Bayesian networks – exercises

Collected by: Jiří Kléma, klema@labe.felk.cvut.cz

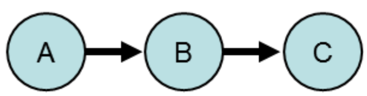
Fall 2013/2014

Goals:

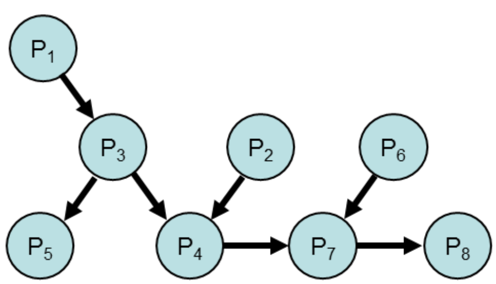
The text provides a pool of exercises to be solved during AE4M33RZN tutorials on graphical probabilistic models. The exercises illustrate topics of conditional independence, learning and inference in Bayesian networks. The identical material with the resolved exercises will be provided after the last Bayesian network tutorial.

1 Independence and conditional independence

Exercise 1. *Formally prove which (conditional) independence relationships are encoded by serial (linear) connection of three random variables.*



Exercise 2. *Having the network/graph shown in figure below, decide on the validity of following statements:*



a) $P_1, P_5 \perp\!\!\!\perp P_6 | P_8$,

b) $P_2 \not\perp\!\!\!\perp P_6 | \emptyset$,

c) $P_1 \perp\!\!\!\perp P_2 | P_8$,

d) $P_1 \perp\!\!\!\perp P_2, P_5 | P_4$,

e) *Markov equivalence class that contains the shown graph contains exactly three directed graphs.*

Exercise 3. *Let us have an arbitrary set of (conditional) independence relationships among N variables that is associated with a joint probability distribution.*

a) *Can we always find a directed acyclic graph that perfectly maps this set (perfectly maps = preserves all the (conditional) independence relationships, it neither removes nor adds any)?*

b) *Can we always find an undirected graph that perfectly maps this set?*

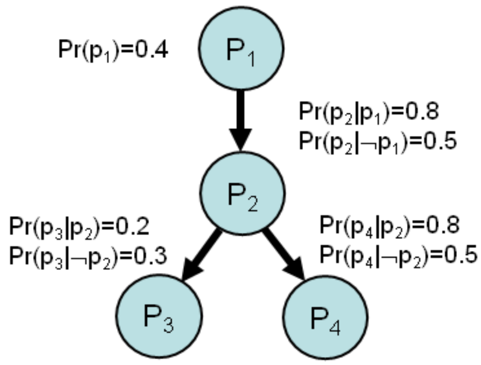
c) *Can directed acyclic models represent the conditional independence relationships of all possible undirected models?*

d) *Can undirected models represent the conditional independence relationships of all possible directed acyclic models?*

e) *Can we always find a directed acyclic model or an undirected model?*

2 Inference

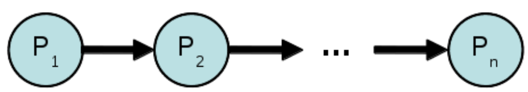
Exercise 4. *Given the network below, calculate marginal and conditional probabilities* $Pr(\neg p_3)$*,* $Pr(p_2|\neg p_3)$*,* $Pr(p_1|p_2, \neg p_3)$ *a* $Pr(p_1|\neg p_3, p_4)$*. Apply the method of* **inference by enumeration***.*



Exercise 5. *For the same network calculate the same marginal and conditional probabilities again. Employ the properties of directed graphical model to* **manually simplify** *inference by enumeration carried out in the previous exercise.*

Exercise 6. *For the same network calculate* $Pr(\neg p_3)$ *and* $Pr(p_2|\neg p_3)$ *again. Apply the method of* **variable elimination***.*

Exercise 7. *Analyze the complexity of inference by enumeration and variable elimination on a chain of binary variables.*



