DISTRIBUTED CONSTRAINT OPTIMIZATION

AE4M36MAS - Multiagent systems

ASSIGNMENT

n-Queens problem in a distributed way

n queens from a $n \times n$ world had
a serious dispute:

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- They don't talk to each other except for few formal messages
  Ok? Nogood AddLink

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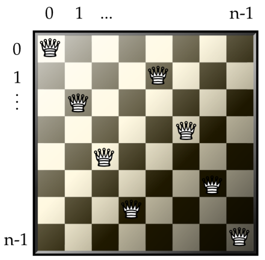
- They don't talk to each other except for few formal messages Ok? Nogood AddLink

Help them to find their place in the world!



n-Queens problem in a distributed way

Every agent controls **one queen** and decides about her position within its row.

In the end, one of the following has to happen:

- One of the agents reports that no solutions exists
- Each queen reports her position in her row (i.e. a column in which it is located)

$\uparrow$ of course correctly ;-)

n-Queens problem in a distributed way

Any **asynchronous** and **distributed** solution is acceptable (e.g. ABT).

$\rightarrow$ No centralized knowledge allowed!

$\rightarrow$ No synchonization!

$\rightarrow$ No hardcoded solutions!

n-Queens problem in a distributed way

Total: **12 points**

- Solve 3×3 chessboard problem with 3 queens (3 points)
- Solve 4×4 chessboard problem with 4 queens (2 points)
- Solve 8×8 chessboard problem with 8 queens (2 points)
- Solve 12×12 chessboard problem with 12 queens (3 points)

n-Queens problem in a distributed way

Guaranteed termination detection (**1 point**)

- How to detect *quiescence* in an algorithmic way?
- You may want to get inspired by other DCSP/DCOP algorithms.

Quiescence should be discovered using **local knowledge** only.

$\rightarrow$ Sending whole solution to a single agent for verification is not an option!

n-Queens problem in a distributed way

Report (**1 point**)

- How is the n-queens problem modeled as a DCSP? (variables, domains, constraints, agents)
- How is the ABT algorithm customized for the n-queens problem?
- How do you determine priorities between agents?
- How do you detect that the search has terminated?

REVISION

Distributed CSP

- $\mathcal{X} = \{x_1, \ldots, x_n\}$ — set of *variables* to assign
- $\mathcal{D} = \{D_1, \ldots, D_n\}$ — set of *domains* ($x_i \in D_i$)
- $\mathcal{C} = \{C_1, \ldots, C_m\}$ — set of *constraints*
- $\mathcal{A} = \{A_1, \ldots, A_k\}$ — set of *agents*

Distributed CSP

Agent i should come up with an assignment for his variable x_i in a **distributed** way.

Tuple $(x_1, \ldots, x_n)$ should satisfy all the constraints.

Asynchronous backtracking

Agents asynchronously decide about their variable and communicate their decisions.

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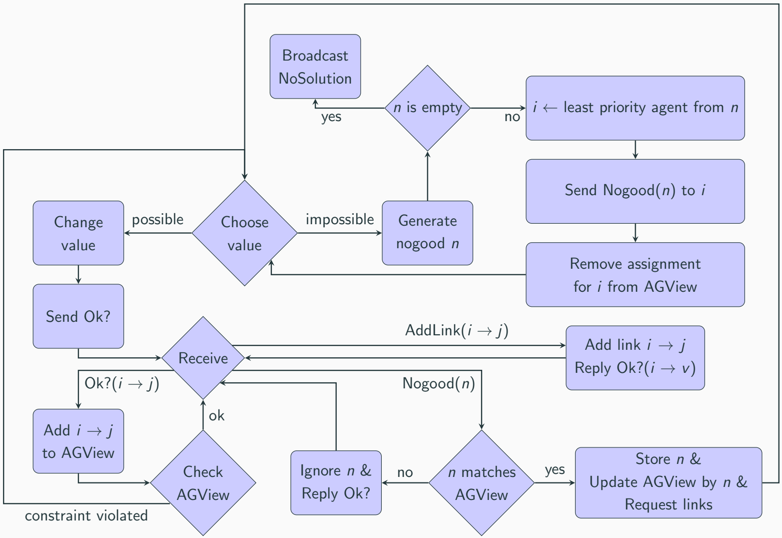
Asynchronous backtracking

Agents asynchronously decide about their variable and communicate their decisions.

