

AE4M33RZN, Fuzzy logic: Examples in fuzzy DL

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Plan of the lecture

Fuzzy DL examples

Concrete data types

Ex: Jim revisited

We will use the Lukasiewicz logic in the following examples ($\sqcap = \sqcap_L, \dots$).

$$\langle \mathit{jim} : \mathit{Male} \mid 0.9 \rangle \quad (1)$$

$$\langle \mathit{jim} : \mathit{Female} \mid 0.2 \rangle \quad (2)$$

$$\langle \mathit{Male} \sqcap \mathit{Female} \sqsubseteq \perp \mid 1 \rangle \quad (3)$$

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The interpretation domain is $\Delta^{\mathcal{F}_1} = \Delta^{\mathcal{F}_2} = \{j\}$, $\mathit{jim}^{\mathcal{F}_1} = \mathit{jim}^{\mathcal{F}_2} = j$.

$$\mathit{Male}^{\mathcal{F}_1} = \{(j; 0.9)\}$$

$$\mathit{Male}^{\mathcal{F}_2} = \{(j; 0.9)\}$$

$$\mathit{Female}^{\mathcal{F}_1} = \{(j; 0)\}$$

$$\mathit{Female}^{\mathcal{F}_2} = \{(j; 0.2)\}$$

Ex: Jim revisited (check your knowledge)

Let's check the interpretation against the definitions...

$\mathcal{I} \models \tau$	$\tau_{(1)}$	$\tau_{(2)}$	$\tau_{(3)}$
\mathcal{I}_1	?	?	?
\mathcal{I}_2	?	?	?

Ex: Jim revisited (check your knowledge)

Let's check the interpretation against the definitions...

$\mathcal{I} \models \tau$	$\tau_{(1)}$	$\tau_{(2)}$	$\tau_{(3)}$
\mathcal{I}_1	yes	no	yes
\mathcal{I}_2	yse	yes	no

Ex: Jim revisited (in fuzzyDL)

Let's change the weights and encode the example in fuzzyDL:

```
(instance jim Male 0.4)
```

```
(instance jim Female 0.2)
```

```
(1-implies (and Male Female) *bottom* 0.9)
```

```
(min-instance? jim Male)
```

```
(max-instance? jim Male)
```

```
(min-instance? jim Female)
```

```
(max-instance? jim Female)
```

Let $\langle jim : \text{Male} | \alpha \rangle$ and $\langle jim : \text{Female} | \beta \rangle$, what are the bounds on α and β ? fuzzyDL shows that $0.4 \leq \alpha \leq 0.9$ and $0.2 \leq \beta \leq 0.7$. Why?

Ex: Smokers

Recall the motivational example from the first lecture:

$$\langle \text{symmetric}(\text{friend}) \rangle \quad (4)$$

$$\langle (\text{anna}, \text{bill}) : \text{friend} \mid 1 \rangle \quad (5)$$

$$\langle (\text{bill}, \text{cloe}) : \text{friend} \mid 1 \rangle \quad (6)$$

$$\langle (\text{cloe}, \text{dirk}) : \text{friend} \mid 1 \rangle \quad (7)$$

$$\langle \text{anna} : \text{Smoker} \mid 1 \rangle \quad (8)$$

$$\langle \exists \text{ friend} \cdot \text{Smoker} \sqsubseteq \text{Smoker} \mid 0.7 \rangle \quad (9)$$

What are the bounds on $\langle i : \text{Smoker} \rangle$ for $i \in \{\text{anna}, \text{bill}, \text{cloe}, \text{dirk}\}$?

Ex: Smokers

What changes if we add

$$\langle \text{dirk} : \neg \text{Smoker} \mid 0.7 \rangle \quad (10)$$

(11)

What are the bounds on $\langle i : \neg \text{Smoker} \rangle$ for $i \in \{\text{anna}, \text{bill}, \text{cloe}, \text{dirk}\}$?

Ex: Smokers (in fuzzyDL)

```
(implies (some friendOf Smoker) Smoker 0.7)
```

```
(symmetric friendOf)  
(related anna bill friendOf)  
(related bill cloe friendOf)  
(related cloe dirk friendOf)
```

```
(instance anna Smoker)  
(instance dirk (not Smoker) 0.7)
```

```
(min-instance? anna Smoker)  
(min-instance? bill Smoker)  
(min-instance? cloe Smoker)  
(min-instance? dirk Smoker)
```

```
(max-instance? anna Smoker)  
(max-instance? bill Smoker)  
(max-instance? cloe Smoker)  
(max-instance? dirk Smoker)
```

Concrete data types

The domain $\Delta^{\mathcal{F}}$ is an unordered set. This is good for modelling categorical data: e.g. colors, people, ...

General idea: Extended interpretation

But we also need to include real numbers \mathbb{R} . The *fuzzy description logic with concrete datatypes* $\mathcal{SHIF}(\mathcal{D})$ uses “abstract objects” and “concrete objects”:

$$\Delta^{\mathcal{F}} = \Delta_a^{\mathcal{F}} \cup \mathbb{R}$$

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All non-concrete notions are called *abstract*.

