Basics of Description Logic $\mathcal{ALC}$

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1 Getting familiar with $\mathcal{ALC}$

Consider the following $\mathcal{ALC}$ theory $\mathcal{K} = (\mathcal{T}, \{\})$, where $\mathcal{T}$ contains the following axioms:

$$
\begin{aligned}
Man &\sqsubseteq Person \\
Woman &\sqsubseteq Person \sqcap \neg Man \\
Father &\equiv Man \sqcap \exists hasChild \cdot Person \\
GrandFather &\equiv \exists hasChild \cdot \exists hasChild \cdot \top \\
Sister &\equiv Person \sqcap \neg Man \sqcap \exists hasSibling \cdot Person
\end{aligned}
$$

1. What is the meaning of these particular axioms ? Do they reflect your understanding of reality ? Formulate them in natural language.

2. Rewrite last axiom into the semantically equivalent FOPL formula.

3. Consider the following interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \bullet^{\mathcal{I}})$:

$$
\begin{aligned}
\Delta^{\mathcal{I}} &= Person^{\mathcal{I}} = \{B, A\} \\
Man^{\mathcal{I}} &= \{B\} \\
Woman^{\mathcal{I}} &= \{A\} \\
Father^{\mathcal{I}} &= GrandFather^{\mathcal{I}} = \{B\} \\
hasChild^{\mathcal{I}} &= \{\langle B, B \rangle\} \\
hasSibling^{\mathcal{I}} &= \{\}
\end{aligned}
$$

First consider $Sister^{\mathcal{I}} = \{B\}$ and next consider $Sister^{\mathcal{I}} = \{\}$. For each option answer the following question:

   a) Is $\mathcal{I}$ a model $\mathcal{K}$? If yes, decide, whether $\mathcal{I}$ reflects reality.

   b) We know that $\mathcal{ALC}$ has the *tree model property* and *finite model property*. In case $\mathcal{I}$ is a model, is $\mathcal{I}$ tree-shaped ? If not, find a model that is tree-shaped.

4. Using other axioms define concepts:
   - "A father having just sons."

- "Someone who has at least one sister, but no brother."

5. Let's consider two roles *hasChild* and *hasSibling*. During knowledge modeling, it is often necessary to specify :

   global domain and range of given role, i.e. statement of the type "By *hasChild* we connect always a person (instance of the *Person* class – domain) with another person (instance of the *Person* class – range)".

   local domain and range of given role, e.g. "Every father having only sons can be connected by *hasChild* just with man (instances of the *Man* class – range)".

   Show, in which way it is possible to model global domain and range of these roles in $\mathcal{ALC}$.

2 Using Protégé

1. Go through the Protégé Crash Course on the tutorial web pages.

2. Create a new ontology in Protégé 4 and insert there all the definitions from Section 1. Verify correctness of your solution of the previous task (e.g. in the DL query tab).