Cheat sheet

AE4M33RZN

1 Bayesian networks

Evaluation functions

- **Bayesian Information Criterion (BIC)**

$$BIC(G:D) = -\frac{K}{2}\log_2 M + \log_2 L(G:D) = -\frac{K}{2}\log_2 M - M\sum_{i=1}^{n} H(P_i|rodice(P_i)^G)$$

$K = \sum_{i=1}^{n} q_i(r_i - 1)$
q_i ...the number of unique instantiations of P_i parents, n ...the number of variables,
r_i ...the number of distinct P_i values, M ...the number of observations,

$$H(P_i|rodice(P_i)^G) = -\sum_{j=1}^{q_i}\sum_{k=1}^{r_i}\frac{N_{ij}}{M}\frac{N_{ijk}}{N_{ij}}\log_2\frac{N_{ijk}}{N_{ij}} = -\sum_{j=1}^{q_i}\sum_{k=1}^{r_i}\frac{N_{ijk}}{M}\log_2\frac{N_{ijk}}{N_{ij}}$$

N_{ij} ...the number of samples, where $parents(P_i)$ take the j-th instantiation of values,
N_{ijk} ...the number of samples, where P_i takes the k-th value and $parents(P_i)$ the j-th instantiation of values.

- **Bayesian score**

$$\ln Pr(D|G) = \ln\prod_{i=1}^{n} g(P_i, rodice(P_i)^G)$$

$$g(P_i, rodice(P_i)) = \prod_{j=1}^{q_i}\frac{(r_i-1)!}{(N_{ij}+r_i-1)!}\prod_{k=1}^{r_i} N_{ijk}!$$

Dynamic Bayesian Networks (DBNs)

- **the recursive definition of filtering task**

$$Pr(X_{t+1}|e_{1:t+1}) = \alpha Pr(e_{t+1}|X_{t+1})Pr(X_{t+1}|e_{1:t}) = \alpha Pr(e_{t+1}|X_{t+1})\sum_{x_t} Pr(X_{t+1}|x_t)Pr(x_t|e_{1:t})$$

X_t -- the set of unobservable state variables at time t,
E_t -- the set of observable evidence variables at time t.

2 Description Logics

$\rightarrow_\sqcap$ **rule**

if $(C_1 \sqcap C_2) \in L_G(a)$ a $\{C_1, C_2\} \nsubseteq L_G(a)$ for some $a \in V_G$.

then $S' = S \cup \{G'\} \setminus \{G\}$, where $G' = (V_G, E_G, L_{G'})$, a $L_{G'}(a) = L_G(a) \cup \{C_1, C_2\}$ and otherwise is the same as L_G.

$\rightarrow_\sqcup$ **rule**

if $(C_1 \sqcup C_2) \in L_G(a)$ a $\{C_1, C_2\} \cap L_G(a) = \emptyset$ for some $a \in V_G$.

then $S' = S \cup \{G_1, G_2\} \setminus \{G\}$, where $G_{(1|2)} = (V_G, E_G, L_{G_{(1|2)}})$ and $L_{G_{(1|2)}}(a) = L_G(a) \cup \{C_{(1|2)}\}$ and otherwise is the same as L_G.

$\rightarrow_\exists$ **rule**

if $(\exists R \cdot C) \in L_G(a)$ and there exists no $b \in V_G$ such that $R \in L_G(a, b)$ and at the same time $C \in L_G(b)$.

then $S' = S \cup \{G'\} \setminus \{G\}$, where $G' = (V_G \cup \{b\}, E_G \cup \{\langle a, b\rangle\}, L_{G'})$ and $L_{G'}(b) = \{C\}, L_{G'}(a, b) = \{R\}$ and otherwise is the same as L_G.

$\rightarrow_\forall$ **rule**

if $(\forall R \cdot C) \in L_G(a)$ and there exists no $b \in V_G$ such that $R \in L_G(a, b)$ and at the same time $C \notin L_G(b)$.

then $S' = S \cup \{G'\} \setminus \{G\}$, where $G' = (V_G, E_G, L_{G'})$, a $L_{G'}(b) = L_G(b) \cup \{D\}$ and otherwise is the same as L_G.