Concrete data types: New concepts

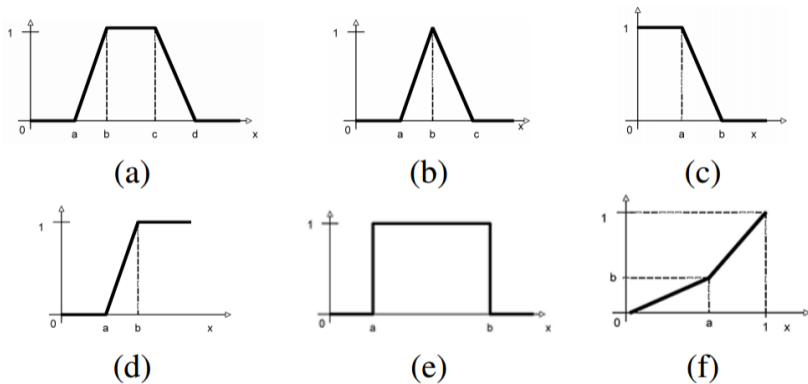


Fig. 1. (a) Trapezoidal function; (b) Triangular function; (c) *L*-function; (d) *R*-function; (e) Crisp interval; (f) Linear function.

Ex: Age of parents

```
(related adam bob parent) (related adam eve parent)
```

```
(define-fuzzy-concept around23 triangular(0,100, 18,23,26))  
(define-fuzzy-concept moreTh17 right-shoulder(0,100, 13,21))  
(instance bob (some age around23) 0.9)  
(instance eve (some age moreTh17))
```

```
(define-fuzzy-concept young left-shoulder(0,100, 17,25))  
(define-concept YoungPerson (some age young))
```

```
(min-instance? eve YoungPerson) (max-instance? eve YoungPerson)  
(min-instance? bob YoungPerson) (max-instance? bob YoungPerson)  
(min-instance? adam (all parent YoungPerson))  
(max-instance? adam (all parent YoungPerson))  
(min-instance? adam (some parent YoungPerson))  
(max-instance? adam (some parent YoungPerson))
```

Ex: Age of parents

1. What are the bounds on α from $\langle \text{eve} : \text{YoungPerson} \mid \alpha \rangle$?

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Start by drawing the concept `around23`, then construct an interpretation. How much freedom do you have when constructing the interpretation?

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1. What are the bounds on α from $\langle \text{eve} : \text{YoungPerson} \mid \alpha \rangle$?

Start by drawing the concept `around23`, then construct an interpretation. How much freedom do you have when constructing the interpretation?

2. Let fuzzyDL reasoner give you both bounds on $\langle i : \text{YoungPerson} \mid \beta_i \rangle$ for $i \in \{\text{eve}, \text{bob}\}$.

How do you infer the bounds on $\langle \text{adam} : \text{YoungPerson} \mid \gamma \rangle$?

Ex: Car dealing

1. The buyer wants a **passenger** that costs **less than €26000**.
2. If there is an **alarm system** in the car, **then** he is satisfied with paying no more than **€22300**, but he can go up to **€22750** with a lesser degree of satisfaction.
3. The **driver insurance**, **air conditioning** and the **black color** are important factors.
4. Preferably the price is no more than **€22000**, but he can go to **€24000** to a lesser degree of satisfaction.

Ex: Car dealing

1. The seller wants to sell no less than **€22000**.
2. Preferably the buyer buys the **insurance plus** package.
3. If the **color is black**, then it is highly possible the car has an **air-conditioning**.

This can be formalized in fuzzy description logic.

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We have the background knowledge:

$\langle \text{Sedan} \sqsubseteq \text{PassengerCar} \mid 1 \rangle$

$\langle \text{InsurancePlus} = \text{DriverInsurance} \sqcap \text{TheftInsurance} \mid 1 \rangle$

Ex: Car dealing

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3. $B_2 = \text{DriverInsurance}, B_3 = \text{AirCondition}, B_4 = \exists \text{color} \cdot \text{Black}$
4. $B_5 = \exists \text{price} \cdot \text{I.sh.}(22000, 24000)$

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1. $S = \text{PassengerCar} \sqcap \exists \text{price} \cdot \geq 22000$
2. $S_1 = \text{InsurancePlus}$
3. $S_2 = (0.5 (\exists \text{color} \cdot \text{Black}) \mapsto \text{AirCondition})$

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$$\text{Buy} = B \sqcap (0.1B_1 + 0.2B_2 + 0.1B_3 + 0.4B_4 + 0.2B_5)$$

and

$$\text{Sell} = S \sqcap (0.6S_1 + 0.4S_2)$$

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and

$$\text{Sell} = S \sqcap (0.6S_1 + 0.4S_2)$$

A good choice of \sqcap can make B a hard constraint.

Ex: Car dealing

Optimal match

$$bsb(K, \text{Buy} \sqcap \text{Sell})$$

Finds the optimal match between a seller and a buyer. (Finds an ideal, imaginary car that maximizes satisfaction of both parties.)

Particular car

$$bdb(K, \langle \text{audiTT} : \text{Buy} \sqcap \text{Sell} \rangle)$$

Finds the degree of satisfaction for a particular car *audiTT*.

Where to find more examples?

- **Simple examples** are bundled with fuzzyDL installation (/opt/fuzzydl / on the heartofgold server).
- **Advanced examples** can be found on the fuzzyDL web site:
<http://gaia.isti.cnr.it/~straccia/software/fuzzyDL/fuzzyDL.html>