AE4M33RZN, Fuzzy description logic: fuzzyDL reasoner

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Finite model property

Definition

A logic is said to have the finite model property if every satisfiable formula of the logic admits a finite model, i.e., a model with a finite domain. [Baader, 2003]

- Why is FMP important? Unless FMP holds, we need to be clever about our reasoning algorithms and avoid creating infinite models.
- Does FMP hold in Fuzzy Description Logic? Unfortunately no.

Witnessed model property

Definition

An interpretation $\mathcal{I}$ is $\circ$-**witnessed** if for all $x \in \Delta$, there is $y \in \Delta$ s.t.

$$(\exists \mathsf{R} \cdot \mathsf{C})^{\mathcal{I}}(x) = \mathsf{R}^{\mathcal{I}}(x, y) \underset{\circ}{\wedge} \mathsf{C}^{\mathcal{I}}(y)$$

and similarly there is a $y \in \Delta$ s.t.

$$(\mathsf{C} \sqsubseteq \mathsf{D})^{\mathcal{I}}(y) = \mathsf{C}^{\mathcal{I}}(y) \overset{\circ}{\underset{\circ}{\Rightarrow}} \mathsf{D}^{\mathcal{I}}(y) .$$

We say that the y is the “witness”, because he is responsible for the particular membership degree of $\exists \mathsf{R} \cdot \mathsf{C}$ (or $\mathsf{C} \sqsubseteq \mathsf{D}$).

Relationship between FMP and WMP

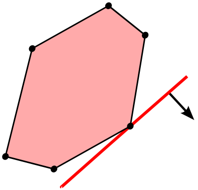
- It is easy to see that every finite model is a witnessed model, because all $\sup()$ can be replaced by $\max()$ in the definition of $\exists$.
- **Example:** Assume $\underset{S}{\neg}$ and $\underset{S}{\wedge}$ logic and a concept

$$C = \neg \forall R \cdot A \sqcap \neg \exists R \cdot \neg A \,.$$

  We will show that C can be satisfied to the degree 0.5 in an **infinite** model, but **no finite model** (and therefore no witnessed model) can satisfy C to 0.5.
- Are we hopeless? **No!** In Łukasiewicz logic ($\underset{S}{\neg}, \underset{L}{\wedge}, \overset{R}{\underset{L}{\Rightarrow}}$) we can restrict our reasoning to witnessed and finite models without loosing any information [Hájek, 2005].

Linear programming in a nutshell

Imagine a 2D space with a convex polygon in the space (x, y). Given constraints $4x + y \geqslant 6, y \leqslant 8, \ldots$, minimize $x - 2y$.



Source: [Wikipedia, 2013]

- Usually written in a matrix form

$$\text{maximize } \mathbf{c}^T \cdot x \quad (1)$$

$$\text{subject to } A\,\mathbf{x} \leqslant 0 \quad (2)$$

- (Mixed) Integer LP allows (some) variables to be **discrete**.
- LP with real values is in P class, ILP is NP-complete.

Linear programming in a nutshell

Solution of a ⟨⟨M⟩I⟩LP

- One solution (a point in the polytope).
- No solution (the polytope is empty).
- Multiple solutions with equal objective function value.

Syntactical notes about fuzzyDL:

- $x \in \mathbb{R}$ will be **real numbers.**
- $y \in \mathbb{N}$ will be **integer numbers.**
- All values x, y will be bounded by $[0, 1]$.

FuzzyDL algorithm overview

- Transforms $\mathcal{K}$ to the **negated-normal-form.**[1]
- Creates an witnessed interpretation of $\mathcal{K}$.
- During its working it creates
  - a **completion forest** and
  - a **list** of **linear** constraints $\mathscr{C}$.
- Linear constraints $\mathscr{C}$ are solved using any **mixed-integer-linear-programming solver.**

Disclaimer: Not going beyond Ł-logic, no concrete data types.