- **Ok?** asks lower priority subscribers whether current assignment is okay for them
- **Nogood** notifies one higher priority agent that he must take some action — otherwise a solution will not be found
- **AddLink** represents the subscription for a variable of a higher priority agent (when I am asked to check something I cannot check at the moment)

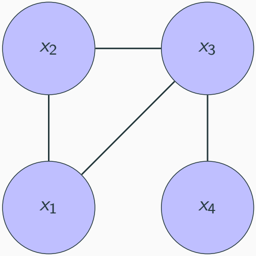
Asynchronous backtracking



DISTRIBUTED OPTIMIZATION

What we had so far?

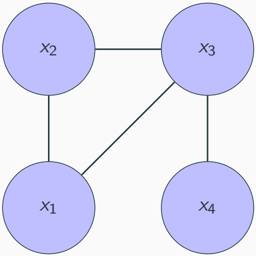
x_i	x_j	
○	○	T
○	●	F
●	○	F
●	●	T



$$C_k : D_i \times D_j \to \{\mathrm{T}, \mathrm{F}\}$$

What we have in DCOPs?

x_i	x_j	
○	○	1
○	●	2
●	○	2
●	●	0



$$C_k : D_i \times D_j \rightarrow \mathbb{N}_0$$

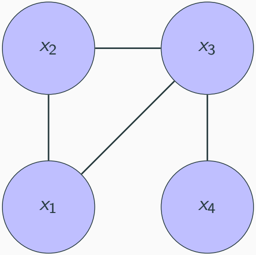
DCOPs

- $\mathcal{X} = \{x_1, \ldots, x_n\}$ — set of *variables* to assign
- $\mathcal{D} = \{D_1, \ldots, D_n\}$ — set of *domains* ($x_i \in D_i$)
- $\mathcal{C} = \{C_1, \ldots, C_m\}$ — set of *constraints*
- $\mathcal{A} = \{A_1, \ldots, A_k\}$ — set of *agents*

Goal

$$\min_{\mathbf{x}} \sum_{C_i \in \mathcal{C}} C_i(\mathbf{x})$$

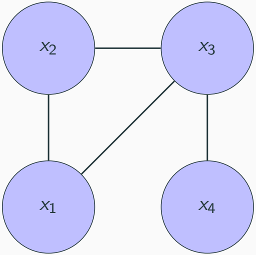
Branch & Bound



x_i	x_j	
○	○	1
○	●	2
●	○	2
●	●	0

Branch & Bound

Agent 1: $x_1 = \circ$
$LB = 0,\ UB = \infty$



x_i	x_j	
$\circ$	$\circ$	1
$\circ$	$\bullet$	2
$\bullet$	$\circ$	2
$\bullet$	$\bullet$	0

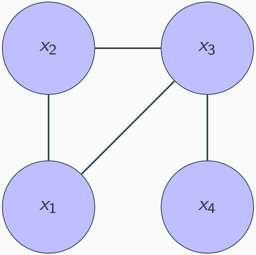
Branch & Bound

Agent 1: $x_1 = \circ$

$LB = 0, \; UB = \infty$

Agent 2: $x_2 = \circ$

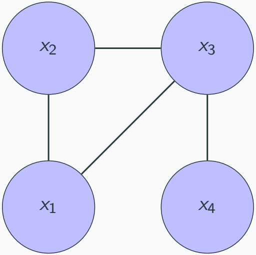
$LB = 1, \; UB = \infty$



x_i	x_j	
$\circ$	$\circ$	1
$\circ$	$\bullet$	2
$\bullet$	$\circ$	2
$\bullet$	$\bullet$	0

