\* 1. What is the minimum and maximum number of keys in the 2-3-4 tree which has the depth 4 and which contains exactly one 4-node?

\* 2. Suppose that a 2-3-4 tree is originally empty. Insert, in the given order, into the tree the keys

23, 31, 15, 24, 36, 20, 32, 18, 59, 60, 58, 57.

Draw the tree after each insertion.

\* 3. Suppose that a B+ tree of order 1 is originally empty. Insert, in the given order, into the tree the keys

32, 18, 31, 59, 20, 23, 24, 36, 60, 58, 15, 57.

Draw the tree after each insertion.

\* 4. Delete the keys from the B+ tree in the previous problem in the order:

23, 31, 15, 24, 36, 20, 32, 18, 59, 60, 58, 57.

Draw the tree after each insertion.

\* 5. Build a k-d tree contining the following points in the plane with coordinates:

(50, 50), (20, 35), (60,15), (30, 40), (50, 40), (40, 50), (15, 60), (15, 35), (35, 40).

Insert the points into the tree in the given order.

6. Two empty 2-3-4 trees are isomorphic. Let T1 and T2 be two unempty 2-3-4 trees with the respective roots R1 and R2. T1 and T2 are isomorphic iff both 1. and 2. holds:

1. The root of T1 contains the same number of keys as the root of T1

2. The leftmost subtree of R1 is isomprphic to the leftmost subtree of R2, the second subtree of R1 from left is isomorphic to the second subtree opf R2 from left, etc., and finaly the rightmost subtree of R1 is isomprphic to the rightmost subtree of R2.

What is the number of non-isomorphic 2-3-4 trees with A) 3, B) 4, C) nodes?

7. Write a pseudocode (or, indeed, a code) which will determine the number of non-isomorphic 2-3-4 trees with fixed depth D. Alternatively tou might derive a closed algebraic fomula which returns the number depending only on the value of D.

8. The outer loop iterates through x-coordinates of the points in the plane starting from x =10 and ending at x = 40 with step = 10. The inner loop iterates through the y-coordinates of the points starting from y = 50 and ending at y = 80 with step = 10. Each point is inserted into an originally empty k-d tree in the order imposed by the progress of the loops. Draw the resulting k-d tree.

9. Delete the points (50, 10), (60, 20), (70, 30), (60, 30) (in this order) from the k-d tree built in the previous problem. Draw the resulting k-d tree.