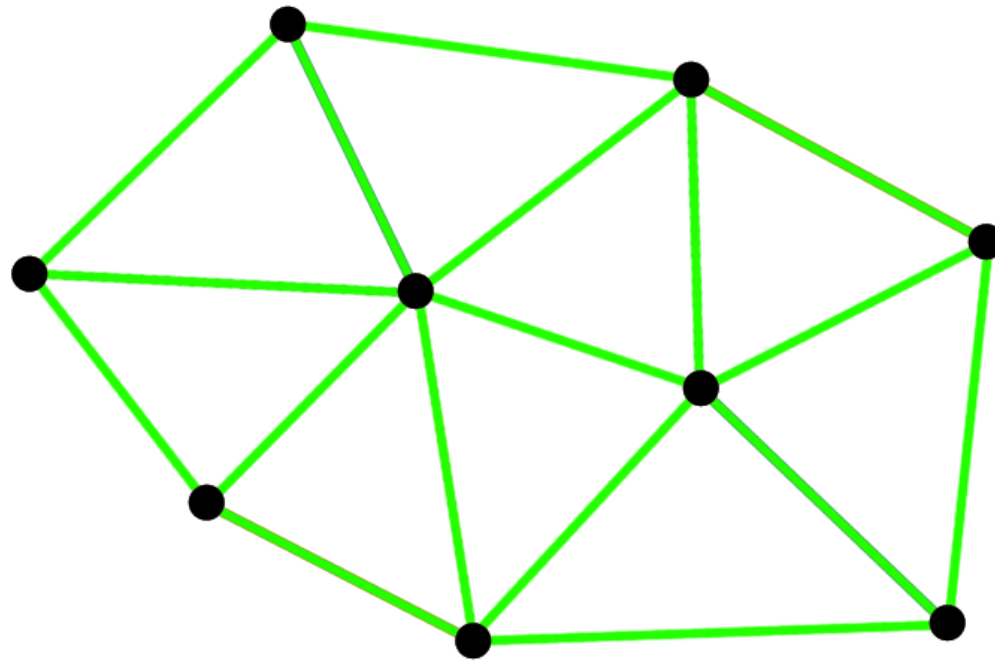
- Pointcloud visualization
- Set of adjacent triangles build over given points P
- No point of P inside the circumcircle of any triangle
- Maximizes the minimum interior a triangles (no skinny triangles)
- Author – Boris Delaunay (1934)



Set of given points...



... and the resulting geometry

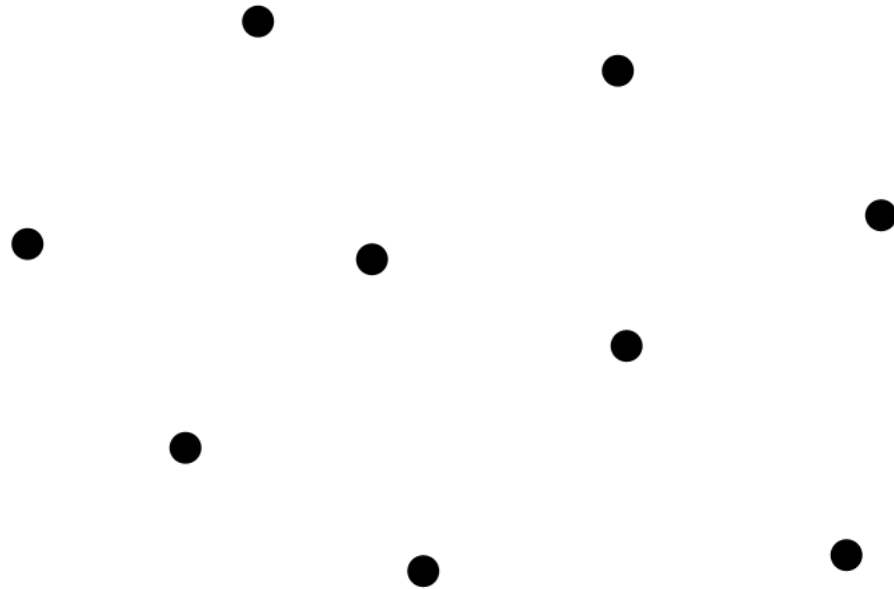


But how??

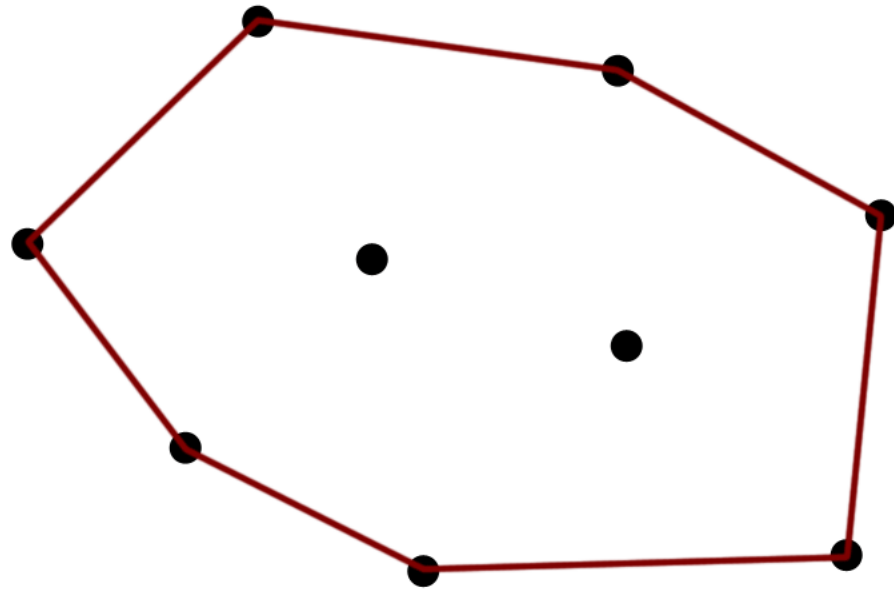
DeWall algorithm (D & C)

- (1) Compute the CH(P)
- (2) Select the splitting plane α
- (3) Split P into subsets P_1 and P_2 and construct triangulation Σ_α
- (4) Recursively apply DeWall on P_1 and P_2 to build triangulation Σ_1 and Σ_2
- (5) Return the union of Σ_α , Σ_1 and Σ_2