Exercise 8. *For the same network calculate the conditional probability* $Pr(p_1|p_2, \neg p_3)$ *again. Apply a sampling approximate method. Discuss pros and cons of rejection sampling, likelihood weighting and Gibbs sampling. The table shown below gives an output of a uniform random number generator on the interval (0,1), use the table to generate samples.*

r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}
0.2551	0.5060	0.6991	0.8909	0.9593	0.5472	0.1386	0.1493	0.2575	0.8407
r_{11}	r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	r_{17}	r_{18}	r_{19}	r_{20}
0.0827	0.9060	0.7612	0.1423	0.5888	0.6330	0.5030	0.8003	0.0155	0.6917

Exercise 9. *Let us have three tram lines – 6, 22 and 24 – regularly coming to the stop in front of the faculty building. Line 22 operates more frequently than line 24, 24 goes more often than line 6 (the ratio is 5:3:2, it is kept during all the hours of operation). Line 6 uses a single car setting in 9 out of 10 cases during the daytime, in the evening it always has the only car. Line 22 has one car rarely and only in evenings (1 out of 10 tramcars).*

Line 24 can be short whenever, however, it takes a long setting with 2 cars in 8 out of 10 cases. Albertov is available by line 24, lines 6 and 22 are headed in the direction of IP Pavlova. The line changes appear only when a tram goes to depot (let 24 have its depot in the direction of IP Pavlova, 6 and 22 have their depots in the direction of Albertov). Every tenth tram goes to the depot evenly throughout the operation. The evening regime is from 6pm to 24pm, the daytime regime is from 6am to 6pm.

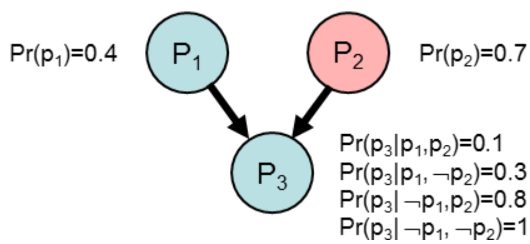
a) *Draw a* **correct, efficient and causal** *Bayesian network.*

b) *Annotate the network with the conditional probability tables.*

c) *It is evening. A short tram is approaching the stop. What is the probability it will go to Albertov?*

d) *There is a tram 22 standing in the stop. How many cars does it have?*

Exercise 10. *Trace the algorithm of* **belief propagation** *in the network below knowing that* $\mathbf{e} = \{p_2\}$. *Show the individual steps, be as detailed as possible. Explain in which way the unevidenced converging node* P_3 *blocks the path between nodes* P_1 *and* P_2.

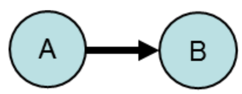


3 (Conditional) independence tests, best network structure

Exercise 11. *Let us concern the frequency table shown below. Decide about independence relationships between* A *and* B.

	c		$\neg c$	
	b	$\neg b$	b	$\neg b$
a	*14*	*8*	*25*	*56*
$\neg a$	*54*	*25*	*7*	*11*

Exercise 12. *Let us consider the network structure shown in the figure below. Our goal is to calculate maximum likelihood (ML), maximum aposteriori (MAP) and Bayesian estimates of the parameter* $\theta = Pr(b|a)$. *4 samples are available (see the table). We also know that the prior distribution of* $Pr(b|a)$ *is Beta(3,3).*



A	B
T	T
F	F
T	T
F	F

4 Dynamic Bayesian networks

Exercise 13. *A patient has a disease N. Physicians measure the value of a parameter P to see the disease development. The parameter can take one of the following values {low, medium, high}. The value of P is a result of patient's unobservable condition/state S. S can be {good, poor}. The state changes between two consecutive days in one fifth of cases. If the patient is in good condition, the value for P is rather low (having 10 sample measurements, 5 of them are low, 3 medium and 2 high), while if the patient is in poor condition, the value is rather high (having 10 measurements, 3 are low, 3 medium and 4 high). On arrival to the hospital on day 0, the patient's condition was unknown, i.e., $Pr(S_0 = good) = 0.5$.*

a) *Draw the transition and sensor model of the dynamic Bayesian network modeling the domain under consideration,*

b) *calculate probability that the patient is in good condition on day 2 given low P values on days 1 and 2,*

c) *can you determine the most likely patient state sequence in days 0, 1 and 2 without any additional computations?, justify.*