AE4M33RZN, Fuzzy logic: Tutorial examples

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Task 2

Assignment

On the universe $\Delta = \{a, b, c, d\}$ there is a fuzzy set

$$\mu_A = \Big\{(a; 0.3), (b; 1), (c; 0.5)\Big\}.$$

Find its horizontal representation.

$$\mathrm{R}_A(\alpha) = \begin{cases} X & \alpha = 0, \\ \{a, b, c\} & \alpha \in (0; 0.3\rangle, \\ \{b, c\} & \alpha \in (0.3; 0.5\rangle, \\ \{b\} & \alpha \in (0.5; 1\rangle. \end{cases}$$

Task 3

Assignment

The fuzzy set A has a horizontal representation

$$\mathrm{R}_A(\alpha) = \begin{cases} \{a, b, c, d\} & \alpha \in \langle 0, 1/3 \rangle \\ \{a, d\} & \alpha \in (1/3, 1/2 \rangle \\ \{d\} & \alpha \in (1/2, 2/3 \rangle \\ \emptyset & \alpha \in (2/3, 1 \rangle \end{cases}$$

Find the vertical representation.

Solution

$$\mu_A(a) = \sup\left\{\alpha \in \langle 0, 1\rangle :\ a \in \mathrm{R}_A(\alpha)\right\} = \sup\langle 0, 1/2\rangle = 1/2$$

$$\mu_A(b) = 1/3, \quad \mu_A(c) = 1/3, \quad \mu_A(d) = 2/3, \quad \text{therefore}$$

$$\mu_A = \left\{(a, 1/2), (b, 1/3), (c, 1/3), (d, 2/3)\right\}$$

Task 5

Assignment

On the universe $\Delta = \mathbb{R}$ there is a fuzzy set A:

$$\mu_A(x) = \begin{cases} x & x \in \langle 0; 1 \rangle \\ 2 - x & x \in (1; 1.5\rangle \\ 0 & \text{otherwise} \end{cases}$$

Find its horizontal represenation.

Solution

$$\mathtt{R}_A(\alpha) = \begin{cases} \mathbb{R} & \alpha = 0 \\ \langle \alpha; 1.5 \rangle & \alpha \in (0; 0.5\rangle \\ \langle \alpha; 2 - \alpha \rangle & \alpha \in (0.5; 1\rangle \end{cases}$$

Task 7

Assignment

The fuzzy set A has a horizontal representation

$$\mathrm{R}_A(\alpha) = \begin{cases} \mathbb{R} & \alpha = 0 \\ \langle \alpha^2, 1) & \text{otherwise.} \end{cases}$$

Find the vertical representation.

$$\mu_A(x) = \begin{cases} \sqrt{x} & x \in (0, 1) \\ 0 & \text{otherwise.} \end{cases}$$

Task 8

Assignment

Decide if the following function is a fuzzy conjunction.

$$\alpha \underset{\circ}{\wedge} \beta = \begin{cases} \alpha & \beta = 1, \\ \beta & \alpha = 1, \\ \alpha\beta & \alpha\beta \geqslant 1/10,\ \max(\alpha, \beta) < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Solution

The first two possibilities ensure the boundary condition. Comutativity and monotonicity are trivially satisfied. If one of the arguments is 1, the associativity as well. For $\alpha, \beta, \gamma < 1$ the associativity follows from

$$\alpha \mathbin{\underset{\circ}{\wedge}} \left(\beta \mathbin{\underset{\circ}{\wedge}} \gamma\right) = \begin{cases} \alpha\beta\gamma & \alpha\beta\gamma \geqslant 1/10. \\ 0 & \text{otherwise}, \end{cases}$$

We get the same for $\left(\alpha \mathbin{\underset{\circ}{\wedge}} \beta\right) \mathbin{\underset{\circ}{\wedge}} \gamma$.

It is always a fuzzy conjunction (interpretable as an algebraic conjunction, in which we ignore small values, e.g. for filtering small values).

Task 10

Assignment

Decide if the following function is a fuzzy conjunction.

$$\alpha \diamond \beta = \begin{cases} \alpha\beta & \alpha\beta \geqslant 0{,}01 \text{ or } \max(\alpha, \beta) = 1, \\ 0 & \text{otherwise} \end{cases}$$

Task 11

Assignment

Decide if the following function is a fuzzy conjunction.