Branch & Bound

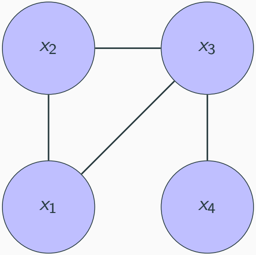
Agent 1: $x_1 = \circ$
$LB = 0,\ UB = \infty$
Agent 2: $x_2 = \circ$
$LB = 1,\ UB = \infty$
Agent 3: $x_3 = \circ$
$LB = 3,\ UB = \infty$



x_i	x_j	
$\circ$	$\circ$	1
$\circ$	$\bullet$	2
$\bullet$	$\circ$	2
$\bullet$	$\bullet$	0

Branch & Bound

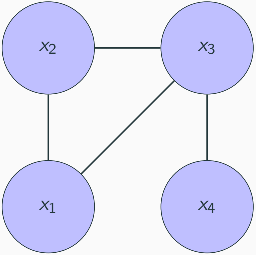
Agent 1: $x_1 = \circ$
$LB = 0,\ UB = 4$
Agent 2: $x_2 = \circ$
$LB = 1,\ UB = 4$
Agent 3: $x_3 = \circ$
$LB = 3,\ UB = 4$
Agent 4: $x_4 = \circ$
$LB = 4,\ UB = 4$



x_i	x_j	
$\circ$	$\circ$	1
$\circ$	$\bullet$	2
$\bullet$	$\circ$	2
$\bullet$	$\bullet$	0

Branch & Bound

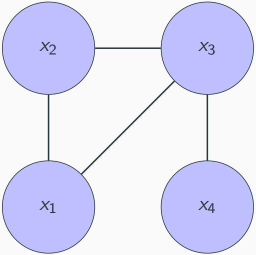
Agent 1: $x_1 = \circ$
$LB = 0,\ UB = 4$
Agent 2: $x_2 = \circ$
$LB = 1,\ UB = 4$
Agent 3: $x_3 = \circ$
$LB = 3,\ UB = 4$
Agent 4: $x_4 = \bullet$
$LB = \mathbf{5},\ UB = 4$



x_i	x_j	
○	○	1
○	●	2
●	○	2
●	●	0

Branch & Bound

Agent 1: $x_1 = \circ$
$LB = 0,\ UB = 4$
Agent 2: $x_2 = \circ$
$LB = 1,\ UB = 4$
Agent 3: $x_3 = \bullet$
$LB = \mathbf{5},\ UB = 4$



x_i	x_j	
$\circ$	$\circ$	1
$\circ$	$\bullet$	2
$\bullet$	$\circ$	2
$\bullet$	$\bullet$	0

Branch & Bound

Agent 1: $x_1 = \circ$
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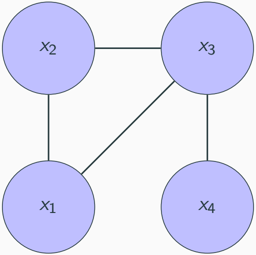
Agent 2: $x_2 = \bullet$
$LB = 2,\ UB = 4$



x_i	x_j	
$\circ$	$\circ$	1
$\circ$	$\bullet$	2
$\bullet$	$\circ$	2
$\bullet$	$\bullet$	0

Branch & Bound

Agent 1: $x_1 = \circ$
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Agent 2: $x_2 = \bullet$
$LB = 2,\ UB = 4$
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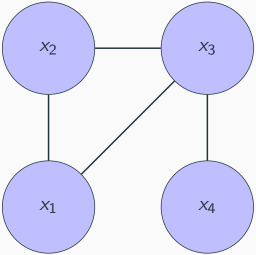

x_i	x_j	
○	○	1
○	●	2
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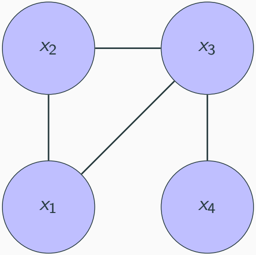

x_i	x_j	
$\circ$	$\circ$	1
$\circ$	$\bullet$	2
$\bullet$	$\circ$	2
$\bullet$	$\bullet$	0

Branch & Bound

Agent 1: $x_1 = \bullet$
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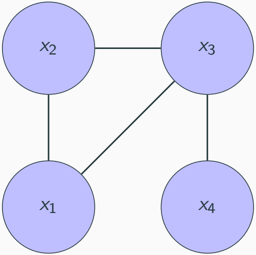
... etc ...

