

Inference in Description Logics

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Our plan

- 1 Inference Problems
- 2 Inference Algorithms
 - Tableau Algorithm for \mathcal{ALC}
- 3 From \mathcal{ALC} to OWL(2)-DL

Inference Problems

Inference Problems in TBOX

We have introduced syntax and semantics of the language \mathcal{ALC} . Now, let's look on automated reasoning. Having a \mathcal{ALC} theory $\mathcal{K} = (\mathcal{T}, \mathcal{A})$. For TBOX \mathcal{T} and concepts $C_{(i)}$, we want to decide whether

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All these tasks can be reduced to unsatisfiability checking of a single concept ...

Reducing Subsumption to Unsatisfiability

Example

These reductions are straightforward – let's show, how to reduce subsumption checking to unsatisfiability checking. Reduction of other inference problems to unsatisfiability is analogous.

$$\begin{aligned}(\mathcal{T} \models C_1 \sqsubseteq C_2) & \text{ iff} \\(\forall I)(I \models \mathcal{T} \implies I \models C_1 \sqsubseteq C_2) & \text{ iff} \\(\forall I)(I \models \mathcal{T} \implies C_1^I \subseteq C_2^I) & \text{ iff} \\(\forall I)(I \models \mathcal{T} \implies C_1^I \cap (\Delta^I \setminus C_2^I) \subseteq \emptyset) & \text{ iff} \\(\forall I)(I \models \mathcal{T} \implies I \models C_1 \sqcap \neg C_2 \sqsubseteq \perp) & \text{ iff} \\(\mathcal{T} \models C_1 \sqcap \neg C_2 \sqsubseteq \perp) & \end{aligned}$$

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All these tasks, as well as concept unsatisfiability checking, can be reduced to consistency checking. Under which condition and how ?

Reduction of concept unsatisfiability to theory consistency

Example

Consider an \mathcal{ALC} theory $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, a concept C and a fresh individual a_f not occurring in \mathcal{K} :

$$\begin{array}{ll} (\mathcal{T} \models C \sqsubseteq \perp) & \text{iff} \\ (\forall I)(I \models \mathcal{T} \implies I \models C \sqsubseteq \perp) & \text{iff} \\ (\forall I)(I \models \mathcal{T} \implies C^I \subseteq \emptyset) & \text{iff} \\ \neg [(\exists I)(I \models \mathcal{T} \wedge C^I \not\subseteq \emptyset)] & \text{iff} \\ \neg [(\exists I)(I \models \mathcal{T} \wedge a_f^I \in C^I)] & \text{iff} \\ (\mathcal{T}, \{C(a_f)\}) \text{ is inconsistent} & \end{array}$$

Note that for more expressive description logics than \mathcal{ALC} , the ABOX has to be taken into account as well due to its interaction with TBOX.

Inference Algorithms

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We will introduce tableau algorithms.

Tableaux Algorithms

- Tableaux Algorithms (TAs) serve for checking theory consistency in a simple manner: **“Consistency of the given ABOX \mathcal{A} w.r.t. TBOX \mathcal{T} (resp. consistency of theory \mathcal{K}) is proven if we succeed in constructing a model of $\mathcal{T} \cup \mathcal{A}$.”** (resp. theory \mathcal{K})

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 - ▶ chosen *strategy* for rule application
- TAs are not new in DL – they were known for FOL as well.

Completion Graphs

completion graph is a labeled oriented graph $G = (V_G, E_G, L_G)$, where each node $x \in V_G$ is labeled with a set $L_G(x)$ of concepts and each edge $\langle x, y \rangle \in E_G$ is labeled with a set of edges $L_G(\langle x, y \rangle)$ ²

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Do not mix with notion of complete graphs known from graph theory.

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Tableau Algorithm for \mathcal{ALC} with empty TBOX

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 - ▶ Sets $V_{G_0}, E_{G_0}, L_{G_0}$ are smallest possible with these properties.

Tableau algorithm for \mathcal{ALC} without TBOX (2)

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- 4 (Rule Application) Find a rule that is applicable to G and apply it. As a result, we obtain from the state S a new state S' . Jump to step 2.