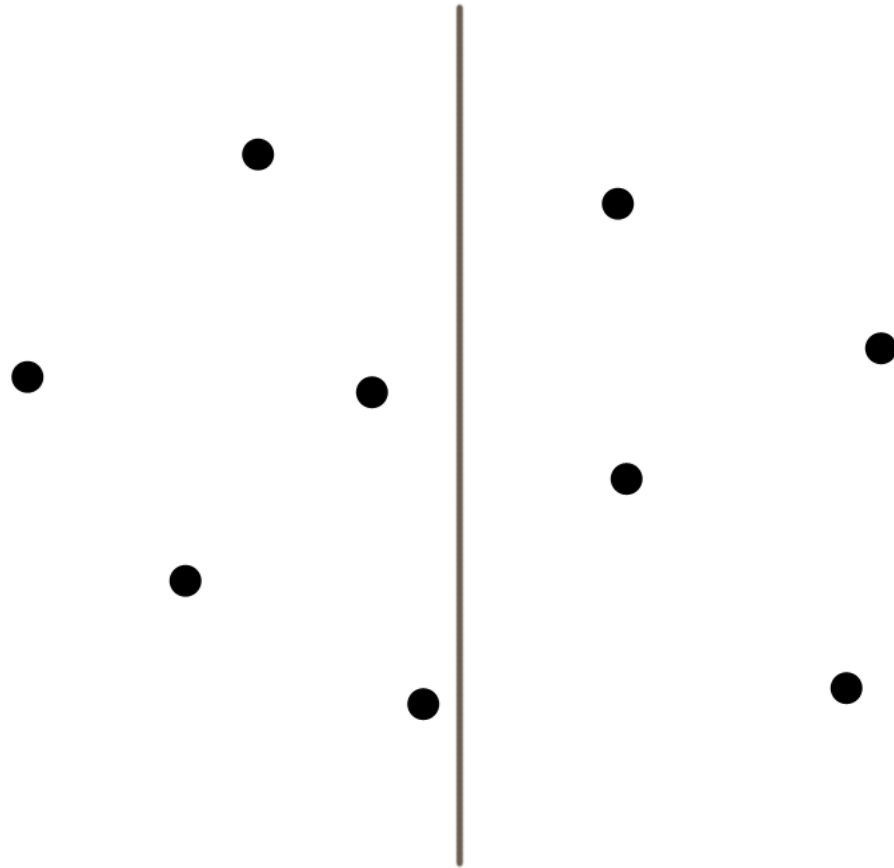
Step by step example



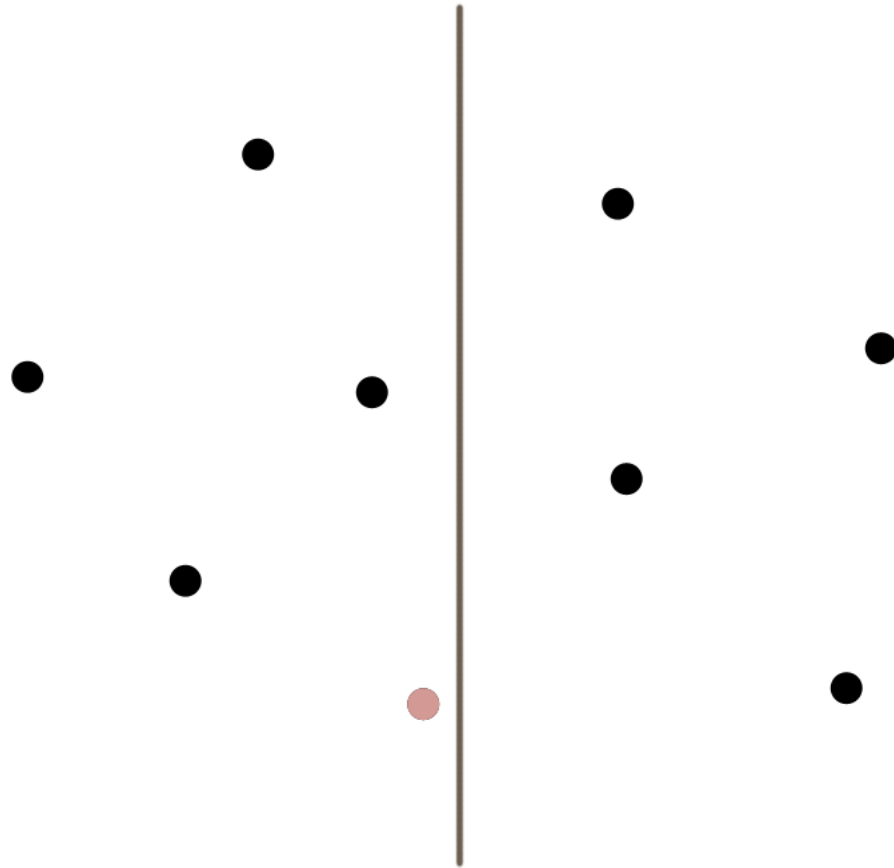
Step by step example



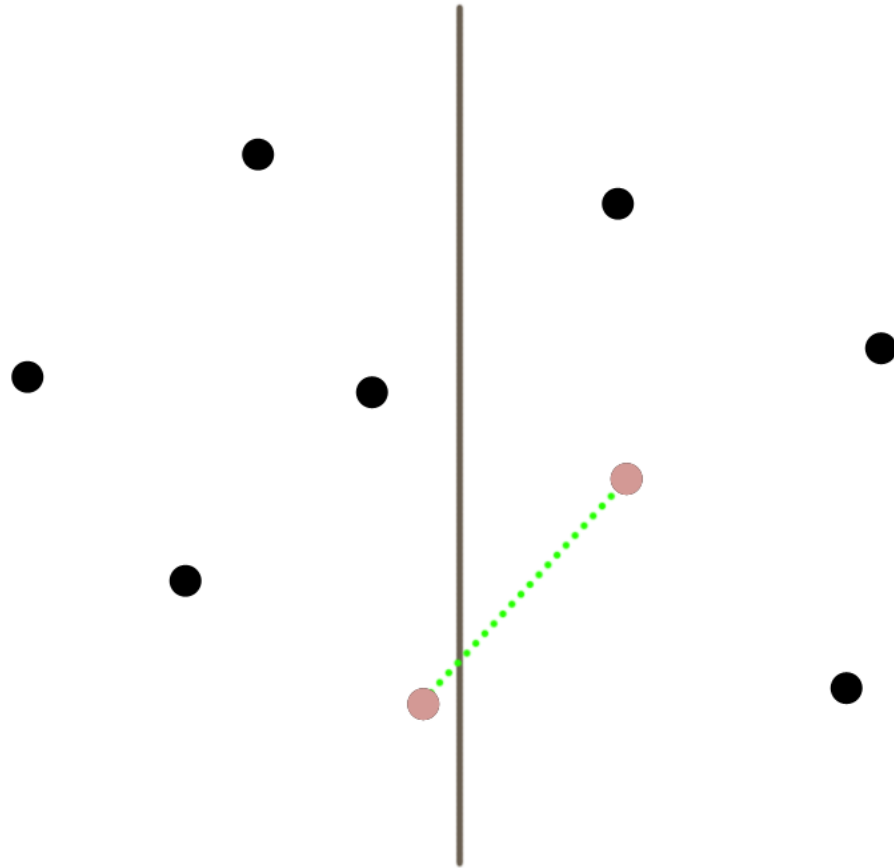
Step by step example



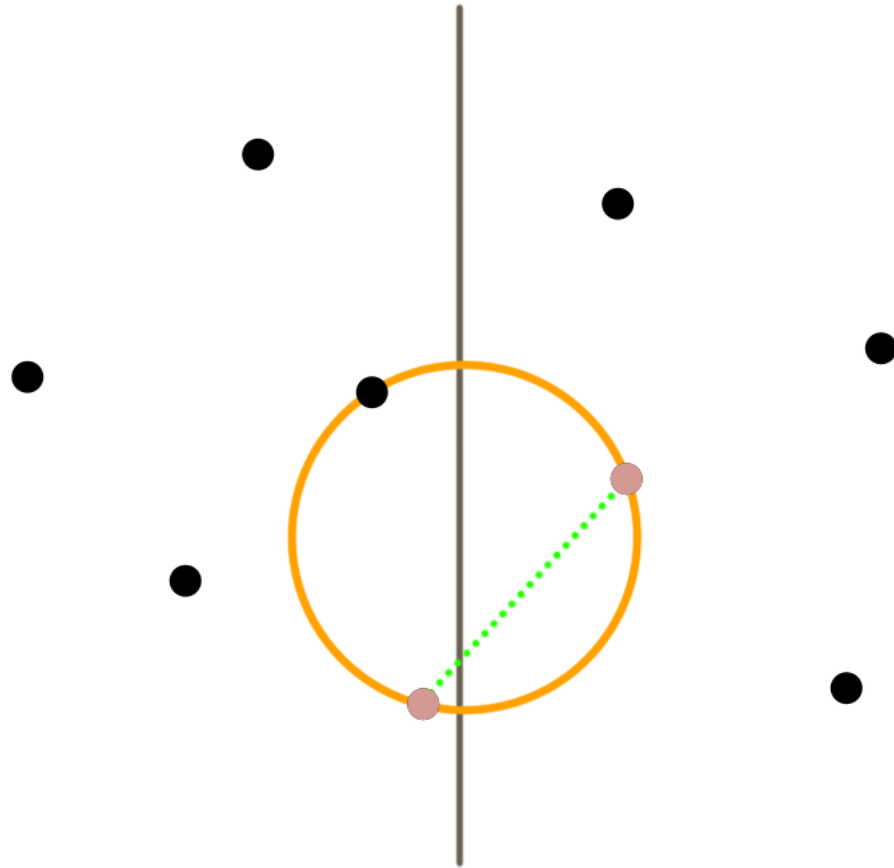
Step by step example



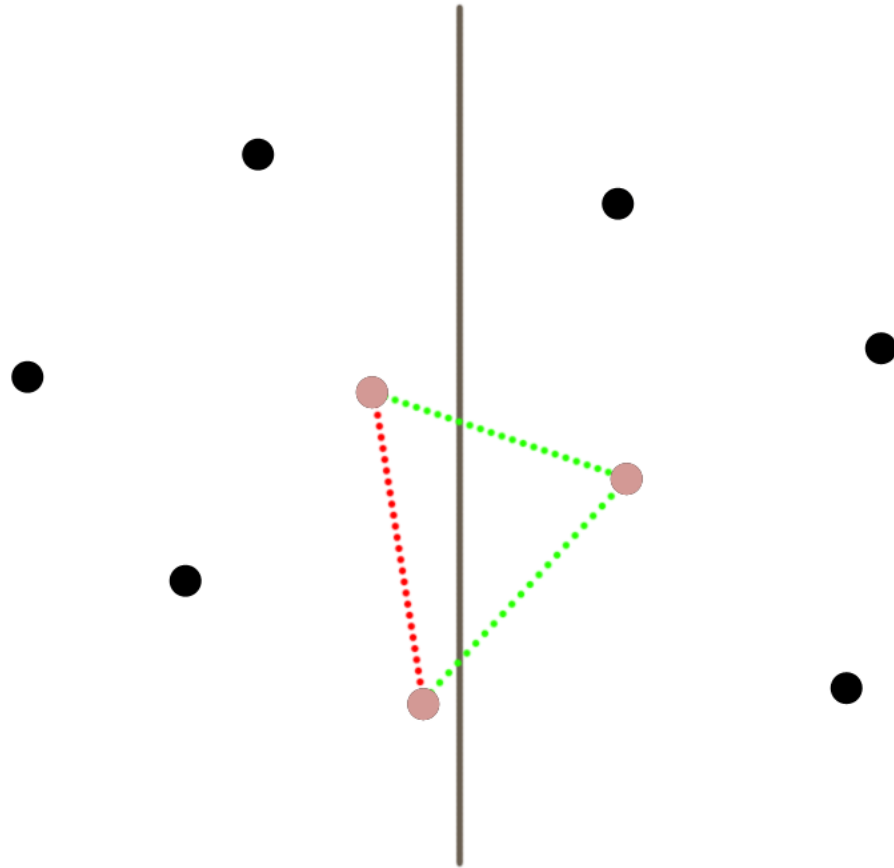
Step by step example



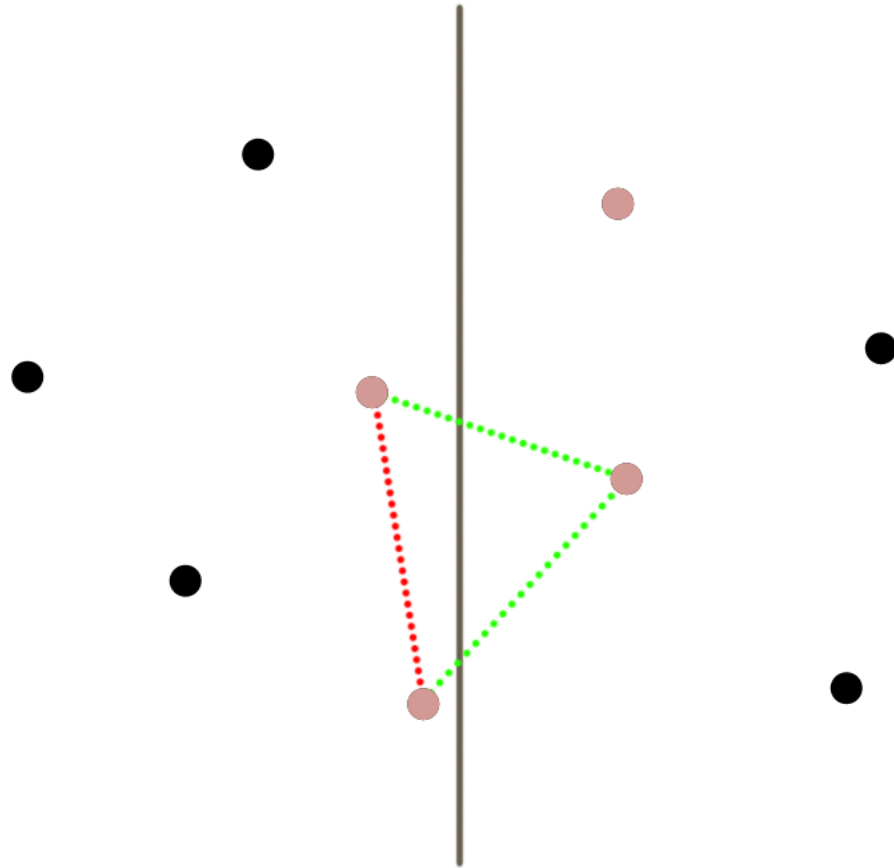
Step by step example



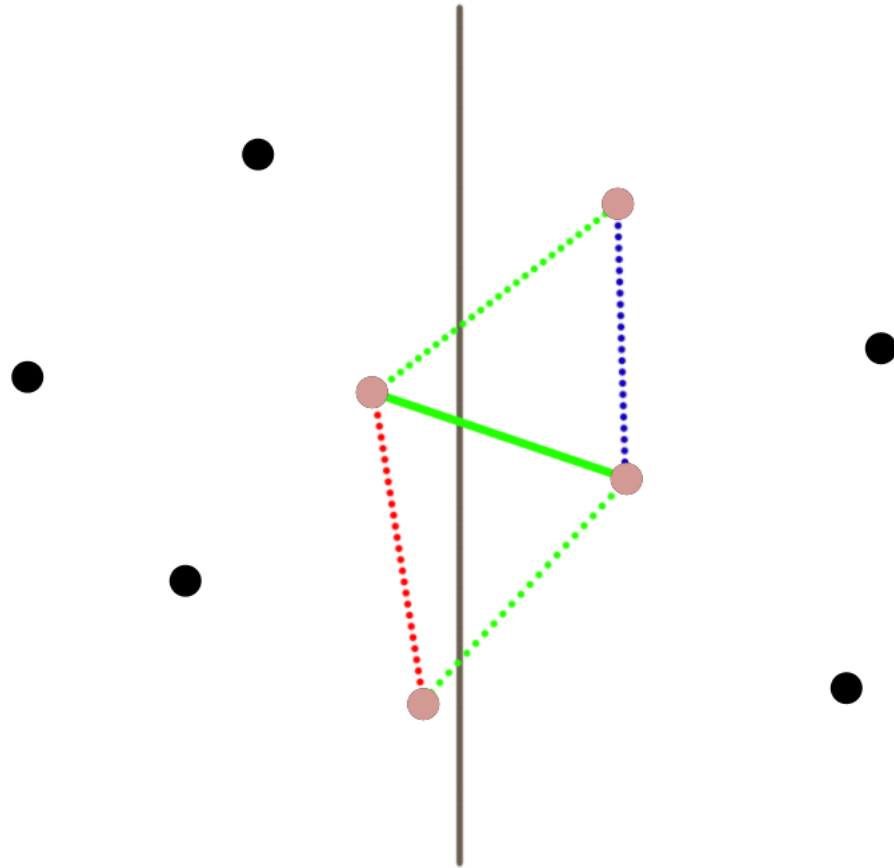
Step by step example



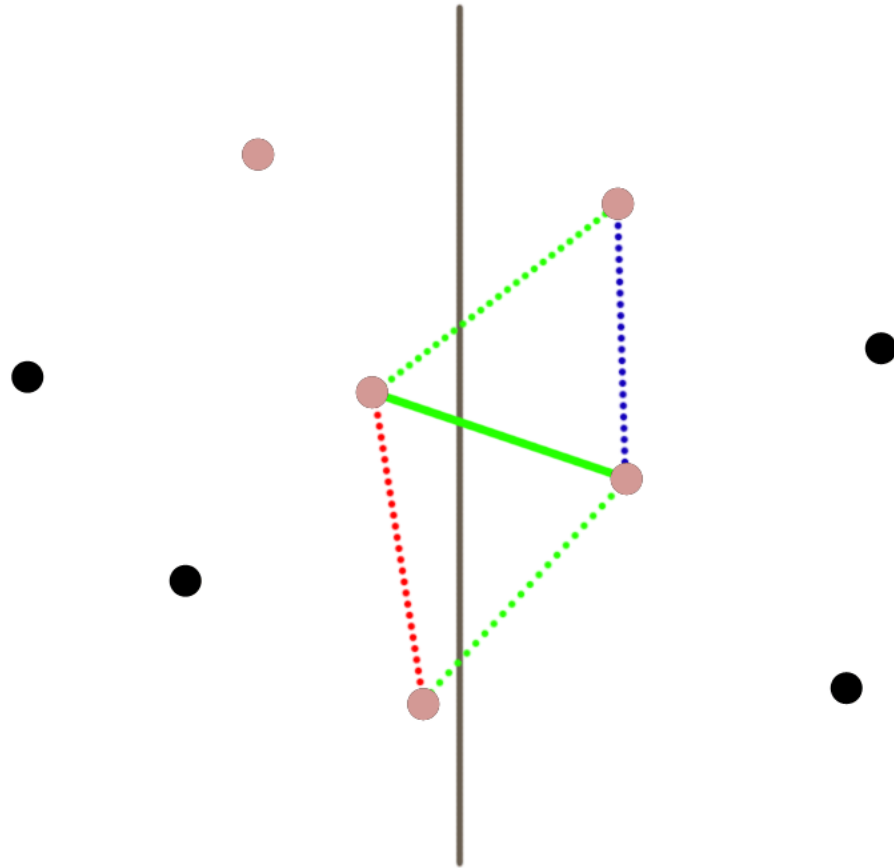
Step by step example



