

# Expressive Description Logics

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From  $\mathcal{ALC}$  to OWL(2)-DL

Final Remarks

# From $\mathcal{ALC}$ to OWL(2)-DL

- We have introduced  $\mathcal{ALC}$ , together with a decision procedure. Its expressiveness is higher than propositional calculus, still it is insufficient for many practical applications.
- Let's take a look, how to extend  $\mathcal{ALC}$  while preserving decidability.

## Extending ... $\mathcal{ALC}$ ... (2)

$\mathcal{N}$  (Number restrictions) are used for restricting the number of successors in the given role for the given concept.

syntax (concept)	semantics
$(\geq n R)$	$\left\{ a \mid \left  \{ b \mid (a, b) \in R^{\mathcal{I}} \} \right  \geq n \right\}$
$(\leq n R)$	$\left\{ a \mid \left  \{ b \mid (a, b) \in R^{\mathcal{I}} \} \right  \leq n \right\}$
$(= n R)$	$\left\{ a \mid \left  \{ b \mid (a, b) \in R^{\mathcal{I}} \} \right  = n \right\}$

### Example

- Concept  $Woman \sqcap (\leq 3 \text{ hasChild})$  denotes women who have at most 3 children.
- What denotes the axiom  $Car \sqsubseteq (\geq 4 \text{ hasWheel})$  ?
- ... and  $Bicycle \equiv (= 2 \text{ hasWheel})$  ?

## Extending ... $\mathcal{ALC}$ ... (3)

$\mathcal{Q}$  (Qualified number restrictions) are used for restricting the number of successors of *the given type* in the given role for the given concept.

syntax (concept)	semantics
$(\geq n R C)$	$\left\{ a \mid \left  \{ b \mid (a, b) \in R^I \wedge b^I \in C^I \} \right  \geq n \right\}$
$(\leq n R C)$	$\left\{ a \mid \left  \{ b \mid (a, b) \in R^I \wedge b^I \in C^I \} \right  \leq n \right\}$
$(= n R C)$	$\left\{ a \mid \left  \{ b \mid (a, b) \in R^I \wedge b^I \in C^I \} \right  = n \right\}$

### Example

- Concept  $Woman \sqcap (\geq 3 \text{ hasChild } Man)$  denotes women who have at least 3 sons.
- What denotes the axiom  $Car \sqsubseteq (\geq 4 \text{ hasPart } Wheel)$  ?
- Which qualified number restrictions can be expressed in  $\mathcal{ALC}$  ?

# Extending ... $\mathcal{ALC}$ ... (4)

- (Nominals) can be used for naming a concept elements explicitly.

syntax (concept)	semantics
$\{a_1, \dots, a_n\}$	$\{a_1^I, \dots, a_n^I\}$

## Example

- Concept  $\{MALE, FEMALE\}$  denotes a gender concept that must be interpreted with at most two elements. Why at most ?
- $Continent \equiv \{EUROPE, ASIA, AMERICA, AUSTRALIA, AFRICA, ANTARCTICA\}$   
?

$\mathcal{I}$  (Inverse roles) are used for defining role inversion.

$$\frac{\text{syntax (role)}}{R^-} \quad \frac{\text{semantics}}{(R^{\mathcal{I}})^{-1}}$$

## Example

- Role  $hasChild^-$  denotes the relationship  $hasParent$ .
- What denotes axiom  $Person \sqsubseteq (= 2 hasChild^-)$  ?
- What denotes axiom  $Person \sqsubseteq \exists hasChild^- \cdot \exists hasChild \cdot \top$  ?



# Extending ... $\mathcal{ALC}$ ... (6)

$\cdot^{trans}$  (Role transitivity axiom) denotes that a role is transitive.  
Attention – it is not a transitive closure operator.

syntax (axiom)	semantics
$trans(R)$	$R^{\mathcal{I}}$ is transitive

## Example

- Role *isPartOf* can be defined as transitive, while role *hasParent* is not. What about roles *hasPart*, *hasPart*<sup>-</sup>, *hasGrandFather*<sup>-</sup> ?
- What is a transitive closure of a relationship ? What is the difference between a transitive closure of *hasDirectBoss* <sup>$\mathcal{I}$</sup>  and *hasBoss* <sup>$\mathcal{I}$</sup> .