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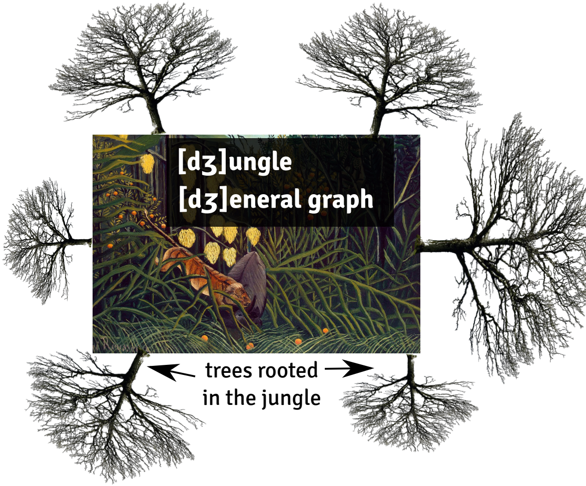
[1] Makes sure that the negation ¬ appears only in front of concepts using:

$\mathrm{nnf}(\neg \forall \mathsf{R} \cdot \mathsf{C}) = \exists \mathsf{R} \cdot \mathrm{nnf}(\neg \mathsf{C})$ and $\mathrm{nnf}(\neg \exists \mathsf{R} \cdot \mathsf{C}) = \forall \mathsf{R} \cdot \mathrm{nnf}(\neg \mathsf{C})$.

Completion-forest informally



Completion forest is a **graph**, that has a general structure (jungle) “in the middle” and many trees, whose root nodes are nodes in the jungle.

Completion-forest formally

The fuzzyDL algorithm starts with creating the “jungle”. It contains all **individuals** (connected by an edge if they are linked by some relation).

Initialization

- Create a new vertex v_a for each **individual** a in the $\mathcal{K}$.
- Create an edge (v_a, v_b) for each role assertion between a and b.
- Add a **label** $\langle \mathsf{C}, n \rangle$ to vertex a for each concept assertion $\langle a : \mathsf{C} \mid n \rangle$.
- Add a label $\langle \mathsf{R}, n \rangle$ to edge (a, b)
  for each role assertion $\langle (a, b) : \mathsf{R} \mid n \rangle$.

Forest completion (1)

The reasoner applies each of the following rules sequentially:

- A If a vertex v is labeled $\langle C, l \rangle$, add $(x_{v:C} \geqslant l)$ into $\mathscr{C}$.
- $\bar{A}$ If a vertex v is labeled $\langle \neg C, l \rangle$, add $(x_{v:C} \leqslant 1 - l)$ into $\mathscr{C}$.
- R If an edge (v, w) is labeled $\langle R, l \rangle$, add $(x_{(v,w):R} \geqslant l)$ into $\mathscr{C}$.
- $\bot$ If a vertex v is labeled $\langle \bot, l \rangle$, add $(l = 0)$ into $\mathscr{C}$.

Forest completion (2)

- $\sqcap$ If a vertex v is labeled $\langle C \sqcap D, l \rangle$, append labels $\langle C, x_1 \rangle, \langle D, x_2 \rangle$ to v and add the following constraints into $\mathscr{C}$ (with fresh x_1, x_2, y):

$$\begin{aligned} y &\leqslant 1 - l \\ x_1 &\leqslant 1 - y \\ x_2 &\leqslant 1 - y \\ x_1 + x_2 &= l + 1 - y \end{aligned}$$

- $\sqcup$ If a vertex v is labeled $\langle C \sqcup D, l \rangle$, append labels $\langle C, x_1 \rangle, \langle C, x_2 \rangle$ to v and add $(x_1 + x_2 = l)$ into $\mathscr{C}$ (with fresh x_1, x_2, y).