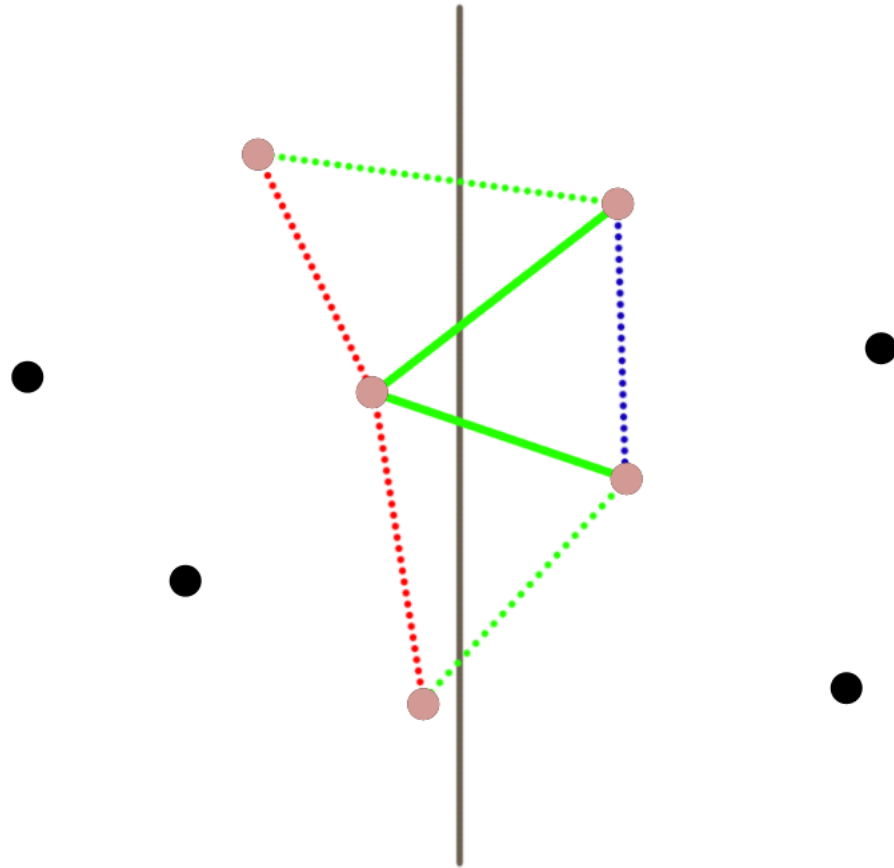
Step by step example



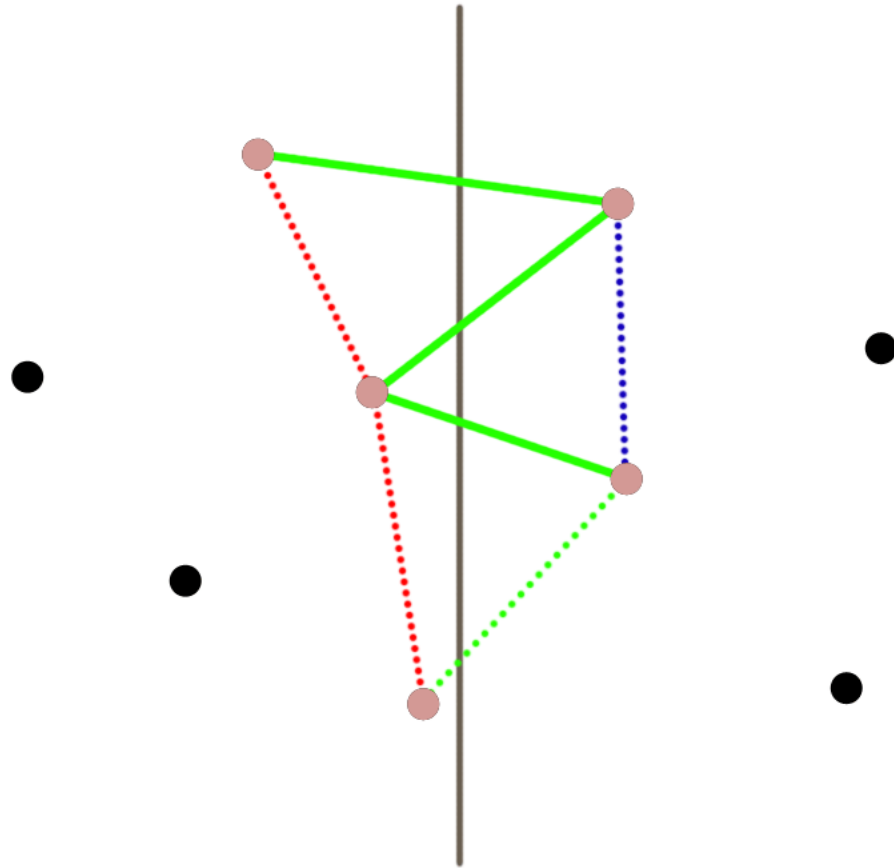
Step by step example



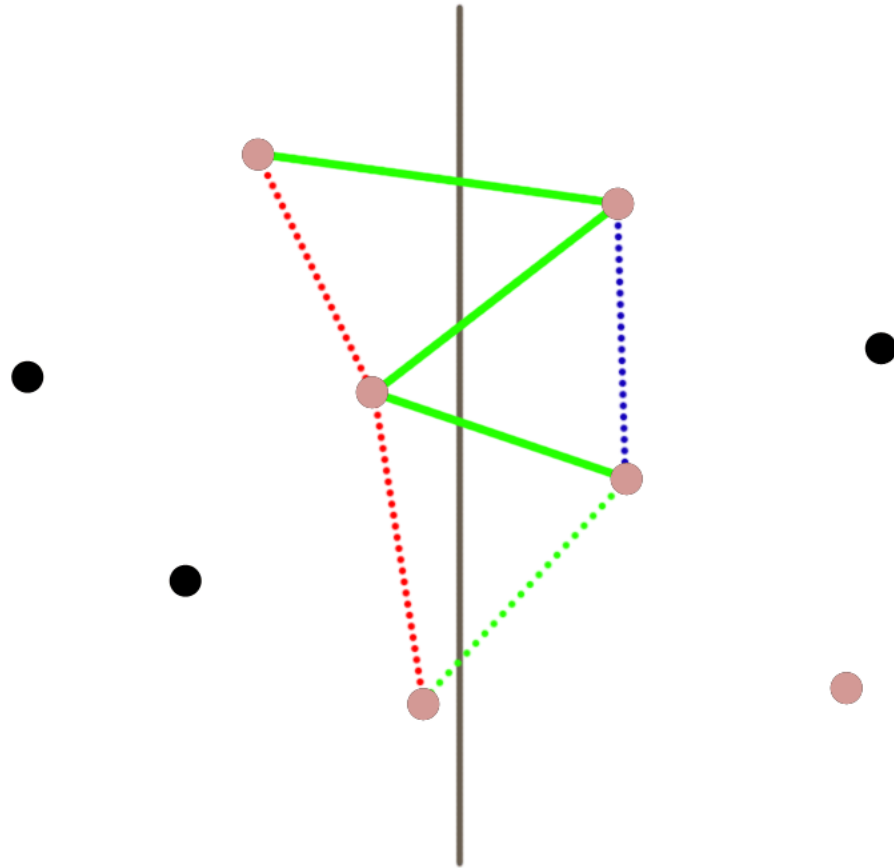
Step by step example



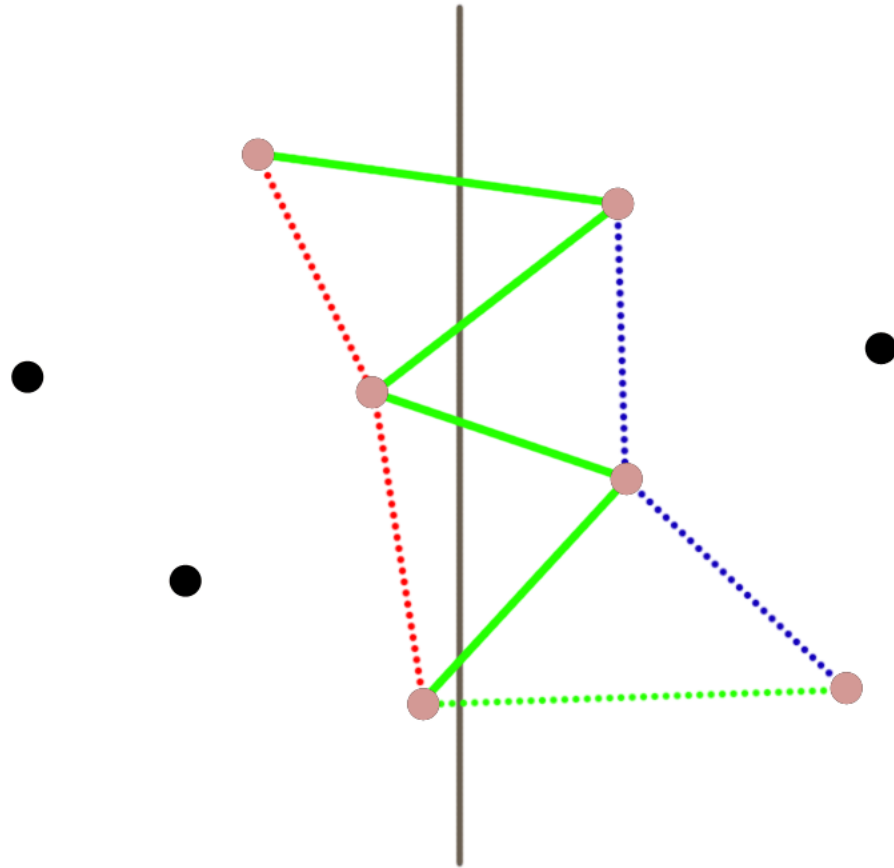
Step by step example



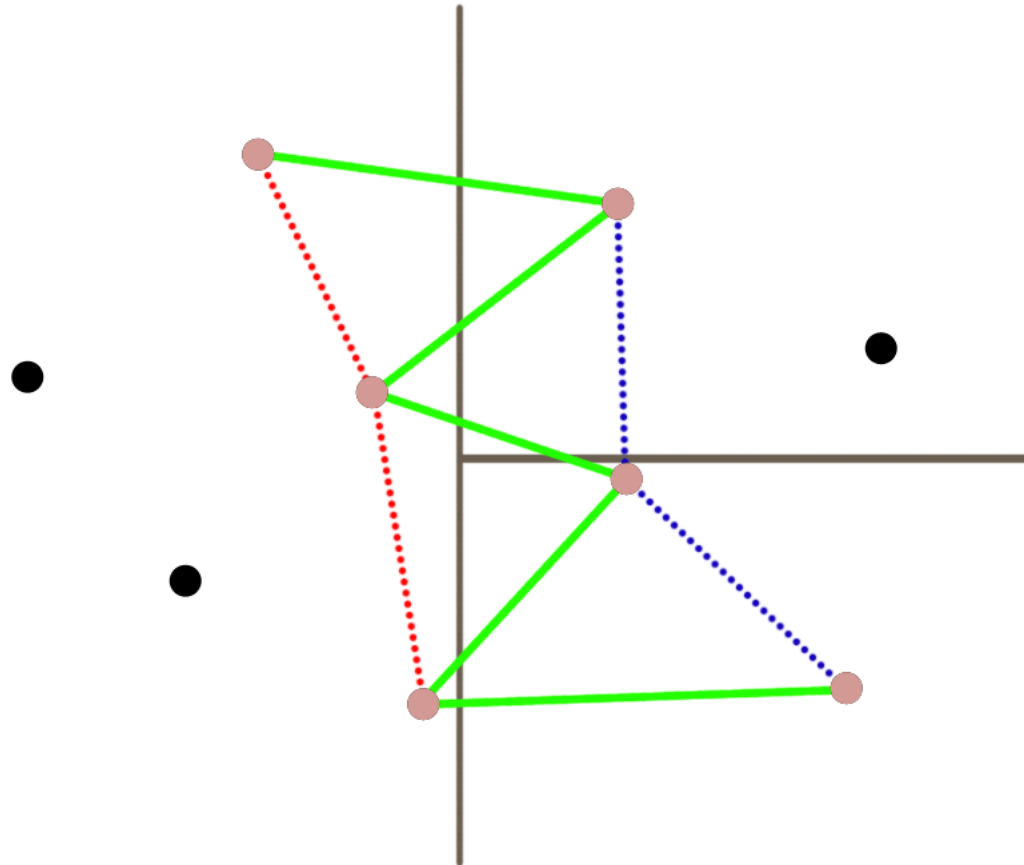
Step by step example



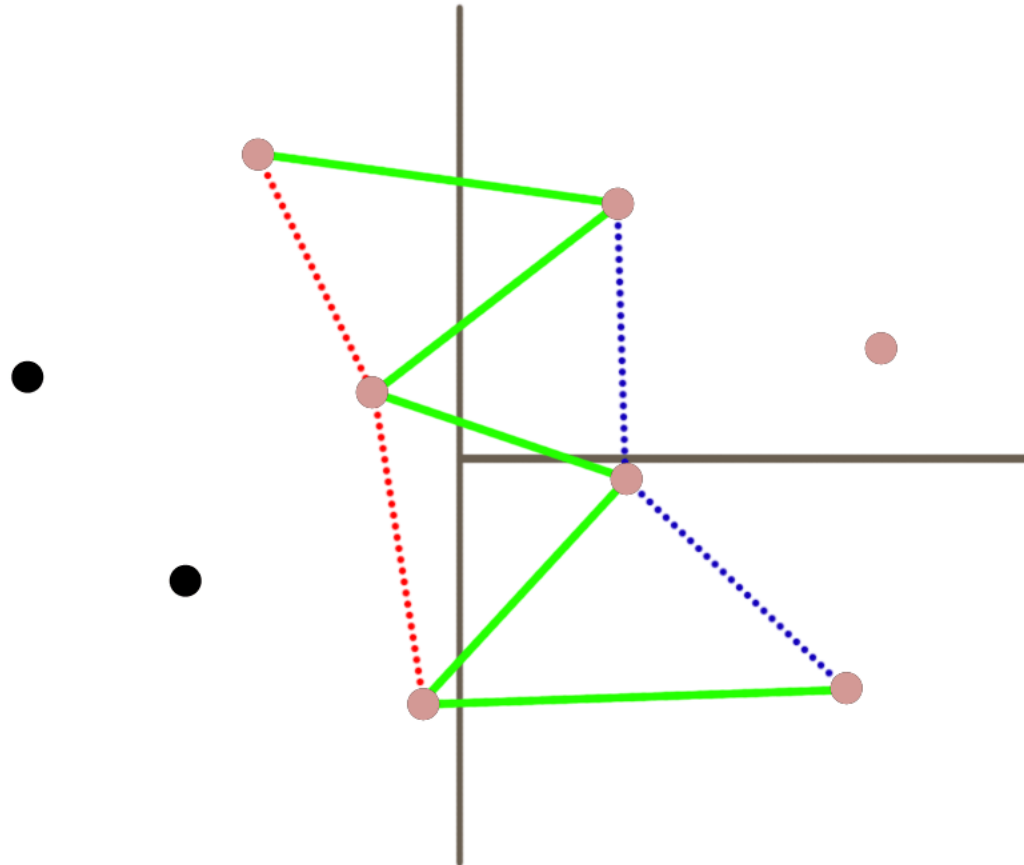
Step by step example



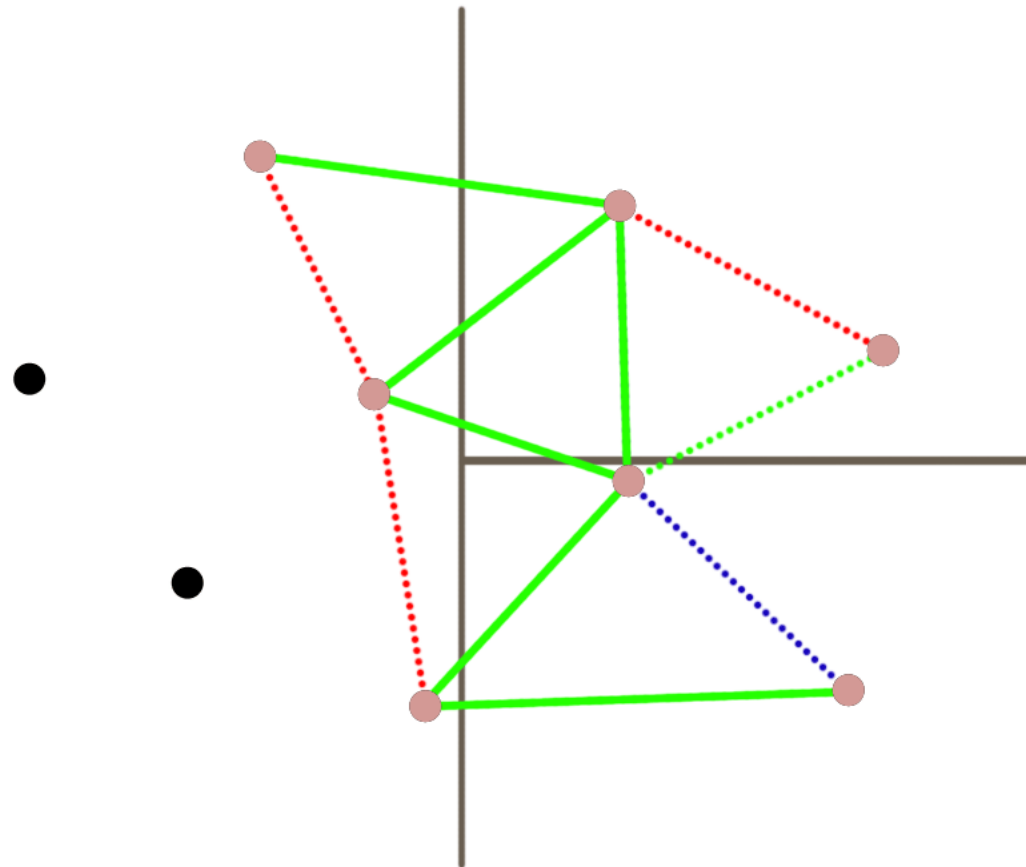
Step by step example



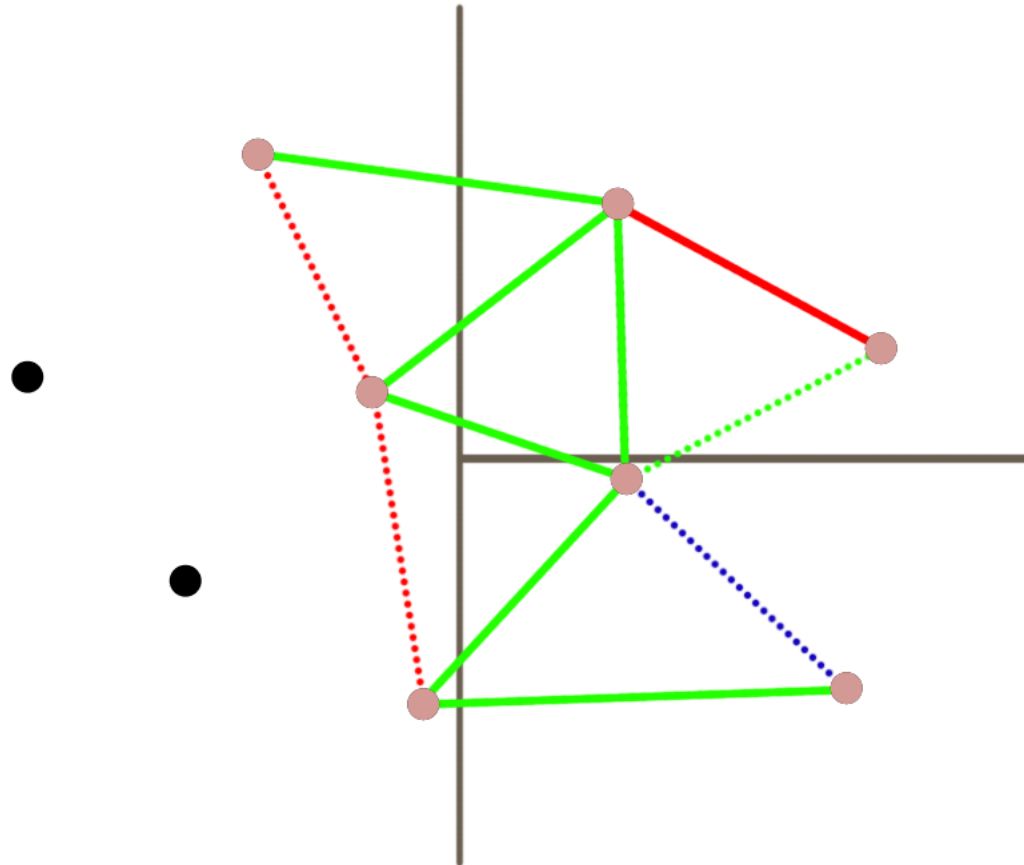