LB=UB
$\rightarrow$ Solution found



x_i	x_j	
○	○	1
○	●	2
●	○	2
●	●	0

Branch & Bound

Why we **do not like** such an approach in MAS?

Branch & Bound

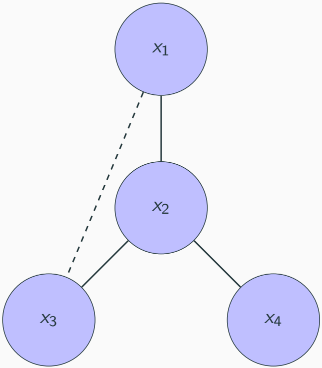
Why we **do not like** such an approach in MAS?

→ We need all agents to take decisions **simulataneously**!

Opportunistic Best-first Search

1. Introduce a hierarchy between agents

   DFS tree (back edges are dashed)



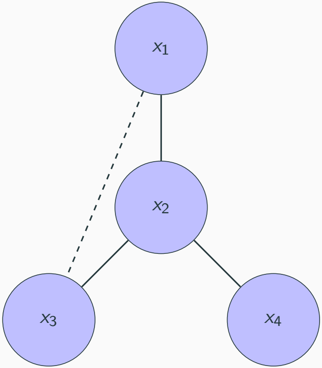
Opportunistic Best-first Search

Let $x_1 = \circ$.

Question

It's Christmas time! Assume that you can get any information about "subtrees" rooted in x_3 and x_4 at no cost.

What is the optimal assignment for x_2?



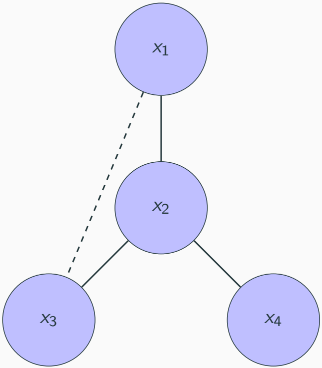
Opportunistic Best-first Search

Let $x_1 = \circ$.

Question

What is the optimal assignment for x_2?

$$\underset{v\in\{\circ,\bullet\}}{\arg\min}\Big[C(x_1=\circ,x_2=v) + OPT_{x_3}(x_1=\circ,x_2=v) + OPT_{x_4}(x_1=\circ,x_2=v)\Big]$$



Opportunistic Best-first Search

More generally:

$$\arg\min_{v \in D_i} \left[\delta_{ctx}(v) + \sum_{c \in child(i)} OPT_c\left(ctx \cup \{x_i = v\}\right) \right]$$

where

ctx	current context (assignment for i's ancestors) (~agent view)
$\delta_{ctx}(v)$	penalty for constraints involving x_i and some ancestor of i when $x_i = v$
$OPT_c(ctx)$	optimal solution of the subtree rooted in c in the given context

Opportunistic Best-first Search

There is a problem — we do not know $OPT_c(ctx)$

(otherwise we wouldn't be here right now ;-))

Inspire yourself in Branch & Bound algorithm!

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(given my assignment)

Opportunistic Best-first Search

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Inspire yourself in Branch & Bound algorithm!

$\rightarrow$ Keep bounds on solutions of subtrees
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Solution: Take the opportunity and pick the value that may lead to the best solution! (i.e. the one with minimal lower bound)

$$LB(v) = \delta_{ctx}(v) + \sum_{c \in child(i)} lb_c(v)$$

What we need to store?

For every my assignment:

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For every my assignment:
For every child of mine:

○	○	●	●
x_3	x_4	x_3	x_4

What we need to store?

For every my assignment:

For every child of mine:

Store bounds:

	$\circ$	$\circ$	$\bullet$	$\bullet$
	x_3	x_4	x_3	x_4
$lb_c(v)$	0	0	0	0
$ub_c(v)$	∞	∞	∞	∞

What we need to store?