TA for \mathcal{ALC} without TBOX – Inference Rules

\rightarrow_{\square} rule

TA for \mathcal{ALC} without TBOX – Inference Rules

\rightarrow_{\sqcap} rule

if $(C_1 \sqcap C_2) \in L_G(\mathbf{a})$ and $\{C_1, C_2\} \not\subseteq L_G(\mathbf{a})$ for some $\mathbf{a} \in V_G$.

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then $S' = S \cup \{G'\} \setminus \{G\}$, where $G' = (V_G, E_G, L_{G'})$, and $L_{G'}(a) = L_G(a) \cup \{C_1, C_2\}$
and otherwise is the same as L_G .

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TA for \mathcal{ALC} without TBOX – Inference Rules

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TA Run Example

Example

Let's check consistency of the ontology $\mathcal{K}_2 = (\emptyset, \mathcal{A}_2)$, where $\mathcal{A}_2 = \{(\exists maDite \cdot Muz \sqcap \exists maDite \cdot Prarodic \sqcap \neg \exists maDite \cdot (Muz \sqcap Prarodic))(JAN)\}$.

- Let's transform the concept into NNF:

$$\exists maDite \cdot Muz \sqcap \exists maDite \cdot Prarodic \sqcap \forall maDite \cdot (\neg Muz \sqcup \neg Prarodic)$$

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"JAN"

$((\forall maDite \cdot (\neg Muz \sqcup \neg Prarodic)) \sqcap (\exists maDite \cdot Prarodic) \sqcap (\exists maDite \cdot Muz))$

TA Run Example (2)

Example

...

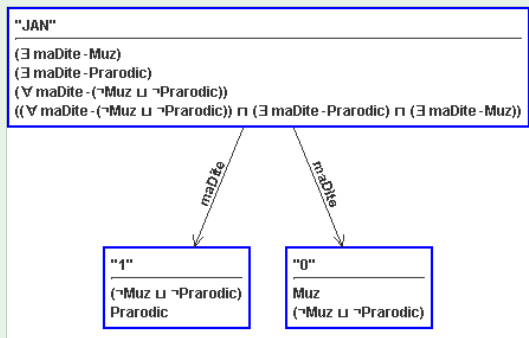
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- $\{G_0\} \xrightarrow{\neg\text{-rule}} \{G_1\} \xrightarrow{\exists\text{-rule}} \{G_2\} \xrightarrow{\exists\text{-rule}} \{G_3\} \xrightarrow{\forall\text{-rule}} \{G_4\}$, where G_4 is



TA Run Example (3)

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...

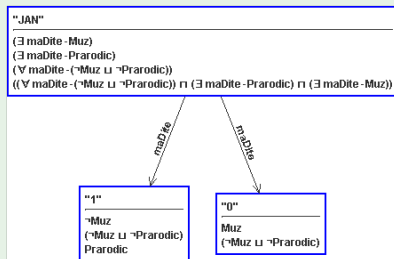
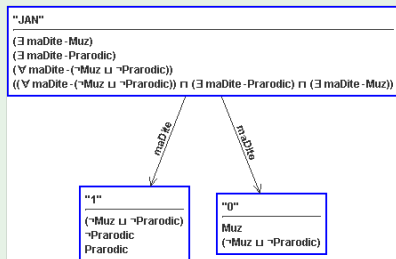
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- Now, we have to apply the \sqcup -rule to the concept $\neg Muz \sqcup \neg Rodic$ either in the label of node "0", or in the label of node "1". Its application e.g. to node "1" we obtain the state $\{G_5, G_6\}$ (G_5 left, G_6 right)

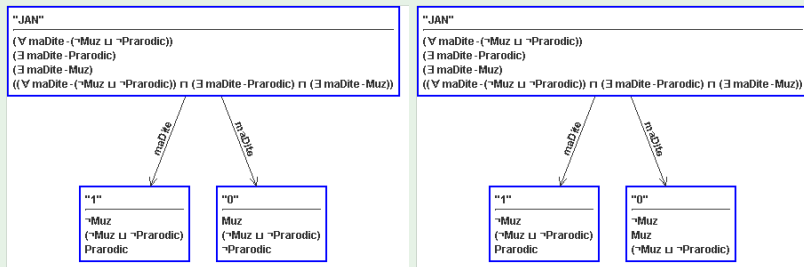


TA Run Example (4)

