

AE4M33RZN, Fuzzy logic: Tutorial examples

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Plan of the lecture

Task 3

Assignment

On the universe $\Delta = \{a, b, c, d\}$ there is a fuzzy set

$$\mu_A = \{(a; 0.3), (b; 1), (c; 0.5)\}.$$

Find its horizontal representation.

$$R_A(\alpha) = \begin{cases} X & \alpha = 0, \\ \{a, b, c\} & \alpha \in (0; 0.3), \\ \{b, c\} & \alpha \in (0.3; 0.5), \\ \{b\} & \alpha \in (0.5; 1). \end{cases}$$

Assignment

The fuzzy set A has a horizontal representation

$$R_A(\alpha) = \begin{cases} \{a, b, c, d\} & \alpha \in \langle 0, 1/3 \rangle \\ \{a, d\} & \alpha \in (1/3, 1/2) \\ \{d\} & \alpha \in (1/2, 2/3) \\ \emptyset & \alpha \in (2/3, 1) \end{cases}$$

Find the vertical representation.

Solution

$$\mu_A(a) = \sup\{\alpha \in \langle 0, 1 \rangle : a \in R_A(\alpha)\} = \sup\langle 0, 1/2 \rangle = 1/2$$

$$\mu_A(b) = 1/3, \quad \mu_A(c) = 1/3, \quad \mu_A(d) = 2/3, \quad \text{therefore}$$

$$\mu_A = \{(a, 1/2), (b, 1/3), (c, 1/3), (d, 2/3)\}$$

Assignment

On the universe $\Delta = \mathbb{R}$ there is a fuzzy set A :

$$\mu_A(x) = \begin{cases} x & x \in \langle 0; 1 \rangle \\ 2 - x & x \in (1; 1.5) \\ 0 & \text{otherwise} \end{cases}$$

Find its horizontal representation.

$$R_A(\alpha) = \begin{cases} \mathbb{R} & \alpha = 0 \\ \langle \alpha; 1.5 \rangle & \alpha \in (0; 0.5) \\ \langle \alpha; 2 - \alpha \rangle & \alpha \in (0.5; 1) \end{cases}$$

Assignment

The fuzzy set A has a horizontal representation

$$R_A(\alpha) = \begin{cases} \mathbb{R} & \alpha = 0 \\ \langle \alpha^2, 1 \rangle & \text{otherwise.} \end{cases}$$

Find the vertical representation.

$$\mu_A(x) = \begin{cases} \sqrt{x} & x \in (0, 1) \\ 0 & \text{otherwise.} \end{cases}$$

Assignment

Decide if the following function is a fuzzy conjunction.

$$\alpha \underset{\circ}{\wedge} \beta = \begin{cases} \alpha & \beta = 1, \\ \beta & \alpha = 1, \\ \alpha\beta & \alpha\beta \geq 1/10, \max(\alpha, \beta) < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Solution

The first two possibilities ensure the boundary condition. Comutativity and monotonicity are trivially satisfied. If one of the arguments is 1, the associativity as well. For $\alpha, \beta, \gamma < 1$ the associativity follows from

$$\alpha \underset{\circ}{\wedge} (\beta \underset{\circ}{\wedge} \gamma) = \begin{cases} \alpha\beta\gamma & \alpha\beta\gamma \geq 1/10. \\ 0 & \text{otherwise,} \end{cases}$$

We get the same for $(\alpha \underset{\circ}{\wedge} \beta) \underset{\circ}{\wedge} \gamma$.

It is always a fuzzy conjunction (interpretable as an algebraic conjunction, in which we ignore small values, e.g. for filtering small values).

Assignment

Decide if the following function is a fuzzy conjunction.

$$\alpha \diamond \beta = \begin{cases} \alpha\beta & \alpha\beta \geq 0,01 \text{ or } \max(\alpha, \beta) = 1, \\ 0 & \text{otherwise} \end{cases}$$

Assignment

Decide if the following function is a fuzzy conjunction.

$$\underset{\circ}{\wedge}: \langle 0, 1 \rangle^2 \rightarrow \langle 0, 1 \rangle$$

$$\alpha \underset{\circ}{\wedge} \beta = \begin{cases} \min(\alpha, \beta) & \alpha + \beta \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