Forest completion (3)

- $\forall$ If a vertex v is labeled $\langle \forall\, \mathsf{R} \cdot \mathsf{C}, l_1 \rangle$, an edge (v, w) is labeled $\langle R, l_2 \rangle$ and the rule has not been applied to this pair, then append the label $\langle \mathsf{C}, x \rangle$ to w and add the following constraints into $\mathscr{C}$ (with fresh x, y):

$$l_1 + l_2 - 1 \leqslant x \leqslant y \leqslant l_1 + l_2$$

- $\sqsubseteq$ If $\langle \mathsf{C} \sqsubseteq \mathsf{D} \mid n \rangle \in \mathscr{K}$, and the rule has not been applied to a node v, then append labels $\langle \mathrm{nnf}(\neg\, \mathsf{C}), 1 - x_1 \rangle$, $\langle \mathsf{D}, x_2 \rangle$ to v and add $(x_1 \leqslant x_2 + 1 - n)$ to $\mathscr{C}$.

Forest completion: Example

Consider $\mathcal{K} = \{\langle \exists \mathsf{R} \cdot \mathsf{C} \sqsubseteq \mathsf{D} \mid 1 \rangle, \langle (a, b) : \mathsf{R} \mid 0.7 \rangle, \langle b : \mathsf{C} \mid 0.8 \rangle\}$.
Show that $\mathrm{glb}(\mathcal{K}, a : \mathsf{D}) = 0.5$.

Termination (1)

Unless the rules are applied repeatedly, the algorithm (as explained so far) terminates.
For defining $\exists$ rule, new nodes are added, which needs to refine the terminating condition.

Equivalence of labels

Two lists of labels $[\langle C_1, l_1\rangle, \ldots, \langle C_n, l_n\rangle]$ and $[\langle C_1, l'_1\rangle, \ldots, \langle C_n, l'_n\rangle]$ are equivalent iff either

- l_i and l'_i are variables or
- l_i and l'_i are negated variables or
- l_i and l'_i are equal rationals.

Termination (2)

Directly blocked node

A node is directly blocked iff

- it is outside the “jungle” and
- none of its ancestors are blocked and
- **it has an ancestor with equivalent labels.**

Blocked node

A node is blocked iff either

- it is directly blocked or
- one of its predecessors is blocked.

Forest completion (4)

- $\exists$ If a vertex v is labeled $\langle \exists R \cdot C, l \rangle$ and **it is not blocked,** add a new vertex w and an edge (v, w), add labels $\langle C, x_2 \rangle$ to w, and $\langle R, x_1 \rangle$ to (v, w) and the following constraints into $\mathscr{C}$ (with fresh x_1, x_2 and y):