Step by step example



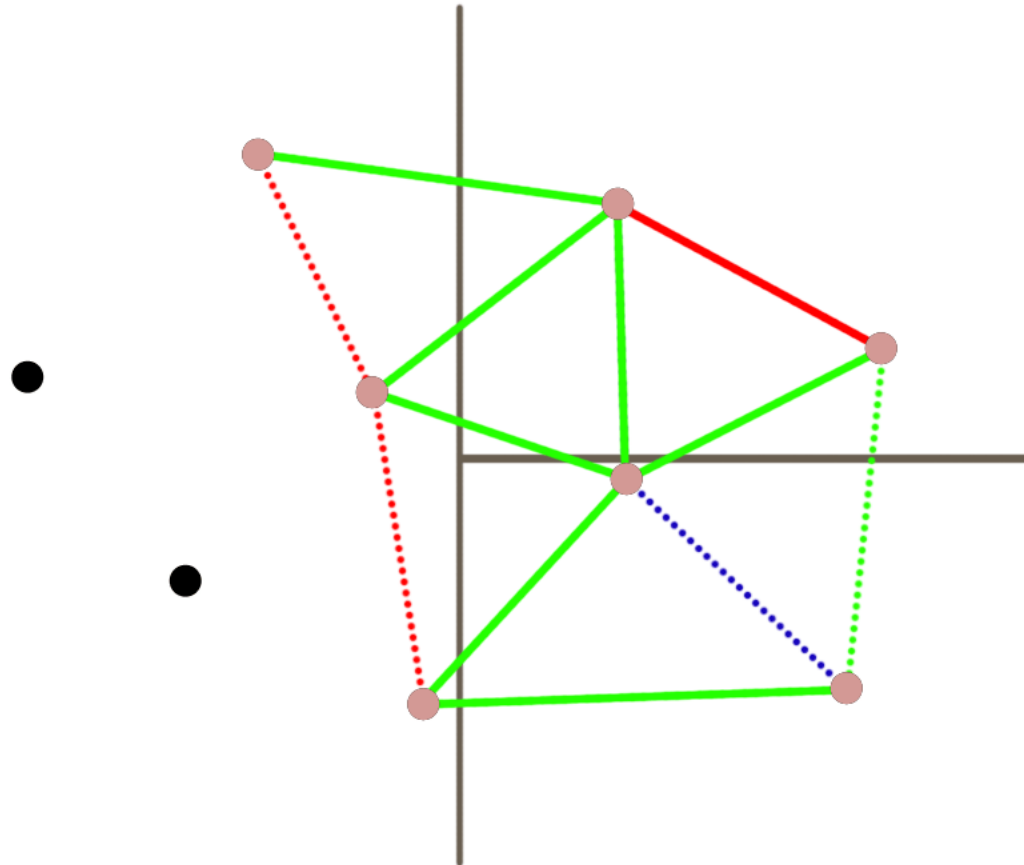
Step by step example



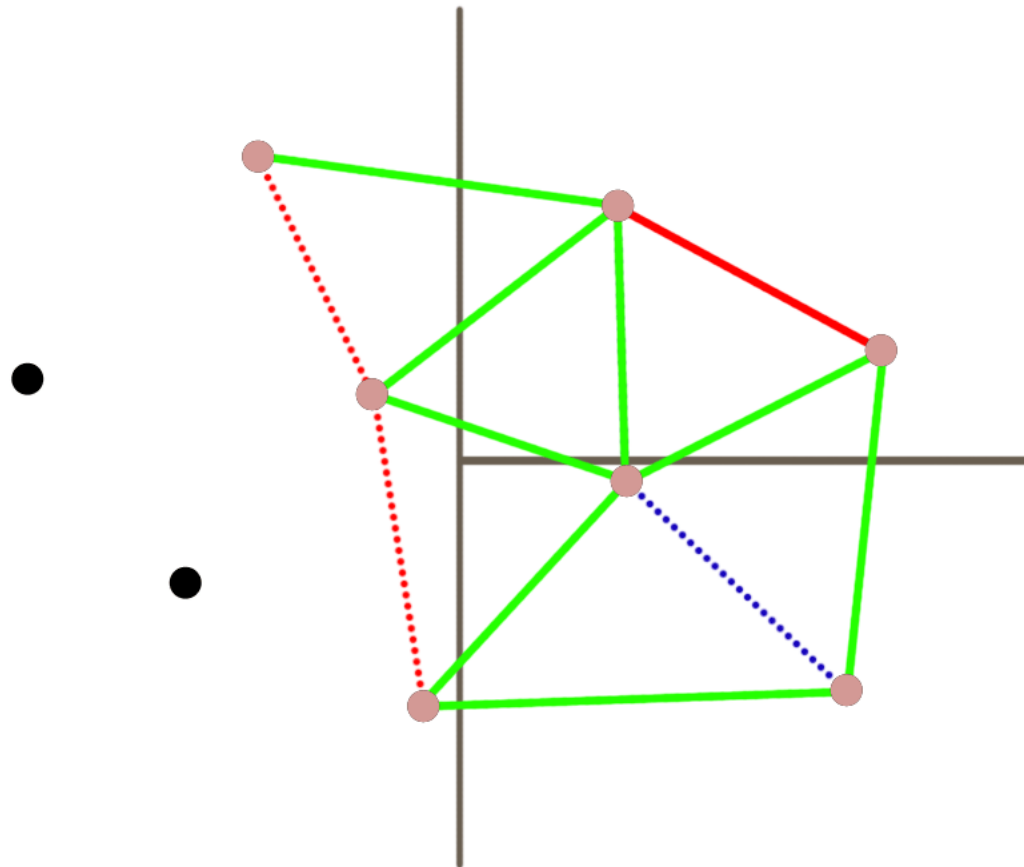
Step by step example



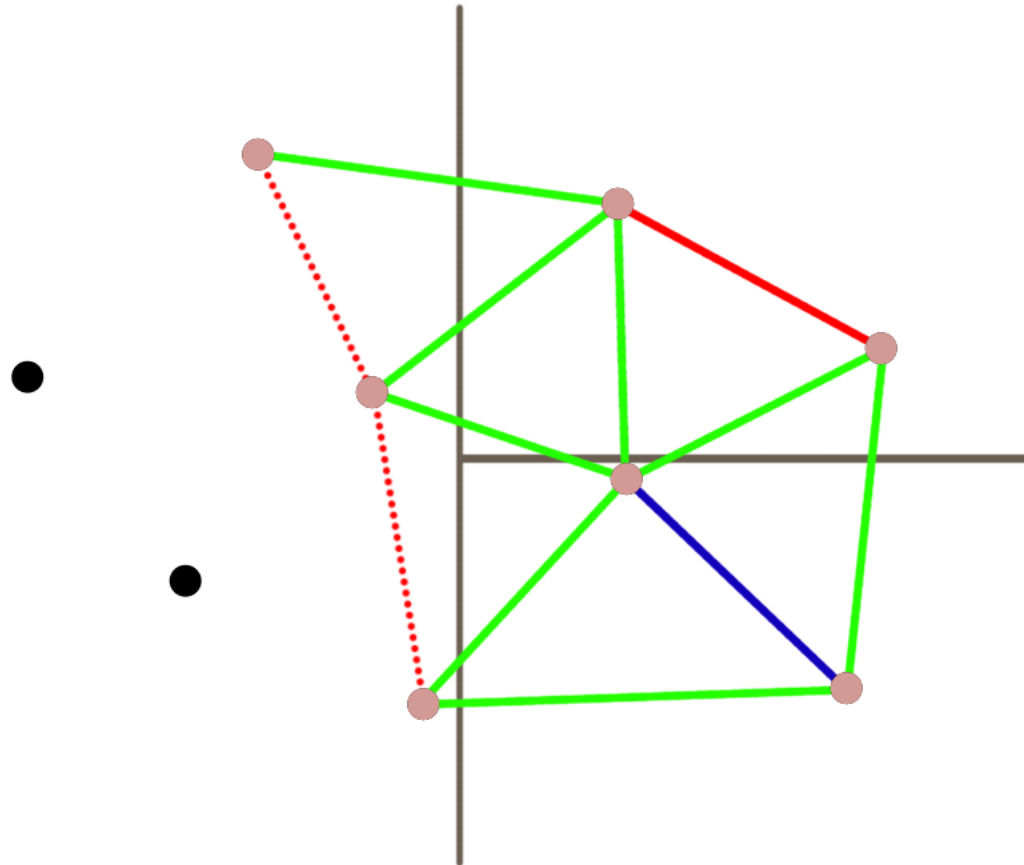
Step by step example



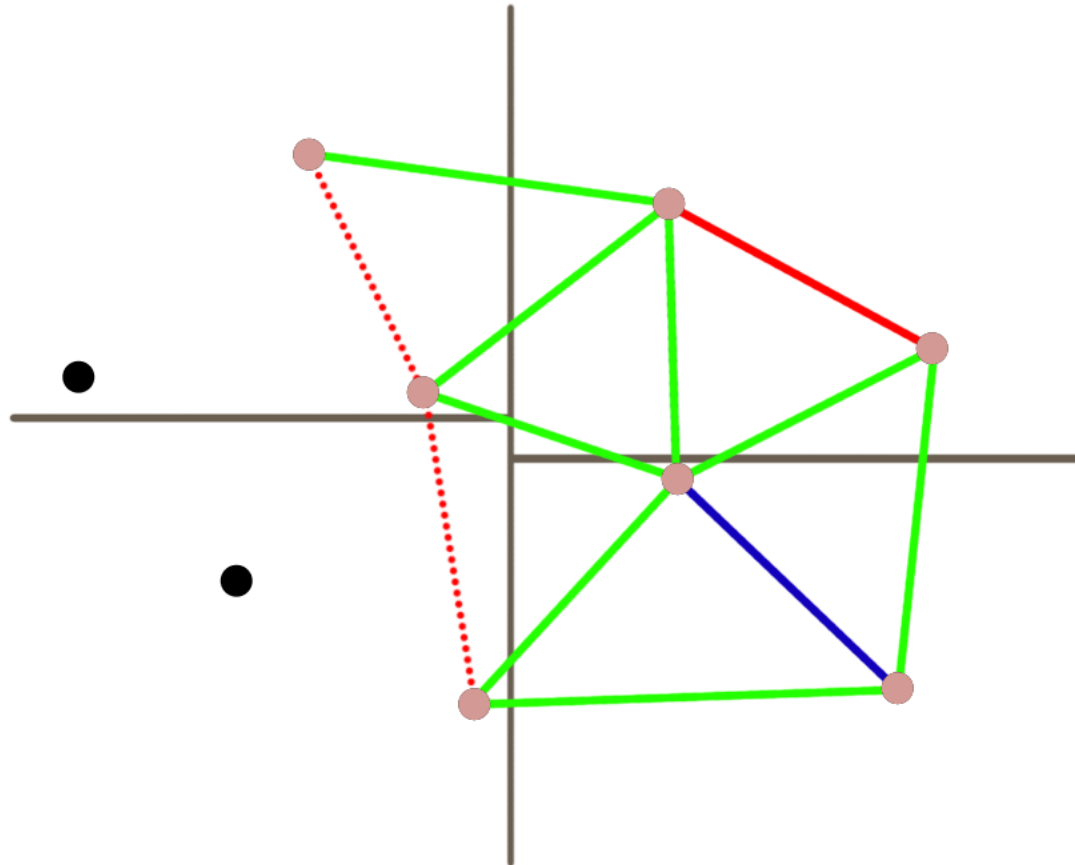
Step by step example



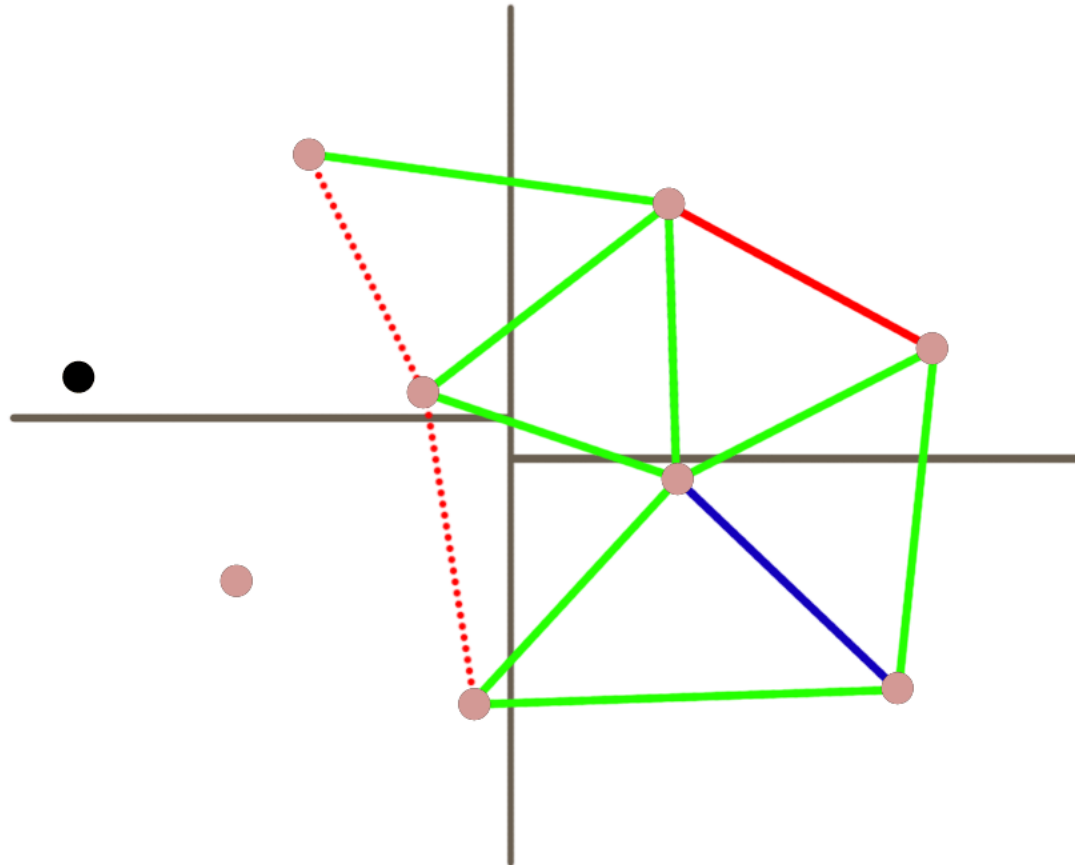
Step by step example



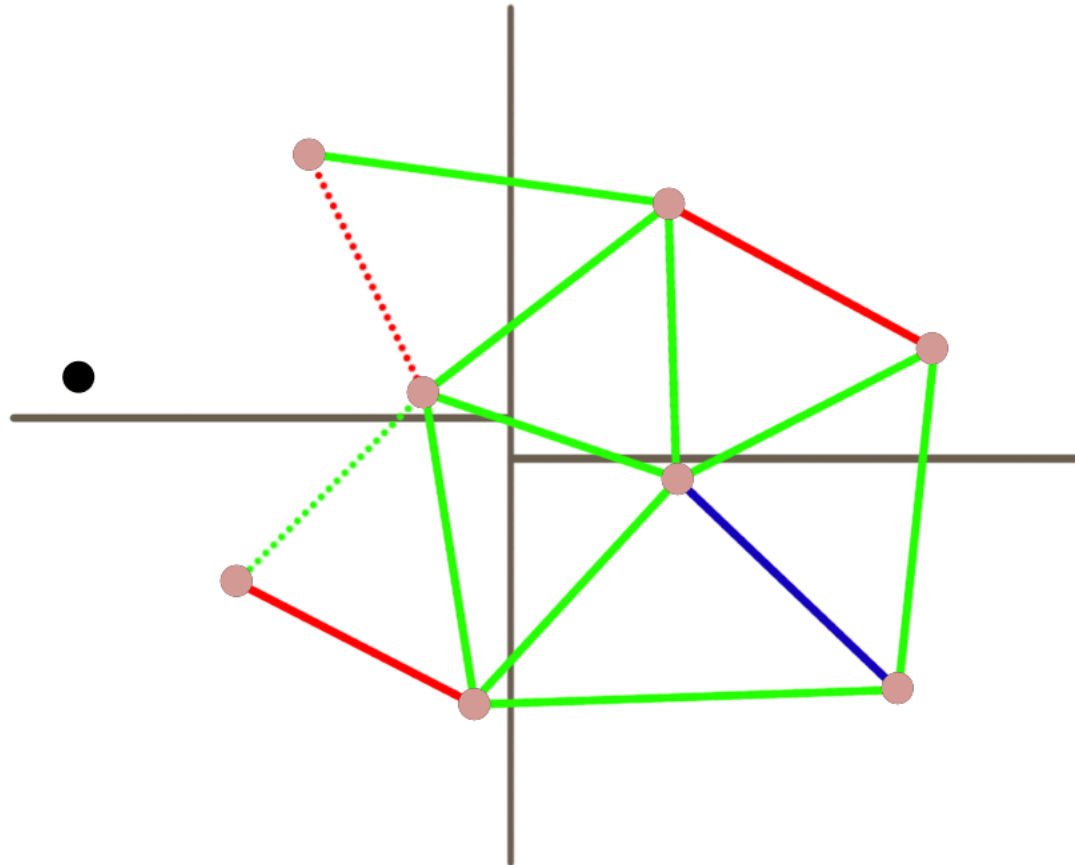
Step by step example



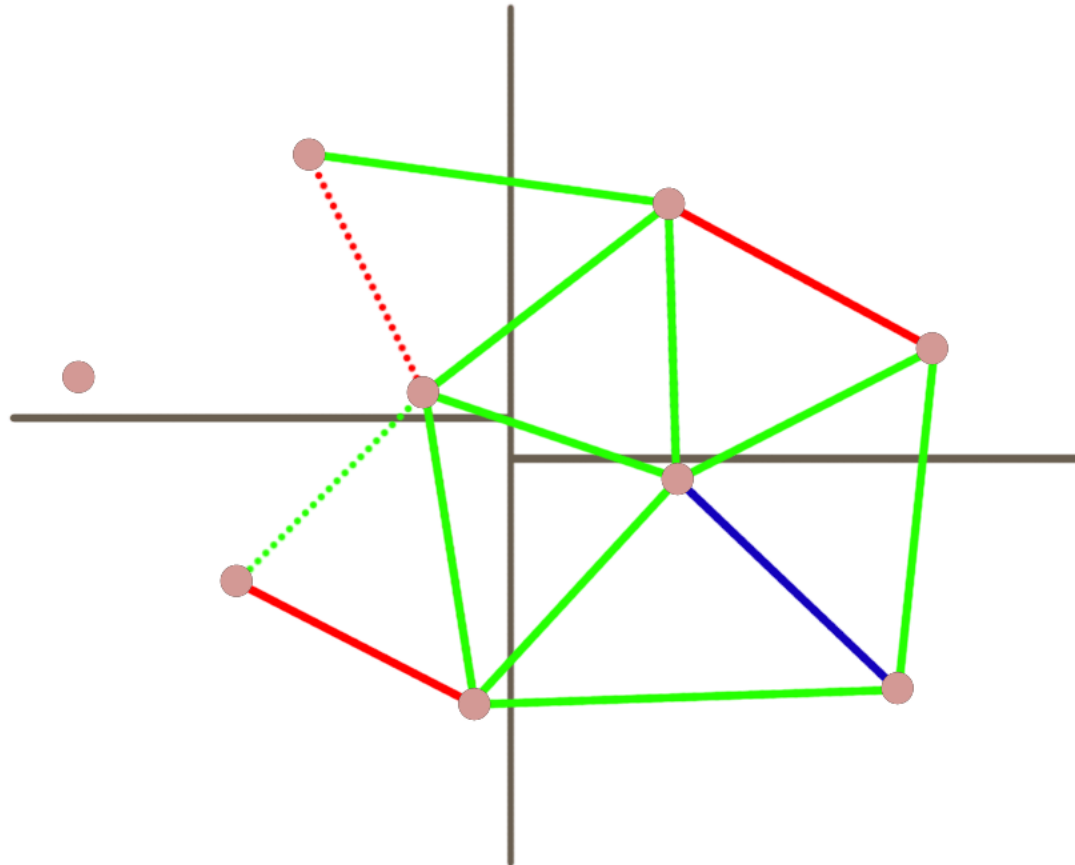
Step by step example



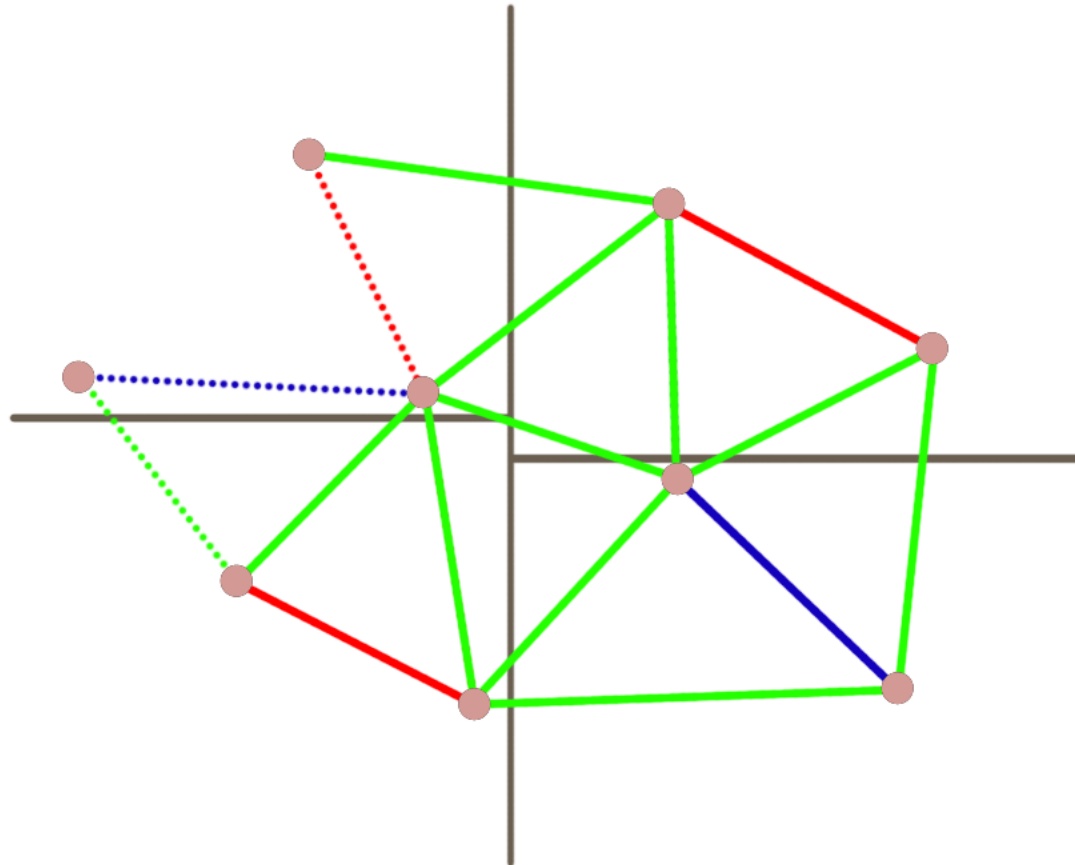