Task 13

Assignment

Decide, if for all $\alpha, \beta \in [0, 1]$ holds $(\alpha \overset{\circ}{\vee} \beta) \underset{L}{\wedge} (\alpha \overset{\circ}{\vee} \neg_S \beta) = \alpha$, where the

disjunction $\overset{\circ}{\vee}$ is

1. standard, $\overset{S}{\vee}$,
2. algebraic, $\overset{A}{\vee}$.
3. Łukasiewicz, $\overset{L}{\vee}$,

Assignment

Decide if the function $\underset{\circ}{\wedge} : \langle 0, 1 \rangle^2 \rightarrow \langle 0, 1 \rangle$

$$\alpha \underset{\circ}{\wedge} \beta = \begin{cases} \alpha\beta & \alpha + \beta \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

is a fuzzy conjunction.

Assignment

Decide if the $(\alpha \wedge \alpha) \vee (\alpha \wedge \alpha) \leq \alpha$ holds for

1. standard, \mathbb{S}
2. algebraic, \mathbb{A}
3. Łukasiewicz, \mathbb{L}

Assignment

Decide, which equalities hold:

$$1. (\alpha \underset{S}{\wedge} \alpha) \underset{V}{\vee} (\alpha \underset{S}{\wedge} \beta) = \alpha \underset{S}{\wedge} (\alpha \underset{V}{\vee} \beta)$$

$$2. (\alpha \underset{L}{\wedge} \alpha) \underset{V}{\vee} (\alpha \underset{L}{\wedge} \beta) = \alpha \underset{L}{\wedge} (\alpha \underset{V}{\vee} \beta)$$

$$3. \alpha \underset{V}{\vee} (\alpha \underset{L}{\wedge} \beta) = \alpha \underset{L}{\wedge} (\alpha \underset{V}{\vee} \beta)$$

Justify your conclusions.

Task 17

Assignment

Decide, which equalities hold:

$$1. (\alpha \underset{S}{\wedge} \beta) \underset{L}{\wedge} \gamma = \alpha \underset{S}{\wedge} (\beta \underset{L}{\wedge} \gamma)$$

$$2. \neg \underset{S}{\wedge} (\alpha \underset{L}{\vee} \beta) = \neg \underset{S}{\wedge} \alpha \underset{L}{\wedge} \neg \underset{S}{\wedge} \beta$$

$$3. (\alpha \underset{L}{\wedge} \alpha) \underset{S}{\vee} \neg \alpha = (\neg \underset{S}{\wedge} \alpha \underset{L}{\wedge} \neg \underset{S}{\wedge} \alpha) \underset{S}{\vee} \alpha$$

Justify your conclusions.

Assignment

Verify that $\alpha \underset{\circ}{\wedge} (\alpha \underset{\circ}{\overset{R}{\Rightarrow}} \beta) = \alpha \underset{S}{\wedge} \beta$ holds for

1. algebraic ops.
2. standard ops.

Solution

We will show the solution for algebraic operations. The other ones are similar.

$$\alpha \underset{A}{\wedge} (\alpha \underset{A}{\overset{R}{\Rightarrow}} \beta) = \left\{ \begin{array}{l} \alpha \cdot 1 = \alpha \text{ for } \alpha \leq \beta \\ \alpha \cdot \frac{\beta}{\alpha} = \beta \text{ for } \alpha > \beta \end{array} \right\} = \min(\alpha, \beta) = \alpha \underset{S}{\wedge} \beta$$

Assignment

Complete the table, so that R is a S -partial order.

| R | a | b | c | d |
|-----|-----|-----|-----|-----|
| a | | | | |
| b | 0.5 | | | |
| c | | 0.3 | | |
| d | | 0.2 | | |

Solution

Reflexivity implies 1's on the diagonal. S-partial order implies 0's to non-zero elements symmetric over the main diagonal.

| <i>R</i> | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|----------|----------|----------|----------|----------|
| <i>a</i> | 1 | 0 | x' | y' |
| <i>b</i> | 0.5 | 1 | 0 | 0 |
| <i>c</i> | x | 0.3 | 1 | z' |
| <i>d</i> | y | 0.2 | z | 1 |

The transitivity implies e.g. $R(3, 2) \wedge_S R(2, 1) \leq R(3, 1)$, which translates

into a condition $\min(0.3, 0.5) \leq x$

Using this and similar conditions, we derive the subspace of all solutions: $z \leq 0.2$, $x \geq 0.3$, $y \geq 0.2$, $\min(y, z') \leq a$, $x' = y' = 0$.