$$y \leqslant 1 - l$$

$$x_1 \leqslant 1 - y$$

$$x_2 \leqslant 1 - y$$

$$x_1 + x_2 = l + 1 - y$$

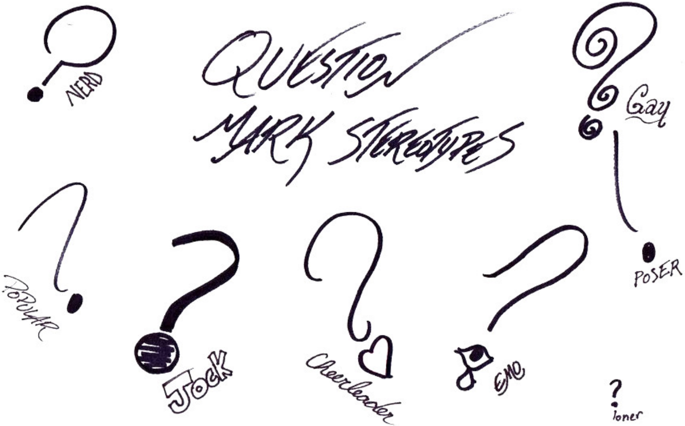
FuzzyDL: Overview

- The instance of MILP is created using constraints $\mathscr{C}$.
- In order to solve $\mathrm{glb}(\mathcal{K}, \langle a : C \rangle)$, the objective function is set to minimize x in the MILP instance created for an augmented knowledge base $\mathcal{K} \cup \langle a : \neg C \mid 1 - x \rangle$.
- Similarly for $\mathrm{glb}(\mathcal{K}, \langle a : C \sqsubseteq D \rangle)$ the augmented knowledge base is $\mathcal{K} \cup \langle a : \neg C \mid 1 - x \rangle$.
- $\mathcal{K}$ is inconsistent iff the MILP instance has no solution.
- Hence the $\mathrm{glb}(\cdot, \cdot)$ is found if MILP instance has a solution.

FuzzyDL: Conclusion

- FuzzyDL is a tableau algorithm with exactly 1 branch.
  The $\sqcup$ does not cause branching.
- Rules are applied deterministically (to ensure termination).
- The complexity of reasoning is caused by the integer (y) variables.

Questions?! Ask, please.



Source: `ragtagdoodles.deviantart.com`

Ex: Jim revisited

We will use the Łukasiewicz logic in the following examples ($\sqcap = \sqcap_{\text{Ł}}$, ...).

$$\langle jim : \text{Male} \mid 0.9 \rangle \tag{3}$$

$$\langle jim : \text{Female} \mid 0.2 \rangle \tag{4}$$

$$\langle \text{Male} \sqcap \text{Female} \sqsubseteq \bot \mid 1 \rangle \tag{5}$$

The interpretation domain is $\Delta^{\mathcal{I}_1} = \Delta^{\mathcal{I}_2} = \{j\}$, $jim^{\mathcal{I}_1} = jim^{\mathcal{I}_2} = j$.

$\text{Male}^{\mathcal{I}_1} = \{(j; 0.9)\}$ $\text{Male}^{\mathcal{I}_2} = \{(j; 0.9)\}$

$\text{Female}^{\mathcal{I}_1} = \{(j; 0)\}$ $\text{Female}^{\mathcal{I}_2} = \{(j; 0.2)\}$

Ex: Jim revisited (check your knowledge)

Let's check the interpretation against the definitions...

$\mathscr{I} \models \tau$	$\tau_{(1)}$	$\tau_{(2)}$	$\tau_{(3)}$
$\mathscr{I}_1$	sǝʎ	ou	sǝʎ
$\mathscr{I}_2$	ǝsʎ	sǝʎ	ou

Ex: Jim revisited (in fuzzyDL)

Let's change the weights and encode the example in fuzzyDL:

```
(instance jim Male 0.4)
(instance jim Female 0.2)

(l-implies (and Male Female) *bottom* 0.9)

(min-instance? jim Male)
(max-instance? jim Male)
(min-instance? jim Female)
(max-instance? jim Female)
```

Let $\langle jim : \mathrm{Male} \,|\, \alpha \rangle$ and $\langle jim : \mathrm{Female} \,|\, \beta \rangle$, what are the bounds on α and β? fuzzyDL shows that $0.4 \leqslant \alpha \leqslant 0.9$ and $0.2 \leqslant \beta \leqslant 0.7$. Why?

Ex: Smokers

Recall the motivational example from the first lecture:

$$\langle \text{symmetric}(\mathsf{friend}) \rangle \tag{6}$$

$$\langle (\mathit{anna}, \mathit{bill}) : \mathsf{friend} \,|\, 1 \rangle \tag{7}$$

$$\langle (\mathit{bill}, \mathit{cloe}) : \mathsf{friend} \,|\, 1 \rangle \tag{8}$$

$$\langle (\mathit{cloe}, \mathit{dirk}) : \mathsf{friend} \,|\, 1 \rangle \tag{9}$$

$$\langle \mathit{anna} : \mathsf{Smoker} \,|\, 1 \rangle \tag{10}$$

$$\langle \exists \mathsf{friend} \cdot \mathsf{Smoker} \sqsubseteq \mathsf{Smoker} \,|\, 0.7 \rangle \tag{11}$$

What are the bounds on $\langle i : \mathsf{Smoker} \rangle$ for $i \in \{\mathit{anna}, \mathit{bill}, \mathit{cloe}, \mathit{dirk}\}$?