$\rightarrow_\sqsubseteq$ **rule**

if $\top_C \notin L_G(a)$ for some $a \in V_G$.

then $S' = S \cup \{G'\} \setminus \{G\}$, where $G' = (V_G, E_G, L_{G'})$, a $L_{G'}(a) = L_G(a) \cup \{\top_C\}$ and otherwise is the same as L_G.

3 Basics of fuzzy logic

Inverse membership	$\mu_A^{-1}(M) = \{x \in X \mid A(x) \in M\}$
Height	$\mathrm{Height}(A) = \sup\{\alpha \mid x \in \Delta, A(x) = \alpha\}$
Support	$\mathrm{Supp}(A) = \{x \in X \mid A(x) > 0\}$
Core	$\mathrm{Core}(A) = \{x \in X \mid A(x) = 1\}$
Vertical to horizontal	$\mathrm{R}_A(\alpha) = \{x \in X \mid A(x) \geqslant \alpha\}$
Horizontal to vertical	$\mu_A(x) = \max\{\alpha \in [0,1] \mid x \in \mathrm{R}_A(\alpha)\}$

Fuzzy inclusion	$A \subseteq B$ if $\mu_A(x) \leqslant \mu_B(x)$ for all $x \in \Delta$
Cutworthiness	$P(A_1, ..., A_n) \Rightarrow P(\mathrm{R}_{A_1}(\alpha), ..., \mathrm{R}_{A_n}(\alpha))$
Cut-consistency	$P(A_1, ..., A_n) \Leftrightarrow P(\mathrm{R}_{A_1}(\alpha), ..., \mathrm{R}_{A_n}(\alpha))$

3.1 Negation

Fuzzy negation	if $\alpha \leqslant \beta$ then $\underset{\circ}{\neg}\beta \leqslant \underset{\circ}{\neg}\alpha$ (N1)
	$\underset{\circ}{\neg}\underset{\circ}{\neg}\alpha = \alpha$ (N2)
Standard negation	$\underset{S}{\neg}\alpha = 1 - \alpha$
Cosine negation	$\underset{\cos}{\neg}\alpha = (\cos(\pi\alpha) + 1)/2$
Sugeno negation	$\underset{S\lambda}{\neg}\alpha = \frac{1-\alpha}{1+\lambda\alpha}$
Yager negation	$\underset{Y\lambda}{\neg}\alpha = (1 - a^\lambda)^{1/\lambda}$
Non-involutive negation	$\underset{\circ}{\neg}\underset{\circ}{\neg}0 = 1$ and $\underset{\circ}{\neg}\underset{\circ}{\neg}1 = 0$ (N0)
	if $\alpha \leqslant \beta$ then $\underset{\circ}{\neg}\beta \leqslant \underset{\circ}{\neg}\alpha$ (N1)
Gödel negation	$\underset{G}{\neg}\alpha = \begin{cases} 1 & \alpha = 0 \\ 0 & \text{otherwise} \end{cases}$