$\mathcal{H}$  (Role hierarchy) serves for expressing role hierarchies (taxonomies) – similarly to concept hierarchies.

syntax (axiom)	semantics
$R \sqsubseteq S$	$R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$

## Example

- Role *hasMother* can be defined as a special case of the role *hasParent*.
- What is the difference between a concept hierarchy  $Mother \sqsubseteq Parent$  and role hierarchy  $hasMother \sqsubseteq hasParent$ .

# Extending ... $\mathcal{ALC}$ ... (8)

$\mathcal{R}$  (role extensions) serve for defining expressive role constructs, like role chains, role disjunctions, etc.

syntax	semantics
$R \circ S \sqsubseteq P$	$R^{\mathcal{I}} \circ S^{\mathcal{I}} \sqsubseteq P^{\mathcal{I}}$
$Dis(R, R)$	$R^{\mathcal{I}} \cap S^{\mathcal{I}} = \emptyset$
$\exists R \cdot Self$	$\{a \mid (a, a) \in R^{\mathcal{I}}\}$

## Example

- How would you define the role *hasUncle* by means of *hasSibling* and *hasParent* ?
- how to express that *R* is transitive, using a role chain ?
- Whom does the following concept denote  $Person \sqcap \exists likes \cdot Self$  ?

- From the previously introduced extensions, two prominent decidable supersets of  $\mathcal{ALC}$  can be constructed:
  - $\mathcal{SHOIN}$  is a description logics that backs OWL-DL.
  - $\mathcal{SROIQ}$  is a description logics that backs OWL2-DL.
  - Both OWL-DL and OWL2-DL are semantic web languages – they extend the corresponding description logics by:
    - syntactic sugar – axioms NegativeObjectPropertyAssertion, AllDisjoint, etc.
    - extralogical constructs – imports, annotations
    - data types – XSD datatypes are used

# Extending $\mathcal{ALC}$ – Reasoning

- What is the impact of the extensions to the automated reasoning procedure ? The introduced tableau algorithm for  $\mathcal{ALC}$  has to be adjusted as follows:
  - additional inference rules reflecting the semantics of newly added constructs ( $\mathcal{O}, \mathcal{N}, \mathcal{Q}$ )
  - definition of  $R$ -neighbourhood of a node in a completion graph.  $R$ -neighbourhood notion generalizes simple tests of two nodes being connected with an edge, e.g. in  $\exists$ -rule. ( $\mathcal{H}, \mathcal{R}, \mathcal{I}$ )
  - new conditions for direct clash detection
  - more strict blocking conditions (blocking over graph structures).
- This results in significant computation blowup – from EXPTIME ( $\mathcal{ALC}$ ) to
  - NEXPTIME for  $\mathcal{SHOIN}$
  - N2EXPTIME for  $\mathcal{SROIQ}$

# Final Remarks

# Other extensions

Modal Logic introduces *modal operators* – possibility/necessity, used in multiagent systems.

## Example

- ( $\Box$  represents e.g. the "believe" operator of an agent)

$$\Box(\text{Man} \sqsubseteq \text{Person} \sqcap \forall \text{hasFather} \cdot \text{Man}) \quad (1)$$

- As  $\mathcal{ALC}$  is a syntactic variant to a multi-modal propositional logic, where each role represents the accessibility relations between worlds in Kripke structure, the previous example can be transformed to the modal logic as:
- 

$$\Box(\text{Man} \Rightarrow \text{Person} \wedge \Box_{\text{hasFather}} \text{Man}) \quad (2)$$

Vague Knowledge - fuzzy, probabilistic and possibilistic extensions (see [HPS05]).

Data Types ( $\mathcal{D}$ ) allow integrating a data domain (numbers, strings), e.g.

$\text{Person} \sqcap \exists \text{hasAge} \cdot 23$  represents the concept describing "23-years old persons".

RacerPro (<http://www.racer-systems.com>) is a commercial LISP-based system for OWL-DL and SWRL (also available in client/server version).

Pellet (<http://www.mindswap.org>) is an open-source Java OWL2-DL engine.

Jena <http://jena.sourceforge.net/> is an open-source Java framework and API for OWL and RDF(S).

FaCT++ <http://owl.man.ac.uk/factplusplus/> is a DL reasoner for *SHOIQ* written in C++.

and other ... KAON2, FOWL, Kris