Ex: Smokers

What changes if we add

$$\langle dirk : \neg \text{Smoker} \,|\, 0.7 \rangle \tag{12}$$

$$\tag{13}$$

What are the bounds on $\langle i : \neg \text{Smoker} \rangle$ for $i \in \{anna, bill, cloe, dirk\}$?

Ex: Smokers (in fuzzyDL)

```
(implies (some friendOf Smoker) Smoker 0.7)

(symmetric friendOf)
(related anna bill friendOf)
(related bill cloe friendOf)
(related cloe dirk friendOf)

(instance anna Smoker)
(instance dirk (not Smoker) 0.7)

(min-instance? anna Smoker)
(min-instance? bill Smoker)
(min-instance? cloe Smoker)
(min-instance? dirk Smoker)

(max-instance? anna Smoker)
(max-instance? bill Smoker)
(max-instance? cloe Smoker)
(max-instance? dirk Smoker)
```

Concrete data types

The domain $\Delta^{\mathscr{I}}$ is an unordered set. This is good for modelling cathegorical data: e.g. colors, people, ...

General idea: Extended interpretation

But we also need to include real numbers $\mathbb{R}$. The *fuzzy description logic with concrete datatypes* $\mathcal{SHIF}(\mathcal{D})$ uses “abstract objects” and “concrete objects”:

$$\Delta^{\mathscr{I}} = \Delta_a^{\mathscr{I}} \cup \mathbb{R}$$

Concrete data types

- *Concrete individuals*, are interpreted as objects from $\mathbb{R}$.
- *Concrete concepts*, are interpreted as subsets from $\mathbb{R}$.
- *Concrete roles*, are interpreted as subsets from $(\Delta_a^{\mathcal{I}} \times \mathbb{R})$.

All non-concrete notions are called *abstract*.

Concrete data types: New concepts

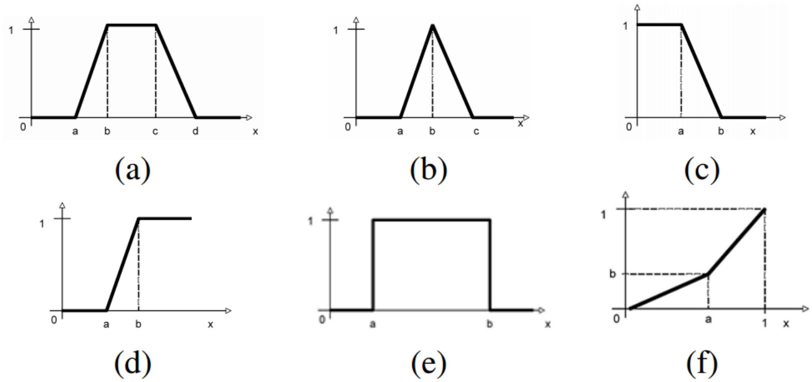


Fig. 1. (a) Trapezoidal function; (b) Triangular function; (c) L-function; (d) R-function; (e) Crisp interval; (f) Linear function.

Ex: Age of parents

```
(related adam bob parent) (related adam eve parent)

(define-fuzzy-concept around23 triangular(0,100, 18,23,26))
(define-fuzzy-concept moreTh17 right-shoulder(0,100, 13,21))
(instance bob (some age around23) 0.9)
(instance eve (some age moreTh17))

(define-fuzzy-concept young left-shoulder(0,100, 17,25))
(define-concept YoungPerson (some age young))

(min-instance? eve YoungPerson) (max-instance? eve YoungPerson)
(min-instance? bob YoungPerson) (max-instance? bob YoungPerson)
(min-instance? adam (all  parent YoungPerson))
(max-instance? adam (all  parent YoungPerson))
(min-instance? adam (some parent YoungPerson))
(max-instance? adam (some parent YoungPerson))
```

Ex: Age of parents

1. What are the bounds on α from $\langle \mathit{eve} : \text{YoungPerson} \,|\, \alpha \rangle$?

   Start by drawing the concept around23, then construct an interpretation. How much freedom do you have when constructing the interpretation?