3.2 Implication

General fuzzy implication	$(x \overset{\circ}{\underset{\circ}{\Rightarrow}} y) = (x \Rightarrow y)$ on $x, y \in \{0, 1\}$
Residue implication	$\alpha \overset{R}{\underset{\circ}{\Rightarrow}} \beta = \sup\{\gamma \mid \alpha \underset{\circ}{\wedge} \gamma \leqslant \beta\}$
R-impl. properties	$\alpha \overset{R}{\underset{\circ}{\Rightarrow}} \beta = 1$ iff $\alpha \leqslant \beta$ (I1)
	$1 \overset{R}{\underset{\circ}{\Rightarrow}} \beta = \beta$ (I2)
	$\alpha \overset{R}{\underset{\circ}{\Rightarrow}} \beta$ is not increasing in α and not decreasing in β (I3)
Standard implication	$\alpha \overset{R}{\underset{S}{\Rightarrow}} \beta = \begin{cases} 1 & \text{if } \alpha \leqslant \beta \\ \beta & \text{otherwise} \end{cases}$
Łukasiewicz implication	$\alpha \overset{R}{\underset{L}{\Rightarrow}} \beta = \begin{cases} 1 & \text{if } \alpha \leqslant \beta \\ 1 - \alpha + \beta & \text{otherwise} \end{cases}$
Algebraic implication	$\alpha \overset{R}{\underset{A}{\Rightarrow}} \beta = \begin{cases} 1 & \text{if } \alpha \leqslant \beta \\ \frac{\beta}{\alpha} & \text{otherwise} \end{cases}$
S-implication	$\alpha \overset{S}{\underset{\circ}{\Rightarrow}} \beta = \underset{S}{\neg}\alpha \overset{\circ}{\vee} \beta$
Kleene-Dienes implication	$\alpha \overset{S}{\underset{S}{\Rightarrow}} \beta = \max(1 - \alpha, \beta)$
Generalized fuzzy inclusion	$A \overset{\circ}{\underset{\circ}{\subseteq}} B = \inf\{A(x) \overset{\circ}{\underset{\circ}{\Rightarrow}} B(x) \mid x \in \Delta\}$

3.3 Conjunction

Definition	
	$\alpha \underset{\circ}{\wedge} \beta = \beta \underset{\circ}{\wedge} \alpha$ (T1)
	$\alpha \underset{\circ}{\wedge} (\beta \underset{\circ}{\wedge} \gamma) = (\alpha \underset{\circ}{\wedge} \beta) \underset{\circ}{\wedge} \gamma$ (T2)
	if $\beta \leqslant \gamma$ then $(\alpha \underset{\circ}{\wedge} \beta) \leqslant (\alpha \underset{\circ}{\wedge} \gamma)$ (T3)
	$(\alpha \underset{\circ}{\wedge} 1) = \alpha$ (T4)
Standard c.	$\alpha \underset{S}{\wedge} \beta = \min(\alpha, \beta)$
Łukasiewicz c.	$\alpha \underset{L}{\wedge} \beta = \max(\alpha + \beta - 1, 0)$
Algebraic product	$\alpha \underset{A}{\wedge} \beta = \alpha \cdot \beta$
Weak conjunction	$\alpha \underset{W}{\wedge} \beta = \begin{cases} \alpha & \text{if } \beta = 1 \\ \beta & \text{if } \alpha = 1 \\ 0 & \text{otherwise} \end{cases}$

3.4 Disjunction

Definition	
	$\alpha \overset{\circ}{\vee} \beta = \beta \overset{\circ}{\vee} \alpha$ (S1)
	$\alpha \overset{\circ}{\vee} (\beta \overset{\circ}{\vee} \gamma) = (\alpha \overset{\circ}{\vee} \beta) \overset{\circ}{\vee} \gamma$ (S2)
	if $\beta \leqslant \gamma$ then $(\alpha \overset{\circ}{\vee} \beta) \leqslant (\alpha \overset{\circ}{\vee} \gamma)$ (S3)
	$(\alpha \overset{\circ}{\vee} 0) = \alpha$ (S4)
Standard d.	$\alpha \overset{S}{\vee} \beta = \max(\alpha, \beta)$
Łukasiewicz d.	$\alpha \overset{L}{\vee} \beta = \min(\alpha + \beta, 1)$
Algebraic sum	$\alpha \overset{A}{\vee} \beta = \alpha + \beta - \alpha \cdot \beta$
Weak disjunction	$\alpha \overset{W}{\vee} \beta = \begin{cases} \alpha & \text{if } \beta = 0 \\ \beta & \text{if } \alpha = 0 \\ 1 & \text{otherwise} \end{cases}$