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- We see that G_5 contains a direct clash in node "1". The only other option is to go through the graph G_6 . By application of \sqcup -rule we obtain the state $\{G_5, G_7, G_8\}$, where G_7 (left), G_8 (right) are derived from G_6 :

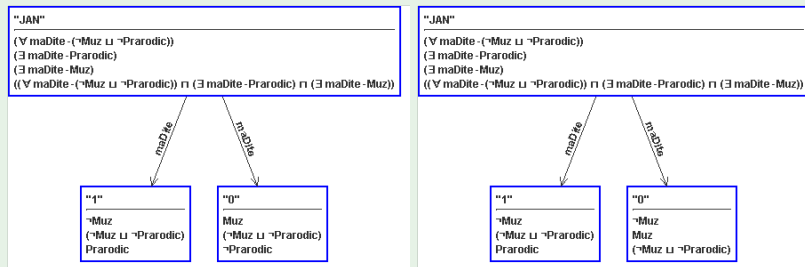


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- G_7 is complete and without direct clash.

TA Run Example (5)

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... A canonical model \mathcal{I}_2 can be created from G_7 . Is it the only model of \mathcal{K}_2 ?

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- after application of any of the following rules $\rightarrow_{\sqcap}, \rightarrow_{\exists}, \rightarrow_{\forall}$ graph G is either enriched with a new node, new edge, or labeling of an existing node/edge is enriched. All these operations are finite.

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- Soundness of the TA can be verified as follows. For any $\mathcal{I} \models \mathcal{A}_{G_i}$, it must hold that $\mathcal{I} \models \mathcal{A}_{G_{i+1}}$. We have to show that application of each rule preserves consistency. As an example, let's take the \rightarrow_{\exists} rule:

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- For other rules, the soundness is shown in a similar way.

Completeness

- To prove completeness of the TA, it is necessary to construct a model for each complete completion graph G that doesn't contain a direct clash. Canonical model \mathcal{I} can be constructed as follows:
 - ▶ the domain $\Delta^{\mathcal{I}}$ will consist of all nodes of G .
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- What about complexity of the algorithm ?
 - ▶ P-SPACE (between NP and EXP-TIME).

General Inclusions

We have presented the tableau algorithm for consistency checking of $\mathcal{K} = (\emptyset, \mathcal{A})$. How the situation changes when $\mathcal{T} \neq \emptyset$?

- consider \mathcal{T} containing axioms of the form $C_i \sqsubseteq D_i$ for $1 \leq i \leq n$.
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$$\top \sqsubseteq \top_C$$

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- for each model \mathcal{I} of the theory \mathcal{K} , each element of $\Delta^{\mathcal{I}}$ must belong to $\top_C^{\mathcal{I}}$. How to achieve this ?

General Inclusions (2)

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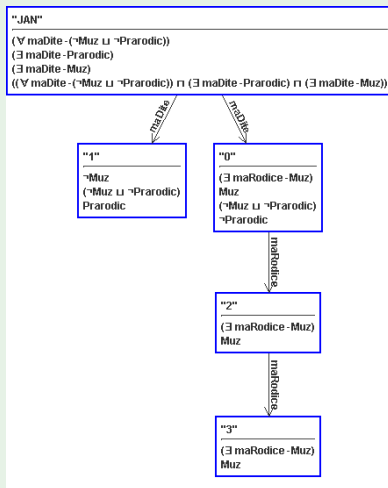
Example

Consider $\mathcal{K}_3 = (\{\text{Muz} \sqsubseteq \exists \text{maRodice} \cdot \text{Muz}\}, \mathcal{A}_2)$. Then \top_C is $\neg \text{Muz} \sqcup \exists \text{maRodice} \cdot \text{Muz}$. Let's use the introduced TA enriched by $\rightarrow_{\sqsubseteq}$ rule. Repeating several times the application of rules $\rightarrow_{\sqsubseteq}$, \rightarrow_{\sqcup} , \rightarrow_{\exists} to G_7 (that is not complete w.r.t. to $\rightarrow_{\sqsubseteq}$ rule) from the previous example we get

...

General Inclusions (3)

Example



... this algorithm doesn't necessarily terminate ☹.

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Blocking in TA (2)

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- **Introduced TA with subset blocking is sound, complete and finite decision procedure for ALC .**

Let's play ...