2. Let fuzzyDL reasoner give you both bounds on $\langle i : \text{YoungPerson} \,|\, \beta_i \rangle$ for $i \in \{\mathit{eve}, \mathit{bob}\}$.

   How do you infer the bounds on $\langle \mathit{adam} : \text{YoungPerson} \,|\, \gamma \rangle$?

Ex: Car dealing

1. The buyer wants a **passenger** that costs **less than €26000**.
2. If there is an **alarm system** in the car, **then** he is satisfied with paying no more than **€22300**, but he can go up to **€22750** with a lesser degree of satisfaction.
3. The **driver insurance**, **air conditioning** and the **black color** are important factors.
4. Preferably the price is no more than **€22000**, but he can go to **€24000** to a lesser degree of satisfaction.

Ex: Car dealing

1. The seller wants to sell no less than **€22000**.
2. Preferably the buyer buys the **insurance plus** package.
3. If the **color is black**, then it is highly possible the car has an **air-conditioning**.

This can be formalized in fuzzy description logic.
We have the background knowledge:

$$\langle \mathsf{Sedan} \sqsubseteq \mathsf{PassengerCar} \,|\, 1 \rangle$$

$$\langle \mathsf{InsurancePlus} = \mathsf{DriverInsurance} \sqcap \mathsf{TheftInsurance} \,|\, 1 \rangle$$

Ex: Car dealing

The buyer's preferences:

1. $B = \text{PassengerCar} \sqcap \exists \text{price} \cdot \leqslant 26000$
2. $B_1 = \text{AlarmSystem} \mapsto \exists \text{price} \cdot l.sh.(22300, 22750)$
3. $B_2 = \text{DriverInsurance}, B_3 = \text{AirCondition}, B_4 = \exists \text{color} \cdot \text{Black}$
4. $B_5 = \exists \text{price} \cdot l.sh.(22000, 24000)$

The buyer's preferences:

1. $S = \text{PassengerCar} \sqcap \exists \text{price} \cdot \geqslant 22000$
2. $S_1 = \text{InsurancePlus}$
3. $S_2 = (0.5\ (\exists \text{color} \cdot \text{Black}) \mapsto \text{AirCondition})$

Ex: Car dealing

We know that S and B are hard constraints and $B_{1..5}$ and $S_{1..2}$ are soft preferences. All the concepts can be “summed up”:

$$\text{Buy} = B \sqcap (0.1B_1 + 0.2B_2 + 0.1B_3 + 0.4B_4 + 0.2B_5)$$

and

$$\text{Sell} = S \sqcap (0.6S_1 + 0.4S_2)$$

A good choice of $\sqcap$ can make B a hard constraint.

Ex: Car dealing

Optimal match

$$\mathrm{glb}(\mathcal{K}, \mathsf{Buy} \sqcap \mathsf{Sell})$$

Finds the optimal match between a seller and a buyer. (Finds an ideal, imaginary car that maximizes satisfaction of both parties.)

Particular car

$$\mathrm{glb}(\mathcal{K}, \langle \mathit{audiTT} : \mathsf{Buy} \sqcap \mathsf{Sell} \rangle)$$

Finds the degree of satisfaction for a particuklar car *audiTT*.

Where to find more examples?

- **Simple examples** are bundled with fuzzyDL installation (`/opt/fuzzydl/` on the `heartofgold` server).
- **Advanced examples** can be found on the fuzzyDL web site: `http://gaia.isti.cnr.it/~straccia/software/fuzzyDL/fuzzyDL.html`

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