Step by step example



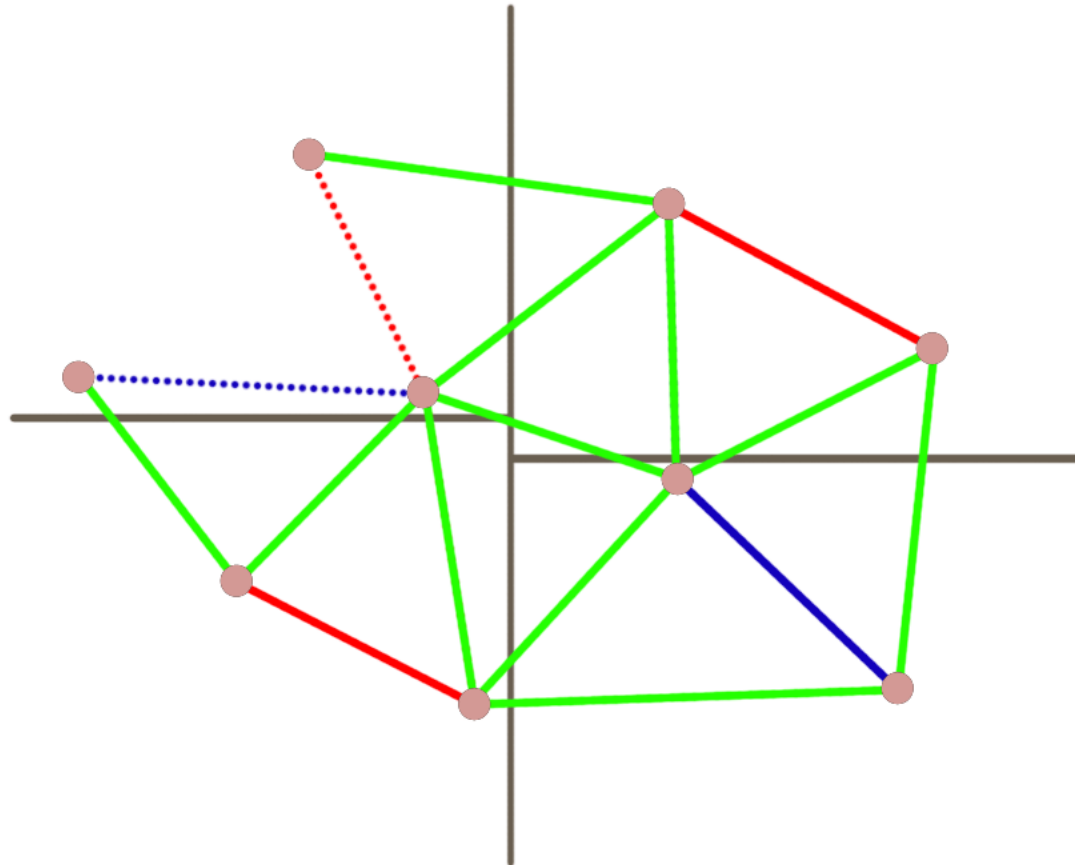
Step by step example



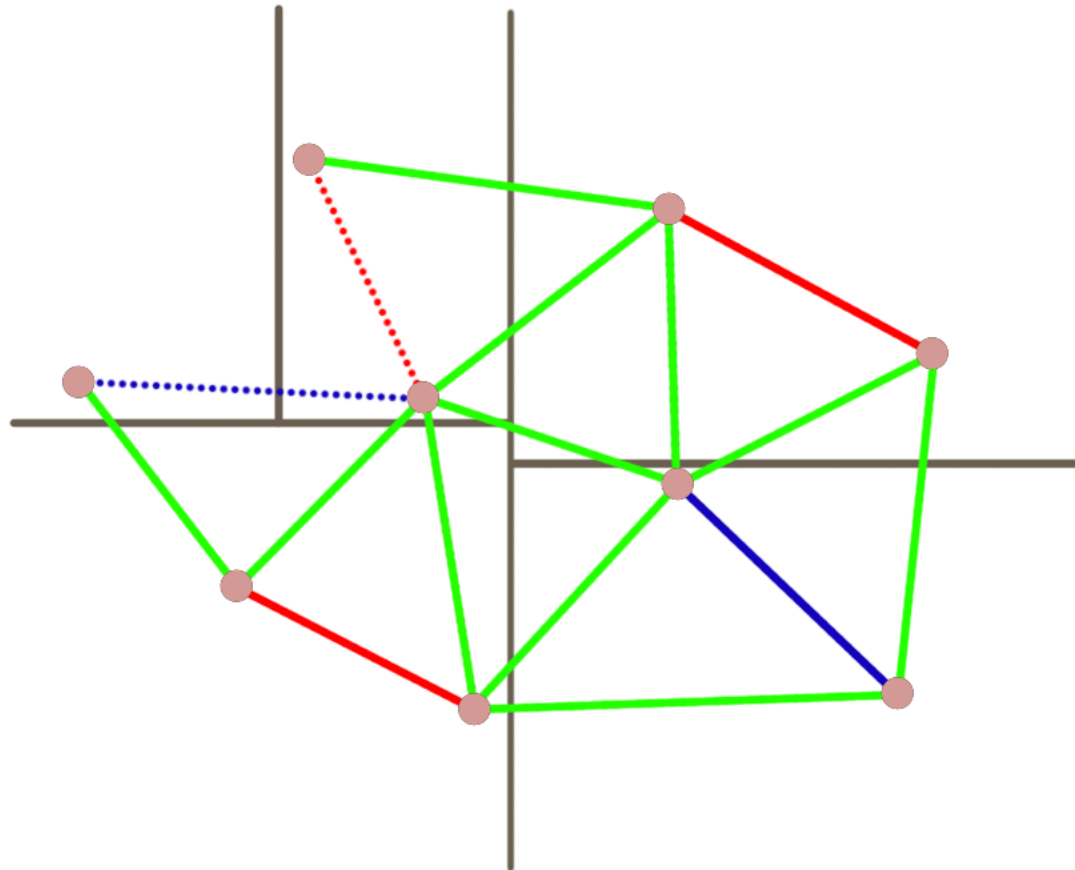
Step by step example



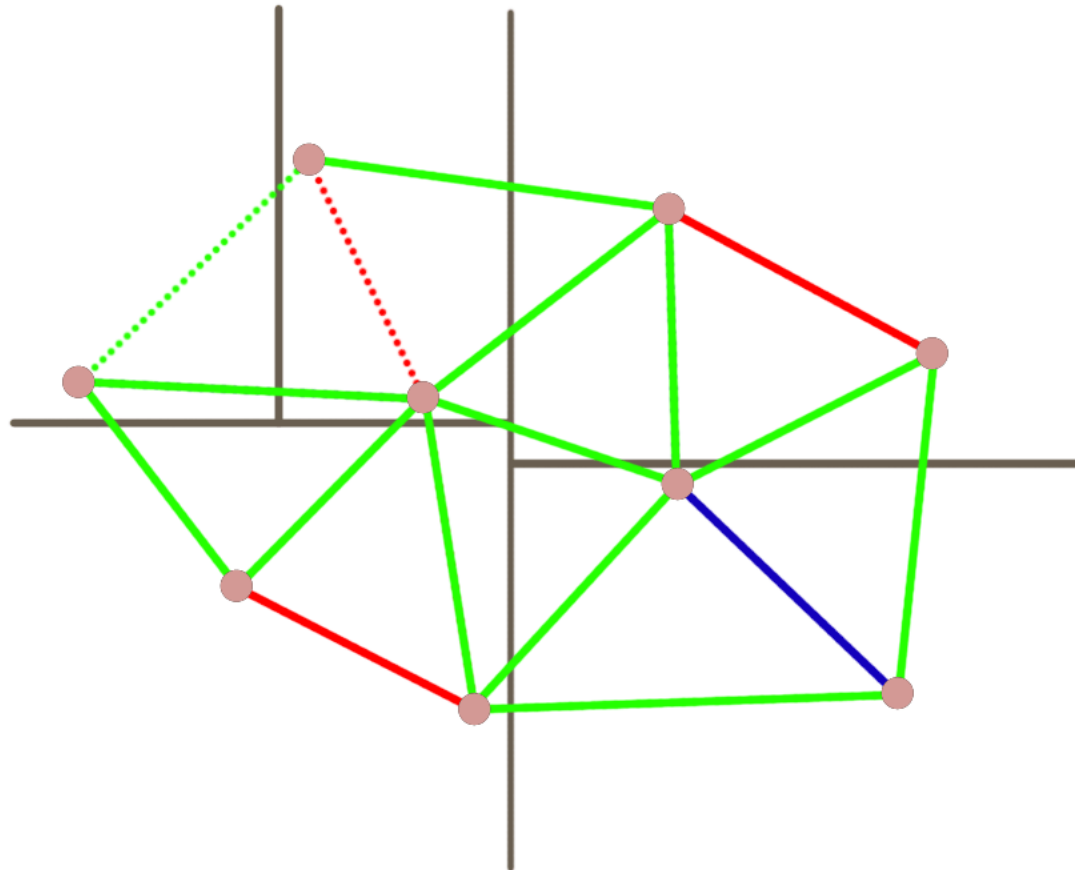
Step by step example



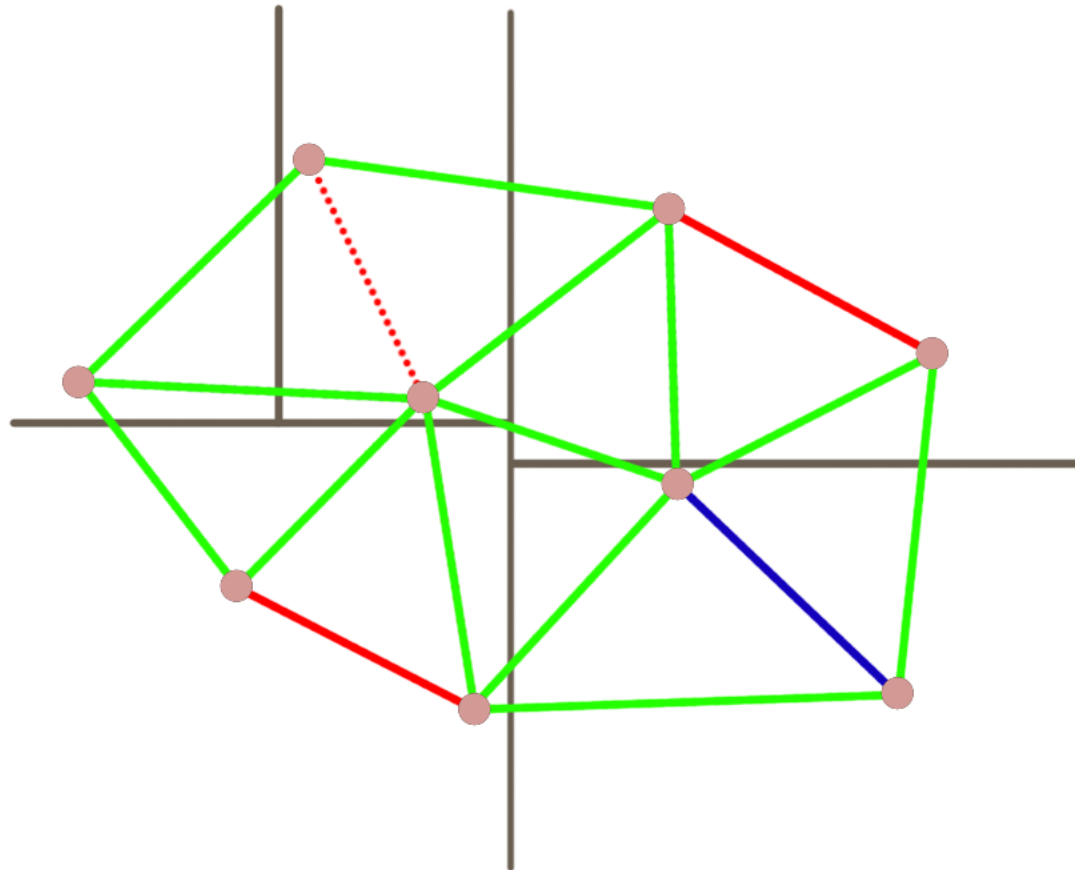
Step by step example



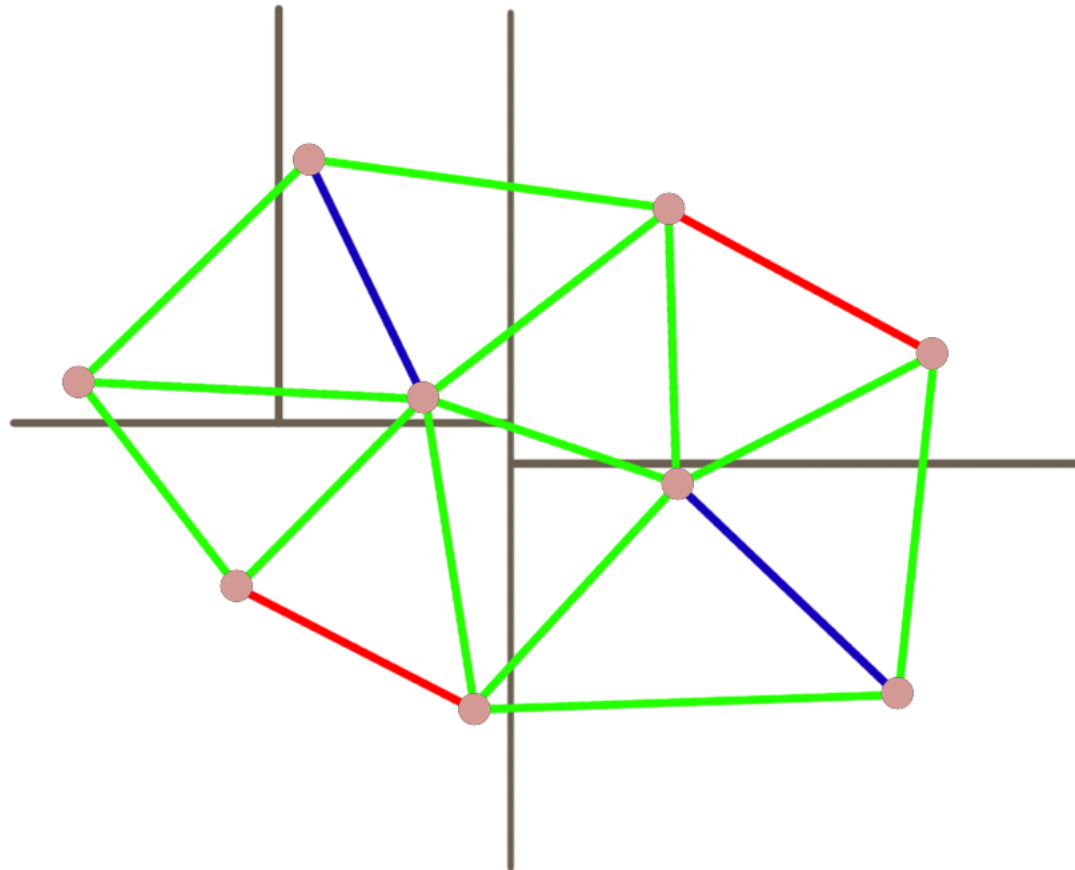
Step by step example



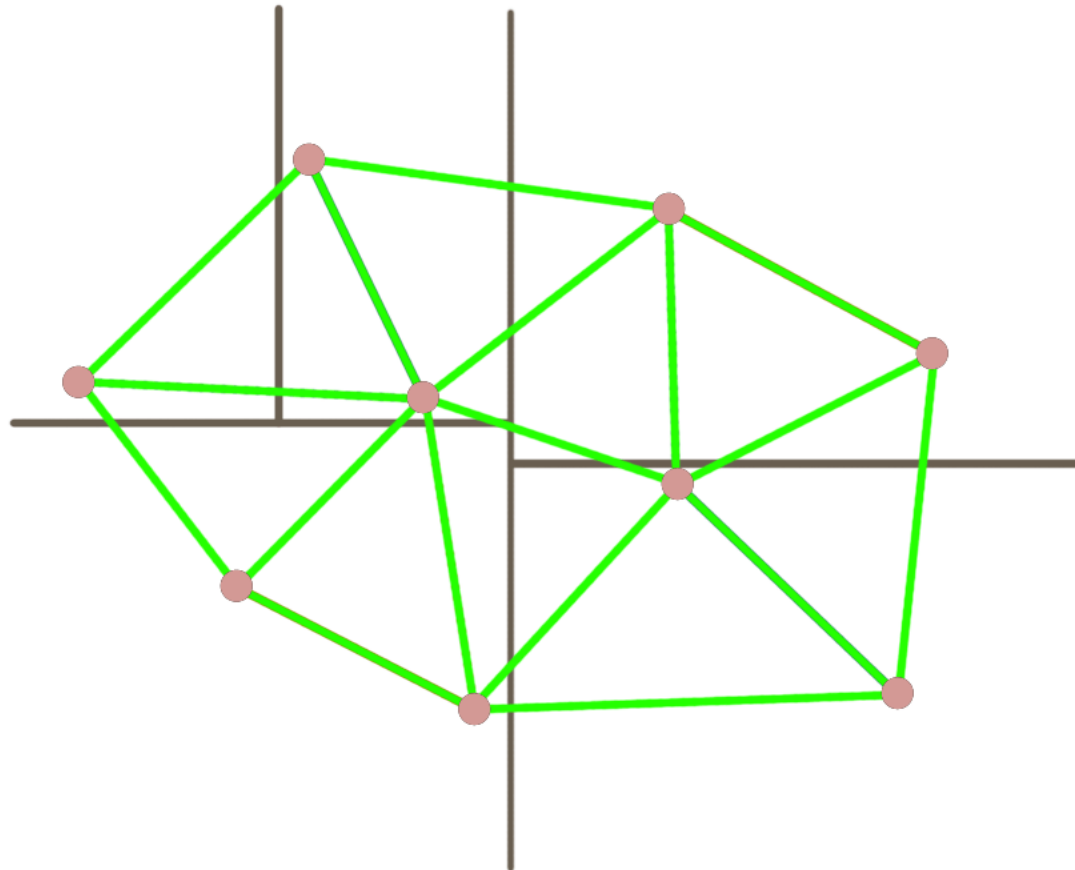
Step by step example



Step by step example



Step by step example



How to achieve it?

- InCoDe algorithm