- <http://krizik.felk.cvut.cz/km/dl/index.html>

From \mathcal{ALC} to OWL(2)-DL

Extending ... \mathcal{ALC} ...

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- Let's take a look, how to extend \mathcal{ALC} while preserving decidability.

Extending ... \mathcal{ALC} ... (2)

\mathcal{N} (Number restrictions) are used for restricting the number of successors in the given role for the given concept.

| syntax (concept) | semantics |
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| $(\geq n R)$ | $\left\{ a \mid \left \{ b \mid (a, b) \in R^{\mathcal{I}} \} \right \geq n \right\}$ |
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- ▶ ... and $Bicycle \equiv (= 2 \textit{hasWheel})$?

Extending ... \mathcal{ALC} ... (3)

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- ▶ Which qualified number restrictions can be expressed in \mathcal{ALC} ?

Extending ... *ALC* ... (4)

- (Nominals) can be used for naming a concept elements explicitly.

| syntax (concept) | semantics |
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| $\{a_1, \dots, a_n\}$ | $\{a_1^I, \dots, a_n^I\}$ |

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- ▶ Concept $\{MALE, FEMALE\}$ denotes a gender concept that must be interpreted with at most two elements. Why at most ?

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- ▶ *Continent* \equiv
 $\{EUROPE, ASIA, AMERICA, AUSTRALIA, AFRICA, ANTARCTICA\}$?

Extending ... \mathcal{ALC} ... (5)

\mathcal{I} (Inverse roles) are used for defining role inversion.

$$\frac{\text{syntax (role)} \quad \text{semantics}}{R^- \quad (R^{\mathcal{I}})^{-1}}$$

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.trans (Role transitivity axiom) denotes that a role is transitive. Attention – it is not a transitive closure operator.

| syntax (axiom) | semantics |
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| $trans(R)$ | $R^{\mathcal{I}}$ is transitive |

Example

- ▶ Role *isPartOf* can be defined as transitive, while role *hasParent* is not. What about roles *hasPart*, *hasPart*⁻, *hasGrandFather*⁻ ?

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- ▶ Role *isPartOf* can be defined as transitive, while role *hasParent* is not. What about roles *hasPart*, *hasPart*⁻, *hasGrandFather*⁻ ?
- ▶ What is a transitive closure of a relationship ? What is the difference between a transitive closure of *hasDirectBoss* ^{\mathcal{I}} and *hasBoss* ^{\mathcal{I}} .

Extending ... \mathcal{ALC} ... (7)

\mathcal{H} (Role hierarchy) serves for expressing role hierarchies (taxonomies) – similarly to concept hierarchies.

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- ▶ What is the difference between a concept hierarchy $Mother \sqsubseteq Parent$ and role hierarchy $hasMother \sqsubseteq hasParent$.

Extending ... \mathcal{ALC} ... (8)

\mathcal{R} (role extensions) serve for defining expressive role constructs, like role chains, role disjunctions, etc.

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- ▶ Whom does the following concept denote $Person \sqcap \exists likes \cdot Self$?

Global restrictions

- *Simple roles* have no (direct or indirect) subroles that are either *transitive* or are defined by means of property chains

hasFather ◊ *hasBrother* \sqsubseteq *hasUncle*

hasUncle \sqsubseteq *hasRelative*

hasBiologicalFather \sqsubseteq *hasFather*

hasRelative and *hasUncle* are not simple.

- Each concept construct and each axiom from this list contains only *simple roles*:
 - ▶ number restrictions – ($\geq n R$), ($= n R$), ($\leq n R$) + their qualified versions
 - ▶ $\exists R \cdot Self$
 - ▶ specifying functionality/inverse functionality (leads to number restrictions)
 - ▶ specifying irreflexivity, asymmetry, and disjoint object properties.

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DL-safe rules

DL-safe rules are decidable conjunctive rules where each variable **only binds individuals** (i.e. representation of domain elements, not domain elements themselves).

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$$\Box(\text{Man} \Rightarrow \text{Person} \wedge \Box_{\text{hasFather}} \text{Man}) \quad (2)$$

Vague Knowledge - fuzzy, probabilistic and possibilistic extensions

Data Types (\mathcal{D}) allow integrating a data domain (numbers, strings), e.g. $\text{Person} \sqcap \exists \text{hasAge} \cdot 23$ represents the concept describing "23-years old persons".