Introduction to Non-Cooperative Game Theory

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Overview

1. Introduction
   - What are games?
   - What is the optimality?
   - What are the representations?
   - What are the strategies?
2. Exercises
   - Find a solution
   - Find a game with desired properties
   - Let's play a game

What is the Non-Cooperative Game Theory Good for?

Describes optimal behavior in non-cooperative scenarios/situations (games).

There are different notions of optimality (we will see later).

What do we need to have a game?

- players (or agents)
- actions (each player can act in the environment)
- utility (how good is the outcome for players when they play certain actions)
- states (if the game evolves in time)
- knowledge (what the players know?)

What does it mean "optimal"?

Defined by *solution concepts* that specify conditions that a strategy profile must satisfy. One common assumption: players are *rational* (they are maximizing their utility).

There are many different solution concepts:

- Maxmin/Minmax strategies
- Nash Equilibrium (and derivatives like Perfect, Proper, Sequential,...)
- Correlated Equilibrium (and derivatives like Coarse, Extensive-Form, ...)
- Stackelberg Equilibrium (and derivatives like Stackelberg Correlated Equilibrium, ...)

Why do we need so much solution concepts?

Different solution concept fits different conditions in the real world/simulation/situation.

How can we represent a game?

There is only a single game. We can reason about and work with this game using *different mathematical representations*.

We can identify 3 main representations (there is a lot more):

- normal-form games (standard computer-science representation, visualized as matrices, typically used for one-shot simultaneous move games (rock-paper-scissors)).
- extensive-form games (typical representation for sequential games with finite states and finite course of the game (chess, poker)).
- stochastic games (typical representation for sequential games with finite states and infinite course of the game (driving a car)).

What is a strategy in a game?

Strategy is a contingency plan what to do in each situation.

What is a difference between planning and solving a game?

We can identify 3 main types of strategies (there is a lot more):

- pure strategies (a decision which action to play in each situation that is possible).
- mixed strategies (a randomized strategy; probability distribution over pure strategies).
- behavioral strategies (a randomized strategy; probability distribution over actions to play in each situation that is possible).

Exercises

Let's try this.

Solution Concepts

Your task is to:

- find all pure NE
- find all pareto optimal outcomes

	X	**Y**	**Z**
A	$(1,3)$	$(6,1)$	$(4,2)$
B	$(0,4)$	$(2,9)$	$(-1,3)$
C	$(2,-1)$	$(5,2)$	$(6,3)$

Construct A Game

Give an example of a zero-sum game that does not have a NE in pure strategies.

Give an example of a general-sum game, in which no pure NE is pareto optimal.

Give an example of a general-sum game, in which all the following solution concepts using pure strategies are different: minmax, maximin, NE.

Give an example of a general-sum game, with two pure NE such that when players play strategies from different equilibria, the expected outcome is worse for both players.

Analyze whether the last situation can happen in a zero-sum game.

Iterated Elimination of Dominated Strategies

Determine whether the process of iterated elimination of **strictly dominated** strategies yields a single possible outcome. If so, verify that this is the only NE of the game.

	X	**Y**	**Z**
A	$(1,0)$	$(3,0)$	$(2,1)$
B	$(3,1)$	$(0,1)$	$(1,2)$
C	$(2,1)$	$(1,6)$	$(0,2)$

	X	**Y**	**Z**
A	$(1,3)$	$(6,1)$	$(4,2)$
B	$(0,4)$	$(2,9)$	$(-1,3)$
C	$(2,-1)$	$(5,2)$	$(6,3)$

How would you prove that the elimination of strictly dominated strategies preserve Nash Equilibria?

Let's play a game

Write down on a piece of paper an integer number from the interval $\{0, 1, \ldots, 100\}$ together with your name.

Send all your papers to me, I will compute the average value x.

The winner is the player(s) who guessed a number that is closest to $\frac{2}{3}x$.

Write down a normal-form representation of this game. What is a NE?