3.5 Relations

Composition	$R \underset{\circ}{\bigcirc} S(x,z) = \sup_{y \in Y} R(x,y) \underset{\circ}{\wedge} S(y,z)$
Comp. properties	$R \underset{\circ}{\bigcirc} (S \underset{\circ}{\bigcirc} T) = (R \underset{\circ}{\bigcirc} S) \underset{\circ}{\bigcirc} T$ (C3)
	$R \underset{\circ}{\bigcirc} E = R, \; E \underset{\circ}{\bigcirc} R = R$ (C1)
	$(R \overset{S}{\cup} S) \underset{\circ}{\bigcirc} T = (R \underset{\circ}{\bigcirc} T) \overset{S}{\cup} (S \underset{\circ}{\bigcirc} T)$ (C4)
	$(R \underset{\circ}{\bigcirc} S)^{-1} = S^{-1} \underset{\circ}{\bigcirc} R^{-1}$ (C2)
	$R \underset{\circ}{\bigcirc} (S \overset{S}{\cup} T) = (R \underset{\circ}{\bigcirc} S) \overset{S}{\cup} (R \underset{\circ}{\bigcirc} T)$ (C5)
First projection	$R^{(1)}(x) = \sup_{y \in Y} R(x,y)$
Second projection	$R^{(2)}(y) = \sup_{x \in X} R(x,y)$
Cylindrical extension	$A \times B(x,y) = A(x) \underset{S}{\wedge} B(y)$
Identity relation	$E = \{(x,x) \mid x \in \Delta\}$
Reflexivity	$E \subseteq R$
Symmetrictity	$R = R^{-1}$
∘-anti-symmetricity	$R \underset{\circ}{\cap} R^{-1} \subseteq E$
∘-transitivity	$R \underset{\circ}{\bigcirc} R \subseteq R$
∘-partial order	reflexive, ∘-transitive and ∘-anti-symmetric
∘-equivalence	reflexive, ∘-transitive and ∘-symmetric

4 Fuzzy description logic

non-atomic c.	interpretation
$\bot$	0
$\top$	1
A	$A^{\mathcal{I}}(x)$
$\neg C$	$\underset{S}{\neg} C^{\mathcal{I}}(x)$
$C \underset{\circ}{\sqcap} D$	$C^{\mathcal{I}}(x) \underset{\circ}{\wedge} D^{\mathcal{I}}(x)$
$C \overset{\circ}{\sqcup} D$	$C^{\mathcal{I}}(x) \overset{\circ}{\vee} D^{\mathcal{I}}(x)$
$C \underset{\circ}{\mapsto} D$	$C^{\mathcal{I}}(x) \underset{\circ}{\Rightarrow} D^{\mathcal{I}}(x)$
$\exists R \cdot C$	$\sup_y R^{\mathcal{I}}(x,y) \underset{\circ}{\wedge} C^{\mathcal{I}}(y)$
$\forall R \cdot C$	$\inf_y R^{\mathcal{I}}(x,y) \underset{\circ}{\overset{\circ}{\Rightarrow}} C^{\mathcal{I}}(y)$
$(n\, C)$	$n \cdot C(x)$
$mod(C)$	$mod(C^{\mathcal{I}}(x))$
$w_1 C_1 + \ldots + w_k C_k$	$w_1 C_1^{\mathcal{I}}(x) + \ldots + w_k C_k^{\mathcal{I}}(x)$

axiom	satisfied if
$\langle i : C \mid \alpha \rangle$	$C^{\mathcal{I}}(i^{\mathcal{I}}) \geqslant \alpha$
$\langle (i,j) : R \mid \alpha \rangle$	$R^{\mathcal{I}}(i^{\mathcal{I}}, j^{\mathcal{I}}) \geqslant \alpha$
$\langle C \sqsubseteq D \mid \alpha \rangle$	$C \underset{\circ}{\overset{\circ}{\subseteq}} D \geqslant \alpha$
$\langle R_1 \sqsubseteq R_2 \rangle$	$R_1^{\mathcal{I}} \subseteq R_2^{\mathcal{I}}$
$\langle$transitive $R\rangle$	R is ∘-transitive
$\langle R_1 = R_2^{-1} \rangle$	$R_1^{\mathcal{I}} = (R_2^{\mathcal{I}})^{-1}$