- `makeFirstSimplex()`
- `makeSimplex()`
- Triangulation construction in Σ_α only

Remarks

- Possible improvement - uniform grid
 - Number of cells is equal to the number of points
 - Restrict search to cells contained in AABB of the circumscribed sphere
 - Parallelization
- $O(n^3)$ worst case, $o(n^2)$ expected

Sources

- D&C Algorithm of Delaunay triangulation: DeWall algorithm. [Maur '02, 15-17]
- DeWall: A Fast Divide & Conquer Delaunay Triangulation Algorithm in E^d ; P. Cignoni, C. Montani, R. Scopigno. Pisa, Italy. 1997
- http://en.wikipedia.org/wiki/Delaunay_triangulation

Questions?

Thank you for your attention!

makeFirstSimplex()

- Pick a point (p_1) closest to the splitting plane
- Pick a point (p_2) closest to the p_1 in the other halfspace
- Pick a point (p_3), so that the circumscribed sphere (p_1, p_2, p_3) has minimal radius
- Repeat until required d-simplex is built

makeSimplex()

- Repeatedly picks a point that minimizes the „Delauney distance“ function
- $dd(f, p) = r$ if c is in $\text{Halfspace}(f, p)$
 $= -r$ otherwise
- r – radius of the circumscribed sphere around f, p
- c – center of the circumscribed sphere around f, p

Triangulation construction in Σ_α only

- We want to triangulate only those simplices, intersected by splitting plane α
- Active Faces List
 - AFL_α : $(d - 1)$ faces, intersected by α
 - AFL_1 : $(d - 1)$ faces with all points in P_1
 - AFL_2 : $(d - 1)$ faces with all points in P_2



**OI-OPPA. European Social Fund
Prague & EU: We invest in your future.**
