Vapnik-Chervonenkis dimension

1 VC-dimension of circles

1. Let us have a set of hypotheses $\mathcal{H}_{circ}$ which contains two-dimensional circles which are positive inside and negative outside. Calculate the VC dimension of $\mathcal{H}_{circ}$.

2. Now, let us have a set of hypotheses $\mathcal{H}_{circles^{\pm}}$ which contains both circles which are positive inside and negative outside and circles which are negative inside and positive outside. Show how to shatter a set of four points using this class of hypotheses. This will give you a lower bound on the VC dimension.

2 VC-dimension of linear half-spaces

Assume the class of linear separators

$$\mathcal{H}_d = \left\{x \mapsto \operatorname{sign}(w^T x + b) | w \in R^d, b \in R\right\}$$

3. Let us have $d+1$ points containing the origin and vectors of the standard basis (i.e. vectors which have 1 in i-th entry and 0 elsewhere). Show that this set can be shattered using $\mathcal{H}_d$.

4. Prove that no set of $d+2$ points can be shattered using $\mathcal{H}_d$. (Hint: Use Radon's theorem which says that any set of $d+2$ points in Rd can be splitted into two disjoint sets with intersecting convex hulls.)

This gives us that VC-dimension of $\mathcal{H}$ is exactly $d+1$.