

Basics of Description Logic \mathcal{ALC}

Petr Křemen

v1.1

1 Understanding \mathcal{ALC}

Consider the following \mathcal{ALC} theory $\mathcal{K} = (\mathcal{T}, \{\})$, where \mathcal{T} contains the following axioms:

$$\begin{aligned} \textit{Man} &\sqsubseteq \textit{Person} \\ \textit{Woman} &\sqsubseteq \textit{Person} \sqcap \neg \textit{Man} \\ \textit{Father} &\equiv \textit{Man} \sqcap \exists \textit{hasChild} \cdot \textit{Person} \\ \textit{GrandFather} &\equiv \exists \textit{hasChild} \cdot \exists \textit{hasChild} \cdot \top \\ \textit{Sister} &\equiv \textit{Person} \sqcap \neg \textit{Man} \sqcap \exists \textit{hasSibling} \cdot \textit{Person} \end{aligned}$$

Ex. 1 — What is the meaning of these particular axioms? Do they reflect your understanding of reality? Formulate them in natural language.

Answer (Ex. 1) — For example, the third axiom defines a concept *Father* as any *Man* that has some *Person* as a child. The fourth axiom is not well defined – it allows grandfathers to be women. More precise version of the fourth axiom might be e.g. $\textit{GrandFather} \equiv \textit{Man} \sqcap \exists \textit{hasChild} \cdot \exists \textit{hasChild} \cdot \top$.

Ex. 2 — Rewrite the last axiom into the semantically equivalent FOPL formula.

Answer (Ex. 2) — Each TBox axiom corresponds to a universally closed FOPL formula. Notice that two different variables are enough for encoding of any \mathcal{ALC} axiom. The encoding of the last axiom is:

$$(\forall x)(\textit{Sister}(x) \equiv \textit{Person}(x) \wedge \neg \textit{Man}(x) \wedge (\exists y)(\textit{hasSibling}(x, y) \wedge \textit{Person}(y)))$$

Ex. 3 — Consider the following interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \bullet^{\mathcal{I}})$:

$$\begin{aligned} \Delta^{\mathcal{I}} &= \textit{Person}^{\mathcal{I}} = \{B, A\} \\ \textit{Man}^{\mathcal{I}} &= \{B\} \\ \textit{Woman}^{\mathcal{I}} &= \{A\} \\ \textit{Father}^{\mathcal{I}} &= \textit{GrandFather}^{\mathcal{I}} = \{B\} \end{aligned}$$

$$\begin{aligned}
hasChild^{\mathcal{I}} &= \{(B, B)\} \\
hasSibling^{\mathcal{I}} &= \{\} \\
Sister^{\mathcal{I}} &= \{B\}
\end{aligned} \tag{1}$$

1. Is \mathcal{I} a model \mathcal{K} ? If yes, decide, whether \mathcal{I} reflects reality.

2. We know that \mathcal{ALC} has the *tree model property* and *finite model property*. In case \mathcal{I} is a model, is \mathcal{I} tree-shaped? If not, find a model that is tree-shaped.

Answer (Ex. 3) — \mathcal{I} is not a model of \mathcal{K} , as it does not satisfy the last axiom: $Sister^{\mathcal{I}} \neq Person^{\mathcal{I}} \cap (\Delta^{\mathcal{I}} \setminus Man^{\mathcal{I}}) \cap \{x \in \Delta^{\mathcal{I}} \mid (\exists y \in \Delta^{\mathcal{I}})((x, y) \in hasSibling^{\mathcal{I}} \wedge y \in Person^{\mathcal{I}})\}$

Ex. 4 — How does the situation change when we consider the same \mathcal{I} , except that $Sister^{\mathcal{I}} = \{\}$?

Answer (Ex. 4) — Now, \mathcal{I} is a model of \mathcal{K} as it satisfies all axioms. However, it does not reflect the reality well, as it states that a person B is his/her own child. This interpretation is finite, yet not tree-shaped. A tree-shaped model ensured by the *tree-model property* of \mathcal{ALC} is e.g. the following infinite model $\mathcal{I} = (\Delta^{\mathcal{I}}, \bullet^{\mathcal{I}})$, where

$$\begin{aligned}
\Delta^{\mathcal{I}} &= Person^{\mathcal{I}} = Man^{\mathcal{I}} = Father^{\mathcal{I}} = GrandFather^{\mathcal{I}} = \{A_1, A_2, \dots\}_{i=1\dots\infty} \\
Woman^{\mathcal{I}} &= Sister^{\mathcal{I}} = \{\} \\
hasChild^{\mathcal{I}} &= \{(A_i, A_{i+1})\}_{i=1\dots\infty} \\
hasSibling^{\mathcal{I}} &= \{\}
\end{aligned} \tag{2}$$

Ex. 5 — Using the vocabulary from \mathcal{K} , define the concept “A father having just sons.”

Answer (Ex. 5) — $FatherOfBoys \equiv Father \sqcap \forall hasChild \cdot Man$

Ex. 6 — Using the vocabulary from \mathcal{K} , define the concept “A man who has no brother, but at least one sister with more than one child.”

Answer (Ex. 6) — $HappyUncle \equiv Man \sqcap \exists hasSibling \cdot (Woman \sqcap \exists hasChild \cdot \top) \sqcap \forall hasSibling \cdot \neg Man$

Ex. 7 — During knowledge modeling, it is often necessary to specify:

global domain and range of given role, i.e. statement of the type “By *hasChild* we always connect a *Person* (domain) with another *Person* (range)”.

local range of given role, e.g. “Every father having only sons (domain) can be connected by *hasChild* (domain) just with a *Man* (range)”.

Show, in which way it is possible to model global domain and range of these roles in \mathcal{ALC} .

Answer (Ex. 7) — Global domain and range can be modeled as:

$$\begin{aligned} \exists hasChild \cdot \top &\sqsubseteq Person \\ \top &\sqsubseteq \forall hasChild \cdot Person \end{aligned} \tag{3}$$

Local range is similar and only replaces the top concepts in the global range axiom:

$$\begin{aligned} \exists hasChild \cdot \top &\sqsubseteq Person \\ FatherOfSons &\sqsubseteq \forall hasChild \cdot Man \end{aligned} \tag{4}$$

2 Using Protégé

1. Go through the Protégé Crash Course on the tutorial web pages.
2. Create a new ontology in Protégé 4 and insert there all the definitions from Section 1. Verify correctness of your solution of the previous task (e.g. in the DL query tab).