For every my assignment:

For every child of mine:

Store bounds:

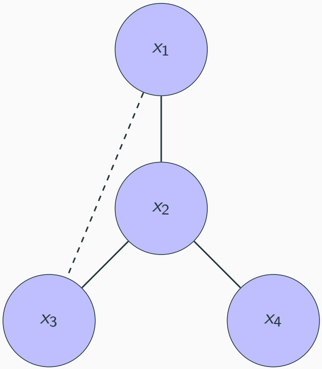
Context:

	$\circ$	$\circ$	$\bullet$	$\bullet$
	x_3	x_4	x_3	x_4
$lb_c(v)$	0	0	0	0
$ub_c(v)$	∞	∞	∞	∞
	$x_1 = \circ$	$x_1 = \circ$	$x_1 = \circ$	$x_1 = \circ$

Challenge

It's pre-2005 era. A complete asynchronous distributed algorithm for solving DCOPs is non-existent...

It's your turn to make ADOPT work!



ADOPT messages

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- **cost!**
  Agent notifies his parent about bounds on the solution of his subtree
  Include context! Otherwise the whole system goes out of sync
- **solution!**
  Broadcasted by root agent in the DFS tree when detecting LB=UB.
- **threshold!** (optional)
  Sent to children not to make them swap their value too often.

ADOPT properties

Optimal and **asynchronous** algorithm for solving DCOPs.

Question: What is the key difference in the way ADOPT backtracks? (compared to ABT / synchronous BnB)

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  (i.e. inconsistency has been proven)

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- ABT backtrack when it has no other option
  (i.e. inconsistency has been proven)
- BnB backtracks when suboptimality is detected
  (i.e. once LB $\geq$ UB)

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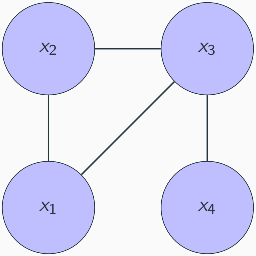
Optimal and **asynchronous** algorithm for solving DCOPs.

Question: What is the key difference in the way ADOPT backtracks? (compared to ABT / synchronous BnB)

- ABT backtrack when it has no other option
  (i.e. inconsistency has been proven)
- BnB backtracks when suboptimality is detected
  (i.e. once LB $\geq$ UB)
- ADOPT keeps informing parent about solution bounds
  (backtrack may happen due to the **opportunity** to change)

Example

x_i	x_j	
○	○	1
○	●	2
●	○	2
●	●	0



Approximate algorithms

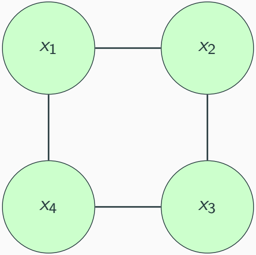
When we need solution **fast** and with **little effort**.

$\rightarrow$ Optimality guarantees are sacrificed

$\rightarrow$ Much better scalability

Approximate algorithms

At least some **coordination is needed**.

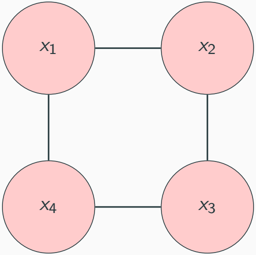


Graph coloring — each agent can decide to be either green or red.

Question: What is the best choice for each of the agents?

Approximate algorithms

At least some **coordination is needed**.



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- *Randomize to decide whether an agent is going to act.*
  $\rightarrow$ DSA-1 algorithm

Approximate algorithms

Recall of mining in Jason. How to solve this issue?

- *Randomize to decide whether an agent is going to act.*
  $\rightarrow$ DSA-1 algorithm
- *Negotiate with neighbors.*
  $\rightarrow$ MGM-1 algorithm

DSA-1 — Distributed stochastic algorithm

Toss a coin to decide whether:

- I will do the greedy step
- I will wait for others to do something

Keep exchanging individual assignments.