$\underset{\circ}{\wedge} : \langle 0, 1 \rangle^2 \longrightarrow \langle 0, 1 \rangle$

$$\alpha \underset{\circ}{\wedge} \beta = \begin{cases} \min(\alpha, \beta) & \alpha + \beta \geqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

Task 12

Assignment

Decide, if for all $\alpha, \beta \in [0,1]$ holds $(\alpha \overset{\circ}{\vee} \beta) \underset{L}{\wedge} (\alpha \overset{\circ}{\vee} \underset{S}{\neg} \beta) = \alpha$, where the disjunction $\overset{\circ}{\vee}$ is

1. standard, $\overset{S}{\vee}$,
2. algebraic, $\overset{A}{\vee}$.
3. Łukasiewicz , $\overset{L}{\vee}$,

Task 13

Assignment

Decide if the function $\underset{\circ}{\wedge} \colon \langle 0, 1\rangle^2 \to \langle 0, 1\rangle$

$$\alpha \underset{\circ}{\wedge} \beta = \begin{cases} \alpha\beta & \alpha + \beta \geqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

is a fuzzy conjunction.

Task 14

Assignment

Decide if the $(\alpha \wedge \alpha) \vee (\alpha \wedge \alpha) \leqslant \alpha$ holds for

1. standard, $\overset{S}{\vee}$
2. algebraic, $\overset{A}{\vee}$
3. Łukasiewicz , $\overset{L}{\vee}$

Task 15

Assignment

Decide, which equalities hold:

1. $(\alpha \underset{S}{\wedge} \alpha) \overset{L}{\vee} (\alpha \underset{S}{\wedge} \beta) = \alpha \underset{S}{\wedge} (\alpha \overset{L}{\vee} \beta)$
2. $(\alpha \underset{L}{\wedge} \alpha) \overset{S}{\vee} (\alpha \underset{L}{\wedge} \beta) = \alpha \underset{L}{\wedge} (\alpha \overset{S}{\vee} \beta)$
3. $\alpha \overset{S}{\vee} (\alpha \underset{L}{\wedge} \beta) = \alpha \underset{L}{\wedge} (\alpha \overset{S}{\vee} \beta)$

Justify your conclusions.

Task 16

Assignment

Decide, which equalities hold:

1. $(\alpha \underset{S}{\wedge} \beta) \underset{L}{\wedge} \gamma = \alpha \underset{S}{\wedge} (\beta \underset{L}{\wedge} \gamma)$
2. $\underset{S}{\neg}(\alpha \overset{A}{\vee} \beta) = \underset{S}{\neg}\alpha \underset{L}{\wedge} \underset{S}{\neg}\beta$
3. $(\alpha \underset{L}{\wedge} \alpha) \overset{L}{\vee} \underset{S}{\neg}\alpha = (\underset{S}{\neg}\alpha \underset{L}{\wedge} \underset{S}{\neg}\alpha) \overset{L}{\vee} \alpha$

Justify your conclusions.

Task 17

Assignment

Verify that $\alpha \underset{\circ}{\wedge} (\alpha \overset{\mathrm{R}}{\underset{\circ}{\Rightarrow}} \beta) = \alpha \underset{\mathrm{S}}{\wedge} \beta$ holds for

1. algebraic ops.
2. standard ops.

Solution

We will show the solution for algebraic operations. The other ones are similar.

$$\alpha \underset{A}{\wedge} (\alpha \underset{A}{\overset{R}{\Rightarrow}} \beta) = \left\{ \begin{array}{c} \alpha \cdot 1 = \alpha \;\; \text{for} \;\; \alpha \leqslant \beta \\ \alpha \cdot \dfrac{\beta}{\alpha} = \beta \;\; \text{for} \;\; \alpha > \beta \end{array} \right\} = \mathbf{min}(\alpha, \beta) = \alpha \underset{S}{\wedge} \beta$$

Task 19

Assignment

Complete the table, so that R is a $\mathbb{S}$-partial order.

R	a	b	c	d
a				
b	0.5			
c		0.3		
d		0.2		

Solution

Reflexivity implies 1's on the diagonal. $\mathbb{S}$-partial order implies 0's to non-zero elements symmetric over the main diagonal.

R	a	b	c	d
a	1	0	x'	y'
b	0.5	1	0	0
c	x	0.3	1	z'
d	y	0.2	z	1

The transitivity implies e.g. $R(3,2) \underset{\mathbb{S}}{\wedge} R(2,1) \leqslant R(3,1)$, which translates into a condition $\min(0.3, 0.5) \leqslant x$

Using this and similar conditions, we derive the subspace of all solutions: $z \leqslant 0.2$, $x \geqslant 0.3$, $y \geqslant 0.2$, $\min(y, z') \leqslant a$, $x' = y' = 0$.